Sudan University of Science & Technology

College of Graduate studies and Scientific Research

Theoretical Verification of Thermal Neutrons Scattering

التحقق النظري من تشتت النيوترونات الحرارية

Thesis submitted for partial fulfillment of M.Sc degree in physics

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الآية

قـال الله تـعـالـى :

إِنَّ اللَّهَ لاَ يُغَيِّرُ مَا بِقَوْمٍ حَتَّى يُغَيِّرُواْ مَا بِأَنفُسِهِمْ وَ إِذَا أَرَادَ اللهُ بِقَوْمٍ سُوءًا فَلاَ مَرَدَّ لَهُ وَ مَا لَهُم مِّن دُونِهِ مِن وَالٍ ﴿ ☀ ﴾

صدق الله الـعـظيم
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Dedication

To my parent, my wife and my sons Nouman and salih

Acknowledgment

Thank to my supervisor Dr. Ahmad Hassan El faki and
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engineering department   in Sudan university. Who give me the
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As well

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The content

Abstract

One of the technologically most important interactions of neutrons with matter is their loss of energy ("slowing down") by a series of elastic collisions. These can be treated by the methods of classical mechanics, assuming the interacting particles as perfectly elastic spheres. The energy loss is an important subject, and is discussed in several books where numerical tables and graphs are presented. Formulas are found semi empirically with several correction coefficients. Despite all efforts, no direct, exact formula has so far been obtained analytically. The purpose of this research is theoretical verification of thermal neutrons scattering by using a recently introduced method of the quantization of neoconservative systems and detailed comparisons with a significant number of measurements of differential and integral neutron cross sections and other relevant data are reported (for the validation of the generated Scattering Law data files $S(\alpha, \beta, T)$), as ENDF data library are reproduced reasonably well, necessary in generating and processing the thermal neutron scattering data.

مستخلص

واحدة من أهم تفاعلات النيوترون مع المادة في الإطار التكنولوجي هي مسألة ضياع الطاقة أو إبطائها من خلال سلسلة من التصادمات المرنة التي يمكن معالجتها بواسطة طرق الميكانيكا الكلاسيكية, وذلك بافتراض الدقائق المتفاعلة على إنها كرات مرنة تماماً .

وفقدان الطاقة في هذه الحالة هو موضوع هام نوقش في العديد من الكتب حيث تم استخدام الجداول والرسوم البيانية وكذلك قد تم الحصول على العديد من الصيغ شبه التجريبية من معاملات التصحيح، وعلى الرغم من كل الجهود المبذولة إلا إنه لا توجد علاقة تم الحصول عليها مباشرة بواسطة الطرق التحليلية.

والغرض من هذا البحث هو التحقق النظري لتشتت النيوترونات الحرارية تحليلياً عن طريق النظم الغير متزنة ومقارنتها تفصيلياً مع القياسات الأساسية بالنسبة للجوانب التفاضلية والتكاملية للمقاطع العرضية وغيرها من البيانات المناسبة التي وردت(لصلاحية قانون التشتت الذي تم توليده من ملف البيانات $(s(\alpha , \beta , \tau))$ كما وردت مكتبة البيانات (ENDE) بشكل معقول وهي ضرورية في توليد ومعالجة البيانات عند تشتت النيوترونات الحرارية .

Chapter one

Introduction

1.1 General

A neutron is a tiny subatomic particle that can be found in practically all form of conventional matter. The only stable exception is the hydrogen atoms, where it is bound closely with protons through the strong nuclear force, the strongest force in nature. Neutrons are responsible for about half the weight of conventional matter by volume. It can be seen as a proton and an electron smashed together. Because both of these particles have opposite charge of the same magnitude [1,3], their fusion result in charge less particle. This lack of charge can make neutron difficult to detect. Neutrons can sometimes behave charged in a limited way because their constituents, quarks, have small charges. The neutron is a baryon and is considered to be composed of two down quarks and one up quarks. A free neutron will decay with a half-life of about 10.3 minutes but it is stable if combined into a nucleus. The decay of the neutron involves the weak interaction. The neutron is about 0.2 percent more massive than proton, which translates to an energy difference of 1:29Mev.The decay of the neutron is associated with a quark transfer motion in which a down quarks is converted to an up by the weak interaction. It is possible for proton to be transformed in to a neutron, but you have to a supply 1:29Mev of energy to reach the threshold for that transformation.

The existence of neutron was first suggested by Rutherford in 1920[1,3]. He thought that an electron could exist in a nucleus and could combine with a proton to form a neutron. Being electrically neutral, the neutron was very difficult to discover by methods of particle detection which depends on the deflection of the particles in a magnetic or electric field or on their ionization of matter.

The field of neutron systems has become an integral part of investigations into an array of important issues that span _fields as diverse as nuclear engineering and particle physics, fundamental symmetries, astrophysics and cosmology, fundamental constants, gravitation, and the interpretation of quantum mechanics. The experiments employ a diversity of measurement strategies and techniques, including condensed matter and low temperature physics, optics, and atomic physics, as well as nuclear and particle physics, and they address a wide range of issues. Nevertheless, the field possesses a coherence that derives from the unique properties of the neutron as an electrically neutral, strongly interacting, long-lived unstable particle that can be used either as the probe or as an object of study. By fundamental neutron concept, we mean that class of experiments using slow neutrons which primarily address issues associated with the Standard Model (SM) of the strong, weak, electromagnetic, and gravitational interactions and their connection with issues in astrophysics and cosmology Neutrons experience all known forces in strengths that make them accessible to experimentation.

It is an amusing fact that the magnitude of the average neutron interaction energy in matter, in laboratory magnetic fields, and near the surface of the Earth is the same order of magnitude for all forces except the weak interaction. The experiments include measurement of neutron-decay parameters, the use of parity violation to isolate the weak interaction between nucleons, and searches for a source of time reversal violation beyond the SM. These experiments provide information that is complementary to that available from existing accelerator-based nuclear physics facilities and high-energy free neutrons are unstable with a 15 minute lifetime but are prevented from decaying while bound in nuclei through the combined effects of energy conservation and Fermi statistics. They must be liberated from nuclei using nuclear reactions with MeV -

scale energies in order to be used and studied. We define slow neutrons to be neutrons whose energy has been lowered well below this scale. The available dynamic range of neutron energies for use in laboratory research is quite remarkable. Thermodynamic language is used to describe different regions; a neutron in thermal equilibrium at 300 K has a kinetic energy of only 0.025 eV. Because its de Broglie wavelength (0.18 nm) is comparable to inter-atomic distances, this energy also represents the boundary below which coherent interactions of neutrons with matter become important. The most intense sources of neutrons for experiments at thermal energies are nuclear reactors, although accelerators can also produce higher energy neutrons.

As the uncharged member of the nucleon pair, the neutron plays a fundamental role in the study of nuclear forces. Unaffected by the Coulomb barrier, neutrons of even very low energy (eV or less) can penetrate the nucleus and initiate nuclear reactions. In contrast to part of our lack of understanding of processes in the interior of stars results from the difficulty of studying proton -induced reactions at energies as low as keV. On the other hand the lack of coulomb interaction presents some experimental problems when using neutrons as a nuclear probe : energy selection and focusing of an incident neutron beam are difficult and neutrons do not produce primary ionization events in detectors(neutrons passing through matter have negligible interactions with the atomic electrons).Basic researches with neutrons goes back almost to the earliest days of nuclear physics, and it continues to be a vital and exciting research field today. For example, interference effects with neutron beams have permitted some basic aspects of quantum mechanics to be demonstrated for the first time. The electric dipole moment of the neutron should vanish if the neutron were an elementary particle or even a

composite particle in which the binding forces were symmetric with respect to the parity and time-reversal operations.

Many careful and detailed experiments have been done and all indicate a vanishing electric dipole moment, but the limit has been pushed so low (10□25e.cm) that it is almost possible to distinguish among certain competing theories for the interactions among the elementary particles. The so-called Grand Unified Theories that attempt to unify the strong (nuclear), electromagnetic and weak(βdecay) interactions predict that the conservation of nucleon number (actually baryon number) can break down and that a neutron could convert in to its anti-particle, the anti-neutron and then back again to a neutr [3]. No evidence has yet been seen for this effect either, but current research is trying to improve the limits in our knowledge of the neutron-anti-neutron conversion frequency

1.2 Problem Statement

The problems so far have to do merely with the realization of the chain reaction. If such a reaction is going to be of use, we must be able to control it. The problem of control is different depending on whether we are interested in steady production of thermal neutrons for an explosion. In general, the steady production of thermal neutrons requires a slow-neutron-induced fission chain reaction occurring in a mixture or lattice of uranium and moderator[3].

1.3 Research Objectives

The purpose of neutron reach facilities is to provide unprecedented experimental capabilities in the areas of neutron scattering materials, so extrapolated quantities include fuel cycle parameters fuel element power distribution neutron fluxes in the reflectors and targets regions so that the development of the response function generator in hexagonal geometry that leads to advance the state-of-the-art for reactor analysis.

1.3 Methodology

The method used to produce approximate solutions to the integral form of the Boltzmann transport equation for thermal neutron fluxes near a moderator temperature discontinuity. Both spherical and plane geometries are considered. The validity of the technique is established by using a synthetic scattering kernel and making comparisons with exact transport theory solutions for two-temperature problems. The relative accuracy and importance of the choice of different weighting functions and of the number of trial modes in the flux expansion is investigated [3]. Generally, it is found that using slowing down distribution in a finite block distributions for the energy trial modes of the flux produces a significant increase in the accuracy compared with a diffusion theory approximation, so that the Boltzmann transport equation describes the transport of neutral particles from one collision with an atom to another [1.3]. It is a 'balance' statement that accounts for additions to and subtractions from the radiation in a given increment of space, energy, direction and time [3].

Chapter Two

Literature Review and governing formulas

2.1 Introduction

The <u>neutrons</u> emitted in <u>nuclear fission</u> reactions have high energies, typically in the range of 1 <u>MeV</u>. But the <u>cross section</u> for neutron capture leading to fission is greatest for neutrons of energy around 1 eV, a million times less. Neutrons with energies less than one electron volt are commonly referred to as "thermal neutrons" since they have energies similar to what particles have as a result of ordinary room-temperature <u>thermal energy</u> [4]. It is necessary to slow down the neutrons for efficient operation of a nuclear reactor, a process called moderation. While neutrons are efficiently slowed by inelastic scattering from U-238 , the non-fissionable isotope of uranium, when their energies are higher than 1 MeV, the remainder of the process of slowing them down must be done by elastic scattering from other nuclei. When a neutron collides elastically with another nucleus at rest in the medium, it transfers some of its energy to it. The maximum transfer of energy occurs when the target nucleus is comparable in mass to the projectile[4,5]. Water and carbon (graphite) are commonly used moderators. Water is a good moderator, but the hydrogen in the water molecule have a fairly high cross section for neutron capture, removing neutrons from the fission process

2.2 Concept of moderation

In materials containing atoms of low atomic mass, neutrons of all energies can lose a significant fraction of their energy in a single elastic collision and such materials are referred to as moderators [4,5]. In heavy nuclei appreciable energy loss in a collision is only possible at high energies where inelastic scattering can occur. The neutron dose rate from a point source of fast neutrons falls off with distance r approximately as $\exp(-\Sigma_{rem}r)/4\pi r^2$, where Σ_{rem} depends on the medium where Σ has been defined earlier. This macroscopic cross-section is called the removal cross-section and since all interactions tend to remove energy from the beam its value is not too different from the total macroscopic cross-section ($N\sigma_{nT}$) of the material, but is slightly lower. This exponential fall off is only approximate and holds less well for media in which hydrogen

is the principal fast neutron attenuator. In the table overleaf the removal cross-section refers to a fission neutron source.

In the slowing-down region the average number of collisions, $\bar{n}$, to slow a neutron from energy E_1 to energy E_2 is equal to $\ln(E_1/E_2)/\xi$, where ξ is the average change per collision in the logarithm of the energy. At energies below that at which scattering becomes entirely elastic, ξ is independent of energy and is approximately equal to $2/(A + \frac{2}{3})$. The spatial distribution of neutrons of energy E_2 which have slowed down from a point source of energy E_1 is of the form $\exp(-r^2/4\tau)^2$ where τ is referred to as the Fermi Age and is the mean square distance a neutron migrates in slowing down from E_1 to E_2. It is given by:

$$\tau = \int_{E2}^{E1} \frac{DdE}{E\xi\sum_{nn}}$$

1.1

Where D is the diffusion coefficient and equal to $(3\Sigma_{nT} - 3b\Sigma_{nn})^{-1}$ and b is the average value of $\cos \Psi$ where Ψ is the angle of scatter of a neutron in a collision. The table refers to the age of neutrons from a fission source slowing down to an energy of 1.46 eV. This value, which is just above the thermal region, is appropriate to the age determined from the measured spatial distribution of the resonance neutrons detected by indium foils.

The root mean square distance a neutron travels from the position where it is etherealized to the point where it is absorbed is the thermal diffusion length, L, and is equal to

$$(4/\pi)^{1/4}(D_{th}/\Sigma_{nA})^{1/2} \qquad (1.2)$$

where D_{th} is the value of the diffusion coefficient averaged over the thermal neutron spectrum and Σ_{nA} is assumed to have a $1/v$ dependence and is evaluated at an energy kT_n where T_n is the temperature of the medium [4,6].

2.3 Neutron Moderators

As a beam of neutron travels through a bulk matter, the intensity will decreases as neutrons are removed from the beam by nuclear reactions. For fast neutrons, many reactions such as (n,p)(n,n) or (n,2n) are possible, but for slow or thermal neutrons the primary cause of their disappearance

is capture, in the form of the (n,)reaction. Often the cross sections for these capture reactions are dominated by one or more resonances, where the cross section becomes very large.

Beams of neutron can be produced from a variety of nuclear reactions. We cannot accelerate neutrons as charged particles, but we can start with high-energy neutron and reduce their energy through collisions with atoms of various materials. This slowing of neutron is called "Moderating" the neutrons[4,5].

In crossing a thickness dx of a material, the neutrons will encounter ndx atoms per unit surface area of the beam of the material. Where n is the number of atoms per unit volume of the material. If $\_t$ is the total cross section (including scattering processes, which will tend to divert neutrons from the beam), then the loss in intensity I is $dI = -I\sigma_t ndx$

And the intensity decreases with absorber thickness according to an exponential relationship

$$I = I_0 e^{-\sigma_t t n x} \qquad\qquad 1.3$$

The expression refers only to mono energetic neutrons-the original intensity of neutrons of a certain energy decreases according to the above equation.

In thermal reactors moderator materials are required to reduce the neutron energies from the fission to the thermal range with as few collisions as possible, thus circumventing resonance capture of neutrons in uranium-238. To be an effective moderator a material must have a low atomic weight. To be an effective moderator a material must have a low atomic weight. Only then is ζ —the slowing down decrement defined by $\zeta =$ ln(E/É)p(E→É)dÉ ,that most widely employed measure of a nuclide's ability to slow neutrons down by elastic scattering is the slowing down decrement. It is defined as the mean value of the logarithm of the energy

loss ratio or ln(E/É) is large enough to slow neutrons down to thermal energies with relatively few collisions[4,5,6].A good moderator, however, must possess additional properties. Its macroscopic scattering cross section must be sufficiently large. Otherwise, even though a neutron colliding with it would lose substantial energy, in the competition with other materials, too few moderator collisions would take place to have a significant impact on the neutron spectrum. Thus a second important parameter in determining a material's value as a moderator is the slowing down power, defined as $\zeta\sum_s$, in other word the average logarithmic energy decrement is the average decrease per collision in the logarithm of the neutron energy.

$$\zeta = \ln E - \ln \acute{E} = \ln(E/\acute{E}) \qquad\qquad 1.4$$

where

ζ= average logarithmic energy decrement

E= average initial neutron energy

É= average final neutron energy

The symbol ζ is commonly called the average logarithmic energy decrement because of the fact that a neutron loses, on the average, a fixed fraction of its energy per scattering collision.

Since the fraction of energy retained by a neutron in a single elastic collision is a constant for a given material, is also a constant.

Because it is a constant for each type of material and does not depend upon the initial neutron energy, is a convenient quantity for assessing the moderating ability of a material. The number of collisions (N) to travel from any energy, E_{high}, to any lower energy, E_{low}, can be calculated as

shown as $\qquad N = {\ln E - \ln E' \over \zeta}$

So the collisions are required to slow a neutron from an energy of 2 MeV to a thermal energy of 0.025 eV, using water as the moderator? Water has

a value of 0.948 for ζ using the above equation is 19.2 collisions, and the average number of elastic scattering collisions needed for 2 MeV neutron to slow down to thermal energies;

$^{1}_{2}H$ =18 collisions

$^{2}_{4}H$ = 25 collisions

He = 40 collisions

U> 2000 collisions

Chapter Three

Theory of slowing down neutron

3.1 Energy Distribution of Neutrons

If the production of neutrons throughout all space is uniform, then the neutron density cannot depend on x[4,7].The general Boltzmann equation reduces to

$$N\sigma F(u,\mu) = -\frac{(1+M)^2}{2M}\sum_{1}\sum_{1'}\cdot\frac{21+1}{2}\frac{21'+1}{2}\int_{u}^{u+\ln\alpha^2}\int_{-1}^{1} N\sigma_{s_0}(u')F(u',\mu')e^{u'-u}$$

$$f_{1'}(u')\ P_{1'}h(u'-u)\ P_1[g(u'-u)]\ P_1(\mu)\ P_1(\mu')\ du'd\mu' + S(\mu)\ \partial(u-u_0) \tag{3.1}$$

The total neutron flux per logarithmic energy interval, $F_0(u)$, and the total number of neutrons produced per C.C. per second, S_0, are

$$F_0(u) = \int_{-1}^{1} F(u,\mu)d\mu$$

$$S_0 = \int_{-1}^{1} S(\mu)d\mu$$

Thus upon integrating Eq. 3.1over u and u' there results

$$N\sigma F_0(u) = -\frac{(1+M)^2}{2M}\int_{u}^{u+\ln\alpha^2} N\sigma_{s_0}\ F_0(u')\ e^{u'-u}\ f[u',h(u'-u)]\ du' + S_0\ \partial(u-u_0) \tag{3.2}$$

In the isotropic scattering case, $f = 1/2$, this reduces to

$$N\sigma F_0(u) = -\frac{(1+M)^2}{4M}\int_{u}^{u+\ln\alpha^2} N\sigma_{s_0}\ F_0(u')\ e^{u'-u}\ du' + S_0\ \partial(u-u_0) \tag{3.3}$$

It is to be understood, as before, that $F_0(u) = 0$ if $u< u_o$.

The distribution $F_0(u)$ for large values of u (i.e. at energies far from the source energy) is easy to calculate provided there is no absorption ($N\sigma= N\sigma s$).

In this case the distribution equation(3.2)

$$N\sigma_{s_0}F_0(u) = -\frac{(1+M)^2}{2M}\int_{u}^{u+\ln\alpha^2} N\sigma_{s_0}F_0(u')e^{u'-u}f[u',h(u'-u)]du' \tag{3.4}$$

Now, since $f(E',E,\underline{\Omega}',\underline{\Omega})$ is probability that collision by a neutron of energy E' and direction $\underline{\Omega}'$ results in an energy E and a direction $\underline{\Omega}$,

$$\int_{\underline{\Omega}}^{\alpha^2} \int_{E,}^{E'} f\left(E', D, \underline{\Omega}'\underline{\Omega}\right) dE d\underline{\Omega} = 1 \qquad (3.5)$$

hence, on transforming to the variable u, and integrating - over angle, we find

$$-\frac{(1+M)^2}{2M} \int_{u}^{u+\ln\alpha^2} e^{u'-u} f\left[(u'-u)\right] du' = 1 \qquad (3.6)$$

Consequently

$N\sigma_{so} F_0(u) = C$ a constants

However, Eq(3.3) satisfies since this solution does not satisfy the initial condition (which is derived from Eq (3.2), namely

$$\underset{u \to u_0}{Lim} N_\sigma F_0(u) = s_0 \partial(u - u_0)$$

it cannot be correct close to the source energy u_o. Thus $N\sigma_{so}, F_0(u) = C$

is only the asymptotic solution of Eq(3.2) correct at energies u which

differ from u_o by several logarithmic slowing down intervals $\ln\alpha^2$.

We compute the value of the constant C in the case of isotropic scattering, i.e., f = 1/2. This is of most practical interest since the asymptotic solution applies only to neutrons which have lost considerable energy, and therefore the scattering by the moderator will usually have become isotropic by the time the asymptotic solution becomes valid [4,7].

To compute C we equate the number of neutrons which cross a given energy E per second per C.C. in the course of slowing down, to the number of neutrons produced per second per c.c. To calculate the number of neutrons which cross E per second per C.C. we observe that all neutrons from logarithmic interval du' which enter a logarithmic energy interval du″ a lying below u(E) will have crossed E. The number of collisions per logarithmic energy interval du' per second is

$$N\sigma_{s_0} F_0(u') \ du' ;$$

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The probability that these collisions will result in neutrons being thrown into energy interval dE" is

$$\frac{(1+M)^2}{2M} f[E', \eta(E', E'')] \, dE''$$

or, in the logarithmic energy variable,,

$$\frac{(1+M)^2}{2M} e^{u'-u} \, f\{u', h(u'-u'')\} \, du''$$

Hence the total number of neutrons thrown across E per second per c.c. is

$$\frac{(1+M)^2}{2M} \int_u^{u+\ln\alpha^2} \int_{u-\ln\alpha^2}^u N_{S0} F_0(u') e^{u'-u''} f[u'; h(u'-u'')] du'' \, du' \tag{3.7}$$

Since the scattering is assumed to be isotropic $f = 1/2$ Hence the number of neutrons crossing energy E per second per c.c., i.e., the slowing down density q(E), which is to be equated to S_o, is

$$q(E) = S_0 = \frac{(1+M)^2}{4M} C \int_u^{u+\ln\alpha^2} \int_{u'-\ln\alpha^2}^u e^{u'-u''} \, du'' du' = C\xi = N\sigma_{s_0} F_0(u)\xi \tag{3.8}$$

Thus

$$C = \frac{S_0}{\xi}$$

and the energy distribution is

$$F_0(u) = \frac{S_0}{N\sigma_{s_0}\xi} \tag{3.9}$$

In terms of E, rather than u, the distribution is

$$F_0(E) = \frac{S_0}{N\sigma_{s_0}\xi E} \tag{3.10}$$

Or

$$n(v) = \frac{S_0}{N\sigma_{s_0}\xi v^{3/2}} \tag{3.11}$$

In the case of isotropic scattering in a mixture, the slowing down density in an infinite medium in which neutrons are produced uniformly satisfies the equation

$$\frac{\partial}{\partial u}\left[N\sigma_{s_0}\bar{\xi}F_0(u)\right] = S_0\,\partial(u)$$ (3.12)

$$\bar{\xi} = \sum_i c^i \xi^i$$

Where

Now the solution of Eq 3.12 is

$$N\sigma_{s_0}\bar{\xi}F_0(u) = S_0 \qquad u > 0$$
$$= 0 \qquad u < 0.$$ (3.13)

and the neutron energy distribution implied by Eq(3.13) is the same as that of Eq.3.10 on the other hand the differential equation which led to Eq(3.13) is an approximation which was valid, because it involved a Taylor series expansion, only if c (in the case of mixtures) varied slowly over one slowing down interval. Actually the energy distribution (Eq. 3.13) is a rigorous asymptotic solution of the space-independent Boltzmann equation only for isotropic scattering in a single substance. For mixtures or anisotropic scattering, Eq. 3.13 is only approximately correct[7,8.

The assumption of a single substance was not necessary to obtain Eq. (3.13)

3.2 Spatial Distribution of Slowed Neutrons; the Slowing Down Kernels

The slowing down density satisfies

$$\Delta q(\underline{r},\tau) = \frac{\partial q(\underline{r},\tau)}{\partial \tau}$$

with the initial condition

$$\lim_{\tau\to 0} \Delta q(\underline{r},\tau) - \frac{\partial q(r,\tau)}{\partial \tau} = S_0(\underline{r})\,\partial(\tau)$$

Suppose a point source emits one fast neutron per second at $\underline{r} = 0$ in an infinite medium. The slowing down density at some lower energy corresponding to age τ will be the solution of

$$\Delta q(\underline{r},\tau) + \partial(\tau)\,\partial(\underline{r}) = \frac{\partial q(\underline{r},\tau)}{\partial \tau} \tag{3.14}$$

This equation is identical in form with the time dependent diffusion equation

The solution, as was found there, is

$$q(\underline{r},\tau) = \frac{e^{-\frac{r^2}{4\tau}}}{(4\pi\tau)^{3/2}} \tag{3.15}$$

i.e., the slowing down density of neutrons from a point monoenergetic source is

Distributed around the point according to a Gaussian function. The range(r_0) of the Gaussian (i.e., the distance at which the density falls to i/e of its value at the source) is

$$r_0 = \sqrt{2\tau} \tag{3.16}$$

For many purposes it is important to know the second spatial moment of the slowing down density. If the slowing down density is Gaussian, then the second moment of neutrons slowed to age which we denote by $\overline{r^2}(\tau)$ is

$$\overline{r^2}(\tau) = \frac{\int_0^\infty q(r,\tau)\,r^2 \cdot 4\pi r^2 dr}{\int_0^\infty q(r,\tau)\cdot 4\pi r^2 dr} = \frac{\int_0^\infty r^4\, e^{-r^2/4\tau}\, dr}{\int_0^\infty r^2\, e^{-r^2/4\tau}\, dr} = 6$$

For a Gaussian distribution, the following relation holds between the age, the second moment, and the range:

$$\tau = r^{-2/6} = r_0^{3/4}$$

The relation between τ and the second moment of the slowing down distribution is the same as the relation between the square of the diffusion

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length, L, and the second moment of the distribution of thermal neutrons around a point source. For this reason $\sqrt{\tau}$ is often called the slowing down length. The age τ is related to the logarithmic energy as

$$\tau(u) = \int_0^{u_i} \frac{du}{3N\sigma_{tr} \, N\sigma_{so}}$$ (3.17)

and is therefore a monotone increasing function of **u**. Thus the spatial distribution of slowed neutrons keeps a Gaussian shape as the neutrons lose energy, but the neutron distribution gradually spreads out since the range r_0 increases with u. The distribution of neutrons slowing down from an energetic source is in this approximation exactly the same as the distribution of heat from an instantaneous heat source. The energy distribution of the neutrons slowed from a point source is

$$F_0(r,u) = \frac{1e^{-\frac{r^2}{4\tau(u)}}}{N_{\sigma_0} \zeta [4\pi(u)]^{3/2}}$$

$\tau(u)$ being the function expressed in Eq. 3.17. If all cross sections are constant, then

$$\tau(u) = \frac{u}{3N\sigma_{tr} \, N\sigma_{s_0} \, \xi}$$

$$F_0(r,u) = \frac{1}{N\sigma_{so} \zeta} \frac{e^{-\frac{3N_{\sigma u}N_{\sigma o}\zeta r^2}{4u}}}{\left(\frac{4\pi u}{3N_{\sigma U} N_{\sigma so}\xi}\right)^{\frac{3}{2}}}$$ **(3,18)**

.......(3.18)

At a given point the flux as the function of **u** waxes and then wanes; the maximum

occurs at the logarithmic energy $\mathbf{u}_{max}$ given by

$$u_{max} = \frac{N\sigma_{tr} \, N\sigma_{so} \xi r^2}{2}$$

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The slowing down density(Eq. 3.15) from a point mono -energetic neutron source may be designated the "point slowing down kernel." Following the procedure used in the discussion of the time dependent diffusion equation, we can write down the corresponding kernels in other geometries.

From the slowing down density arising from a monochromatic energy source it is a trivial matter to compute the slowing down density from a fission neutron source which is poly-energetic. If the number of neutrons emitted per sec between energy E′ and E′ + dE′ from a point source at the origin is f (E′)dE′

Then the slowing down density from such a source is evidently

$$q(r, \mathcal{T}(E)) = \int_0^\infty \frac{\phi e^{\frac{2}{-4[\zeta(E)-\tau(E)]}}}{\{4\pi[\tau(E)-\tau(E)]\}} f(E) dE \qquad (3,19)$$

Where q r τ(E) is the number of neutrons crossing energy E per second per c.c at r. Since in a chain reaction the neutrons originate from a fission spectrum, then slowing down distribution as given by Eq. 3.19 is the one appropriate to a chain reactor in which the moderator is non-hydrogenous.

3.3   Elementary Improvements on Age Theory

The age approximation, and the Gaussian slowing down distribution which it yields, resulted from a spherical harmonic expansion of the angular distribution, and a Taylor's series expansion of the energy distribution.   As has already been pointed out, the Taylor's series expansion is valid only if the mean free path varies slowly over one slowing down interval, while the spherical harmonic expansion could be expected   to be good only fairly near the source.   Thus the age approximation is poor in hydrogenous media (where the mean free path changes rapidly), or at large distances from the soke hi any medium.

That the Gaussian cannot be correct at large distances is evident from the following physical argument: Consider neutrons which have made no collisions

at all.[7,8] These will be distributed like S_O ($e^{-N\sigma r}/4ar^2$) where $N\sigma$ is the macroscopic scattering cross section and So is the source strength. Now at small distances the Gaussian slowing down distribution will exceed this exponential; at large distances, however, the ratio

$$\frac{\text{source neutrons}}{\text{Gaussian moderated neutrons}} = \left(\frac{e^{-N\sigma r}}{4\pi r^2}\right)\left(\frac{(4\pi\tau)^{3/2}}{e^{-r^2/4\tau}}\right)$$

approaches α, since the Gaussian fall off faster than exponential Thus at large distances, the distribution is more exponential than Gaussian. This "first collision paradox" can be expected if the "aging" process, which leads to the Gaussian, is assumed to begin only after the neutrons have made their first collision. The points at which first collisions occur act as "sources" for the slowing down process .

An improvement on the age theory distribution which at least is free from the first collisions are distributed as

$$\frac{e^{-N\sigma(0)r}}{4\pi r^2}$$

the 0 referring to $\mathbf{u} = 0$, the source energy.

According to this picture the slowing down distribution should therefore be

$$q(r,\tau) = \int_0^\infty \frac{e^{-N\sigma(0)|\underline{r}'|}}{4\pi r'^2} \frac{e^{-\frac{|r'-r|^2}{4\tau}}}{(4\pi\tau)^2} d\underline{r}''$$

A further improvement can be made by taking into account the fact that after a neutron has suffered a collision which throws it across energy E, it experiences a "free ride," without changing its energy, until it suffers its next collision. To take this free ride after the last collision into account, it is plausible to include another exponential with mean free path appropriate to the lower energy. Thus the slowing down distribution, including both first and last collisions is

$$q(r,\tau) = \int_0^\infty \int_0^\infty \frac{e^{-N\sigma(0)|r'|}}{4\pi r'^2} \frac{e^{-\frac{|r''-r'|^2}{4\tau}}}{(4\pi\tau)^{3/2}} \frac{e^{-N\sigma(U)|r''-r|}}{4\pi|r''-r|} dr' d\underline{r}'' \qquad (3.20)$$

where $\sigma(0)$ and $\sigma(u)$ are cross sections at the initial and final log energies respectively. Formulas like (3.20) are of course not rigorous; they are rather more plausible than the simple Gaussian and have been used to represent the slowing down distribution from a point monoenergetic source.

In order to compute the second moment of the distribution (3.20) we first state the well known result that the second moment of the distribution from a plane source is just 1/3 the second moment from a point. This

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follows from the relation between a point kernel and the corresponding plane kernel,

$$P_p(r) = -\frac{1}{2\pi r} P'_{pl}(r).$$

Hence

$$\overline{r^2} = \frac{\int_0^\infty r^2 P_p(r) r^2 dr}{\int_0^\infty P_p(r) r^2 dr} = \frac{\int_0^\infty z^3 P'_{pl}(z) dz}{\int_0^\infty z P'_{pl}(z) dz}$$

Upon integrating by-parts, and using the fact that $r^3 P_{pl}(r) \to 0$ as $r \to \alpha$ for any kerels of interest, we obtain

$$\overline{r^2} = 3\overline{z^2}$$

$\overline{z^2}$ being the second moment of the plane distribution.

With this preliminary we compute $\overline{r^2}$ for the distribution (3.20) by computing the corresponding plane second moment, and multiplying by 3. The plane distribution corresponding to (3.20) we write as

$$q(z,\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_1(z') \, P_{pl}(|z' - z''|) \, K_2(|z'' - z|) \, dz' \, dz'' \qquad \ldots\ldots\ldots\ldots(3.21)$$

where $K_1(z)$ and $K_2(z)$ are-plane transport kernels and P_{PI} is the plane Gaussian kernel. The quantity $q(z,\tau)$ is the convolution of the three kernels K_1, P_{Pl}, and K_2.

Now if $\overline{K_1}B_2)$ is the Fourier transform of $K_1(z)$, i.e., if

$$\overline{K_1}(B^2) = \int_{-\infty}^{\infty} K_1(z) \, e^{iBz} dz$$

then z^2_{k1} the second moment of the distribution defined by $K_1(z)$ is

$$\overline{z^2_{K_1}} = \left. \frac{\dfrac{d^2\overline{K_1}(B^2)}{dB^2}}{\overline{K_1}(B^2)} \right|_{B=0} \qquad\qquad (3.22)$$

Furthermore, the Fourier transform of $q(z,\tau)$ is

$$\overline{q}(B^2,\tau) = \overline{K_1}(B^2) \, \overline{P}_p(B^2) \, \overline{K_2}(B^2) \qquad\qquad (3.23)$$

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As follow from equation(3.20) and the definition of Fourier transform .
Hence

$$\left.\frac{\frac{d^2\bar{q}(B^2,\tau)}{dB^2}}{\bar{q}(B^2,\tau)}\right|_{B=0} = \left.\frac{\frac{d^2\bar{K}_1(B^2)}{dB^2}}{\bar{K}_1(B^2)}\right|_{B=0} + \left.\frac{\frac{d^2\bar{P}_{p1}(B^2)}{dB}}{\bar{P}_{p1}(B^2)}\right|_{B=0} + \left.\frac{\frac{d^2\bar{K}_2(B^2)}{dB^2}}{\bar{K}_2(B^2)}\right|_{B=0}$$

That is

$$\overline{z^2_q} = \overline{z^2_{K_1}} + \overline{P^2_{p1}} + \overline{z^2_{K_2}} \qquad (3.24)$$

And

$$\overline{r^2_q} = \overline{r^2_{K_1}} + \overline{r^2_{p1}} + \overline{r^2_{K_2}}$$

In other words, the second moment of a distribution which is the convolution of several kernels is the sum of the second moments of each kernel.

We now apply this result to the distribution (Eq. 3.21). The second moment of the transport kernel

$$e^{-N\sigma(0)r}/4\pi r^2 \qquad \text{is}$$

$$\frac{2}{[N\sigma(0)]^2}$$

and the second moment of the Gaussian is 6τ. Hence the second moment of the slowing down distribution corrected for first and last collisions is

$$\overline{r^2_{(\tau)}} = \frac{2}{[N\sigma(0)]^2} + 6\tau + \frac{2}{[N\sigma(u)]^2} \qquad (3.25)$$

or, upon using the formula for τ

$$\overline{r^2_{(\tau)}} = \frac{2}{[N\sigma(0)]^2} + 2\int_0^u \frac{du}{N\sigma_{tr} N\sigma_{s_0}\xi} + \frac{2}{N\sigma(u)^2} \qquad (3.26)$$

The distribution (Eq. 3.21) is unwieldy analytically, and it has therefore been customary to replace it by a single Gaussian

$$\frac{e^{-r^2/4\tau'}}{(4\pi\tau')^{3/2}}$$

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where τ', the corrected age, is chosen so as to give the same second moment as (Eq. 3.18). Thus the corrected age is

$$\tau'(u) = \frac{\overline{r^2}}{6} = \frac{1}{3\,[N\sigma(0)]^2} + \frac{1}{3}\int_0^u \frac{du}{N\sigma_{tr}\,N\sigma_{s_0}\xi} + \frac{1}{3\,[N\sigma(u)]^2} \qquad (3,27)$$

and it is this age, together with the -simple Gaussian, which is usually used to represent the slowing down density in a heavy moderator [7,8].

3.4 The Group Picture

. The Gaussian slowing down distribution with the corrected age(Eq. 3.27) is a fairly satisfactory representation of the slowing down process in heavy moderator.

However, for certain problems, e.g those involving slowing down process in composite media, even the elegant Gaussian age theory becomes very unwieldy.

The analytical difficulties arise because the age theory equation is a partial differential equation. To avoid these complications a simplified formulation of the slowing down problem which describes the process by a sequence of ordinary differential equations has been used very widely in pile theory.

The general idea of this method, called the method of groups, is to divide the total logarithmic energy interval through which the neutrons pass into a finite number of energy subintervals. Neutrons in a given energy group are supposed to diffuse 'without energy loss until they have experienced a number of collisions equal to the average number of collisions actually required to pass through the energy interval; at this time they pass into the next lower energy interval. Thus removal from an energy interval is treated as an "adsorption" process, the "absorption"

cross-section,($\sigma*_a$), being determined from the relation $\sigma_s/\sigma*_a$ =number of collisions before removal from energy range.

The cross-section for removal of neutrons from one group is also the cross- section for creation of neutrons in the next lower group. The slowing down density, i.e., the number of neutrons passing from the vth energy group to the v^{+}1st group, is therefore

$$\sigma_a^* = \frac{\sigma_{sv}\, \xi}{u_v}$$

(3.28)

where σ_{av} is the average scattering cross-section in the u_v energy interval.

If D, is the average diffusion coefficient in the vth energy group, then the neutron flux $\phi_{v,}(r)$ in the vth group satisfies the diffusion equation

$$D_\nu \Delta \Phi_\nu \, (\underline{r}) - N\sigma_{a\nu}^* \, \Phi \, (\underline{r}) + N\sigma_{a\nu-1} \, \Phi_{\nu-1}(\underline{r}) + S_\nu \, (\underline{r}) = 0 \tag{3.29}$$

$S_v(\underline{r})$ being the number of neutrons produced by an external source per unit volume at $\underline{r}$ in the vth energy interval. In a one group picture, $v = 1$, in which the external source is a δ-function at the origin, the group equation is

$$D_1 \Delta \Phi_1 - N\sigma_{a_1}^* \, \Phi_1 + \partial(r) = 0 \tag{3.30}$$

this has the solution

$$N\sigma_{a_1}^* \, \Phi_1(r) = \frac{e^{-r/L_1^*}}{4\pi r L_1^{*2}} \tag{3.31}$$

Where

$$L_1^{*2} = D_1/N_1\sigma_{a_1}^* \tag{3.32}$$

The second moment of this distribution is $6L^{*2}$, and the slowing down length is given by

$$\frac{\overline{r^2}}{6} = L_1^{*2} = \frac{D_1^*}{N\sigma_{a_1}^*} \tag{3.33}$$

If we substitute for $N\sigma^*_{a1}$ and D_l their expressions in terms of cross-sections, we obtain

$$L_1^{*2} = \frac{u_1}{3N\sigma_{tr_1} \, N\sigma_{s_1}} \tag{3.34}$$

This is identical with the age theory expression for the second moment provided the product $N\sigma_{tr_1}N\sigma_{sl}$ is chosen as

$$\frac{1}{N\sigma_{tr_1} N\sigma_{s_1}} = \frac{1}{u_1} \int_0^{u_1} \frac{du}{N\sigma_{tr_1}(u) N\sigma_{s_1}(u)}$$

(3.35)

With this choice of average cross sections, the one-group picture is seen to give a spatial distribution which is exponential instead of Gaussian which has the same second moment as the age theory, but neutrons is readily found by solving Eq. 3.29. For simplicity we deal with the problem plane symmetry; the solution from a point source is then found by differentiating. The differential equations to be solved are In the n-group picture the distribution from a localized source of high energy

$$\left. \begin{array}{l} D_1 \dfrac{d^2\Phi_1}{dx^2} - N\sigma^*_{a_1}\,\Phi_1 + \partial(x) = 0 \\[2ex] D_\nu \dfrac{d^2\Phi_\nu}{dx^2} - N\sigma^*_{a\nu}\Phi_\nu = N\sigma^*_{a\nu-1}\,\Phi_{i-1} = 0 \qquad \nu = 2 \ldots n. \end{array} \right\}$$

(3.36)

To solve these equations we make a Fourier transformation,

$$\Phi_\nu(x) = \int_{-\infty}^{\infty} \overline{\Phi}_\nu(\omega)\, e^{i\omega x}\, d\omega, \quad \partial(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega x}\, d\omega$$

(3.37)

The transforms satisfy

$$(D_1\omega^2 + N\sigma^*_{a_1})\,\overline{\Phi}_1 = \tfrac{1}{2}\pi$$

$$-(D_\nu\,\omega^2 + N\sigma^*_{a\nu})\,\overline{\Phi}_\nu + N\sigma^*_{a\nu-1}\overline{\Phi}_{\nu-1} = 0 \qquad \nu > 1$$

Hence

$$\overline{\Phi}_1(\omega) = \frac{1}{2\pi(D_1\omega^2 + N\sigma^*_{a1})}; \overline{\Phi}(\omega) = \overline{\Phi}(\omega)..\overline{\Phi}_{k-1}(\omega) = \frac{1}{2\pi(D_1\omega^2 + N\sigma^*_{a1})} \prod_{\nu=2}^{k} \frac{N\sigma^*_{a\nu-1}}{(D_\nu\omega^2 + N\sigma^*_{a\nu})}$$

And therefore

$$\Phi_1(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-i\omega x}}{D_1\omega^2 + N\sigma^*_{a1}} d\omega$$

$$\Phi_1(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-i\omega x}}{D_1\omega^2 + N\sigma^*_{a1}} \prod_{\nu=2}^{k} \frac{N\sigma^*_{a\nu-1}}{(D_V\omega^2 + N\sigma^*_{a\nu})} d\omega \qquad (3.38)$$

Since $\Phi_k(\omega)$ is the product of the Fourier transforms of preceding (Φ_v) the distribution $\Phi_k(X)$ in a given group be the convolution of the distribution of the previous groups. Physically this means that each group acts as the source for succeeding group.

To actually compute $\phi_v(x)$, it is necessary to evaluate the integrals in Eq. 3.38. The integrands have poles in the upper half plane at

$$\omega = i\sqrt{\frac{N\sigma_{av}}{D_v}} = i/L_v^*$$

(3.39)

and we assume for simplicity that all roots are simple. Hence, according to the residue theorem,

$$\Phi_k(X) = \frac{1}{N\sigma_{ak}^*} \sum_{j=1}^{k} \frac{1}{2Lj^*} \frac{e^{-x/L_{j}^*}}{\left(1 - L_1^{*2}\big/L_j^{*2}\right)\left(1 - L_2^{*2}\big/L_j^{*2}\right)\cdots\left(1 - L_k^{*2}\big/L_j^{*2}\right)}$$

(3.40)

where the term $(I - L_j^{*2}/L_j^{*2})$ is omitted from the sum. If $k = I$ (one group picture), Eq. 3.40 reduces to

$$N\sigma_{a_1}^* \Phi_1(x) = \frac{e^{-\dot{x}/L_j^*}}{2L_j^*}$$

(3.41)

which is the plane equivalent of Eq. 3.31. Since $\phi_k(x)$ is the convolution of all the previous $\phi_v(x)$, the total slowing down length for the neutrons slowed out of the kth group must be the sum of the squares of the slowing down lengths in each group individually:

$$\frac{\overline{r_k^2}}{6} = L_1^{*2} + L_2^{*2} + \ldots + L_k^{*2}$$

(3.42)

If the number of groups becomes infinite, but each L_v, is reduced so that

$$\frac{\overline{r_\infty^2}}{6} = \sum_{v=1}^{\infty} L_v^{*2} \equiv \tau$$

(3.43)

where τ remains finite, then the group picture should go over into the continuous
age theory. The slowing down density in the w-group case can be
computed most readily by first passing to the limit in the integrand of
(3.38) and then evaluating the integral. From Eq. 3.38

$$N\sigma_{ak}^* \; \Phi_k(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{i\omega x}}{(1 + L_1^{*2}\omega^2)(1 + L_2^{*2}\omega^2)\dots(1 + L_k^{*2}\omega^2)} \, d\omega$$

Now

$$\lim_{\substack{L_\nu^* \to 0 \\ \Sigma L_\nu^{*2} \to \tau}} \prod_{\nu=1}^{\infty} \frac{1}{(1 + L_\nu^{*2}\omega^2)} = e^{-\omega^2 \Sigma_\nu L_\nu^{*2}} = e^{-\omega^2 \tau}$$

$$(3.44)$$

As this verified by taking logarithm of both sides .Hence

$$\lim_{k \to \infty} N\sigma_{ak}^* \; \Phi_k(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega x} \, e^{-\omega^2 \tau} = \frac{e^{-x^2/4\tau}}{(4\pi\tau)^{1/2}} \qquad (3.45)$$

that is, the group picture and the age picture merge when the number of
groups becomes infinite.

The great merit of the group method is that it involves ordinary instead of
partial differential equations. By taking enough groups it is possible to
approximate the age theory slowing down function to any degree of
accuracy, and still deal only with ordinary equations. The approximate
slowing down functions which are construct&-out of group picture
exponentials are called "synthetic" kernels. In pile problems involving
H_2O as !moderator, it is customary to use one- or two fast neutron

groups in addition, to the thermal neutron group; in piles moderated by heavier materials as many as five or six groups have been used.[7,8,9] In assessing the relative accuracy of the group method and the age theory, it must be remembered that the slowing down function from a point fission source, even in, say, graphite, is not a Gaussian because of the energy spread of the source neutrons. Thus in graphite the three group model is only slightly less accurate than the single Gaussian while in H_2O, because of the very long mean free path at high energies, the slowing down is more nearly represented by a single group picture than by a Gaussian.

3.5 Average Transport Cross Section in Group Method

In order to obtain a one group distribution which has the same second moment as the Gaussian, it is necessary to average the product of the transport and scattering cross sections according to Eq. 3.35. In problems involving only one medium it is only this product which determines the neutron distribution.

However, in problems involving composite media, since one of the boundary conditions across an interface is continuity of the net current, and the current is proportional to the transport mean free path, it is necessary to find an appropriate average for the transport mean free path separately.

To calculate an average transport mean free path which will ensure continuity of the net neutron current in a group, it is necessary to make some assumption with regard to the actual energy distribution of the neutrons in a given group. Evidently the energy distribution will depend on the particular arrangement and properties of the slowing down media on each side of the boundary.[7,8,9]

However, as a simple approximation, it is useful to assume that the energy distribution of the neutrons is the asymptotic distribution

$$\Phi(x,E)\ dE = f(x)\ \frac{dE}{N\sigma_{s_0}\xi E}$$

(3,46)

where a. is the scattering cross section.

The total flux of neutrons in a group from energy E_1 to E_2 is

$$\Phi(x) = \int_{E_1}^{E_2} \Phi(x,E)dE = f(x) \int_{E_1}^{E_2} \frac{dE}{N\sigma_{s_0}\xi E}$$

(3.47)

and the net current is

$$\frac{1}{3}\bar{\lambda}_{tr}\frac{d\Phi(x)}{dx} = \frac{1}{3}\int_{E_1}^{E_2}\lambda_{tr}(E)\frac{d\Phi}{dx}(x,E)\ dE = \frac{1}{3}\frac{df}{dx}\int_{E_1}^{E_2}\frac{\lambda_{tr}(E)dE}{N\sigma_{s_0}\xi E}$$

(3.48)

where-)$\overline{\lambda_{tr}}$ is the correct average transport mean free path .Thus combining Eqs. 3.47 and 3.48, we obtain

$$\overline{\lambda}_{tr} = \frac{\int_{E_1}^{E_2} \lambda_{tr}(E) \frac{dE}{N\sigma_{s_0} E}}{\int_{E_1}^{E_2} \frac{dE}{N\sigma_{s_0} E}}$$

(3.49)

if ζ is constant, i.e., the average transport mean free path which will give continuity of flow and density in a group in which the asymptotic energy distribution holds is an average over $1/N\sigma_s E$.

3.6 The Energy Transfer Distribution of Slowed Neutrons

It is a matter of some practical importance to calculate the manner in which the energy transferred to a moderator by elastic collisions of fast neutrons is distributed in space as the neutrons slow down from a plane source. If the flux of neutrons of log energy $u = \ln E_0/E$ is $F(x,u)$ (plane symmetry), then the number

of elastic collisions per c.c. per second at energy $E = e^u$ is

$$N\sigma_{s_0} F(x,u)$$

Since the logarithm of the ratio of the average energies E' and E after two successive collisions is

$$\ln E'/E = \xi$$

the average energy loss per collision, AE, is

$$\Delta E = E' - E = E(e^\xi - 1)$$

(3.50)

i.e., if the moderator is heavy,

$$\Delta E \approx \xi E$$

(3.51)

This energy increment appears as kinetic energy of the moderator atom. Hence E(x), the energy released per c.c. per second to the moderator by elastic collisions, is for heavy moderators,

$$E(x) = \int_0^\infty N\sigma_{s_0}(u) \, \xi E \, F(x,u) \, du = E_0 \int_0^\infty N\sigma_{s_0}(u) \, \xi e^{-u} \, F(x,u) \, du \qquad (3.52)$$

To evaluate this integral an assumption must be made with respect to the neutron distribution F(x,u). This we take to be Gaussian:

$$F(x,u) = \frac{q(x,u)}{N\sigma_{s_0}\xi} \quad \frac{S}{N\sigma_{s_0}\xi} \frac{e^{-K^2/4\tau(u)}}{[4\pi\tau(u)]^{\frac{1}{2}}}$$

. $\qquad\qquad (3.53)$

where S_0 is the number of neutrons emitted per sq. cm. per second by the source.

$$E(x) = SE_0 \int_0^\infty \frac{e^{-\left[u = x^2/4\tau(u)\right]}}{[4\pi\tau(u)]^{\frac{1}{2}}} du \qquad (3.54)$$

This integral can in general be evaluated only by numerical methods. However, if all cross sections are constant, then, in simplest approximation,

$$\tau(u) = \frac{\lambda_{tr}\lambda}{3} u \qquad (3.55)$$

And

$$E(x) = S_0 E_0 \int_0^\infty \frac{e^{-\left[u - \frac{3x^2}{\lambda_u \lambda tr}\right]}}{\left[\frac{4}{3}\pi\lambda_{tr}\lambda u\right]^{\frac{1}{2}}} du \qquad (3.56)$$

.

Evaluating the integral according to Watson's Bessel Functions we obtain

$$E(x) = \frac{\alpha S_0 E_0}{2} e^{-\alpha x} \qquad (3.57)$$

Where

$$\alpha = \left[\frac{3\xi}{\lambda\lambda_u}\right]^{\frac{1}{2}} \qquad (3.58)$$

The total energy emitted from one side of the source plane cm^2 per second is $S_0E_0/2$; thus, according to Eq. 3.57, the fractional energy

release in each cubic centimeter falls off exponentially with length constant α.

3.7 Slowing Down Distribution in a Finite Block

In order to measure the slowing down distribution from a source it is customary to place the source on the axis of a long parallelepiped and measure the activity of Cd covered In foils placed along the long axis of the parallelepiped. Since **In** has a deep resonance at 1.44 eV, the activity of such a foil will in good part be proportional to the flux of 1.44 eV neutrons. Actually, because of higher resonances, the reading of the **In** foil is not quite proportional to the 1.44 eV flux; according to Hill and Roberts, at points close to a source of 30 kv neutrons in graphite, almost 40% of the activation of **In** is due to absorption above 1.44 eV. Farther from the source the perturbation due to higher resonances becomes less so that the mean square distance to 1.44 eV as measured by **In** foils is in error by much less than 40%. The theory of this experiment is a good illustration of the usefulness of the age approximation, and we give the details in the following paragraphs.

Suppose a monoenergetic unit source is placed at the point x = 0, y = 0, z = 0 in an infinitely long moderating prism of sides 2a . The slowing down density satisfies

$$\Delta q(x,y,z,\tau) = \frac{\partial q(x,y,z,\tau)}{\partial \tau}$$

.
.(3.59)

$$\lim_{\tau \to 0} q(x,y,z,\tau) = \partial(x,y,z)$$

.
(3.60)

where we have assumed the long direction is along z. The boundary conditions may be taken with sufficient accuracy (provided the width of the block is much larger than the mean free path).

q = 0 on the extrapolated boundary.

the extrapolated' boundary being the geometric boundary a' augmented by the extrapolation distance $0.71\lambda_{tr}$. We {denote a' + .71 λ_{tr}, by a. It is convenient to assume λ_{tr} independent of energy; again this is an unimportant assumption provided the block dimension is large compared to a mean free path.

The solution of Eq. 3.19 which satisfies the boundary conditions is

$$q(x,y,z,\tau) = \frac{1}{a^2} \sum_{m,n} \cos B_m x \cos B_n y \; e^{-[B_m^2+B_n^2]\tau} \frac{e^{-z^2/4\tau}}{(4\pi\tau)^{\frac{1}{2}}}$$

. (3.61)

Where

$$B_m = (2m + 1)\frac{\pi}{2a}, \quad B_n = (2n + 1)\frac{\pi}{2a} \qquad (3.62)$$

The sine solution is not used because of the symmetry of the source distribution.

The shape of the distribution along the z-direction is the same as from an infinite plane. As the neutrons age (τ increases) the intensity of the distribution falls because of the exponential factor. This factor accounts for leakage out of the block. Its dependence of τ arises from the circumstance that neutrons with large τ must have diffused for a relatively long time and therefore must have had a good chance to leak out of the sides. The magnitude of the leakage is determined by the ratio τ/a^2.

The distribution (Eq. 3.61) is represented as a sum of characteristic function

The slowing down density can, of course, also be computed by observing that the neutron distribution from a point source in a finite block can be viewed as the superposition of distributions from point sources and sinks appropriately distributed in an infinite medium. The mathematical relation between the source wise and characteristic function

representations of the distributions is established by means of the Poisson summation formula

$$\sum_{m}\sum_{n}\varphi(m,n)=\sum_{\lambda}\sum_{\mu}\int\int_{-\infty}^{\infty}\varphi(u,v)e^{-2\pi i(\lambda u+\mu v)}dudv \qquad (3.63)$$

.

Upon applying this transformation to the series (3.61) with

$$\varphi(m,n)=\cos B_m x\cos B_n y$$

We obtain

$$q(x,y,z,\tau)=\frac{1}{(4\pi\tau)^{\frac{3}{2}}}\sum_{-\infty}^{\infty}\sum(a)^{\lambda+u}e^{-\left[(x-\lambda a)^2+(y-\mu a)^2+z^2\right]/4\tau} \qquad (3.64)$$

Each term in Eq(3.64) represents a source or sink of unit strength situated at the point (λa, μa, 0). The source wise representation of the slowing down distribution converges better than the characteristic function representation at points close to the source; at points far from the source the characteristic function form is the better converging.

3.8 Measurement of Slowing Down Length

The second moment of the distribution (3.61) is

$$\overline{z^2}=\frac{\displaystyle\int_0^{\infty}q(x,y,z,\tau)z^2 dz}{\displaystyle\int_0^{\infty}q(x,y,z,\tau)dz}=2\tau$$

That is, the second moment in a finite block is the same as in an infinite block.

Hence foil measurements in a block of finite width yield the same second moment as measurements in an infinite medium. This result is independent of the relative importance of the various harmonics contained in Eq (3.61) and holds provided only that the distribution is strictly Gaussian.

Most neutron sources are not monoenergetic, nor is the slowing down intrinsically Gaussian. For both these reasons the mean square distance measured in a finite block is not strictly the same as the mean square distance in an infinite system. For example, if the energy distribution of the source is $f(\tau')d\tau'$, then

$$q(x,y,z,\tau) = \frac{1}{a^2} \sum_{m,n} \cos B_m x \, \cos B_n y \int_0^\infty \frac{e^{-[B_m^2+B_n^2](\tau-\tau')} f(\tau') \, e^{-z^2/4(\tau-\tau')} \, d\tau'}{[4\pi(\tau-\tau')]^{1/2}}$$

The second moment of this distribution along the z axis ($x = y = 0$) is

$$\overline{z^2} = \frac{\sum\limits_{m,n} \int_0^\infty \int_0^\infty \frac{e^{-[B_m^2+B_n^2](\tau-\tau')}f(\tau')z^2 \, e^{-z^2/4(\tau-\tau')}}{[4\pi(\tau-\tau')]^{1/2}} dz \, d\tau'}{\sum\limits_{m,n} \int_0^\infty \int_0^\infty \frac{e^{-[B_m^2+B_n^2](\tau-\tau')}f(\tau')e^{-z^2/4(\tau-\tau')}}{[4\pi(\tau-\tau')]^{1/2}} dz \, d\tau'}$$

............(3.65)

In general this second moment will differ from the second moment $\overline{z^2}$ measured in an infinite medium:

$$\overline{z^2}_\infty = \frac{\int_0^\infty \int_0^\infty f(\tau') \, z^2 \frac{e^{-z^2/4(\tau-\tau')}}{[4\pi(\tau-\tau')]^{1/2}} dz \, d\tau'}{\int_0^\infty \int_0^\infty f(\tau') \frac{e^{-z^2/4(\tau-\tau')}}{4\pi(\tau-\tau')} dz \, d\tau'} \qquad (3.66)$$

Corrections must therefore be made to the observed infinite system $\overline{z^2}$ in order to obtain the true infinite system $\overline{z^2}\alpha$. It is possible to compute these corrections for a completely general kernel and this will be done in the remainder of this section, The corrections will be made by observing that the neutron distribution in a finite block can be considered as the sum total of effects from a suitable distribution of positive and negative sources in an infinite medium, provided as we shall assume, the extrapolation distance can be neglected compared to the block size, or is independent of neutron energy. Now a point source at the center of the z = 0 plane in a long block of sides 2a is equivalent to a sequence of

positive and negative sources spaced at intervals of 2a in the z = 0 plane. Such a sequence can be represented as

$$\partial(x)\, \partial(\gamma,\alpha) = S(x,y)\, \partial(z) = \partial(z) \sum_{m.n} \cos B_m x \cos B_n y$$

………………(3.67)

where x and y are allowed to have any value from -α to +α. We consider the function

$$q(x,y,z) = \int_{-\infty}^{\infty}\int S(x',y')\, P[\sqrt{(x-x)^2 + (y-y)^2 + z^2}]\, dx'dy' = \sum_{m,n} \int_{-\infty}^{\infty}\int \cos B_m x' \cos B_n y'$$

$$\times P[\sqrt{(x-x')^2 + (y-y')^2 + z^2}]\, dx'dy' = \sum_{m.n} \cos B_m x \cos B_n y\, \overline{P}(B_m^2 + B_n^2, z)$$

where $P(B_m^2 + B_n^2, z)$ is the two dimensional Fourier transform of the point slowing down kernel, P (r): -

$$\overline{P}(B_m^2 + B_n^2, z) = \int_{-\infty}^{\infty}\int e^{-i[B_m \xi + B_n \eta]}\, P[\sqrt{\xi^2 + \eta^2 + z^2}]\, d\xi d\eta. \qquad (3.69)$$

The function q(x, y, z) can be viewed as the slowing down density in an infinite medium in which the infinite array of !positive and negative sources defined by Eq. 3.67 is situated. Since according to Eq. 3.68 q(x, y, z) vanishes on the boundary of the block, it can also be viewed as the slowing down density in the finite system due to a single point source at x = y = z = 0, provided the extrapolation distance is energy independent. A range measurement results in the observed 2kth moment $\overline{z^{2k}}$

$$\overline{z^{2k}} = \frac{\int_{-\infty}^{\infty} z^{2k} q(o,o,z)dz}{\int_{-\infty}^{\infty} q(o,o,z)dx}$$

$$= \frac{\sum_{m,n} \iiint_{-\infty} z^{2k} \cos B_m x' \cos B_n yp\left(\sqrt{x'^2 + y'^2 + z'^2}\right) dx'\,dy'\,dz'}{\sum_{m,n} \int_{-\infty}^{\infty}\int\int \cos B_n y' \cos B_m x'\, p\left(\sqrt{x'^2 + y'^2 + z'^2}\right) dx'\,dy'\,dz'} \qquad (3.70)$$

We now show how the moment in an infinite system can be expressed in terms of the observed moments in the finite system. Since P(x, y, z) is an even function of x, y, z, we can replace

$CosB_m x'$ $cosB_n y'$ by $cos(B_m x + B_n y)$ in equation 3.70 now

$$\cos(B_m x + B_n y) = \sum_v \sum_{s=0}^{v} \frac{(-1)^v}{2\tau} 2_v c_{2s} B_m^{2(v-s)} B_n^{2s} x^{2(v-s)} y^{2s} \qquad (3.71)$$

Where $_{2v}C_2$ is binomial coeffient

upon substituting equation 3.71 into 3.70 we find

$$\overline{Z^{2K}} = \frac{\sum_m \sum_n \sum_v \sum_{s=0}^{v} \frac{(-1)^v}{[2(v-s)]!} B_m^{2(v-s)} B_n^{2s} \int\int\int_{-\infty}^{\infty} x^{2(v-s)} y^{2s} z^{2k} p(\underline{r}) dxdydz}{\sum_m \sum_n \sum_v \sum_{s=0} \frac{(-1)^v}{[2(v-s)]!} B_m^{2(V-S)} B_n^{2s} \int\int\int_{-\infty}^{\infty} x^{2(v-s)} y^{2s} p(\underline{r}) dxdydz} \qquad (3.72)$$

The integrals which appear in Eq. (3.72) are of the form

$$x_\infty^{2i} y_\infty^{2j} z_\infty^{2l} = \int\int\int_{-\infty}^{\infty} x^{2i} y^{2j} z^{2l} p(\underline{r}) dxdydz \qquad (3.73)$$

and can be evaluated by shifting to polap coordinates. Thus

$$x_\infty^{2i} y_\infty^{2j} z_\infty^{2l} = \frac{(2i)!(2j)!(2l)!(i+j+l)!}{i!j!l!} \overline{z_\infty^{2i+2j+2l}} \qquad (3.73)$$

upon substituting equation (3.73) into(3.72) we obtain

$$\overline{z^{2k}} = \frac{\sum_m \sum_n \sum_v \sum_{s=0} (-1)^v B_m^{2(V-S)} B_n^{2s} \frac{(2k)!(v+k)! z^{2k(v+1)}}{s!(v-s)k![2(v+k)]!}}{\sum_m \sum_n \sum_v \sum_{s=0} (-1)^v B_m^{2(v-s)} B_n^{2s} \frac{v! z_\infty^{2v}}{s!(v-s)!(2v)!}} \qquad (3.74)$$

which is an infinite system of linear equations relating the observed moments $\overline{z^2}k$ to the infinite system moments $\overline{z^2}k\alpha$ The system can be solved for each $\overline{z^2}k\alpha$ in terms of the $\overline{z^2}k$ by successive approximations, in which, at each stage of the approximation only a finite number of equations and unknowns are used[12,13]. Such a process will converge well if the block dimension is large compared to the slowing down range. If the source instead of being concentrated in a point is distributed

over the z = 0 plane   like cosB₀xcosB₀y,   only   the term m = n = 0 appears in Eq. 3.74.

The infinite second moment can then be expressed explicitly in terms of the measured finite system moments:

$$
\overline{z^2_\infty} = \frac{\overline{z^2} + \frac{1}{3} \overline{B_0^2 z^4} + \frac{1}{30} \overline{B_0^4 z^6} \ldots}{1 + \overline{B_0^2 z^2} + \frac{1}{6} \overline{B_0^4 z^4} + \frac{1}{90} \overline{B_0^6 z^6} \ldots}
$$

(3.75)

and this expression gives   the correction for converting $\overline{z^2}$ into $\overline{z^2}\alpha$. Equation 3.75 is of practical importance since measurement of fission neutron ranges are sometimes performed by using the thermal neutrons from a thermal column which are distributed like cos **B**ox cos**B**oy to produce   fissions in a flat plate of fissionable material.   The fission neutrons   in   such an arrangement will be distributed   also as cos**B**oxcos**B**oy [7,8,9].

Chapter Four

Result and discussion

The neutrons produced in the fission reactions emerge with the average energy being around 2 MeV; therefore neutron moderation is required to achieve well thermalized neutron flux. Their usage is spread over various theoretical concepts of the neutron physics field as it carried out in chapter three, and with applications exploiting several physical processes like neutron capture, elastic and inelastic scattering, up scattering, etc. So the use evaluated nuclear data format [ENDF] library neutron transport code is used to evaluate the sample being bombarded with neutrons, causing one of the most common neutron matter nuclear reactions or neutron capture, where a neutron interacts with the target nucleus via non-elastic and elastic interaction and compound nucleus is formed in an excited state. The excitation energy of the compound nucleus is due to the binding energy of the neutron with the nucleus[2,10,11].

Frequently can this new configuration also include radioactive nucleus which also de-excites and emits characteristic delayed gamma rays, but with much longer half lives that can range from part of a second to several years. However ENDF-format libraries are computer readable file of nuclear data that describes nuclear reaction cross-sections, the distribution in energy and angle reaction products, which are intended to be used for a wide variety of application that require calculations of transport of neutrons and charged –particles through materials, so that nuclear data evaluation that utilizes developments in nuclear theory (chapter three), modeling, simulation, and experiment. The ENDF/B-VII.1 library is the latest recommended evaluated nuclear data file for use in nuclear science and technology applications, and incorporates advances made in the five years since the release of ENDF/B-VII.0.

These advances focus on neutron cross sections, covariance's, fission product yields and decay data, and represent work by the US Cross Section Evaluation in nuclear data evaluation that utilizes developments in nuclear theory, modeling, simulation, and experiment[2,9,16].
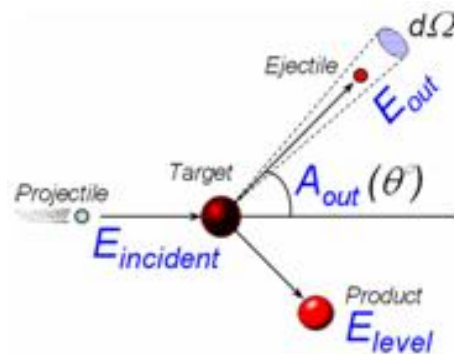
Differential data with respect to angle and energy $d^2\sigma/d\Omega/dE$(E-out)

Cross Section
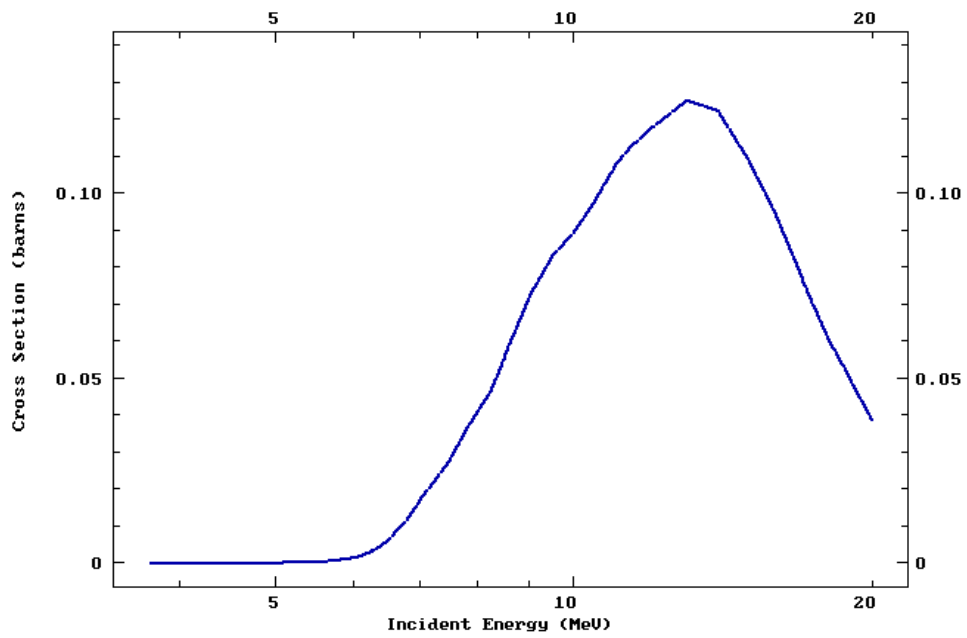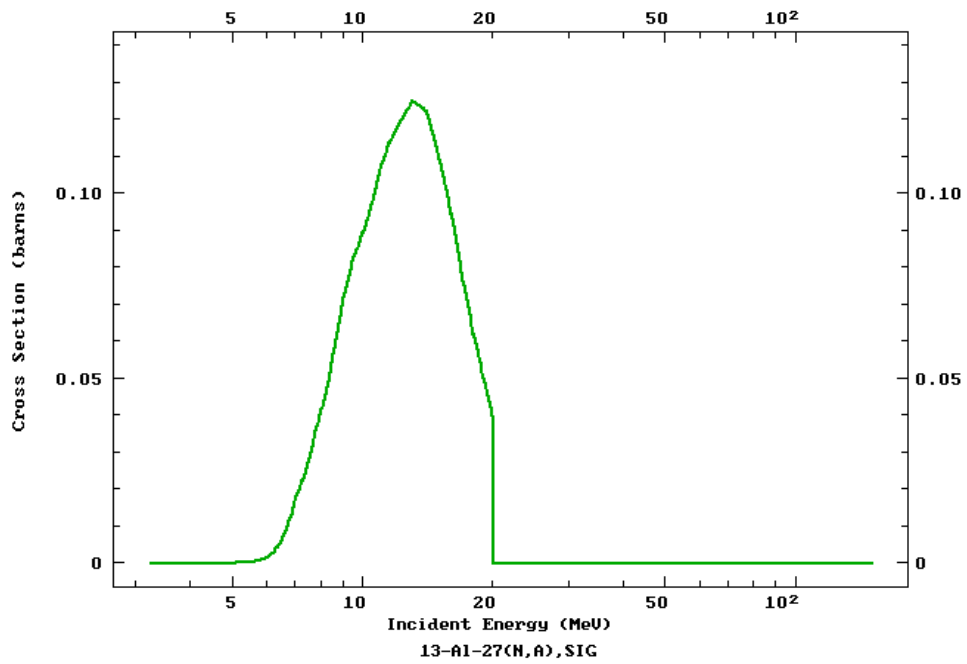
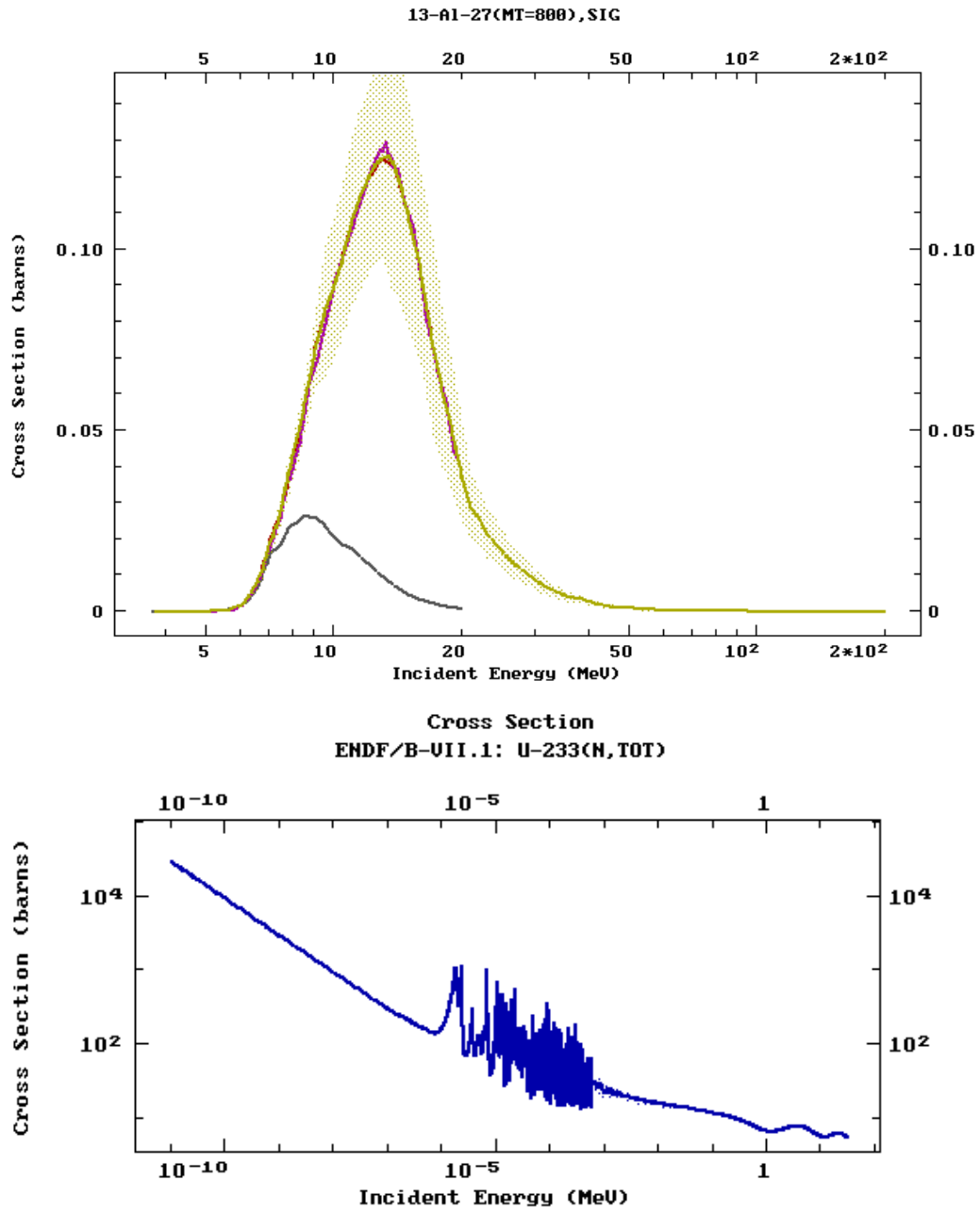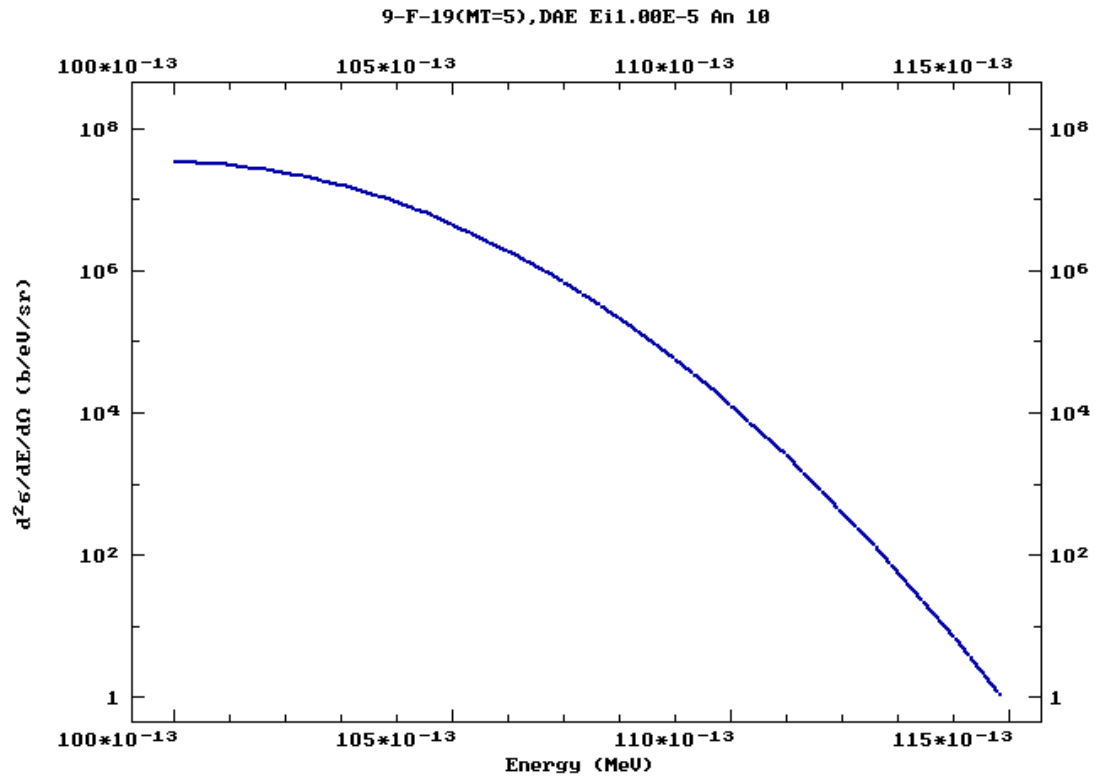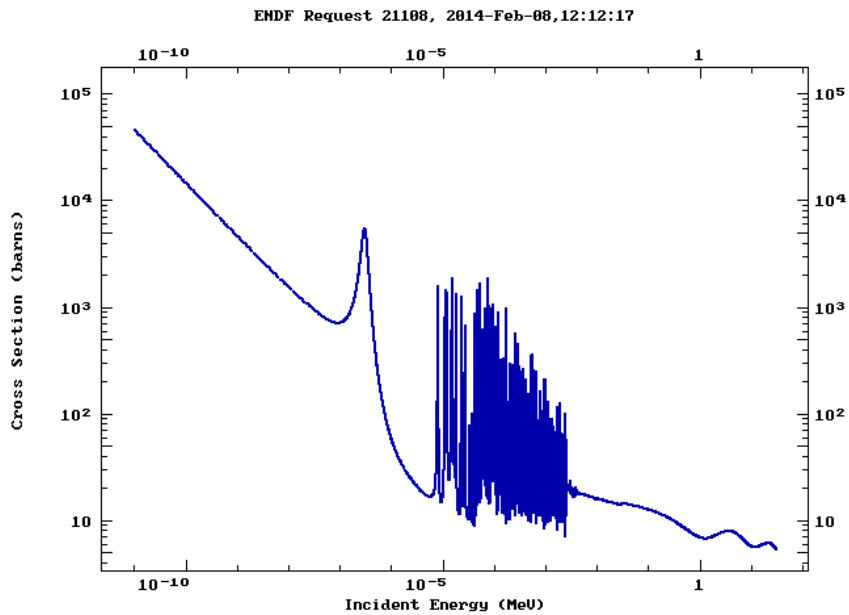Typical Neutron Absorption Cross Section vs. Neutron Energy

Figure 2: Energy dependence of the absorption cross section

9-F-19(MT=5),DAE Ei1.00E-5 An 10

The various cross sections varies with the energy/speed of the neutrons as shown in the graph below.

ENDF Request 21108, 2014-Feb-08,12:12:17

The fission reactions of thermal neutron

**U-235(N,TOT) U-235 PU-239**   The Targets

N,TOT   Total continuum reaction. The reaction

 SIG   Cross sections the quantity

The unusually high absorption cross sections of these two materials make them use fid as thermal-neutron poisons.

At higher energies the cross section may have large peaks superimposed on the I/v trend These peaks are called resonances and occur at neutron energies where reactions with nuclei are enhanced. For example, a resonance will occur if the target nucleus and the captured neutron form a "compound" nucleus, and the energy contributed by the neutron is close to that of an excited state of the compound nucleus.

In heavy nuclei, large and narrow resonances appear for neutron energies in the eV range. For energies in the keV region the resonances can be too close together to resolve. In the MeV region the resonances are more sparse and very broad, and the cross sections become smooth and rolling. For light nuclei, penances appear only in the MeV region and are broad and relatively small. For nuclei with intermediate weights (such as
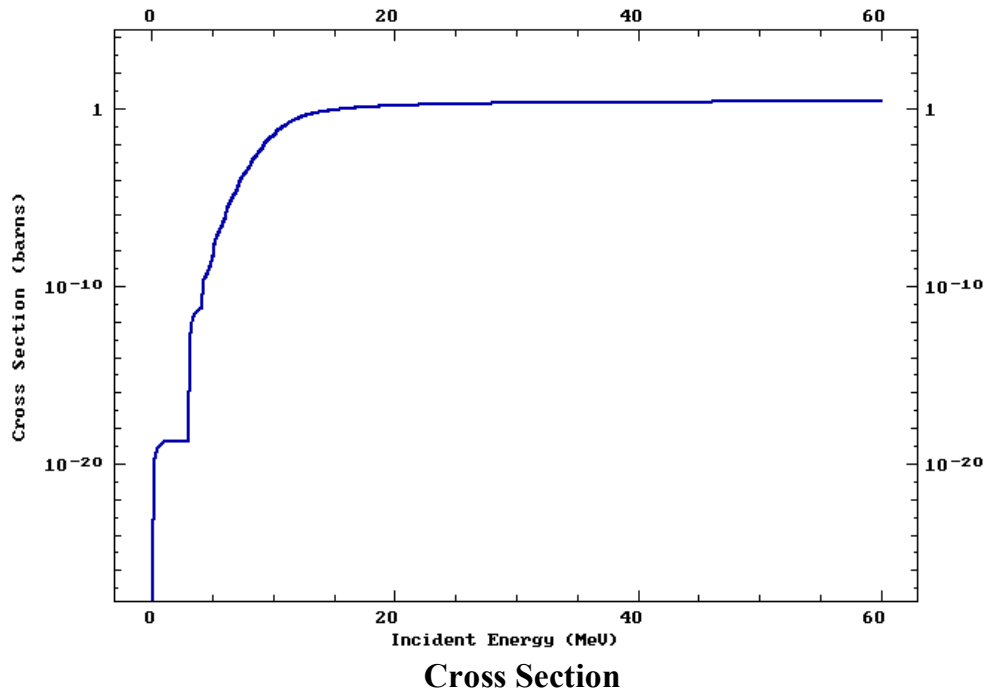
Cross Section

The reaction of thermal neutron with Uranium 235M

U-235M is   the Target

N,TOT is   the Total continuum reaction.

 SIG is   Cross sections quantity

cross sections with reconstructed resonances and applied Doppler

broadening at the temperature 293°K =20°C

cross section from file MF3 as is (sometimes presents only

"background" data without resonances in low energy region)

dσ/dΩ   - angular distributions,

dσ/dE   - energy distributions,

d²σ/dE/dΩ - double differential cross sections,

σ ± Δσ   - cross sections with uncertainties (if given)

Conclusion & Remarks

- -Analytical transport equations exist that describe the exact behavior of neutrons in matter. However, only approximate numerical solutions to these equations can be obtained for complicated systems. Procedures for obtaining these numeral solutions are classified as discrete ordinates techniques

- -A standard basis for comparing moderating abilities of different materials is the moderating power. If one material has a larger moderating power than another, less of that material is needed to achieve the same degree of moderation. Two factors * important

The probability of a scattering interaction and the average change 'in kinetic energy of the neutron after such an interaction.

  - -To be an effective moderator, both the probability of an interaction and the average energy loss in one scatter should be high.

  - -A material with a large moderating power might nevertheless be useless as a practical moderator if it has a large absorption cross section. Such a moderator would effectively reduce the speeds of those neutrons that are not absorbed.

  - Once the neutrons are produced and moderated to appropriate temperatures, they have to be transported with the right characteristics to the sample.

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