# Chapter Three

# Analysis of Bridge Decks

## 3.1 General

Skewed bridges are often encountered in highway design when the geometry cannot accommodate straight bridges. Highway bridges are characterized by the angle formed with the axis of the crossed highway. The skew angle can be defined as the angle between the normal to the centerline of the bridge and the centerline of the abutment or pier cap, due to high traffic speeds road or railway schemes can seldom be modified in order to eliminate the skew of their bridges. Therefore, a considerable number of skew bridge decks are constructed.

For decks with skew less than 25° a simple unit strip method of analysis is generally satisfactory. For skews greater than 25° then a grillage or finite element method of analysis will be required. Skew decks develop twisting moments in the slab which become more significant with higher skew angles. Computer analysis will produce values for Mx, My and M xy where M xy represents the twisting moment in the slab. Due to the influence of this twisting moment, the most economical way of reinforcing the slab would be to place the reinforcing steel in the direction of the principal moments. However these directions vary over the slab and two directions have to be chosen in which the reinforcing bars should lie.

Extensive tests on various steel arrangements have shown the best positions as illustrated in Fig (3.1) below

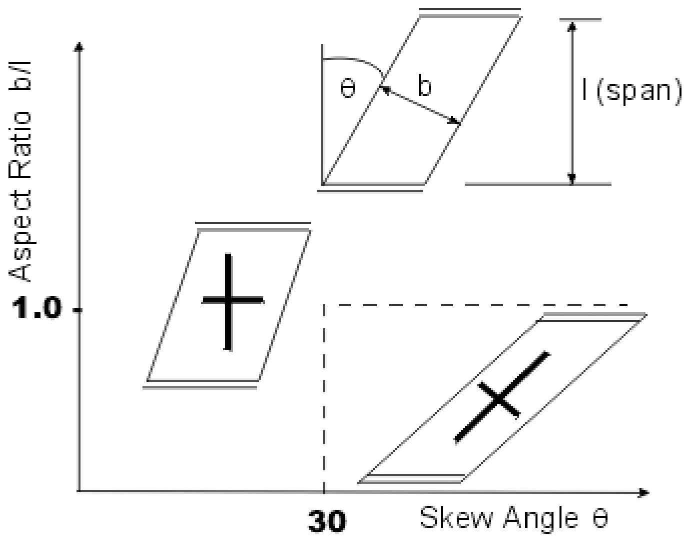


**Fig. (3.1): Skew angle vs. Aspect ratio**

## 3.2 Structural analysis method

In practice, structural analysis can be understand more theoretically as a method of engineering design process to prove the serviceability and safety of a design without a dependence on directly testing it. To obtain an accurate analysis, an engineer must determine all data needed such as structural loads, geometry, support conditions, and material properties of the structures.

As a result, from the analysis include support reactions, stresses, displacements, bending moment and torsion. This term of information is very important to be compared to the criteria that indicate the conditions of failure. All structures have long been designed with the objective to avoid failure under static or moving loads and in time provide a safe and economical construction.

This aspect of safety is appropriately designed with specifications such to the British standard.

In civil engineering fields, a lot of method being used to analyze structures. In most cases, a number of approximate analysis techniques can be used to determine the response of bridge structure due to loadings. Advanced structural analysis may study dynamic response, stability and non-linear behavior.

From a theoretical viewpoint, the main goal of structural analysis is to compute the deformation, internal forces, and stresses that obtained in structure due to loads applied to it. Example of methods being used to analyze the structure, are analytical method, Classical method, elasticity method and finite elements method.

### 3 .2.1 Finite elements method in structural analysis

In conducting this study, the method used to analyze the bridge deck is to use the finite element method. Finite element method is a very useful method to explain for the numerical solution for lots of engineering problems. With the support of computer technology, complex problems can be modeled to ease the analyst.

To work with a finite element analysis via software technology, ones must have a good understanding on the basic theory, techniques of modeling and the computational aspects of the finite element method.

Structure will be divided into elements and every element will be discredited into a simpler geometric shape called finite elements. Another parameter such as material properties and geometry are also considered and expressed in terms of unknown value at element corner [9].

This type of method has been used extensively to analyze bridge decks. This type of method is very familiar to most of bridge designers, and finite element method is the only- method that capable of dealing with certain bridge forms. The methods primarily used by bridge engineers for plate bending problems and lead to in-plane analysis of two dimensional elastic structures [9].

Finite element method then extended to the bending of slabs by Zienkiewicz and Cheung (1964) who was also demonstrated the ability of the method to deal with various boundary conditions, variable slab thicknesses and orthography of slab. Recent years, software has become more practical which allows the use of finite element models for everyday analysis in the office [4] .

### 3.2.2 General finite element method

Finite element method is similar to matrix stiffness method to analyze structures. By using equilibrium equation, we can determine the reaction, displacement and the stiffness of the element.

$$F=KD \qquad \text{--------------------------} \ (3.1)$$

Where;

K= Stiffness matrix of system.

D= Nodal deflection (vertical, horizontal, rotation).

F= Nodal forces.

In finite element method, the element can be line, surface or volume. Truss beam and frame often modeled using lines element. 2D problems such as plane stress, plain strain, plate and shell are modeled as surface element. For 3D analysis, volume element is used [4].

Structures such as beam, truss and frame can be analyzed using stiffness method by utilizing exact mathematical models. However, there are other cases, which involve complex structures, those methods are not suitable to be used. Such complex structure can be solved by using finite element method.

By applying the FEM analysis using software it is more economical in term of cost and time comparing to laboratory test, which are expensive and can be time consuming even though it is the most suitable method to study the static behavior of structure [4].

In this study, SAFE finite element software is used. The model geometry in terms of features is discredited into finite elements in order to perform the analysis. Using the increasing of the discrimination numbers of the features will result in the increment of the accuracy of the solution.

Complete finite element analysis will involve three stages. Figure (3.2) shows steps involved in finite element analysis [4].

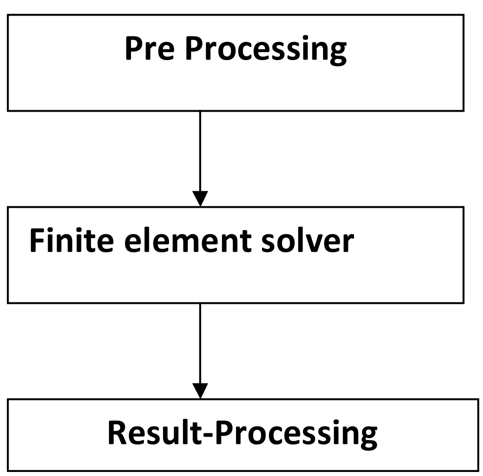


**Fig. (3.2): Steps of finite element analysis.**

### 3.2.2.1 Pre-processing

The first step in using FEA, pre-processing, is constructing a finite element model of the structure to be analyzed. This can be in either 1D, 2D, or 3D form, modeled by line, shape, or surface representation.

The primary objective of the model is to realistically replicate the important parameters and features of the real model. The simplest mechanism to achieve modeling similarity in structural analysis is to utilize pre-existing digital blueprints, design files, CAD models, or data by importing that into an FEA.

Once the finite element geometric model has been created, a meshing procedure is used to define and break up the model into smaller element.

In general, a finite element model is defined by a mesh network, which is made up of the geometric arrangement of elements and nodes. Nodes represent points at which features such as displacements are calculated. FEA packages use node numbers to serve as an identification tool in viewing solutions in structures such as deflections.

Elements are bounded by sets of nodes, and define localized mass and stiffness properties of the model. Elements are also defined by mesh numbers, which allow references to be made to corresponding deflections or stresses at specific model locations [4] .

### 3.2.2.2 Finite element solver

The next stage of the FEA process is analysis. The FEM conducts a series of computational procedures involving applied forces, and the properties of the elements which produce a model solution. Such a structural analysis allows the determination of effects such as deformations, strains, and stresses which are caused by applied structural loads such as force, pressure and gravity [4].

#### 3.2.2.3 Result processing

These results can then be studied using visualization tools in FEA to view and to fully identify the implications of the analysis. Numerical and graphical tools allow the precise location of data such as stresses and deflections to be identified [4].

## 3.3 Bridge deck analysis model using finite element method (FEM)

There are several finite element models that can be used. All models can be used to analyze bridge decks. Using general purpose finite element software. In finite element modeling, different elements are used to cater for different types of structures. Types of models that can be used in analyzing a bridge decks are: [4]

i. Grillage analysis,

ii. Orthotropic plate model,

iii. Beam and shell model, and

iv. 3d solid model.

### 3.3.1 Grillage model

Application of grillage model for analysis bridge deck has been widely practice in the 1960s with the availability of computer technology. This type of analysis is inexpensive and easy to use. The application of grillage analysis is well established and has been discussed by many of researchers [10].

Grillage analysis results have been compared to models of bridges and The method has been prove to be reasonably accurate for many shapes of structures, loading condition and support arrangements [10] .

Plane grillage method often involves the modeling of bridge slab as a skeletal structure made up of a mesh of beams lying in one plane. Each grillage member represents a portion of the slab, with the longitudinal beams representing the longitudinal stiffness while the transverse grillage members representing the transverse stiffness. Nevertheless, it is more practical nowadays to replace it with finite element method [10].

## 3.3.2 Orthotropic plate model

Material that has a behavior, direction is independent of the others is known as anisotropic. Orthotropic material is a material that its behavior varies in mutually perpendicular direction (X and Y) only. This type of model is best suite for bridge decks.

For isotropic material, the behavior in all directions is the same. This type of material not frequently use in bridge construction, but isotropic plate theory can be used with logical accuracy and often use for the analysis of many bridge decks.

There are two types of orthotropic plate identified as:

(a) Materially (naturally) orthotropic

Composed of a homogeneous material which has different elastic properties in two orthogonal directions, but the same geometric properties.

Plate has a uniform thickness, with same second moment of area in both directions, but different in modulus of elasticity, such as timber. This type of plate is not usual to found in bridge deck but is frequently used as approximation of actual condition [10] .

(b) Geometrically (technically) orthotropic

Posse's different second moments of area in two directions, such as reinforced concrete or voided slab [10].

Orthotropic bridge deck is often modeled using materially orthotropic finite elements. In such cases, Ix= Iy, but only one depth can be specified. This problem can be overcome by determining an equivalent plate depth and altering the modulus of elasticity of the element to allow the differences in second moment of inertia [4].

### 3.3.3 Beam and shell model

Beam and shell are used widely for modern bridges. Beam and shell bridges are commonly suitable for the same span length. Slab bridges are frequently chosen because of their availability in inaccessible areas such as deep valleys or railways.

A lot of method in constructing beam and slab bridges, the most known is an in-situ concrete slab on steel or pre-cast concrete beams, or steel beams with a composite steel or concrete slab, a pre-cast concrete slab or even an entire of in-situ beam and slab.

In construction, the beam usually works alone and must capable of carrying the available load including self weight or any other load that present above. The deck is considered to be two dimensional upon completion [10].

Beam and shell models are implemented in two different element formulations, a shell element (Guttmann et al.1995) and a beam element [4] .

### 3.3.3.1 Beam element

Beam element is used to model plane and space frame structures. A variety of thin and thick beams in 2 and 3 dimensions are available. Beam element may be straight or curved and may model axial force, bending and torsional behavior.

Material use such as metal, steel, aluminum, polymer, composite, fiber reinforced composite, timber, concrete, reinforced concrete, pre-stressed concrete, etc

#### ❖ Equations for beams

A beam possesses geometrically similar dimensional characteristics as a truss member, as shown in Figure 3.3. The difference is that the forces applied on beams are transversal; meaning the direction of the force is perpendicular to the axis of the beam. Therefore, abeam experiences bending, this is the deflection in the y direction as a function of x.

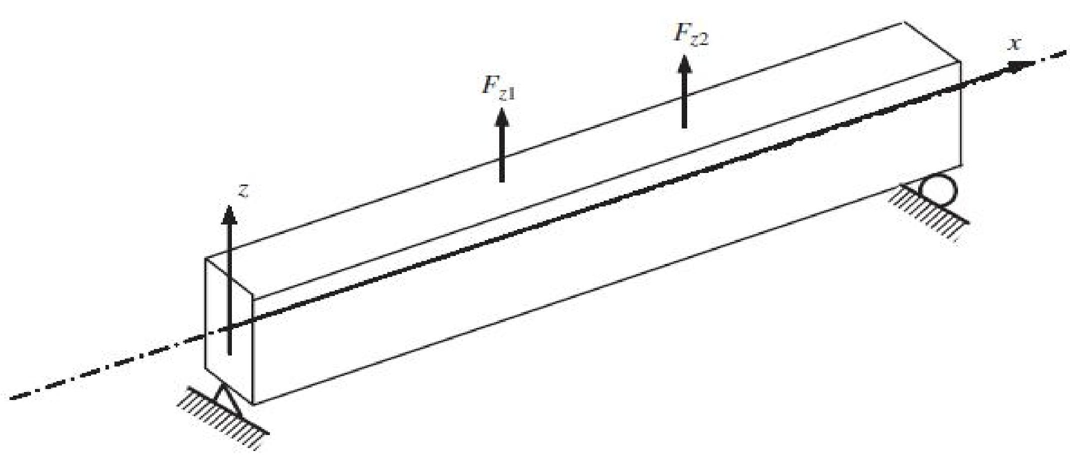


**Fig. (3.3): Simply supported beam.**

- **Stress and Strain**

The stresses on the cross-section of a beam are the normal stress, $\sigma_{xz}$, and shear stress, $\sigma_{xz}$. There are several theories for analyzing beam deflections. These theories can be basically divided into two major categories: a theory for thin beams and a theory for thick beams. This book focuses on the thin beam theory, which is often referred to as the Euler–Bernoulli beam theory. The Euler–Bernoulli beam theory assumes that the plane cross-sections, which are normal to the un deformed, centroidal axis, remain plane after bending and remain normal to the deformed axis, as shown in Figure 3.4. With this assumption, one can first have

$$\varepsilon_{xz} = 0$$ ……………… (3.2)

Which simply means that the shear stress is assumed to be negligible? Secondly, the axial displacement, u, at a distance z from the centroidal axis can be expressed by

$$u = -z\theta$$ ………..(3.3)

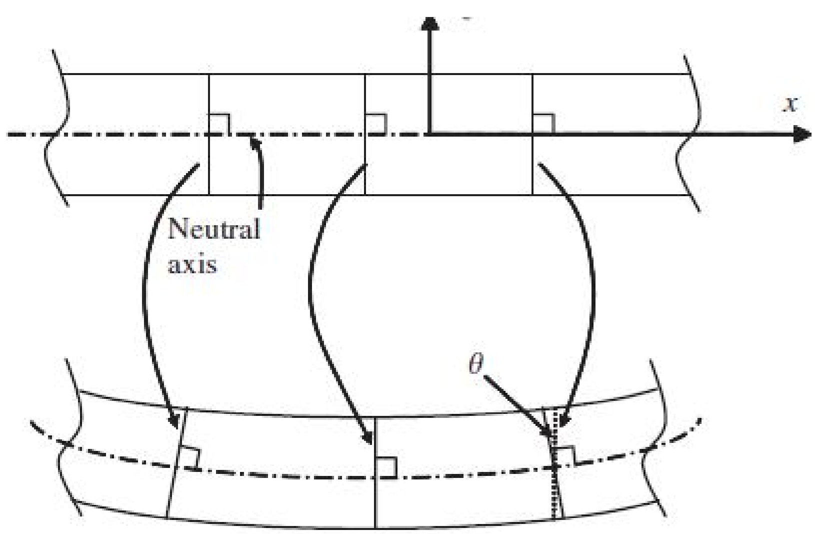


**Fig. (3.4): Euler–Bernoulli assumption for thin beams.**

Where θ is the rotation in the x–z plane. The rotation can be obtained from the deflection of the centroidal axis of the beam, w, in the z direction:

$$\theta = \frac{\partial w}{\partial x} \qquad \text{……..……(3.4)}$$

The relationship between the normal strain and the deflection can be given by

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} = -zLw \qquad \text{…………….(3.5)}$$

Where L is the differential operator given by

$$L = \frac{\partial^2}{\partial x^2} \qquad \text{….. ………. (3.6)}$$

- **Constitutive Equations**

Similar to the equation for truss members, the original Hooke's law is applicable for beams:

$$\sigma_{xx} = E\varepsilon_{xx} \qquad \text{….. ………. (3.7)}$$

- **Moments and Shear Forces**

Because the loading on the beam is in the transverse direction, there will be moments and corresponding shear forces imposed on the cross-sectional plane of the beam. On the other hand, bending of the beam can also be achieved if pure moments are applied instead of transverse loading. Figure 3.5 shows a small representative cell of length dx of the beam. The beam cell is subjected to external force, fz, moment, M, shear force, Q, and inertial force, $\rho A\ddot{w}$, where ρ is the density of the material and A is the area of the cross-section

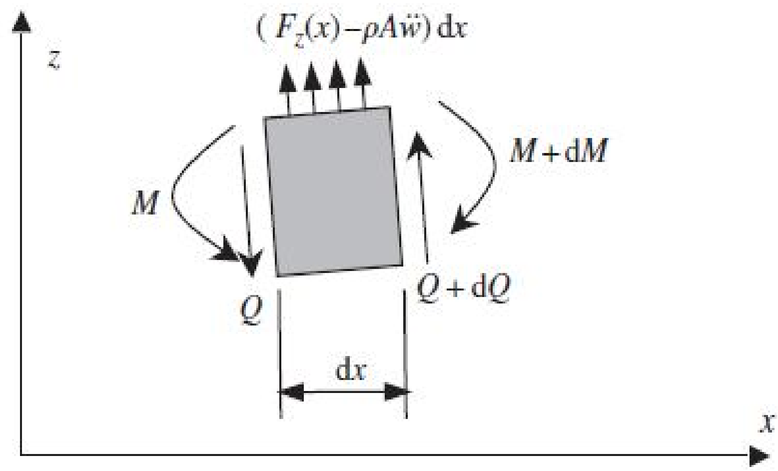


**Fig. 3.5: Isolated beam cell of length dx**

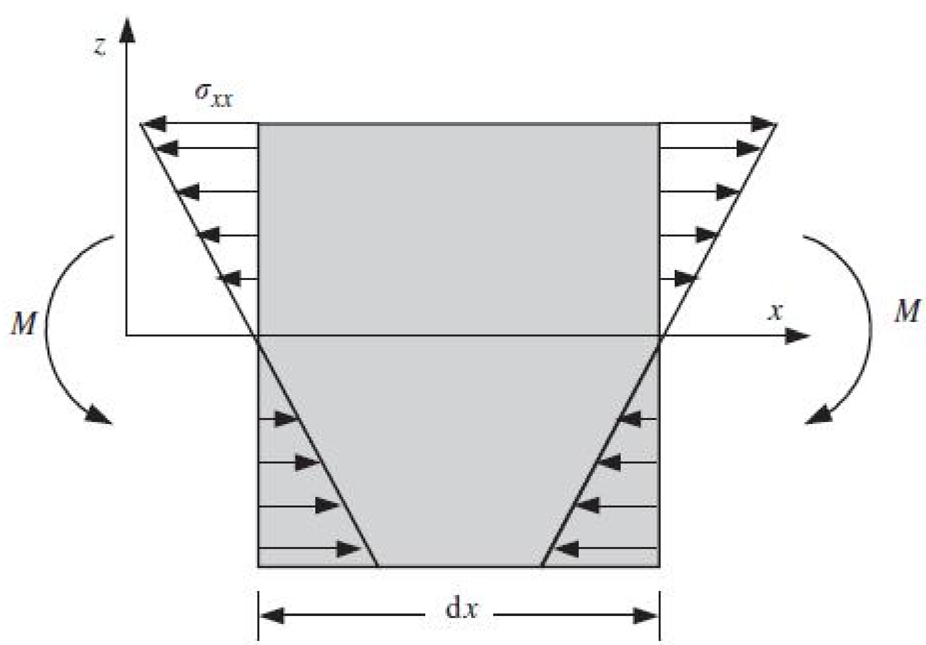


**Fig. 3.6: Normal stresses those results in moment.**

The moment on the cross-section at x results from the distributed normal stress σxx, as shown in Figure 3.6. The normal stress can be calculated by substituting Eq. (3.5) into Eq. (3.7):

$$\sigma_{xx} = -zELw \qquad \text{.................. (3.8)}$$

It can be seen from the above equation that the normal stress σxx varies linearly in the vertical direction on the cross-section of the beam. The moments resulting from the normal stress on the cross-section can be calculated by the following integration over the area of the cross-section:

$$M = \int_A \sigma_{xx} z \, \mathrm{d}A = -E\left(\int_A z^2 \, \mathrm{d}A\right) Lw = -EILw = -EI\frac{\partial^2 w}{\partial x^2} \qquad \text{..(3.9)}$$

Where I is the second moment of area (or moment of inertia) of the cross-section with respect to the y-axis, which can be calculated for a given shape of the cross-section using the following equation

$$I = \int_A z^2 \, \mathrm{d}A \qquad \text{........... (3.10)}$$

We now consider the force equilibrium of the small beam cell in the z direction

$$\mathrm{d}Q + (F_z(x) - \rho A\ddot{w})\,\mathrm{d}x = 0 \qquad \text{... (3.11)}$$

or

$$\frac{\mathrm{d}Q}{\mathrm{d}x} = -F_z(x) + \rho A\ddot{w} \qquad \text{....(3.12)}$$

We would also need to consider the moment equilibrium of the small beam cell with respect to any point at the right surface of the cell,

$$\mathrm{d}M - Q\,\mathrm{d}x + \tfrac{1}{2}(F_z - \rho \mathrm{A}\ddot{w})\,(\mathrm{d}x)^2 = 0$$

………… (3.13)

Neglecting the second order small term containing (dx) 2 leads to

$$\frac{\mathrm{d}M}{\mathrm{d}x} = Q$$ …………………. (3.14)

And finally, substituting Eq. (3.9) into Eq. (3.14) gives

$$Q = -EI\frac{\partial^3 w}{\partial x^3}$$ ………. (3.15)

Equations (3.14) and (3.15) give the relationship between the moments and shear forces in a beam with the deflection of the Euler–Bernoulli beam.

- **Dynamic Equilibrium Equations**

The dynamic equilibrium equation for beams can be obtained simply by substituting Eq. (3.15) into Eq. (3.12):

$$EI\,\frac{\partial^4 w}{\partial x^4} + \rho A\ddot{w} = F_z$$ …………….. (3.16)

The static equilibrium equation for beams can be obtained similarly by dropping the dynamic term in Eq. (3.16):

$$EI\,\frac{\partial^4 w}{\partial x^4} = F_z$$ ………….. (3.17)

- **Shape Function Construction**

Consider a beam element of length l = 2a with nodes 1 and 2 at each end of the element, as shown in Figure 3.8. The local x-axis is taken in the axial direction of the element with its origin at the middle section of the beam. Similar to all other structures, to develop the FEM equations, shape functions for the interpolation of the variables from the nodal variables,

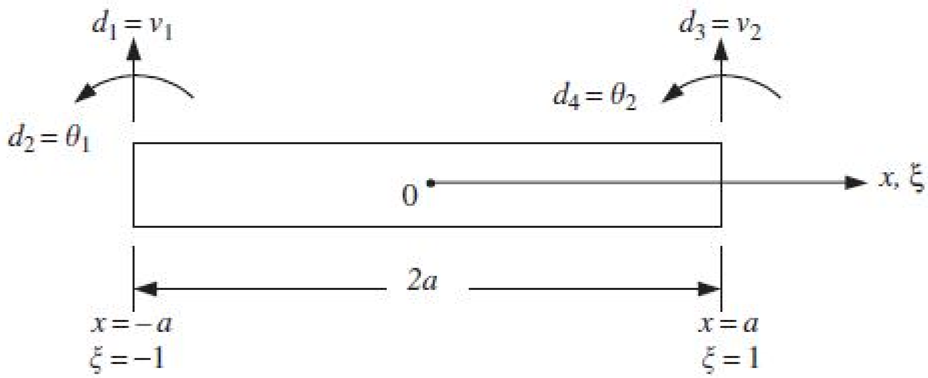


**Fig. (3.7): Beam element and its local coordinate systems.**

would first have to be developed. As there are four DOFs for a beam element, there should be four shape functions. It is often more convenient if the shape functions are derived from a special set of local coordinates, which is commonly known as the natural coordinate system. This natural coordinate system has its origin at the centre of the element, and the element is defined from −1 to +1, as shown in Figure 3.8. The relationship between the natural coordinate system and the local coordinate system can be simply given as

$$\xi = \frac{x}{a} \qquad \text{……….. (3.18)}$$

To derive the four shape functions in the natural coordinates, the displacement in an element is first assumed in the form of a third order polynomial of ξ that contains four unknown constants:

$$v(\xi) = \alpha_0 + \alpha_1\xi + \alpha_2\xi^2 + \alpha_3\xi^3 \quad \text{……………. (3.19)}$$

Where α0 to α3 are the four unknown constants. The third order polynomial is chosen because there are four unknowns in the polynomial, which can be related to the four nodal DOFs in the beam element. The above equation can have the following matrix form:

$$v(\xi) = \begin{bmatrix} 1 & \xi & \xi^2 & \xi^3 \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} \quad \text{… (3.20)}$$

or

$$v(\xi) = \mathbf{p}^T(\xi)\boldsymbol{\alpha} \quad \text{…. (3.21)}$$

Where p is the vector of basic functions and α is the vector of coefficients, as discussed in Chapters 3 and 4. The rotation θ can be obtained from the differential of Eq. (3.19) with these of Eq. (3.18):

$$\theta = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial \xi}\frac{\partial \xi}{\partial x} = \frac{1}{a}\frac{\partial v}{\partial \xi} = \frac{1}{a}(\alpha_1 + 2\alpha_2\xi + 3\alpha_3\xi^2) \quad \text{…… (3.22)}$$

The four unknown constants α0 to α3 can be determined by utilizing the following four conditions:

At x = −a or ξ = −1:

$$\begin{aligned} &(1) \quad v(-1) = v_1 \\ &(2) \quad \left.\frac{\mathrm{d}v}{\mathrm{d}x}\right|_{\xi=-1} = \theta_1 \end{aligned}$$ ….. (3.23)

At $x = +a$ or $\xi = +1$:

$$\begin{aligned} &(3) \quad v(1) = v_2 \\ &(4) \quad \left.\frac{\mathrm{d}v}{\mathrm{d}x}\right|_{\xi=1} = \theta_2 \end{aligned}$$ ….. (3.24)

The application of the above four conditions gives

$$\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1/a & -2/a & 3/a \\ 1 & 1 & 1 & 1 \\ 0 & 1/a & 2/a & 3/a \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix}$$ …. (3.25)

or

$$\mathbf{d}_e = \mathbf{A}_e \boldsymbol{\alpha}$$ …. (3.26)

Solving the above equation for α gives

$$\boldsymbol{\alpha} = \mathbf{A}_e^{-1} \mathbf{d}_e$$ …. (3.27)

where

$$\mathbf{A}_e^{-1} = \frac{1}{4} \begin{bmatrix} 2 & a & 2 & -a \\ -3 & -a & 3 & -a \\ 0 & -a & 0 & a \\ 1 & a & -1 & a \end{bmatrix}$$ …. (3.28)

Hence, substituting Eq. (3.27) into Eq. (3.21) will give

$$v = \mathbf{N}(\xi)\mathbf{d}_e \qquad \text{………… (3.29)}$$

Where N is a matrix of shape functions given by

$$\mathbf{N}(\xi) = \mathbf{PA}_e^{-1} = \begin{bmatrix} N_1(\xi) & N_2(\xi) & N_3(\xi) & N_4(\xi) \end{bmatrix} \qquad \text{………. (3.30)}$$

In which the shape functions are found to be

$$\begin{aligned} N_1(\xi) &= \tfrac{1}{4}(2 - 3\xi + \xi^3) \\ N_2(\xi) &= \tfrac{1}{4}a(1 - \xi - \xi^2 + \xi^3) \\ N_3(\xi) &= \tfrac{1}{4}(2 + 3\xi - \xi^3) \\ N_4(\xi) &= \frac{a}{4}(-1 - \xi + \xi^2 + \xi^3) \end{aligned} \qquad \text{….. (3.31)}$$

- **Strain Matrix**

Having now obtained the shape functions, the next step would be to obtain the element strain matrix. Substituting Eq. (3.29) into Eq. (3.5), which gives the relationship between the strain and the deflection, we have

$$\varepsilon_{xx} = \mathbf{B}\,\mathbf{d}_e \qquad \text{…………. (3.32)}$$

Where the strain matrix B is given by

$$\mathbf{B} = -yL\mathbf{N} = -y\frac{\partial^2}{\partial x^2}\mathbf{N} = -\frac{y}{a^2}\frac{\partial^2}{\partial \xi^2}\mathbf{N} = -\frac{y}{a^2}\mathbf{N}'' \qquad \text{….. (3.33)}$$

In deriving g the above equation, Eqs. (3.6) and (3.18) have been used. From Eq. (3.31), we have

$$\mathbf{N}'' = \begin{bmatrix} N_1'' & N_2'' & N_3'' & N_4'' \end{bmatrix} \qquad \text{...... (3.34)}$$

where

$$N_1'' = \frac{3}{2}\xi, \quad N_2'' = \frac{a}{2}(-1+3\xi)$$
$$N_3'' = -\frac{3}{2}\xi, \quad N_4'' = \frac{a}{2}(1+3\xi) \qquad \text{....... (3.35)}$$

- **Element Matrices**

Having obtained the strain matrix, we are now ready to obtain the element stiffness and mass matrices. By substituting Eq. (3.33), the stiffness matrix can be obtained as

$$\mathbf{k}_e = \int_V \mathbf{B}^T \mathbf{c} \mathbf{B} \, \mathrm{d}V = E \int_A y^2 \, \mathrm{d}A \int_{-a}^{a} \left(\frac{\partial^2}{\partial x^2}\mathbf{N}\right)^T \left(\frac{\partial^2}{\partial x^2}\mathbf{N}\right) \mathrm{d}x$$
$$= E I_z \int_{-1}^{1} \frac{1}{a^4} \left[\frac{\partial^2}{\partial \xi^2}\mathbf{N}\right]^T \left[\frac{\partial^2}{\partial \xi^2}\mathbf{N}\right] a \, \mathrm{d}\xi = \frac{E I_z}{a^3} \int_{-1}^{1} \mathbf{N}''^T \mathbf{N}'' \, \mathrm{d}\xi \qquad \text{... (3.36)}$$

Where Iz =_A y2 dA is the second moment of area (or moment of inertia) of the cross section of the beam with respect to the z axis. Substituting Eq. (3.34) into (3.36), we obtain

$$\mathbf{k}_e = \frac{EI_z}{a^3}\int_{-1}^{1}\begin{bmatrix} N_1''N_1'' & N_1''N_2'' & N_1''N_3'' & N_1''N_4'' \\ N_2''N_1'' & N_2''N_2'' & N_2''N_3'' & N_2''N_4'' \\ N_3''N_1'' & N_3''N_2'' & N_3''N_3'' & N_1''N_4'' \\ N_4''N_1'' & N_4''N_2'' & N_4''N_3'' & N_1''N_4'' \end{bmatrix}\mathrm{d}x \quad \text{.... (3.37)}$$

Evaluating the integrals in the above equation leads to

$$\mathbf{k}_e = \frac{EI_z}{2a^3}\begin{bmatrix} 3 & 3a & -3 & 3a \\ & 4a^2 & -3a & 2a^2 \\ & & 3 & -3a \\ sy. & & & 4a^2 \end{bmatrix} \quad \text{.... (3.38)}$$

To obtain the mass matrix, we substitute Eq. (3.34) into Eq:

$$\mathbf{m}_e = \int_V \rho\mathbf{N}^T\mathbf{N}\,\mathrm{d}V = \rho\int_A \mathrm{d}A\int_{-a}^{a}\mathbf{N}^T\mathbf{N}\,\mathrm{d}x = \rho A\int_{-1}^{1}\mathbf{N}^T\mathbf{N}a\,\mathrm{d}\xi$$

$$= \rho Aa\int_{-1}^{1}\begin{bmatrix} N_1N_1 & N_1N_2 & N_1N_3 & N_1N_4 \\ N_2N_1 & N_2N_2 & N_2N_3 & N_2N_4 \\ N_3N_1 & N_3N_2 & N_3N_3 & N_3N_4 \\ N_4N_1 & N_4N_2 & N_4N_3 & N_4N_4 \end{bmatrix}\mathrm{d}x \quad \text{...... (3.39)}$$

Where A is the area of the cross-section of the beam. Evaluating the integral in the above equation leads to

$$\mathbf{m}_e = \frac{\rho Aa}{105}\begin{bmatrix} 78 & 22a & 27 & -13a \\ & 8a^2 & 13a & -6a^2 \\ & & 78 & -22a \\ sy. & & & 8a^2 \end{bmatrix} \quad \text{.......... (3.40)}$$

The other element matrix would be the force vector. The nodal force vector for beam elements can be obtained. Suppose the element is loaded by an external distributed force fy along the x-axis, two concentrated forces fs1 and fs2, and concentrated moments ms1 and ms2, respectively, at nodes 1 and 2; the total nodal force vector becomes[14].

$$\mathbf{f}_e = \int_V \mathbf{N}^T f_b \, \mathrm{d}V + \int_{S_f} \mathbf{N}^T f_s \, \mathrm{d}S_f$$

$$= f_y a \int_{-1}^{1} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \mathrm{d}\xi + \begin{Bmatrix} f_{s1} \\ m_{s1} \\ f_{s2} \\ m_{s1} \end{Bmatrix} = \begin{Bmatrix} f_y a + f_{s1} \\ f_y a^2/3 + m_{s1} \\ f_y a + f_{s2} \\ -f_y a^2/3 + m_{s1} \end{Bmatrix} \quad \text{……… (3.41)}$$

### 3.3.3.2 Shell element

Shell element is also developed assuming that the thickness of the component is small relative to the other two dimensions and is also modeled by their middle surface. They differ from plate elements in that they are considered to have six degrees of freedom at each node, three translations and three rotations.

Typically the rotation about the axis perpendicular to the surface at a node is eliminated leaving only five degree of freedoms per node. Shell element may be used to model two dimensional (plate) components or three-dimensional (shell) components. Commercially available computer programs typically allow three-node and four-node elements [4].

The typical output includes the moments (usually given as moment per unit width of the face of the elements) and the shear and axial loads in the element. This form of output is convenient because the moments may be directly used to design the deck.

Due to the inclusion of the translations in the plane of the elements, shell elements may be used as part of a three-dimensional model to analyze both the deck and the girders [4].

When the supporting components are modeled using beam elements, only the stiffness of the non-composite beams is introduced when defining the stiffness of the beams. The effect of the composite action between the deck and the supporting components is automatically included due to the presence of the in-plane stiffness of the shell elements representing the deck [4].

Shell is a family of shell elements for the analysis of arbitrarily thick and thin curved shell geometries. The quadratic elements can accommodate generally curved geometry while all elements account for varying thickness.

Anisotropic and composite material properties can be defined. Shell elements are also capable of modeling bend structure. The element formulation takes account of membrane, shear and flexural deformations. The quadrilateral elements use an assumed strain field to define transverse shear which ensures that the element does not lock when it is thin.

Shell elements are used to model 3-dimensional structures whose behavior is dependent upon both flexural and membrane effects. Both thin and thick shell elements are available to be used. Materials use such as metals, steel, concrete, reinforced concrete, composites, laminated composites, fiber-reinforced composites, rubber, isotropic, orthotropic, and anisotropic, Fig (3.9) shows 3D thick shell element [4].

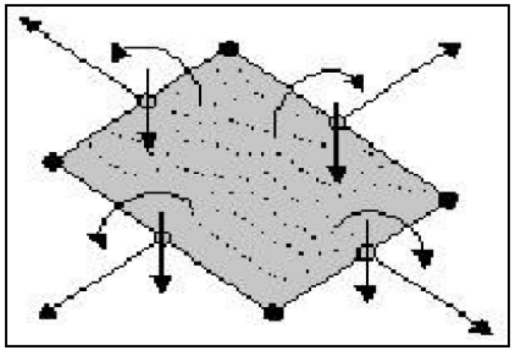

**Fig: (3.8): Thick shell element**

- **Elements in Local Coordinate Systems**

Shell structures are usually curved. We assume that the shell structure is divided into shell elements that are flat. The curvature of the shell is then followed by changing the orientation of the shell elements in space. Therefore, if the curvature of the shell is very large, a fine mesh of elements has to be used. This assumption sounds rough, but it is very practical and widely used in engineering practice. There are alternatives of more accurately formulated shell elements, but they are used only in academic research and have never been implemented in any commercially available software packages. Therefore, this book formulates only flat shell elements. Similar to the frame structure, there are six DOFs at a node for a shell element: three translational displacements in the x, y and z directions, and three rotational deformations with respect to the x, y and z axes. Figure 3.10 shows the middle plane of a rectangular shell element and the DOFs at the nodes. The generalized displacement vector for the element can be written as:

$$\mathbf{d}_e = \begin{Bmatrix} \mathbf{d}_{e1} \\ \mathbf{d}_{e2} \\ \mathbf{d}_{e3} \\ \mathbf{d}_{e4} \end{Bmatrix} \begin{matrix} \text{node 1} \\ \text{node 2} \\ \text{node 3} \\ \text{node 4} \end{matrix}$$

………… (3.42)

Where dei (i = 1, 2, 3, 4) are the displacement vector at node i:

$$\mathbf{d}_{ei} = \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{xi} \\ \theta_{yi} \\ \theta_{zi} \end{Bmatrix} \begin{matrix} \text{displacement in } x \text{ direction} \\ \text{displacement in } y \text{ direction} \\ \text{displacement in } z \text{ direction} \\ \text{rotation about } x\text{-axis} \\ \text{rotation about } y\text{-axis} \\ \text{rotation about } z\text{-axis} \end{matrix}$$

………….. (3.43)

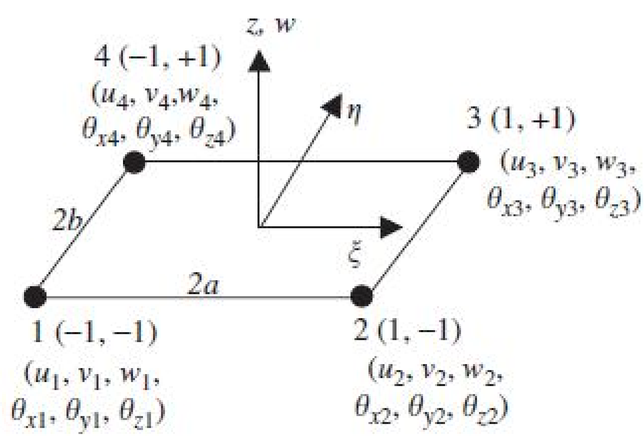


**Fig. (3.9): The middle plane of a rectangular shell element.**

The stiffness matrix for a 2D solid, rectangular element is used for dealing with the membrane effects of the element, which corresponds to DOFs of u and v. The membrane stiffness matrix can thus be expressed in the following form using sub-matrices according to the nodes:

$$
\mathbf{k}_e^m = \begin{array}{c} \begin{array}{cccc} \text{node1} & \text{node 2} & \text{node 3} & \text{node 4} \end{array} \\ \begin{bmatrix} \mathbf{k}_{11}^m & \mathbf{k}_{12}^m & \mathbf{k}_{13}^m & \mathbf{k}_{14}^m \\ \mathbf{k}_{21}^m & \mathbf{k}_{22}^m & \mathbf{k}_{23}^m & \mathbf{k}_{24}^m \\ \mathbf{k}_{31}^m & \mathbf{k}_{32}^m & \mathbf{k}_{33}^m & \mathbf{k}_{34}^m \\ \mathbf{k}_{41}^m & \mathbf{k}_{42}^m & \mathbf{k}_{43}^m & \mathbf{k}_{44}^m \end{bmatrix} \end{array} \begin{array}{l} \\ \text{node 1} \\ \text{node 2} \\ \text{node 3} \\ \text{node 4} \end{array} \qquad \text{……. (3.44)}
$$

Where the superscript m stands for the membrane matrix. Each sub-matrix will have a dimension of 2×2, since it corresponds to the two DOFs u and v at each node. Note again that the matrix above is actually the same as the stiffness matrix of the 2D rectangular, solid element, except it is written in terms of sub-matrices according to the nodes.

The stiffness matrix for a rectangular plate element is used for the bending effects, corresponding to DOFs of w, and θx, θy . The bending stiffness matrix can thus be expressed in the following form using sub-matrices according to the nodes:

$$
\mathbf{k}_e^b = \begin{array}{c} \begin{array}{cccc} \text{node 1} & \text{node 2} & \text{node 3} & \text{node 4} \end{array} \\ \begin{bmatrix} \mathbf{k}_{11}^b & \mathbf{k}_{12}^b & \mathbf{k}_{13}^b & \mathbf{k}_{14}^b \\ \mathbf{k}_{21}^b & \mathbf{k}_{22}^b & \mathbf{k}_{23}^b & \mathbf{k}_{24}^b \\ \mathbf{k}_{31}^b & \mathbf{k}_{32}^b & \mathbf{k}_{33}^b & \mathbf{k}_{34}^b \\ \mathbf{k}_{41}^b & \mathbf{k}_{42}^b & \mathbf{k}_{43}^b & \mathbf{k}_{44}^b \end{bmatrix} \end{array} \begin{array}{l} \\ \text{node 1} \\ \text{node 2} \\ \text{node 3} \\ \text{node 4} \end{array} \qquad \text{…….. (3.45)}
$$

Where the superscript b stands for the bending matrix. Each bending sub-matrix has a dimension of 3 × 3. The stiffness matrix for the shell element in the local coordinate system is then formulated by combining Eqs. (3.44) and (3.45):

$$\mathbf{k}_e = \begin{array}{c} \begin{array}{cccc} \text{node 1} & \text{node 2} & \text{node 3} & \text{node 4} \end{array} \\ \left[\begin{array}{cccccccccccc}
\mathbf{k}_{11}^m & \mathbf{0} & 0 & \mathbf{k}_{12}^m & \mathbf{0} & 0 & \mathbf{k}_{13}^m & \mathbf{0} & 0 & \mathbf{k}_{14}^m & \mathbf{0} & 0 \\
\mathbf{0} & \mathbf{k}_{11}^b & 0 & \mathbf{0} & \mathbf{k}_{12}^b & 0 & \mathbf{0} & \mathbf{k}_{13}^b & 0 & \mathbf{0} & \mathbf{k}_{14}^b & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{k}_{21}^m & \mathbf{0} & 0 & \mathbf{k}_{22}^m & \mathbf{0} & 0 & \mathbf{k}_{23}^m & \mathbf{0} & 0 & \mathbf{k}_{24}^m & \mathbf{0} & 0 \\
\mathbf{0} & \mathbf{k}_{21}^b & 0 & \mathbf{0} & \mathbf{k}_{22}^b & 0 & \mathbf{0} & \mathbf{k}_{23}^b & 0 & \mathbf{0} & \mathbf{k}_{24}^b & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{k}_{31}^m & \mathbf{0} & 0 & \mathbf{k}_{32}^m & \mathbf{0} & 0 & \mathbf{k}_{33}^m & \mathbf{0} & 0 & \mathbf{k}_{34}^m & \mathbf{0} & 0 \\
\mathbf{0} & \mathbf{k}_{31}^b & 0 & \mathbf{0} & \mathbf{k}_{32}^b & 0 & \mathbf{0} & \mathbf{k}_{33}^b & 0 & \mathbf{0} & \mathbf{k}_{34}^b & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{k}_{41}^m & \mathbf{0} & 0 & \mathbf{k}_{42}^m & \mathbf{0} & 0 & \mathbf{k}_{43}^m & \mathbf{0} & 0 & \mathbf{k}_{44}^m & \mathbf{0} & 0 \\
\mathbf{0} & \mathbf{k}_{41}^b & 0 & \mathbf{0} & \mathbf{k}_{42}^b & 0 & \mathbf{0} & \mathbf{k}_{43}^b & 0 & \mathbf{0} & \mathbf{k}_{44}^b & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] \end{array} \begin{array}{l} \text{node 1} \\ \\ \text{node 2} \\ \\ \text{node 3} \\ \\ \text{node 4} \end{array} \tag{3.46}$$

The stiffness matrix for a rectangular shell matrix has a dimension of 24 × 24. Note that in Eq. (3.10), the components related to the DOF θz, are zeros. This is because there is no θz in the local coordinate system. If these zero terms are removed, the stiffness matrix would have a reduced dimension of 20×20. However, using the extended 24×24 stiffness matrix will make it more convenient for transforming the matrix from the local coordinate system into the global coordinate system.

Similarly, the mass matrix for a rectangular element can be obtained in the same way as the stiffness matrix. The mass matrix for the 2D solid element is used for the membrane effects, corresponding to DOFs of u and v. The membrane mass matrix can be expressed in the following form using sub-matrices according to the nodes:

$$\mathbf{m}_e^m = \begin{array}{c} \begin{array}{cccc} \text{node 1} & \text{node 2} & \text{node 3} & \text{node 4} \end{array} \\ \begin{bmatrix}
\mathbf{m}_{11}^m & \mathbf{m}_{12}^m & \mathbf{m}_{13}^m & \mathbf{m}_{14}^m \\
\mathbf{m}_{21}^m & \mathbf{m}_{22}^m & \mathbf{m}_{23}^m & \mathbf{m}_{24}^m \\
\mathbf{m}_{31}^m & \mathbf{m}_{32}^m & \mathbf{m}_{33}^m & \mathbf{m}_{34}^m \\
\mathbf{m}_{41}^m & \mathbf{m}_{42}^m & \mathbf{m}_{43}^m & \mathbf{m}_{44}^m
\end{bmatrix} \end{array} \begin{array}{l} \text{node 1} \\ \text{node 2} \\ \text{node 3} \\ \text{node 4} \end{array} \quad \ldots\ldots\ (3.47)$$

Where the superscript m stands for the membrane matrix. Each membrane sub-matrix has a dimension of 2 × 2. The mass matrix for a rectangular plate element is used for the bending effects, corresponding o DOFs of w, and θx, θy. The bending mass matrix can also be expressed in the following form using sub-matrices according to the nodes:

$$\mathbf{m}_e^b = \begin{array}{c} \begin{array}{cccc} \text{node 1} & \text{node 2} & \text{node 3} & \text{node 4} \end{array} \\ \begin{bmatrix} \mathbf{m}_{11}^b & \mathbf{m}_{12}^b & \mathbf{m}_{13}^b & \mathbf{m}_{14}^b \\ \mathbf{m}_{21}^b & \mathbf{m}_{22}^b & \mathbf{m}_{23}^b & \mathbf{m}_{24}^b \\ \mathbf{m}_{31}^b & \mathbf{m}_{32}^b & \mathbf{m}_{33}^b & \mathbf{m}_{34}^b \\ \mathbf{m}_{41}^b & \mathbf{m}_{42}^b & \mathbf{m}_{43}^b & \mathbf{m}_{44}^b \end{bmatrix} \end{array} \begin{array}{l} \\ \text{node 1} \\ \text{node 2} \\ \text{node 3} \\ \text{node 4} \end{array} \quad \ldots\ldots \; (3.48)$$

Where the superscript b stands for the bending matrix. Each bending sub-matrix has a dimension of 3 × 3. The mass matrix for the shell element in the local coordinate system is then formulated by combining Eqs. (3.47) and (3.48):

$$\mathbf{m}_e = \begin{array}{c} \begin{array}{cccc} \overbrace{\qquad\qquad}^{\text{node 1}} & \overbrace{\qquad\qquad}^{\text{node 2}} & \overbrace{\qquad\qquad}^{\text{node 3}} & \overbrace{\qquad\qquad}^{\text{node 4}} \end{array} \\ \begin{bmatrix} \mathbf{m}_{11}^m & \mathbf{0} & 0 & \mathbf{m}_{12}^m & \mathbf{0} & 0 & \mathbf{m}_{13}^m & \mathbf{0} & 0 & \mathbf{m}_{14}^m & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{m}_{11}^b & 0 & \mathbf{0} & \mathbf{m}_{12}^b & 0 & \mathbf{0} & \mathbf{m}_{13}^b & 0 & \mathbf{0} & \mathbf{m}_{14}^b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{m}_{21}^m & \mathbf{0} & 0 & \mathbf{m}_{22}^m & \mathbf{0} & 0 & \mathbf{m}_{23}^m & \mathbf{0} & 0 & \mathbf{m}_{24}^m & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{m}_{21}^b & 0 & \mathbf{0} & \mathbf{m}_{22}^b & 0 & \mathbf{0} & \mathbf{m}_{23}^b & 0 & \mathbf{0} & \mathbf{m}_{24}^b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{m}_{31}^m & \mathbf{0} & 0 & \mathbf{m}_{32}^m & \mathbf{0} & 0 & \mathbf{m}_{33}^m & \mathbf{0} & 0 & \mathbf{m}_{34}^m & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{m}_{31}^b & 0 & \mathbf{0} & \mathbf{m}_{32}^b & 0 & \mathbf{0} & \mathbf{m}_{33}^b & 0 & \mathbf{0} & \mathbf{m}_{34}^b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{m}_{41}^m & \mathbf{0} & 0 & \mathbf{m}_{42}^m & \mathbf{0} & 0 & \mathbf{m}_{43}^m & \mathbf{0} & 0 & \mathbf{m}_{44}^m & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{m}_{41}^b & 0 & \mathbf{0} & \mathbf{m}_{42}^b & 0 & \mathbf{0} & \mathbf{m}_{43}^b & 0 & \mathbf{0} & \mathbf{m}_{44}^b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{node 1} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{node 2} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{node 3} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{node 4} \end{array} \quad (3.49)$$

Similarly, it is noted that the terms corresponding to the DOF θz are zero for the same reasons as explained for the stiffness matrix.

- **Elements in Global Coordinate System**

The matrices for shell elements in the global coordinate system can be obtained by performing the transformations

$$\mathbf{K}_e = \mathbf{T}^T \mathbf{k}_e \mathbf{T} \qquad \text{........ (3.50)}$$

$$\mathbf{M}_e = \mathbf{T}^T \mathbf{m}_e \mathbf{T} \qquad \text{........ (3.51)}$$

$$\mathbf{F}_e = \mathbf{T}^T \mathbf{f}_e \qquad \text{........ (3.52)}$$

Where T is the transformation matrix, given by

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 \end{bmatrix} \qquad \text{........ (3.53)}$$

In which

$$\mathbf{T}_3 = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix} \qquad \text{............... (3.54)}$$

Where lk, mk and nk (k = x, y, z) are direction cosines, which can be obtained in exactly the same way of space frame. The difference is that there is no need to define the additional point 3, as there are already four nodes for the shell element. The local coordinates x, y, z can be conveniently defined under the global coordinate system using the four nodes of the shell element.

The global matrices obtained will not have zero columns and rows if the elements joined at a node are not in the same plane. If all the elements joined at a node are in the same plane, then the global matrices will be singular. This kind of case is encountered when using shell elements to model a flat plate. In such situations, special techniques, such as a ‘stabilizing matrix’, have to be used to solve the global system equations [13].

### 3.3.4 3D solid model

The use of two dimensional analysis methods with effective flange widths is approximate at best and does not address the issue of up stands, which are often provided at the edges of bridge cantilevers. When the effects of shear lag are significant, some form of three dimensional models is necessary to achieve an accurate representation of the behavior of structure [10] .

The technique of three dimensional finite element analysis using solid ‘brick’ type elements is best used. The benefit of this type of model is that it can be used to illustrate the geometry of very complex bridge decks with high accuracy. The use of such models is only narrow to research and specialized applications due to too much of run times and computer storage requirements because of large quantities of output data generated [10].

Solid elements may be used to model both thin and thick components. The thickness of the component may be divided into several layers or, for thin components such as decks, may be modeled using one layer. The solid elements are developed assuming three translations at each node and the rotations are not considered in the development.

The typical output includes the forces in the direction of the three degrees of freedom at the nodes. Most computer programs have the ability to determine the surface stresses of the solid elements. This form of output is not convenient because these forces or stresses need to be converted to moments that may be used to design the deck. Notice that, theoretically, there should be no force perpendicular to the free surface of an element. However, due to rounding off errors, a small force is typically calculated.

Similar to shell elements, due to the inclusion of all translations in the development of the elements, solid elements may be used as part of three-dimensional model to analyze both the deck and the girders. When the supporting components are modeled using beam elements, only the stiffness of the non composite beams is introduced when defining the stiffness of the beams [4].

## 3.3.4.1 3D Solid element

3D continuum elements are used to model fully 3-dimensional structures. Tetrahedral, heptahedral and hexahedral solid elements are available to model full 3- dimensional stress fields. Materials use such as metals, isotropic, orthotropic, and anisotropic.

3D solid is a family of 3D isoperimetric solid continuum elements with advanced order models capable of modeling curved boundaries. The number of nodes of the elements is numerically integrated 4 or 10 (tetrahedral). 6, 12 or 15 (heptahedral). 8, 16 or 20 (hexahedral). The elements are numbered according to a right-hand screw rule in the local z-direction, Fig (3.11) shows a 3D solid element [4].

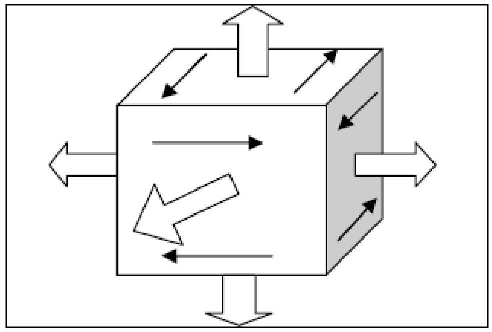

**Fig. (3.10): 3D solid element.**

## ❖ TETRAHEDRON ELEMENT

- **Strain Matrix**

Consider the same 3D solid structure, whose domain is divided in a proper manner into a number of tetrahedron elements (Figure 3.12) with four nodes and four surfaces, as shown in (Figure 3.13). A tetrahedron element has four nodes, each having three DOFs

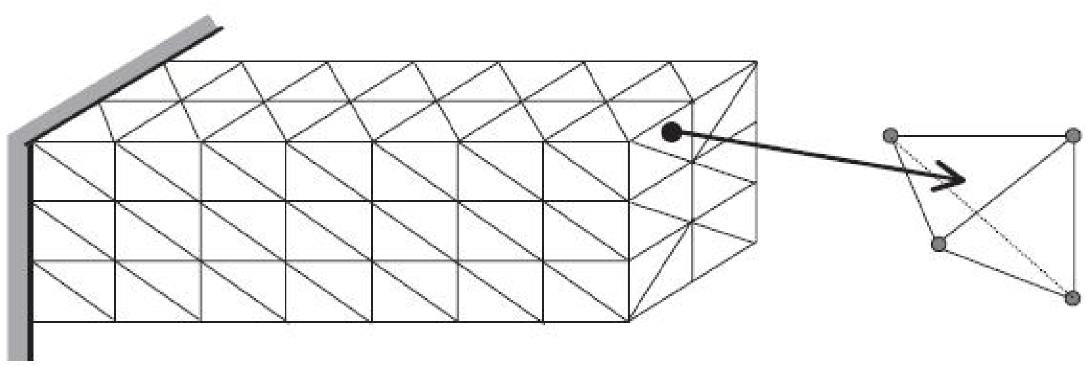

**Fig. 3.11: Solid block divided into four-node tetrahedron elements.**

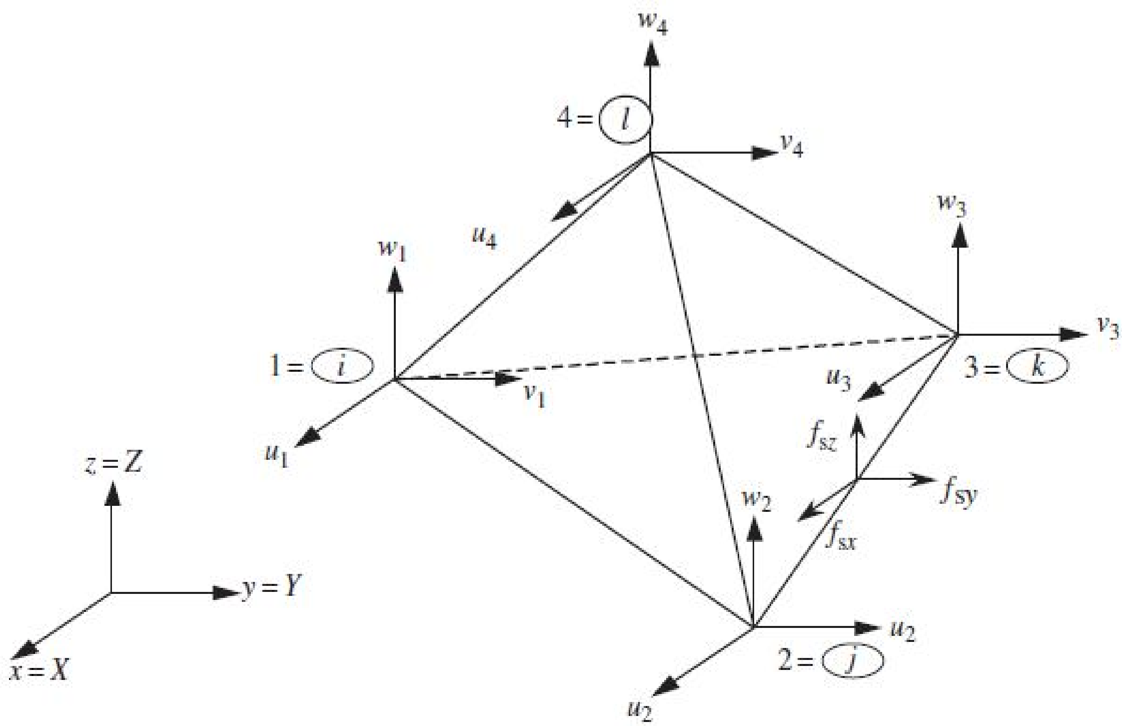


**Fig. 3.12: A tetrahedron element.**

(u, v and w), making the total DOFs in a tetrahedron element twelve, as shown in Figure 3.13. The nodes are numbered 1, 2, 3 and 4 by the right-hand rule. The local Cartesian coordinate system for a tetrahedron element can usually be the same as the global coordinate system, as there are no advantages in having a separate local Cartesian coordinate system. In an element, the displacement vector U is a function of the coordinate x, y and z, and is interpolated by shape functions in the following form, which should by now be shown to be part and parcel of the finite element method:

$$\mathbf{U}^h(x, y, z) = \mathbf{N}(x, y, z)\mathbf{d}_e \qquad \text{............. (3.55)}$$

Where the nodal displacement vector, de, is given as

$$\mathbf{d}_e = \left\{ \begin{array}{c} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ u_3 \\ v_3 \\ w_3 \\ u_4 \\ v_4 \\ w_4 \end{array} \right\} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{displacements at node 1} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{displacements at node 2} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{displacements at node 3} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{displacements at node 4} \end{array}$$

……….. (3.56)

And the matrix of shape functions has the form

$$\mathbf{N} = \begin{bmatrix} \overbrace{N_1 \quad 0 \quad 0}^{\text{node 1}} & \overbrace{N_2 \quad 0 \quad 0}^{\text{node 2}} & \overbrace{N_3 \quad 0 \quad 0}^{\text{node 3}} & \overbrace{N_4 \quad 0 \quad 0}^{\text{node 4}} \\ 0 \quad N_1 \quad 0 & 0 \quad N_2 \quad 0 & 0 \quad N_3 \quad 0 & 0 \quad N_4 \quad 0 \\ 0 \quad 0 \quad N_1 & 0 \quad 0 \quad N_2 & 0 \quad 0 \quad N_3 & 0 \quad 0 \quad N_4 \end{bmatrix}$$

… (3.57)

To develop the shape functions, we make use of what is known as the volume coordinates, which is a natural extension from the area coordinates for 2D solids. The use of the volume coordinates makes it more convenient for shape function construction and element matrix integration. The volume coordinates for node 1 is defined as

$$L_1 = \frac{V_{P234}}{V_{1234}}$$

……………….. (3.58)

Where VP234 and V1234 denote, respectively, the volumes of the tetrahedrons P234 and 1234, as shown in Figure 3.14. The volume coordinate for node 2-4 can also be defined in the same

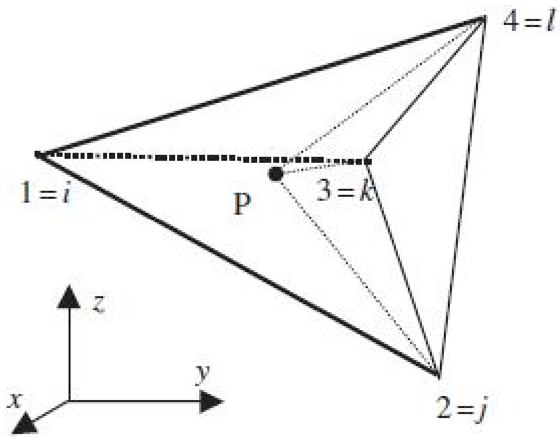


**Fig. 3.13: Volume coordinates for tetrahedron elements.**

manner:

$$L_2 = \frac{V_{P134}}{V_{1234}}, \quad L_3 = \frac{V_{P124}}{V_{1234}}, \quad L_4 = \frac{V_{P123}}{V_{1234}} \quad . (3.59)$$

The volume coordinate can also be viewed as the ratio between the distance of the point P and point 1 to the plane 234:

$$L_1 = \frac{d_{P-234}}{d_{1-234}}, \quad L_2 = \frac{d_{P-134}}{d_{1-234}}, \quad L_3 = \frac{d_{P-124}}{d_{1-234}}, \quad L_4 = \frac{d_{P-123}}{d_{1-234}} \quad \ldots (3.60)$$

It can easily be confirmed that

$$L_1 + L_2 + L_3 + L_4 = 1 \qquad \text{.....(3.61)}$$

since

$$V_{P234} + V_{P134} + V_{P124} + V_{P123} = V_{1234} \qquad \text{.....(3.62)}$$

It can also easily be confirmed that

$$L_i = \begin{cases} 1 & \text{at the home node } i \\ 0 & \text{at the remote nodes } jkl \end{cases} \qquad \text{............ (3.63)}$$

Using Eq. (3.63), the relationship between the volumes coordinates and Cartesian coordinates can be easily derived:

$$\begin{aligned} x &= L_1 x_1 + L_2 x_2 + L_3 x_3 + L_4 x_4 \\ y &= L_1 y_1 + L_2 y_2 + L_3 y_3 + L_4 y_4 \\ z &= L_1 z_1 + L_2 z_2 + L_3 z_3 + L_4 z_4 \end{aligned} \qquad \text{............ (3.64)}$$

Equations (3.61) and (3.64) can then be expressed as a single matrix equation as follows:

$$\begin{Bmatrix} 1 \\ x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{Bmatrix} \qquad \text{........ (3.65)}$$

The inversion of Eq. (3.65) will give

$$\begin{Bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{Bmatrix} = \frac{1}{6V} \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} \begin{Bmatrix} 1 \\ x \\ y \\ z \end{Bmatrix}$$

……… (3.66)

Where

$$a_i = \det \begin{bmatrix} x_j & y_j & z_j \\ x_k & y_k & z_k \\ x_l & y_l & z_l \end{bmatrix}, \quad b_i = -\det \begin{bmatrix} 1 & y_j & z_j \\ 1 & y_k & z_k \\ 1 & y_l & z_l \end{bmatrix}$$ ……(3.67)

$$c_i = -\det \begin{bmatrix} y_j & 1 & z_j \\ y_k & 1 & z_k \\ y_l & 1 & z_1 \end{bmatrix}, \quad d_i = -\det \begin{bmatrix} y_j & z_j & 1 \\ y_k & z_k & 1 \\ y_l & z_l & 1 \end{bmatrix}$$ …… (3.68)

In which the subscript i varies from 1 to 4, and j, k and l are determined by a cyclic permutation in the order of i, j , k, l. For example, if i = 1, then j = 2, k = 3, l = 4.

When i = 2, then j = 3, k = 4, l = 1. The volume of the tetrahedron element V can be obtained by

$$V = \frac{1}{6} \times \det \begin{bmatrix} 1 & x_i & y_i & z_i \\ 1 & x_j & y_j & z_j \\ 1 & x_k & y_k & z_k \\ 1 & x_l & y_l & z_l \end{bmatrix}$$

…………… (3.69)

The properties of Li, as depicted in Eqs. (3.60) to (3.63), show that Li can be used as the shape function of a four-nodal tetrahedron element:

$$N_i = L_i = \frac{1}{6V}(a_i + b_i x + c_i y + d_i z)$$
……….. (3.70)

It can be seen from above that the shape function is a linear function of x, y and z, hence, the four-nodal tetrahedron element is a linear element. Note that from Eq. (3.68), the moment matrix of the linear basis functions will never be singular, unless the volume of the element is zero (or the four nodes of the element are in a plane). Based on Lemmas 2 and 3, we can be sure that the shape functions given by Eq. (3.69) satisfy the sufficient requirement of FEM shape functions.

It was mentioned that there are six stresses in a 3D element in total. The stress components are {σxx σyy σzz σyz σxz σxy}. To get the corresponding strains, {εxx εyy εzz εyz εxz εxy}, we can substitute Eq. (3.55) :

$$\varepsilon = \mathbf{LU} = \mathbf{LNd}_e = \mathbf{Bd}_e$$
…………. (3.71)

Where the strain matrix B is given by

$$\mathbf{B} = \mathbf{LN} = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ 0 & \partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & \partial/\partial x \\ \partial/\partial y & \partial/\partial x & 0 \end{bmatrix} \mathbf{N}$$
………… (3.72)

Using Eq. (3.57), the strain matrix, B, can be obtained as

$$\mathbf{B} = \frac{1}{2V}\begin{bmatrix} b_1 & 0 & 0 & b_2 & 0 & 0 & b_3 & 0 & 0 & b_4 & 0 & 0 \\ 0 & c_1 & 0 & 0 & c_2 & 0 & 0 & c_3 & 0 & 0 & c_4 & 0 \\ 0 & 0 & d_1 & 0 & 0 & d_2 & 0 & 0 & d_3 & 0 & 0 & d_4 \\ c_1 & b_1 & 0 & c_2 & b_2 & 0 & c_3 & b_3 & 0 & c_4 & b_4 & 0 \\ 0 & d_1 & c_1 & 0 & d_2 & c_2 & 0 & d_3 & c_3 & 0 & d_4 & c_4 \\ d_1 & 0 & b_1 & d_2 & 0 & b_2 & d_3 & 0 & b_3 & d_4 & 0 & b_4 \end{bmatrix} \quad \text{…. (3.73)}$$

It can be seen that the strain matrix for a linear tetrahedron element is a constant matrix. This implies that the strain within a linear tetrahedron element is constant, and thus so is the stress. Therefore, the linear tetrahedron elements are also often referred to as a constant strain element or constant stress element, similar to the case of 2D linear triangular elements Element Matrices.

Once the strain matrix has been obtained, the stiffness matrix ke for 3D solid elements can be obtained by substituting Eq. (3.72) . Since the strain is constant, the element strain matrix is obtained as

$$\mathbf{k}_e = \int_{V_e} \mathbf{B}^T \mathbf{c} \mathbf{B} \, \mathrm{d}V = V_e \mathbf{B}^T \mathbf{c} \mathbf{B} \quad \text{……….. (3.74)}$$

The mass matrix can similarly be obtained using:

$$\mathbf{m}_e = \int_{V_e} \rho \mathbf{N}^T \mathbf{N} \, \mathrm{d}V = \int_{V_e} \rho \begin{bmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} & \mathbf{N}_{13} & \mathbf{N}_{14} \\ \mathbf{N}_{21} & \mathbf{N}_{22} & \mathbf{N}_{23} & \mathbf{N}_{24} \\ \mathbf{N}_{31} & \mathbf{N}_{32} & \mathbf{N}_{33} & \mathbf{N}_{34} \\ \mathbf{N}_{41} & \mathbf{N}_{42} & \mathbf{N}_{43} & \mathbf{N}_{44} \end{bmatrix} \mathrm{d}V \quad \text{….. (3.75)}$$

Where

$$\mathbf{N}_{ij} = \begin{bmatrix} N_i N_j & 0 & 0 \\ 0 & N_i N_j & 0 \\ 0 & 0 & N_i N_j \end{bmatrix}$$

………. (3.76)

Using the following formula [Eisenberg and Malvern, 1973],

$$\int_{V_e} L_1^m L_2^n L_3^p L_4^q \, \mathrm{d}V = \frac{m!n!p!q!}{(m+n+p+q+3)!} 6V_e$$

…….. (3.77)

We can conveniently evaluate the integral in Eq. (3.74) to give

$$\mathbf{m}_e = \frac{\rho V_e}{20} \begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ & & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ & & & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ & & & & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ & & & & & 2 & 0 & 0 & 1 & 0 & 0 & 1 \\ & & & & & & 2 & 0 & 0 & 1 & 0 & 0 \\ & & & & & & & 2 & 0 & 0 & 1 & 0 \\ & & & & & & & & 2 & 0 & 0 & 1 \\ & & sy. & & & & & & & 2 & 0 & 0 \\ & & & & & & & & & & 2 & 0 \\ & & & & & & & & & & & 2 \end{bmatrix}$$

…. (3.78)

An alternative way to calculate the mass matrix for 3D solid elements is to use a special natural coordinate system, which is defined as shown in Figures 3.15–3.17. In Figure 3.15, the plane of $\xi$ = constant is defined in such a way that the edge P–Q stays parallel to the edge 2–3 of the element, and point 4 coincides with point 4 of the element. When P moves to point 1, $\xi = 0$, and when P moves to point 2, $\xi = 1$. In Figure 3.16, the plane of $\eta$ = constant is defined in such a way that the edge 1–4 on the triangle coincides with the edge 1–4 of the element, and point P stays on the edge 2–3 of the element. When P moves to point 2, η = 0, and when P moves to point 3, η = 1. The plane of ζ = constant is defined in Figure 3.17, in such a way that the plane P–Q–R stays parallel to the plane 1–2–3 of the element, and when P moves to point 4, ζ = 0, and when P moves to point 2, ζ = 1. In addition,

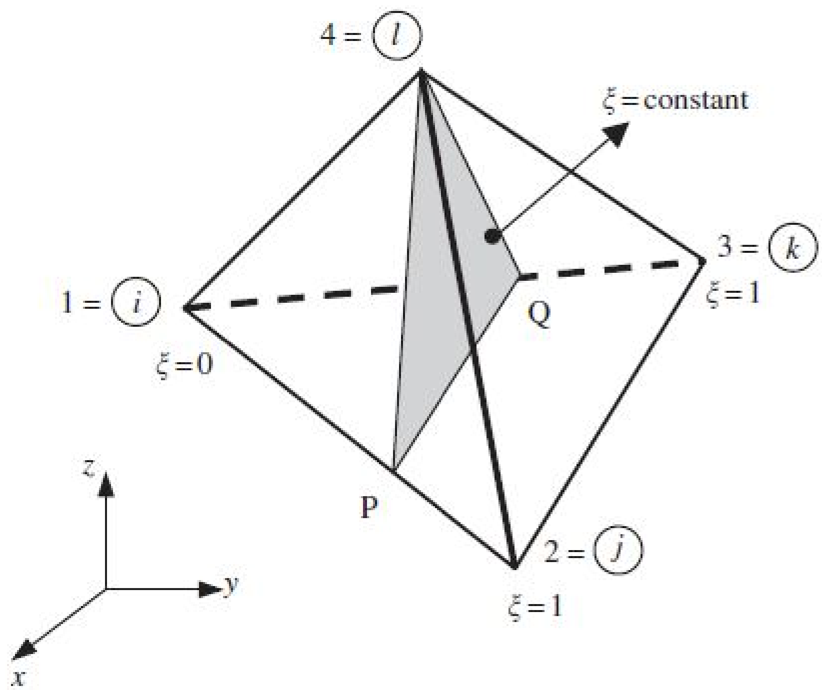


**Fig. (3.14): Natural coordinate, where ξ = constant**

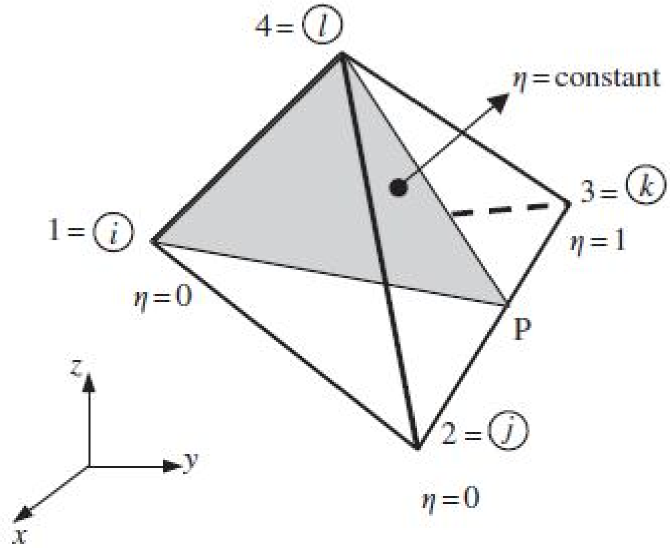


**Fig. (3.15): Natural coordinate, where η = constant**.

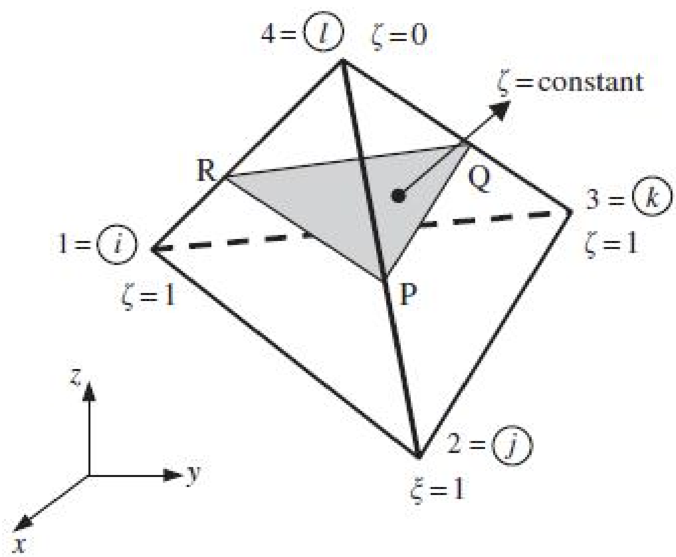


**Fig. (3.16): Natural coordinate, where ζ = constant.**

The plane 1–2–3 on the element sits on the x–y plane. Therefore, the relationship between xyz and ξηζ can be obtained in the following steps:

In Figure 3.18, the coordinates at point P are first interpolated using the x, y and z coordinates at points 2 and 3:

$$x_P = \eta(x_3 - x_2) + x_2$$
$$y_P = \eta(y_3 - y_2) + y_2$$
$$z_P = 0 \quad \text{………… (3.79)}$$

The coordinates at point B are then interpolated using the x, y and z coordinates at point's 1and P:

$$x_B = \xi(x_P - x_1) + x_1 = \xi\eta(x_3 - x_2) + \xi(x_2 - x_1) + x_1$$
$$y_B = \xi(y_P - y_1) + y_1 = \xi\eta(y_3 - y_2) + \xi(y_2 - y_1) + y_1$$
$$z_B = 0$$

…… (3.80)

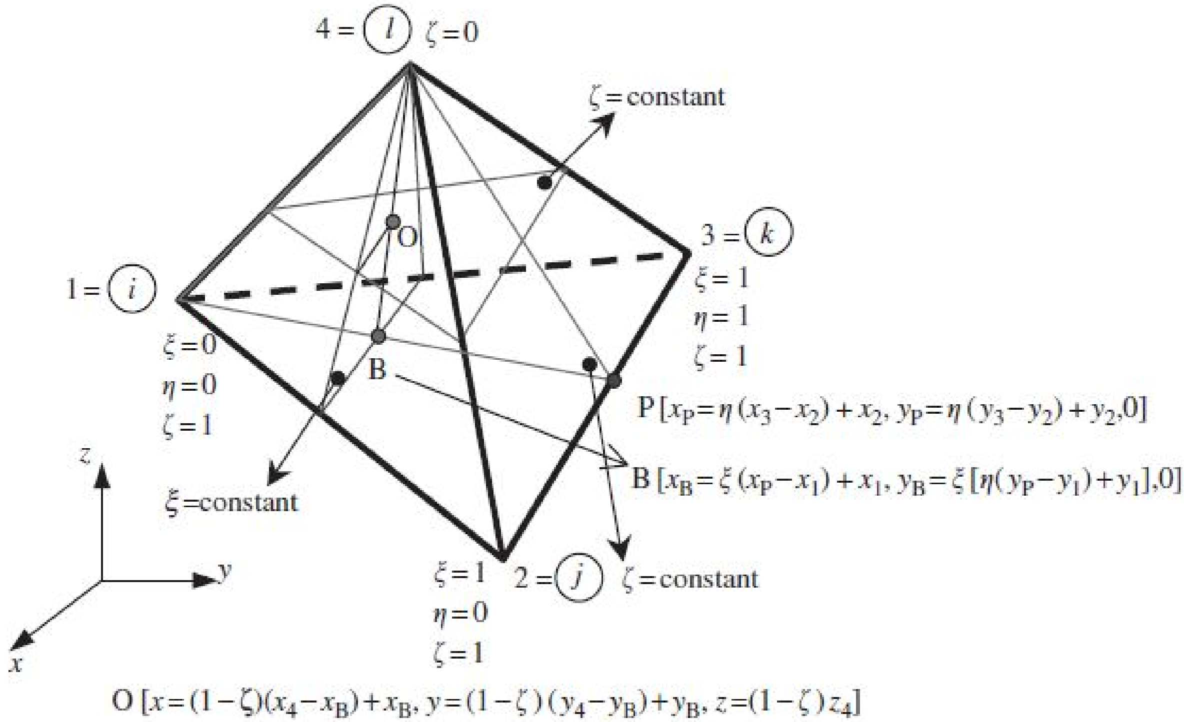


**Fig. (3.17): Cartesian coordinates xyz of point O in term of ξηζ.**

The coordinates at point O are finally interpolated using the x, y and z coordinates at points 4 and B:

$$x = x_4 - \zeta(x_4 - x_B) = x_4 - \zeta(x_4 - x_1) + \xi\zeta(x_2 - x_1) - \xi\zeta(x_2 - x_3)$$
$$y = y_4 - \zeta(y_4 - y_B) = y_4 - \zeta(y_4 - y_1) + \xi\zeta(y_2 - y_1) - \xi\zeta(y_2 - y_3)$$
$$z = (1 - \zeta)z_4$$

..(3.81)

With this special natural coordinate system, the shape functions in the matrix of Eq. (3.57) can be written by inspection as

$$N_1 = (1 - \xi)\zeta$$
$$N_2 = \xi\eta\zeta$$
$$N_3 = \xi\zeta(1 - \eta)$$
$$N_4 = (1 - \zeta)$$

…………. (3.82)

The Jacobian matrix between xyz and ξηζ is required, and is given as

$$\mathbf{J} = \begin{bmatrix} \partial x/\partial\xi & \partial x/\partial\eta & \partial x/\partial\zeta \\ \partial y/\partial\xi & \partial y/\partial\eta & \partial y/\partial\zeta \\ \partial z/\partial\xi & \partial z/\partial\eta & \partial z/\partial\zeta \end{bmatrix}$$

………. (3.83)

Using Eqs. (3.80) and (3.81), the determinate of the Jacobian can be found to be

$$\det[\mathbf{J}] = \begin{vmatrix} \zeta x_{21} + \eta\zeta x_{31} & \xi\zeta x_{31} & -x_{41} + \xi x_{21} + \xi\eta x_{31} \\ \zeta y_{21} + \eta\zeta y_{31} & \xi\zeta y_{31} & -y_{41} + \xi y_{21} + \xi\eta y_{31} \\ 0 & z_4 & 0 \end{vmatrix} = -6V\xi\zeta^2$$

.. (3.84)

The mass matrix can now be obtained as

$$\mathbf{m}_e = \int_{V_e} \rho \mathbf{N}^T \mathbf{N}\, \mathrm{d}V = \int_0^1 \int_0^1 \int_0^1 \rho \mathbf{N}^T \mathbf{N} \det[\mathbf{J}]\, \mathrm{d}\xi\, \mathrm{d}\eta\, \mathrm{d}\zeta$$

….. (3.85)

Which gives

$$\mathbf{m}_e = -6V_e\rho \int_0^1 \int_0^1 \int_0^1 \xi\zeta^2 \begin{bmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} & \mathbf{N}_{13} & \mathbf{N}_{14} \\ \mathbf{N}_{21} & \mathbf{N}_{22} & \mathbf{N}_{23} & \mathbf{N}_{24} \\ \mathbf{N}_{31} & \mathbf{N}_{32} & \mathbf{N}_{33} & \mathbf{N}_{34} \\ \mathbf{N}_{41} & \mathbf{N}_{42} & \mathbf{N}_{43} & \mathbf{N}_{44} \end{bmatrix} \mathrm{d}\xi\, \mathrm{d}\eta\, \mathrm{d}\zeta$$

.. (3.86)

Where Nij is given by Eq. (3.75), but in which the shape functions should be defined by Eq. (3.81). Evaluating the integrals in Eq. (3.85) would give the same mass matrix as in Eq. (3.77).

The nodal force vector for 3D solid elements can be obtained by Suppose the element is loaded by a distributed force fs on the edge 2–3 of the element as shown in Figure 3.13; the nodal force vector becomes

$$\mathbf{f}_e = \int_l [\mathbf{N}]^{\mathrm{T}}\Big|_{3\text{–}4} \begin{Bmatrix} f_{sx} \\ f_{sy} \\ f_{sz} \end{Bmatrix} \mathrm{d}l$$

……….. (3.87)

If the load is uniformly distributed, fsx, fsx and fsz are constants, and the above equation Becomes

$$\mathbf{f}_e = \frac{1}{2} l_{3-4} \begin{Bmatrix} \{\mathbf{0}\}_{3\times 1} \\ \{\mathbf{0}\}_{3\times 1} \\ \begin{Bmatrix} f_{sx} \\ f_{sy} \\ f_{sz} \end{Bmatrix} \\ \begin{Bmatrix} f_{sx} \\ f_{sy} \\ f_{sz} \end{Bmatrix} \\ \{\mathbf{0}\}_{3\times 1} \\ \{\mathbf{0}\}_{3\times 1} \\ \{\mathbf{0}\}_{3\times 1} \\ \{\mathbf{0}\}_{3\times 1} \end{Bmatrix} \qquad \text{…………… (3.88)}$$

Where l3–4 is the length of the edge 3–4. Equation (3.87) implies that the distributed forces are equally divided and applied at the two nodes. This conclusion also applies to evenly distribute surface forces applied on any face of the element, and to evenly distributed body force applied on the entire body of the element. Finally, the stiffness matrix, ke, the mass matrix, me, and the nodal force vector, fe, can be used directly to assemble the global FE equation [14].

## 3.4 Bridge standard

It is a common practice for every engineer to design a structure with accordance to the standard that is needed to ensure the structural is built using the high quality of workmanship for safety, durability and long time benefits. For Sudan, it is a familiarity to engineer to refer to the British Standard. Such British Standard that currently practices in Sudan is: [4]

i. BS 5400: Steel, Concrete and Composite Bridges.

Part 4: Code of Practice for Design of Concrete Bridges.

ii. BS 8110 : Structural Use Of concrete

Part 1: Code of Practice for Design and Construction

Part 2: Code of Practice for Special Circumstances.

iii. BD 37/01 : Design Manual for Road and Bridges
Load for Highway Bridges.

## 3.5 Bridge Loading

The primary function of a bridge is to carry traffic loads: heavy trucks, cars, and trains. Engineers must estimate the traffic loading. On short spans, it is possible that the maximum conceivable load will be achieved—that is to say, on spans of less than 30 meters (100 feet), four heavy trucks may cross at the same time, two in each direction. On longer spans of a thousand meters or more, the maximum conceivable load is such a remote possibility (imagine the Golden Gate Bridge with only heavy trucks crossing bumper-to-bumper in each direction at the same time) that the cost of designing for it is unreasonable. Therefore, engineers use probable loads as a basis for design.

In order to carry traffic, the structure must have some weight, and on short spans this dead load weight is usually less than the live loads. On longer spans, however, the dead load is greater than live loads, and, as spans get longer, it becomes more important to design forms that minimize dead load. In general, shorter spans are built with beams, hollow boxes, trusses, arches, and continuous versions of the same, while longer spans use cantilever, cable-stay, and suspension forms. As spans get longer, questions of shape, materials, and form become increasingly important. New forms have evolved to provide longer spans with more strength from less material. [7]

Loads are classified as external forces applied to the structure and imposed deformations for example deformations occurred caused by restraint of movement due to changes in temperature. Loads are classified either permanent or temporary.

Permanent loads including dead loads, superimposed dead loads, loads due to filling material, differential settlement and loads derived from the nature of structural material such as creep and shrinkage.

Temporary loads including live loads, wind loads, temperature loads, erection loads, primary and secondary highway loadings, footway and cycle track loadings.

Primary loadings are vertical live loads. Secondary loadings are due to changes in speed or direction (e.g. centrifugal, braking, skidding & collision loads). [4]

## 3.5.1 Dead load

Dead load is the mass of the materials and parts of the structure that are structural elements and not including superimposed materials such as premix, rail track ballast, parapets, main, ducts, miscellaneous furniture and etc. Those material that are not structural form but it contribute loads to the bridge is called superimposed dead load.

Dead load can be calculated using the formula (3.2) [4]:

Structure weight, W = Structure volume V * Concrete density p …………. (3.89)

## 3.5.2 Live load

Standard highway live loading consists of HA and HB loading [8]:

### 3.5.2.1 HA Loading

Normal traffic, or C&U (Construction and Use) regulation vehicles are represented by an ‘HA’ load. HA loading consists of:

- A uniform distributed load (UDL), plus a knife edge load (KEL)

Or

- A single wheel load.

HA loading (UDL) is calculated for the ‘loaded length’ of the span members.

- For loaded length up to and including 50m.

$$W=336\,(1/L)^{0.67} \quad (3.90)$$

- For loaded length in excess of 50m but less than 1600m

$$W=36(1/L)^{0.1} \quad (3.91)$$

Where:

W= load per meter of notional lane (kN/m)

L= loaded length in meters.

- The HA loading (KEL) is always taken as 120 KN per notional lane.
- The UDL and KEL are taken to occupy one notional lane uniformly distributed over the full width of the lane.
- A value of 100 KN is always taken for the single wheel load.

## 3.5.2.2 HB Loading

HB loading represents the loads that result from abnormally heavy road vehicles. These are exceptional industrial loads such as:

- Electrical transforms.
- Generators.

- Pressure vessels.
- Machine presses.

The HB vehicle, which is always applied parallel to the carriage-way, can have different lengths since the distance between the two central axles is variable.

This variable spacing allows the worst effects at all design points to be calculated. For simple supported spans, the smallest (6m) is the most critical.

Nominal HB loading is specified by ‘units of loading’, where 1 unit is taken as 10 kN per axle. Different types of road are designed for different numbers of units of HB. Normally 45 units are used for trunk roads and motorways, 37.5 units for principle road and 30 units for all other public roads [8].

### 3.5.3 Other Loading:-

There are other loadings that are stated in the BD 37/01, the loadings are used to design and analyze bridge to get for results that close to the real condition. The loadings are [4]:

i. Wind Load,

ii. Standard footway and cycle track loading,

iii. Accidental wheel loading,

iv. Loads due to vehicle collision with parapets,

v. Vehicle loads in highway bridge supports and superstructures,

vi. Centrifugal loads,

vii. Accidental Loads due to skidding,

viii. Loadings for fatigue, and

ix. Foot/ cycle track bridge loading.

### 3.5.4. Application of HA and HB Loads:

The structure and its elements must be designed to resist the more severe effects of either:

- Design HA loading alone or
- Design HA loading combined with design HB loading.

### 3.5.5. Types HA and HB loading combined:

Types HA and HB loading shall be combined and applied as follows:

- **Case (1):-** ***Where the HB vehicle lies wholly within the notional lane.***

The HB vehicle replaces one lane of HA for a distance extending from 25 m in front of the vehicle to 25m behind. If there is any lane length left to be loaded with

HA, the KEL is not applied. The remaining lanes are loaded with HA including the KEL in the normal way.

**- Case (2a):-** ***The HB vehicle straddling two notional lanes.***

Where the HB vehicle lies partially within a notional lane and the remaining width of that lane (measured from the side of HB vehicle to the edge of that lane) is less than 2.5m, the HB vehicle replaces the HA loadings in the straddled lanes for a distance extending from 25m in front of the vehicle to 25m behind. If there is any lane length left to be loaded with HA, the KEL is not applied. The remaining lanes are loaded with HA including the KEL in the normal way

**- Case (2b):-** ***The HB vehicle straddling two notional lanes.***

Where the HB vehicle lies partially within a notional lane and the remaining width of that lane (measured from the side of HB vehicle to the edge of that lane) is greater than or equal 2.5m, the HA loadings in the that lane remains, but is multiplied by an appropriate lane factor for a notional lane of width 2.5 meters – irrespective of the actual lane width. The HA KEL for that lane is omitted. The remaining lanes are loaded with HA including the KEL in the normal way [8].

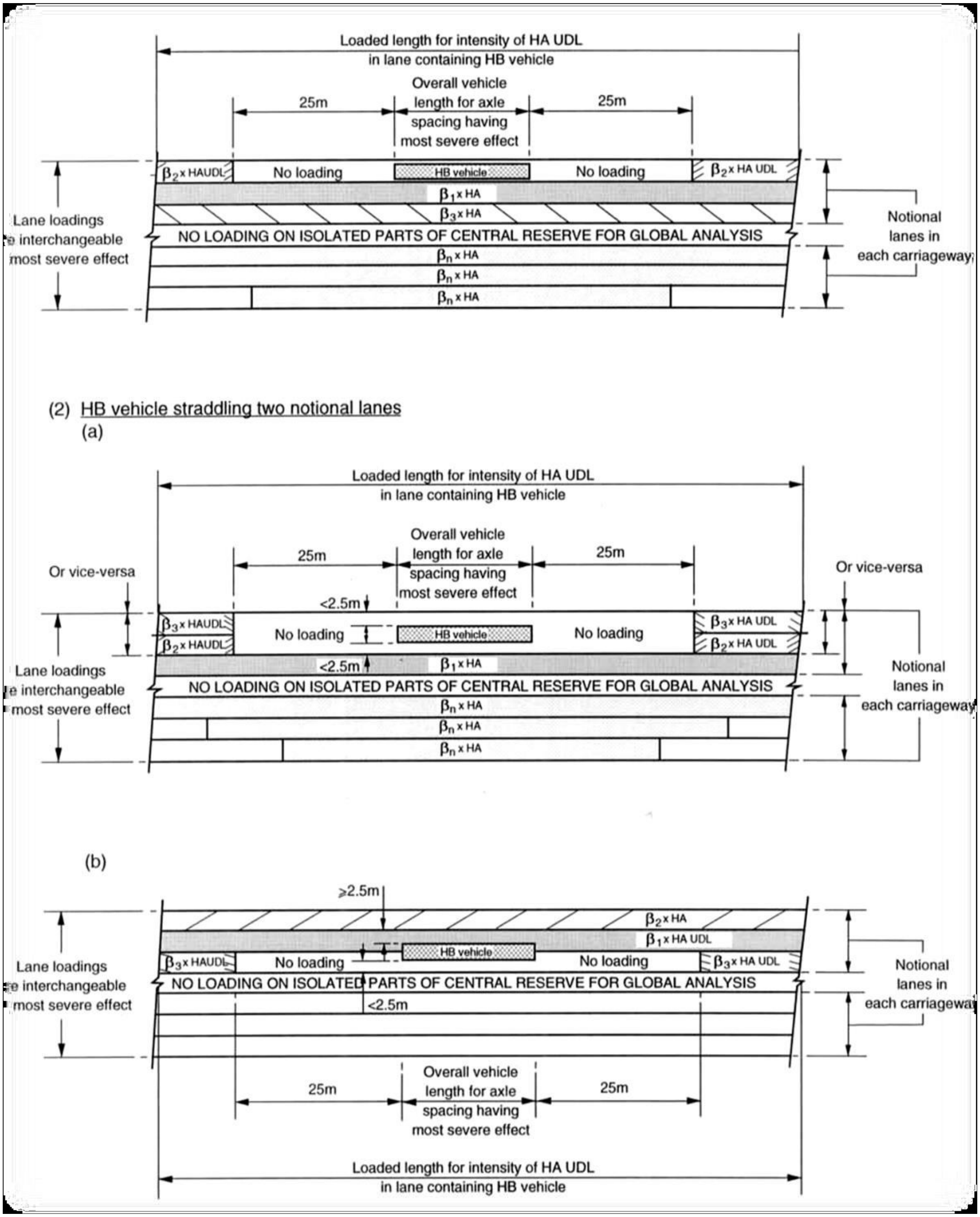


Fig :( 3.18): Types HA and HB loading combined.

## 3.5.6 Highway carriageway and lanes

Carriageway Width - This includes all traffic lanes, hard shoulders, hard strips and marker strips. The carriageway width between raised Krebs or the distance between safety fences minus the ‘set-back’ for the fences. The fences must not less than 0.6 m or more than 1.0 m from the traffic that facing each fence.

Traffic Lanes - Lanes marked on the running surface of the bridge. They have a maximum width of 3.65 meters.

Notional Lanes - Parts of the carriageway road for deriving the intensity of the live loads. Width of notional shall be measured in a direction at right angles to the line of the raised Krebs, lane markers, or edge marking. Notional lanes should be taken not less than 2.5 m, and not more than 3.65 m wide when the numbers of notional lanes exceed two lanes. For carriageway > 5.00 m and above, numbers of notional lanes are as shown in table 1.3 [4]:

**Table (3.1): Number of notional lanes.**

| Carriageway width ,m lanes | Number f notional |
|---|---|
| 5 m up to and including 7.5 | 2 |
| above 7.5 up to and including 10.95 | 3 |
| above 10.95 up to and including 14.6 | 4 |
| above 14.6 up to and including 18.25 | 5 |
| above 18.25 up to and including 21.90 | 6 |

**. National lanes shall be taken to be not less than 2.50 m wide.**