Dedication

To my Mother, Father, Brothers and Sisters

Acknowledgments

MY grate fully thanked to:

Dr : Emad Aldeen Abdallah Abdelrahim

To my teachers

Abstract

In this research we study the curvature of almost Hermition manifolds and their special analogues via intrnsic torsion and representation theory. We also investigate that the compact 7-dimensional manifold equipped with G_2-Structure with closed fundamental form is Einstein if and only if the Riemannian holonomy of the induced metric is contained in G_2. Also we illustrate that Bryant's weak holonomy metric on the homology seven-sphere is the unique weak metric arising from a rational curve.

الخلاصه

في هذا البحث درسنا معظم منحنيات متعددات الطيات لهرمشن ونظائرها الخاصة بواسطة الالتواء الجوهرى ونظرية التمثيل. أيضاً ناقشنا أن متعددات الطيات ذو البعد 7- المتراصة المجهزة طبقاً لتركيب G_2- طبقاً للصيغة الاساسية المغلقه هي انشتاين اذا وفقط اذا كان هلونومي المتري ريمان المحدث محتوى في G_2. ايضاً وضحنا أن مترك الهلونومي الضعيف لبريانك على كرة السبعه الهومولوجيه هو المترك الضعيف الوحيد الناجم عن المنحني الكسري.

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On Hermitian Manifolds and G_2-Structures

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GL(2, $\mathbb{R}$) Structures and G_2 Geometry

INTRODUCTION

In this research we are dealing with a7-dimensional Riemannian manifold that is G_2-Structures, and it is organized as follows:-

Firstly, we begin with abreife introduction of Lie Groups and Lie Algebras via discussing of Leibnitz formula, differential forms and the orientable manifolds with some applications .We Also study the notations of Lie bracket and the action of a matrix group with a Preliminaries results.

In chapter 2 we discuss the develop of the general theory of Cartan Connections with some applications, Also we describe two situations when a Cartan Connection arises naturally, the first one under a reduction of a principal bundle with a principal connection to another Principle sub bundle, and the second when a G_2-Structure with a connection is given, Also we study the relations between principal Cartan connections of a principal G-bundle and a principal connections on the H-bundle.

In chapter 3 we present some preliminary definitions , results and notation of the almost Hermitian manifold and it is structures, and we derive some formula relating curvature and intrinsic torsion with some applications, We Also discuss identities derived from the exterior algebra, We Also introduce a second symmetric Ricci-type tensor and we investigate that the Einstein conditions applied to the two Ricci tensor on a closed G_2-structure again imply that the induced metric has holonomy group contained in G_2.

Finally, we consider the 7-dimensional space of holomorphic parabolas in $\mathbb{C}^2$ and we discuss for a given 7-dimensional family of rational curves the conformal complexified G_2-structure on the module on these curves, We Also discuss the

characterization of the curves and the corresponding G_2-structures in terms of a 7-th order ODE, We Also investigate that if the component of the intrinsic G_2-torsion taking value in the 27-dimensional irreducible representation vanishes, then the resulting G_2 geometry admits a Riemannian real form and it is either flat, or is given by Bryant's weak G_2 holonomy on a seven-sphere or is given by a seven-parameter family of curves with some applications.