Dedication

To my Father

Mother

Husband

Brothers

Sisters

To my best family and my best friends

Acknowledgment

Firstly, I thank Alla almighty who gave me health, Strength, and Patience to complete this Work.

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Abstract

The methods of complex variables functions are extensively and effectively employed in the solution of a great variety of mathematical problems that arise in diverse fields of science. For example, the use of analytic functions in many cases yields sufficiently simple methods of solving boundary-value problems for the Laplace equation, to which various problems of hydro-and aerodynamics, the theory of elasticity, electrostatics and so forth reduce. This is due to the close connection between analytic functions of a complex variable and the harmonic functions of two real variables. We will examine certain general problems of the employment of analytic functions in the solution of boundary value problems for the Laplace equation and will give a number of value problems of the solution of problems in physics and mechanics. And a systematic investigation of basic boundary value problem for complex partial differential equations of arbitrary order restricted to model equations.

الخلاصة

إن طرق دوال المتغيرات المركبة واسعة وفعالة في حل المسائل الرياضية التي تنشا في مختلف المجالات العلمية. على سبيل المثال إستخدام الدوال التحليلية في حالات متعددة تؤدي إلى طرق بسيطة بما فيه الكفاية على حل مسائل القيم الحدية لمعادلة لابلاس، والتي تستنتج من استاتيكا الموائع وديناميكا الهواء ونظرية المرونة والكهرباء الساكنة. وهذا يرجع إلى العلاقة اللصيقة بين الدوال التحليلية في المتغيرات المركبة والدوال التوافقية لمتغيرين حقيقين.

تم دراسة وتحديد المسائل العامة لإستخدام الدوال التحليلية في حل مسائل القيمة الحدية لمعادلة لابلاس وأعطت عدداً من الطرق لحل المسائل الفيزيائية والميكانيكية. كما تم دراسة نظام مسالة القيمة الحدية الأساسية في المعادلات التفاضلية الجزئية المركبة ذات رتبة إختيارية لنموذج المعادلات .

The Contents

Introduction

Complex analysis is one of the most influence areas in mathematics. It has consequences in many branches like algebra, algebraic geometry, number theory, potential theory, differential equations, dynamical systems, integral equations, integral transformations, harmonic analysis, global analysis, operator theory and many others. It also has a lot of applications e.g in physics classical ones are elasticity theory, fluid dynamics, shell theory, underwater acoustics, quantum mechanics etc.

In particular the theory of boundary value problems for analytic functions as the Riemann problem of linear conjugacy and the general Riemann-Hilbert problem has had a lot of influence and even has initiated the theory of singular integral equations and index theory.

Complex analysis is one of the main subjects in university Curricula in mathematics. It is in fact a simply accessible theory with more relations to other subjects in mathematics than other topics. In complex analysis all structural concepts in mathematics are stressed. Algebraic, analytic and topological concepts occur and even geometry is involved. Also questions of ordering sets may be discussed in connection with complex analysis. Gauss, Cauchy and

Riemann were the main initiators of complex analysis and there was more than a century of rapid development. Nowadays complex analysis is not anymore in the center of mathematics 1st research. But there are still activities in this area and problems not yet solved. One of these subjects, complex methods for partial differential equations, will be presented in this research.

Some non-elementary results will be used as properties of some singular integral operators. They will be just quoted and somebody interested in the background has to consult references given. Everything else is just combinatorics. Hierarchies of differential equations, of integral representation formulas of Kernel functions of Green and Neumann functions arise by iterating processes leading from lower to higher order subjects. In this sense everything is evident. As Kronecker ones has expressed it, mathematics is the science where everything is evident. The beautiness of mathematics is partly reflected by esthetic formulas. All this will be seen below.

. In chapter one, we will examine certain general problems of the employment of analytic functions in the solution of boundary value problems for the Laplace equation and will give a number of value problems of the solution of problems in physics and mechanics.

A systematic investigation of basic boundary value problem for complex partial differential equations of arbitrary order is started in chapters two, three and four restricted to model equations. In chapter two and chapter three the Schwarz, the Dirichlet, and the Neumann problems are treated for the inhomogeneous Cauchy-Riemann equation. The fundamental tools are the Gauss theorem and the Cauchy-Pompeiu representation. The principle of iterating these representation formulas are introduced which will enable treating higher order equations. Chapter four is the continuation of an investigation of basic boundary value problems for first order complex model partial differential equations. Model second order equations are the Poisson and the inhomogeneous Bitsadze equations. Different kinds of boundary conditions are posed as combinations of the Schwarz, the Dirichlet, and the Neumann conditions. Solvability conditions and the solutions are given in explicit form for the unit disc. Exemplarily the inhomogeneous polyanalytic equation is investigated as a model equation of arbitrary order.

Boundary Value Problems in Complex Analysis

مسائل القيمة الحدية في التحليل المركب

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