

## Dedication

To my parents, brothers and sisters.

## **Acknowledgements**

I would like to express my gratitude to everyone who supported me during the course of this work.

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## **Abstract**

The finite difference approximation is a numerical method for solving differential equations. The basic idea for the solution is to approximate a differential equation by a system of algebraic equations and is to replace the derivatives in the equation by finite difference. We use a programming language, for example, MATLAB, to solve resulting systems numerically. We present in this thesis the finite difference approximation for solving parabolic differential equations in one dimension. We discuss an extension of the finite difference approximation to solve parabolic systems in higher dimensions. We present some theorems for the convergence of the numerical approximation and we analyze some schemes for their stability and convergence. We consider finite difference schemes in two spatial dimensions. One difficulty associated with schemes in more than one dimension is that the Van Neumann Stability analysis can become formidable. We also introduce the alternating direction implicit method (ADI) which is among the most useful of the methods for multidimensional problems.

## Abstract (Arabic)

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