Dedication

This research is

dedicated with love

and affection to my parents,

Brothers and sister

A know lodgments

Thank the God for this unprecedented love and grace upon me and gave me resole to finish this work. I am grateful to several people for helping , in one way or other ,I would like to the Sudan University of science and Technology (SUST) and my teacher in the department of mathematics realizing that those whom ,owe the most I cannot Thant enough ,and than the things for which I am most ,I cannot put into words, I am indebted to so many sources that it hardly possible to a c know lodge them all.

Beyond that ,I am very happy to have opportunity to express my deep gratitude to my **Supervisor: Dr: Emad Eldeen Abdalah Abdal Rahim**

Abstract

The solution of the equivalence problem of curves with values in Riemannian manifold is discussed. Also domain of validity of Frents theorem is shown to be the spaces of constant curvature.

Also we investigate that there are two important classes of curves, namely , Frents curves and curves in normal position which are coincide in dimensions ≤ 4 only.

A sharp bound for asymptotic stabillity of differential invariants is illustrated, also the complete system of invariants is characterized, and different examples are analyzed in detail.

الخلاصة

تمت مناقشة حل مسألة تكافؤ المنحنيات مع القيم في متعدد سطوح ريمان . ايضآ تم توضيح مجال صحة نظرية فرينت ليكون فضاء الانحناء الثابت . ايضأ وضحنا انه توجد فئتين من المنحنيات ،اسمياً،منحنيات فرينت و منحنيات الوضع الاعتيادي التي تتطابق فقط في الابعاد اقل من 4 فقط.

تم عرض الحد القاطع للاستقرار العرضي للامتغيرات التفاضلية ، كما تم تميز نظام الامتغيرات التام وتم تحليل أمثله مشختلفه بالتفصيل .

The contents

Chapter one

Riemannian Manifolds and geodesics

Chapter two

Exponential map and locally symmetric spaces

Chapter three

Frenent curve and equivalence problem

Chapter Four

Congruence on symmetric manifolds and

Congruence on constant curvature

Introduction

A fundamental problem in Riemannian geometry is that of equivalence of object in a determined class, namely to provide a criterion to know whether two given object in this class are congruent under isometric or not. in this work the solution of this problem is discussed in full generality for the simplest case , that of curves with values in Riemannian manifold . The out line of this Dissertation is as follows:

Firstly we being with a brief introduction to Riemannian geometry and manifold, and the important remark of levi-civita as an intrinsic object on Riemannian manifold, also the Geodesic and the functional curve of the critical point for energy is discussed. And for these notions, we present some applications and examples.

In chapter 2 we explain the concepts of Local existence and Uniqueness of Geodesics, the exponential map and complete Riemannian manifold with some application . In addition we study the Transvection and Holonomy , killing field and locally symmetric space in a brief detail .

In chapter 3 we explain that by the means of curvatures, a given curve can be reconstructed , and the solutions with some results of the equivalence problem by means of function that are invariant under the isometry group of the Riemannian manifold are investigated . Also the developing solution of invariants , where the main question are studied for an arbitrary Riemannian manifold .

Finally we discuss the Frents theorem that classfies curves in Riemannian manifold if and only if it is of constant curvature , we then illustrate that the difficulty of the equivalences problem is closed related to

the size of the isometry group .In addition the study of the computation of a basis of differential invariant with their geometric meaning for Riemannian homogeneous complete 3- dimensional manifold is present