

Dedication

To my family and husband

Acknowledgement

I would like to express my deep sense of gratitude , and thanks to my supervisor Dr. Shawgy Hussein Abd Alla for useful comments, suggestions and constructive criticism.

Especial thanks due to my brother Hashim and sister Tahani Hassan .

I am grateful to Sudan University for Science & Technology.

Finally my thanks are due to brother Imad Ali for his excellent typing.

Abstract

We show that there exist a ball of non zero radius within which one can express a certain subset of variables , in a system of analytic equation , as analytic functions of the remaining variables . We derive a nontrivial lower bound on the radius of such a ball on the domain of validity of the implicit function theorem .

We obtain implicit function theorem for mappings from arbitrary topological vector spaces to Banach spaces.

We prove Newton approximation with parameters , as the basis , for the inverse function theorem . Also we prove inverse and implicit function theorems for SC^k -map over complete valued fields, which parallel to the classical theorems for continuously Fre'chet differentiable mapping in the real case.

الخلاصة

وضح أنه توجد كرة لها نصف قطر غير صفري فى داخلها يمكن أن نعبر عن فئات جزئية معنية من المتغيرات ، فى نظام معادلات تحليلية ، كدوال تحليلية من المتغيرات المتبقية . اشتق حد أسفل غير بدائي على نصف قطر مثل هذه الكرة على مجال الشرعية لمبرهنة الدالة الضمنية .

تحصل على مبرهنات الدالة الضمنية لأجل التطبيقات من فضاءات المتجه التبولوجى الاختيارى الى فضاء باناخ .

أثبت تقريب نيوتن مع مميزات ، كأساس لمبرهنة الدالة العكسية ، أيضاً أثبتت مبرهنات الدالة العكسية والدالة الضمنية لأجل تطبيقات SC^k - فوق حقول القيم التامة ، والتي توازى المبرهنات القديمة لأجل التطبيق التفاضلي لفرشت المستمر فى الحالة الحقيقية .

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Implicit Functions From Topological Vectors Spaces To Banach Spaces

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A thesis Submitted in partial fulfillment for the Degree of M.Sc.
in Mathematics

Sudan University of Science & Technology
College of Science

2004