## Abstract

We show the characterization of hyponormal Toeplitz operators . We study algebraic and spectral properties of Toeplitz operators on the Hardy space , under certain assumptions concerning the symbols in Lebesgue space . Among the algebraic results is a characterization of normal Toeplitz operators with polynomial symbols , and a characterization of hyponormal Toeplitz operators with polynomial symbols of a prescribed form .

We examine under what condition a classic result of H – Weyl , which has extensions to hyponormal and Toeplitz operator , holds for all analytic functions of a single Toeplitz operators with continuous symbol. We prove that all finite normal Toeplitz matrices are either generalized circulant or are obtained from Hermitian Toeplitz matrices by rotation and translation .

We characterize those hyponormal Toeplitz operators on the Hardy space of unit circle among all Toeplitz operators that have polynomial symbols with circulant – type sets of coefficients . Answering a question of C .Cowen for the case of analytic polynomials we show that the reduced Cowen set for an analytic polynomial is strictly convex . We provide a complete criterion for hyponormality when the symbols satisfy partial symmetry condition .

We characterize hyponormal trigonometric Toeplitz pairs ,which are pairs of Toeplitz operators on the Hardy space of the unit circle , with trigonometric polynomial symbols . Moreover weak hyponormality and hyponormality for the Toeplitz pair are equivalent properties . This characterization can be extended to trigonometric Toeplitz n-tuples .

We present a note concering subnormality and k- hyponormality of Toeplitz operators . We provide an example of a Toeplitz operator which

is 2-hyponormal but not subnormal and to consider 2- hyponormal Toeplitz operators with finite rank self –commuators .

We show the quadratic hyponormality and 2 hyponormality for Toeplitz operators . We also discuss the gap between 2 hyponormality and subnormality for Toeplitz operators , and we give applications to flatness hyponormal of Toeplitz operators pairs , Toeplitz extensions of positive moment matrices , and hyponormality of single Toeplitz operators .

We explore finite rank perturbations of unilateral weighted shifts . We show the gap between k-hyponormality and (k+1) - hyponormality for Toeplitz operators . We find that the minimal normal extension of a subnormal operator is exactly the inductive limit of its minimal partially normal extensions .We characterize joint k-hyponormality for two variable weighted shift .

We give two versions to characterize the almost subharmonicity , a pointwise version and a weak – star version . We give applications of those results to hyponormal Toeplitz operators on the Bergman space. We show that two analytic Toeplitz operators essentially doubly commute if they doubly commute on the Bergman space of the polydisk .

We show that if two Toeplitz operators on a Bergman space commute and the symbol of one of them is analytic and nonconstant , then the other is also analytic . It is shown that an operator on the Hardy space commute with all analytic Toeplitz operators modulo the finite rank operators.