CHAPTERONE

COMPRESSIBLE :POTENTIALFLOW

THE FULL POTENTIAL EQUATION

:Introduction *(1.1)*

- Recall that for **incompressible flow conditions**, velocity is not large enough to cause density changes, so **density** is known. Thus the unknowns are **velocity** and **pressure**. We need two equations. The continuity equation is enough to solve for .velocity as a function of **time** and **space**
- Pressure can be obtained from the Bernoulli equation, which comes from momentum .Conservation
- If the flow is irrotational as well, we can define a potential, and reduce the continuity equation to the :form of the Laplace equation

:Here is defined such that

.High-speed flows can also be irrotational

:Loads on an Airfoil

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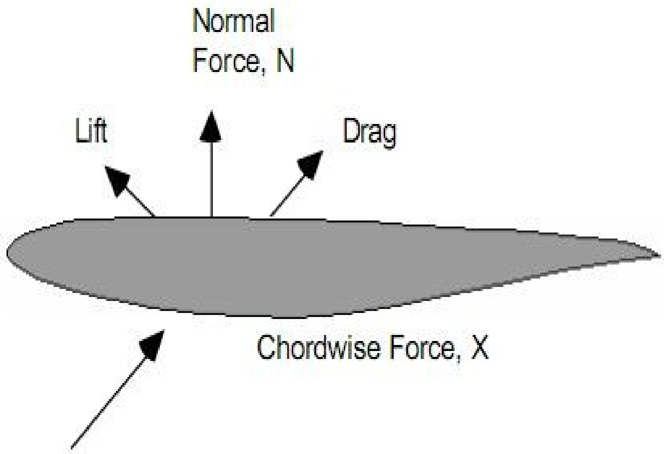

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Where is the airfoil and is the angle of attack

:Continuity Equation

:Here, as we recall

:Steady

:Momentum conservation in deferential form

:U-component

(2)

:V-component

:Energy equation

Energy equation for steady in viscid flows may be :written as

When continuity is used the above equation :becomes

:In terms of substantial derivative where

i.e., specific total enthalpy is constant along a streamline, assuming adiabatic flows (no heat .addition), andnobodyforces doing work on the fluid

:Objectives

Derive the potential equation for compressible .flow -1

Reduce to linearized form for small-perturbation .analysis -2

Apply results to thin airfoils and slender .axisymmetric bodies -3

:Note

Generally, flight vehicles are designed to create as little "perturbation" as possible, because large disturbances cause large drag. We will derive the potential equation for **2-D** flows. At the end, it will be obvious how to extend it to **3-D** flow, so we will just write .down the **3-D** equation

:Assume

.Isentropic flow -1

.Steady flow -2

:Speed of sound ***(1.2)***

.Constant of entropy

:.i.e

(3a)

:(Substituting (3a) in (2

(4)

:For homework (1.3)

-1 Multiply (4a) by and (4b) by and add, call this (equation (5

-2 (Expand equation (1) and substitute in equation (5

:Solution

-1

:Add

-2 (From equation (5

:Dividing this equation by wehave

:Put

Substitute this equation in we have

:Irrotational flow

If this is true, then a potentialfunaction can be defined :such that

(7)

:Substituting in equation (6) or using the notation

What is, the local speed of sound? To see this, go to the energy equation for steady adiabatic flow.

But:

Or:

Thus:

Or:

Extension to 3-D flow:

By inspection, equation (8) and (11) can be written for 3-D potential flow as:

Where:

(1.4) Linearized Potential Equation:

(1-4-1) applications:

Analyze performance of high-speed airfoils and slender, pointed body shape at small angles of attack in the subsonic and supersonic regimes.

(1-4-2) perturbation potentials:

Let us define the potential such that:

Where:

)

(14

:We can put

-:Substitute the equation (14) in equation (8) we have

:Note

This is still an exact equation, no different from equation :(8) using (14), (11) becomes

:Linearization

:Suppose that

:If in addition

This is true if is not too large, i.e excluding hypersonic
:flow. Equation (15) with replaced by

Let us look at each the terms. The sound derivatives of function are of the same order magnitude (no reason to .(expect otherwise

: The coefficient of can be written as -1

: If

This is true if we exclude the transonic regime :where

:The coefficient of can be written as

If is not too large

Exclude hypersonic flow. But then, we already did) (that

:The coefficient of is

We have to throw this out, or we won't get a linear .equation

So, let us assume that: the other coefficients. This means that the airfoil must be very thin if is high, or .else must be very low if the airfoil is not so thin

:The linearized equation then is

:Similarly for 3D

These equations can be used to describe small perturbations in subsonic (but not transonic) and .(supersonic (but not transonic or hypersonic flow

pressure coefficient for small (1-5) :perturbations

Recall the definition of pressure coefficient for :incompressible flows

When the flow is compressible it makes more sense to use as the normalizing parameter. Thus can be written :as

Show this for homework, and also define limiting values) :(and the corresponding conditions

Let us derive a linearized form of, to us along with out small-perturbation from the isentropic flow relations

:Solution

:First we are proving that

Then

:Substitute in we have

.This is a prove it

,From the energy equation

:Where

Or

:Where

:The second term on the RHS is. Thus

:is of the form ,

where ,

.This can be expanded using the Binomial theorem

:As

:Smaller terms where

: Or

: For

: Thus

.This is valid for a flow over flat wings

similarity" Relation for subsonic"(1-6) :Compressible Flow

:(Airfoils (2-D flow (1-6-1)

Method: transform the linearized potential equation into an equation describing a "related" incompressible flow. It solution will give the pressure distribution around the "related" airfoil in compressible flow. Alternately, the pressure distribution in incompressible flow can be obtained from the extensive wind tunnel data available in the literature. Transform this solution back to the .compressible-flow case

Linearized potential equation for **2-D** subsonic flow

(and y subscripts denote derivatives)

Or

Where

The Laplace equation describing incompressible flow
:is

:.i.e

:Can then be transformed to (1.6.2) by setting

And

Where is some constant

:**Note**

The text by Anderson refers to as and

,Thus, equation (1.6.1) becomes

Or

Note that 'm' is still undermined. We can decide later
.what we want it to be

:Boundary conditions (1.6.2)

At infinity

Surface tangency

- Subsonic compressible flow

.Where is the surface

- .Incompressible flow where describes the surface

Not that same is used in both flows. If we used :different ones, everything cancels out eventually

.It makes no difference

,Now

Or

However the boundary condition in the :incompressible flow is

:.i.e

Or **,** relation between surface slopes of the original airfoil .and the "related" incompressible flow airfoil

Note that the slope is a function of chord wise location on the airfoil. Different choices of will give different .relations between the slopes

:pressure coefficients (1.6.3)

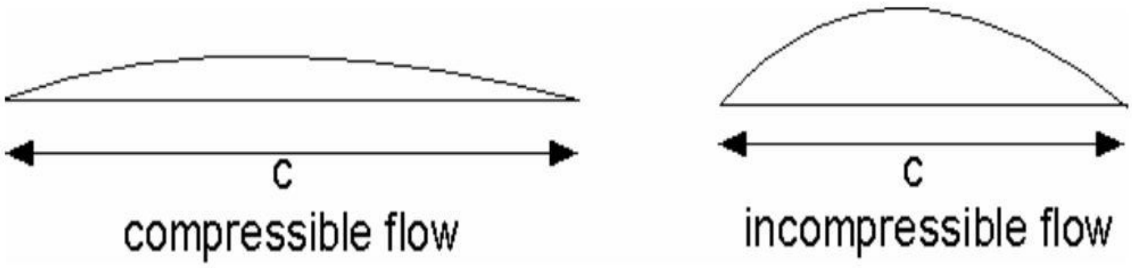
:Incompressible flow

Depending on the problem to be solved, we can make . various choices of

: Consider the choice

This implies that i.e the chord wise pressure distributions in the incompressible and compressible flows are the :same. However, we see also that it implies that

The compressible flow airfoil must have a lower surface . slope than the incompressible. Flow airfoil, in order for the .pressure distributions to be the same



The other usual question is: "How does the pressure distribution over a given airfoil change as the Mach # is " ?increased

. To answer this make a choice of that will give

This is

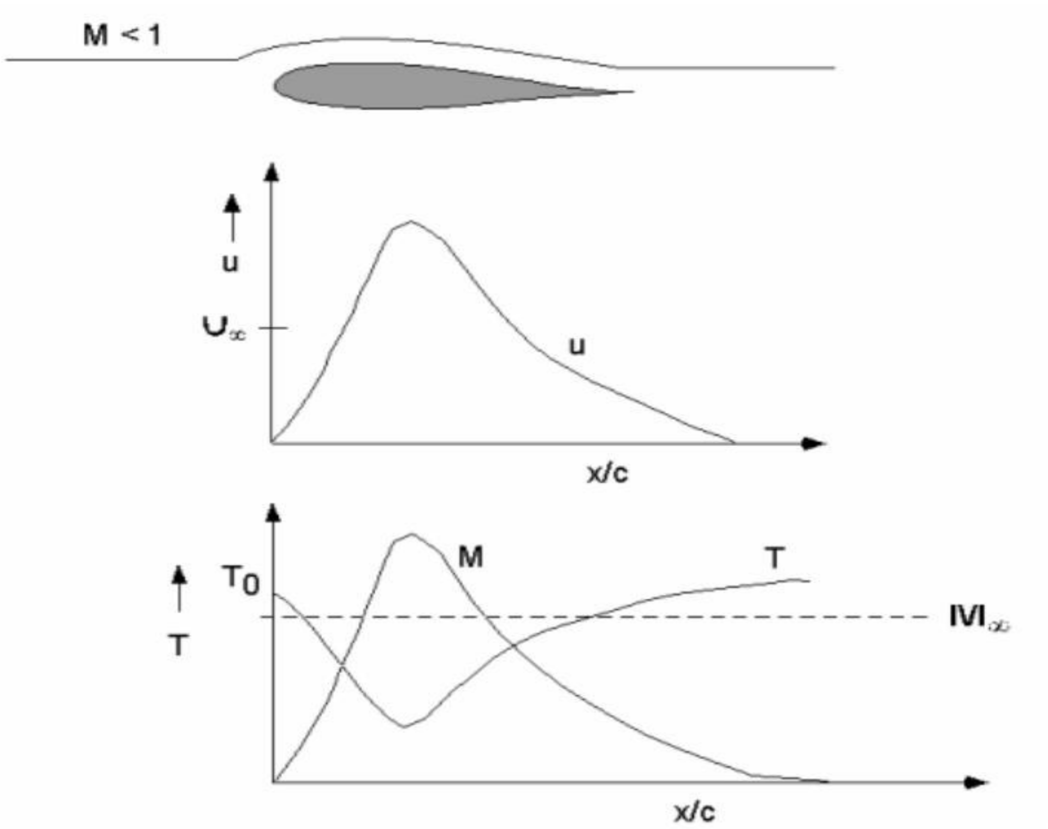
.Prandtl-Glauert transformation

.# Increases with Mach

:Critical Mach number(1-7)

Consider the following situation as subsonic air how over an airfoil (or wing). It accelerates, reaches a maximum speed and then decelerates toward the .trailing edge

Since a



Thus, the Mach number of the flow increases and .then decreases

: calculation of (1-7-1)

:We know that

Or

At M reaches 1 somewhere. The value of at this

. point can be found by setting

:Supercritical Airfoils(1-8)

The preceding discussion shows that if   drag rises ?greatly. How can be increased

-1 Use a very thin airfoil with a sharp leading edge. .This is impractical for airlines

-2 Reduce the curvature on the upper surface. This reduces the acceleration and deceleration of the flow, so that any shocks formed will be relatively .weak

Such airfoils were among the first to be designed using .detailed mathematical computation

CHAPTER TWO

TWO-DIMENTIAL SUBSONIC AND SUPERSONIC SOLUTION OF THE LINEARIZED EQUATION

:Effects of Compressibility(2-1)

In the case of a cylinder moving through an incompressible fluid the stream lines at time t are determined throughout the whole fluid solely by the motion of the cylinder at time t and are independent of .the previous motion of cylinder

The implication of this is that changes of pressure occasioned by the motion of the cylinder are transmitted instantaneously throughout the fluid. In the case of a compressible fluid the situation is guite different since small disturbances such as pressure changes are propagated throughout the fluid with the velocity of .sound which as will be show in fact c or

This means that in the case of a moving cylinder the flow at a distance from the cylinder at time t is dependent on the previous motion of the cylinder and is independent of the actual motion of the cylinder at time .t

The existence of this velocity of propagation is also responsible for the important distinction between .subsonic and supersonic flow

Asourceof small disturbances which moves within each of the expanding wave fronts of the disturbances which it creates, whereas so long as a disturbances is moving whit supersonic velocity, it is outside each disturbances which .it has previously created

If we superimpose uniform flow on the above flow patterns with the object of bringing the sources of the disturbances to rest we get the situation illustrated in .(figures.(2-1) and (2-2

In the case of subsonic flow (figure.2-1), a disturbance at the origin will eventually have an effect at each point of space, but in the supersonic case (figure .2-2), the disturbance can never affect the fluid is outside the enveloping (half) cone of the spherical wave fronts. This half cone is called the Mach Cone of the center of disturbance o and semi-vertical angle, known as the :Mach angle, is given by

.Where M is the Mach number

(Figure (2-1

.(Figure (2-2

Such points must lie within the upstream half cone with . vertex, and the same semi-vertical angle

Thus at any time, space is divided into three distinct regions in relation to a given point in space. first , there is the upstream half cone consisting of points the flow at which will affect the flow at ; second , the downstream half cone consisting of points the flow at which will be affected by the flow at ; third , the remainder of the fluid in which the flow can neither affect the flow at O nor be . affected by it

In the case of two – dimensional flow the Mach drawn at a given point, each of which makes the Mach angle with the direction of flow at that point. A consequence of these arguments is that supersonic problems are often easier to .deal with than subsonic ones

For instance in the case of an aero plane wing in the shape of a cylinder with an aerofoil section , we can argue that at supersonic speeds the whole effect of the wing tips must be contained within the Math cones of the . points on the wing tips

The flow over the rest of the wing is unaffected by the fact that the wing is of finite span and is the same as it . would be if the wing were of infinite span

For wings of large span the flow is therefore two–dimensional in character over all but a very small portion of wing. In subsonic flow, on the other hand, the presence of the wing tips affects the flow at all points of the wing and the problem is other fore 'a' strictly three-dimensional .one

In the subsonic case it is difficult to see whether a two–dimensional treatment would give a reasonably good case we can be certain that theapproximation is good provided .of the span of the wing is sufficiently large

It is also worth mentioning that the symmetry of the .problem

Consider a cylindrical aerofoil whose cross–section is .symmetrical with respect to both the x- and y-axes

When it is placed in a uniform stream in the x-direction without circulation, we expect in the incompressible non-viscous case to find a flow pattern which is symmetrical .with respect to both these coordinate axes

If however the fluid is compressible and the flow is supersonic, the flow must be uniform within the zone of silence of the leading edge. We cannot, however, expect it to be uniform within the refection

In the YZ plane) of this zone.Thus the compressible non-) .viscous flow possesses

Another important feature of compressible flow which we must mention here is the equations, if they exist, for the stream function and the potential, are not linear differential equations as they are in the case of .incompressible flow

Note only does this mean that it is extremely difficult to obtain any solutions at all in closed form, but it also means that we cannot apply the principle of superposition, since the sum of two solutions will not in general be itself a solution of the differential equation concernedas one might expect, therefore most practical problems concerning compressible flow have to be attacked by methods of approximation involving finite difference or expansions in series, or else hodograph .methods

:Linearized theory (2-2)

In the case of two-dimensional, adiabatic, non-viscous .irrotational flow the velocity potential satisfies

:Velocity Potential

For irrotational motion q= - the continuity equation then becomes or by differentiating Bernoulli's equation in the :form

:Partially with respect to in turn, we obtain

:And the similar equations

:Multiply these respectively by

:And add the results together. Then

:But

:Then the equation (2.2.4) becomes

= .

=

:Now the left – hand side of this equation is

= +) + (+

+ +

:Hence satisties

= + + +)

:One can write this equation as

+ + = +

, :As has mentioned, this is a non-linear equation since

If, however, the How is only slightly different from a uniform stream of velocity U in the x-direction, we can :write

Where is the velocity potential of a small perturbation then

Where , while the second derivatives of and must be :identical. We conclude therefore that

Assuming then that and are very small compared with U, :we have

:According to the same assumption, we find that

)

:Dividing by equation (2.2.5) we have

:But

:The equation becomes

-1)

Where is the Mach number of uniform stream , is approximately true subject , to the assumptions mode , provided is appreciably different from unity .Here too we see that there is an essential difference between the case of subsonic flow in which and in which the above equation for is of laplace's form

Where =1- , and the case of supersonic flow in which :and the equation for is a wave .like equation

Where = -1

Consider first the subsonic case. The velocity potential of a small perturbation in a uniform stream U of :incompressible fluid in the plane would satisfy

:Whereas in the compressible case the equation is

Clearly the one equation implies the other if we make the parental– Glauert transformation

Where is any constant .Any solution of the incompressible equation for a small perturbation will afford a solution in the compressible case. Suppose now that the boundaries in the two cases are =g () and

:respectively. Then on the solid boundary we have

:Whereas for a similar reason

On the solid boundary in the incompressible case. It follows therefore that if the boundaries correspond under :the above transformation we must have

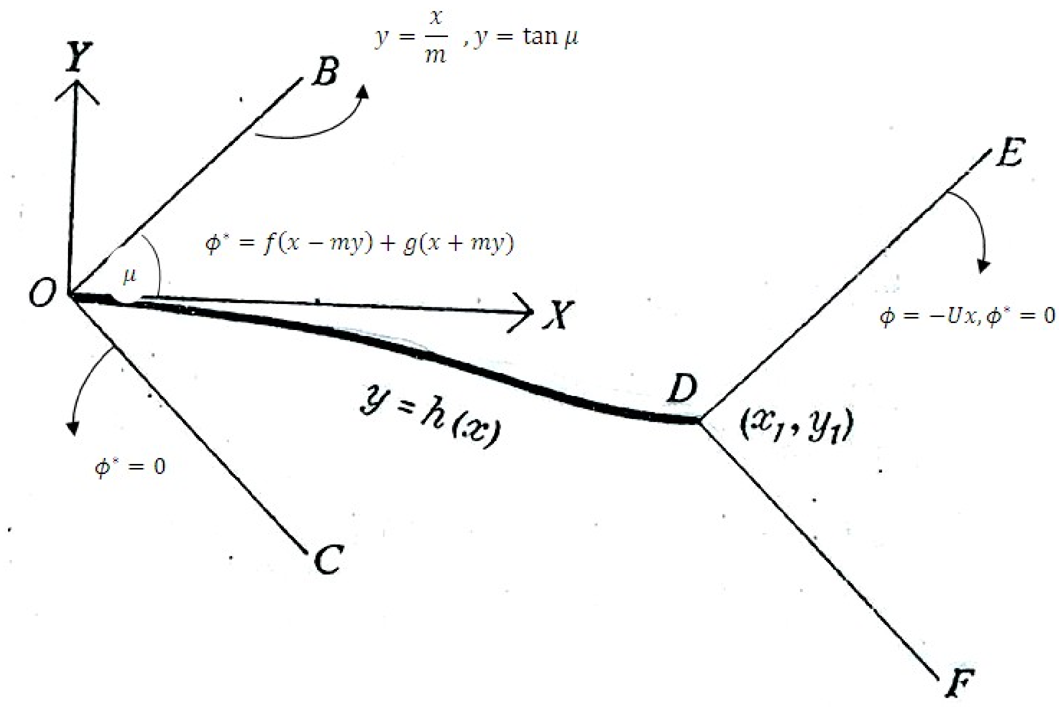
Hence if (,gives a solution for incompressible flow past a body of the form =g(, then

gives a solution for the compressible flow past a body of the form provided .

If the bodies are identical, then and = , from which it :follows that

Alternately,if , then = 1 and

Since , this means that in the compressible solution



(Figure (2-3

The slope of the body and,indeed, its thickness, is less .than it is in the corresponding incompressible solution

Turning now to the supersonic case, we have the :equation

:for which the general solution is of the form

Where and are arbitrary function. We shall describe the application of this formula in Ackeret'stheory for the airfoil of zero thickness (fig(2-3)). The lines are drown at the Mach angle to the impinging uniform stream. That is to

sons, are the Mach lines at while are the Mach lines at .
The region to the left of is the zone of silence in which
.there is no perturbation

In this region then and on the Mach lines and we may take , On the other hand , in the region to the right of and on it self we know that (2.2.6) holds . Since , or is ,the equation of the Mach lines

on this line .Comparison of these two values for on shows that g is a constant which we may choose to be zero by stipulating that the constant be induced in the function g . It follows that in the region we may take g= :and

:If is the equation of the aerofoil then

) -)

= :But

: Assuming that is small compared with, we have

We may there take in the region

The constant of integration being zero if is the origin and .if we assume that is continuous across

:Likewise in the region we may take

If is the point () , the Mach line has the equation

So that on this line has the constant value

Since the motion at the aerofoil is essentially small .disturbance

CHAPTER THREE

SMALL DISTURBANCE THEORY

:Introduction(3-1)

By taking a Taylor series expansion of the relationship between   and about , for fixed and it can be shown : that

:Note that when that

After a good deal of algebra and trigonometry it can also be shown that the pressure change, change in velocity magnitudeand change in entropy for flow over a thin wedge arein terms of change:

Note that a small positive gives rise to

- An increase in pressure.
- A decrease in velocity magnitude.
- A very small change in entropy.

This has an analog in one-dimensional unsteady flow. Consider a piston with an initial velocity of zero accelerating into a tube.

- Lead compression wave travels at sound speed.
- Lead wave increases temperature (and sound speed) of disturbed flow.
- Eventually acoustic waves catch and form a shock.

Consider a piston with zero initial velocity which .decelerates

Centered PrandtlMeyer (3-2) :Rereflections

If we let, the entropy changes become negligibly small relative to pressure and velocity changes, and the flow is isentropic. The relations can be replaced :by differential relations

:Recall now that for adiabatic flow

(Also: (3.2.10

(3.2.11)

Now positive corresponds to compression and negative corresponds to expansion. Let’s define gives and ,expansion

(3.2.17)

(3.2.18)

Now integrate the expression

Let correspond to . This effectively selects

The function is called Prandtl–Meyer function. Many texts tabulate the Prandtl–Meyer function .for a known turning angle; one can find the Mach number. As the flow is entirely isentropic, all other flow variables can be :obtained through the isentropic relations Note

- .As, correspond to vacuum condition
- .Given, one can calculate
- isentropic relations give
- .Prandtl-Meyer function tabulated in many texts

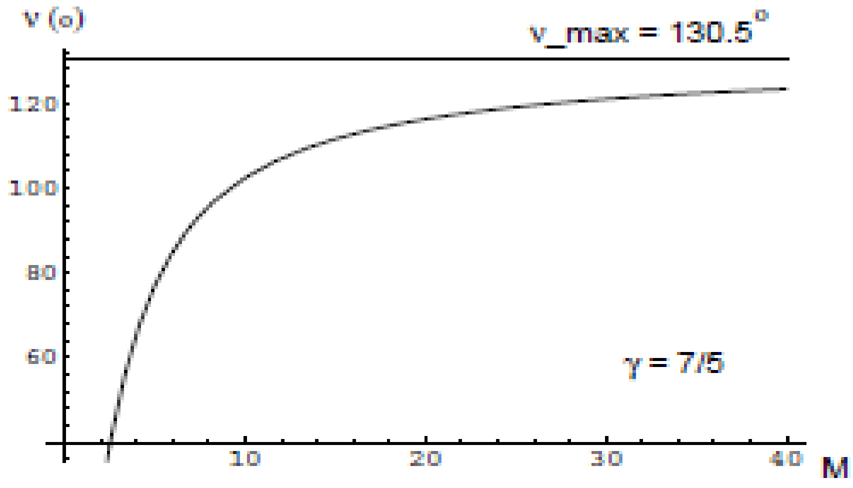


Figure (3-1): Prandtl –Mayer function

:Example

Given calorically perfect, ideal air with =100kpa,,
:turned through a expansion corner

.Find: Fluid properties after the expansion

:Analysis

Now calculate the Prandtl–Meyer function for the freest
:ream

(3.2.28) (3.2.27)

(3.2.30)

The interpretation here is that an initially sonic flow would have had to had turned to achieve a Mach number of .=1.4401

:Now add on the actual turning

(3.2.32)

.

A trial and error solution gives the which corresponds to

:Wave Interactions and Reflections (3-3)

Shocks and rarefactions can interest and in a variety of .ways

:oblique shock Reflected From a wall (3-3-1)

An oblique shock which reflects from a wall is .(represented in figure (3-2

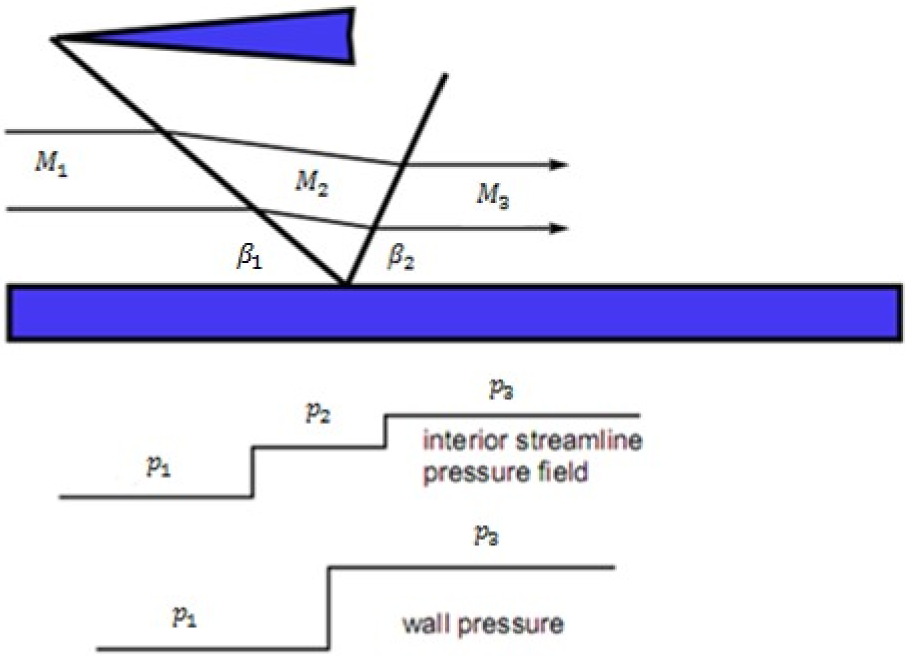


Figure (3-2): Reflection of an oblique shock from a wall

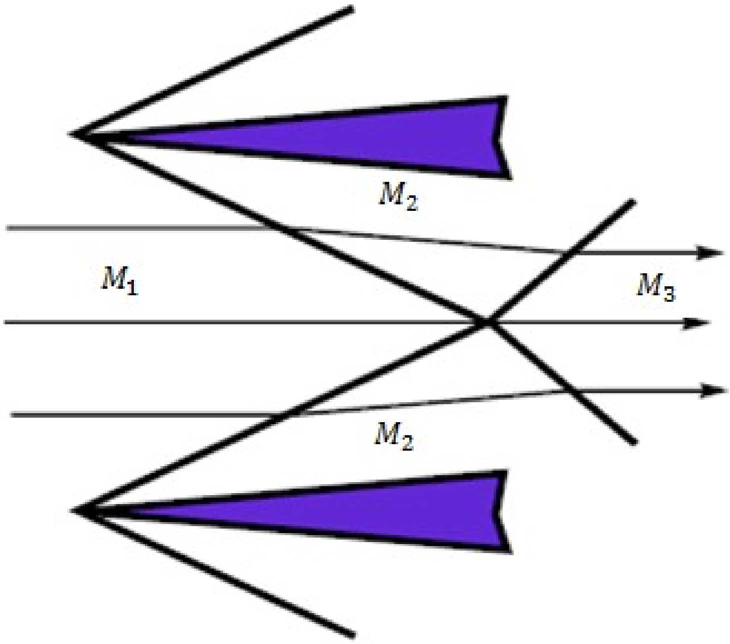
:Notes

- .Analysis just that of two oblique shocks
- .Flow always to be parallel to wall
- Angle of incidence not equal of relation due to non-linear effects.
- .Interior pressure profile has two steps
- .Wall pressure profile has single step

- that is the pressure is higher than that obtained in , .the acoustic limit

:oblique shock Intersection (3-3-2)

Two oblique shocks intersect as Sketched in .(Figure (3-3



:Note

- .Flow always turns to be parallel to wall
- When shocks intersect, flow turns again to be parallel .to itself

:shock strengthening (3-3-3)

A flow turned by a corner through an oblique shock can be strengthened by a second turn as sketched in .(figure (3-4

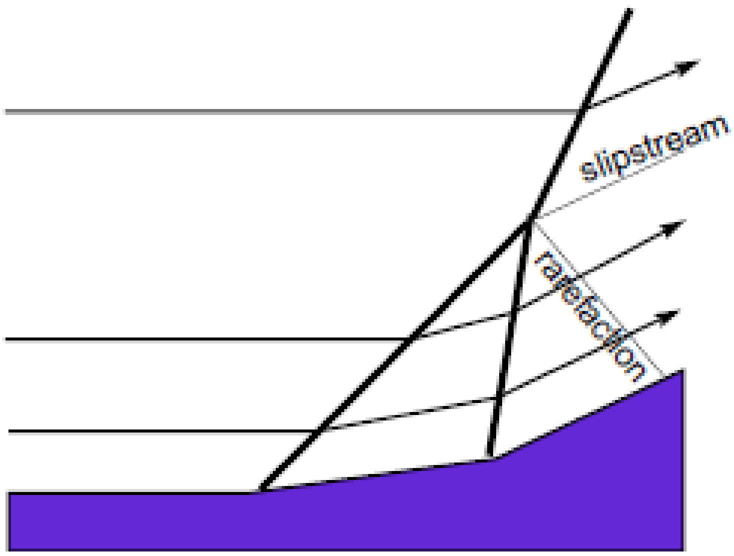


Figure (3-4): Shock strengthening sketch

Notes: three new waves generated

- .Strengthened shock
- Slipstream in which pressures mach, velocity .directions match, but velocity magnitudes differ
- .Weak rarefaction wave

:Supper sonic Flow over Airfoils (3-4)

The standard problem in flow over an airfoil is to determine the lift and the drag. While in actual design it is the magnitude of the lift force , and drag force, that is most crucial, three exists dimensionless numbers the lift coefficient and the drag coefficient which give good relative measures of airfoil . performance

Though this is the traditional formula, it is probably not the best for interpreting how that force varies when flight speed is varied. This is because when , fight speed is varied both numerator and denominator change. To remedy this, we can instead scale by the ambient sound speed to define a dimensionless lift force and dimensionless drag force :

CHAPTER FOUR

Conclusion

In this research study, I conclude with this: The Hypersonic and the Supersonic on an airfoil is a wide big science could be more studied and more research could be done, so I did my best to add more new study and research may be other coming students do .their share as well

I studied the flow characteristics on a 2-dimantional airfoil including solving the linearized equations and normal and oblique .shocks

Their effects on the flow temperature and pressure are investigated. It is important in .aircraft wing design

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