Bitriangular Operators of Jordan form and Inverse Spectral Theory for Symmetric Operators on Joint Invariant Subspaces

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By

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Dedication

To my parents, wife, sons

Acknowledgement

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Abstract

We show the sum rules and their applications of special form for Jacobi matrices, and the spectral properties of self-adjoint extensions of Weyl functions are considered. The representation and Jordan form of biquasitriangular operators are studied, and we determined the homogeneous shift with operators on Hilbert spaces and, also show the inverse spectral theory for symmetric operators with several gaps. We obtained the characteristic operator function of the class of n-hypercontractions on joint invariant subspaces.

الخلاصة

تم توضيح قواعد الجمع وتطبيقاتها من النوع الخاص لمصفوفات الجاكوبيان واعتبار الخواص الطيفية لتمديدات المرافق الذاتي لدوال ويل، تم دراسة التمثيل وصيغة جوردان للمؤثرات الثنائية شبه المثلثية، وتحديد الازاحة المتجانسة مع المؤثرات على فضاءات هلبرت، ونظرية الطيف العكسي للمؤثرات المتماثلة طبقا للمرات المتعددة، وايضا إعطاء دالة المؤثر المميز لعائلة المؤثرات الفوقية -n على الفضاءآت الجزئية اللا متغيرة المقترنة .

Introduction

In the last two decades, there has been exicting progress in multivariable dilation theory, in the attempt to extend the classical Nagy-Foias theory of contractions .Several years ago, P. R. Halmas introduced the remarkable class of quasitriangular operators on the complex separable infinite-dimensional Hilbert space, the study of this class was the spectral characterization of non-quastriangular operators and Jordan form of a Bitriangular operator. Spectral properties of a self-adjoint realizations of the Laplacian on a bounded domain have been investigated for a very long time. .Originally these investigation had been motivated by the important rule of such operators in a wide variety of models in acoustics, electromagnetism and thermodynamics. In the present century the interest gradually shift to fascinating new research areas. In non-relativistic quantum mechanics realizations of the Laplacian on a bounded domain and in probability theory. The connection between the invariant subspaces of an operator and the corresponding characteristic function was first considered for certain particular classes of operators.

In chapter (1) we use sume rules of a special form to study spectral properties of Jacobi matrices. We characterize the spectra of self- adjoint extensions of a symmetric operator with equal deficiency indices in terms of boundary values of their Weyl functions. We develop a dilation for row contractions subjected to constraints. A characteristic function associated with a row contraction is proved.

In chapter 2:are present a matrix representation for biquasitriangular operators and their existence and show the Jordan form of a bitriangular operator. We determine the homogeneous weighted shifts which they consist

of the unweighted bilateral shift and two one parameter families of unilateral shift.

In chapter 3, we study the homogenous operators and projective representations of the Möbius group. We constract a class of multiplication operators on reproducing Kernel Hilbert spaces which are homogenous with respect to the action of the Möbius group consisting of bi-holomorphic automorphism of the unit disc.

In chapter 4, we discuss the systematic application of sum rules for Jacobi matrices. We study the complete classification of the spectral measures of all Jacobi matrices and work with sum values of higher order.

In chapter 5, we show that if there exist a self-adjoint extension of the orthogonal sum of infinitely many pair wise unitarily equivalent symmetric operators with non-zero deficiency indices such that the open subset of a real line is contained in the resolvent set of the self-adjoint extension and the associated Weyl function of the pair of the orthogonal sum and the self-adjoint extension is monotone with respect to the open subset, then for any self-adjoint operator there exists a self-adjoint extension such that the spectral parts are unitarily equivalent , we investigate that the spectral properties of the self-adjoint extension of a symmetric operator in a separable Hilbert space can have inside the gab and provide prescribed spectral properties inside the gab.

In the last chapter we establish the existence of bijectivity between the joint invariant subspaces under n-tuple of operators on a Hilbert space and the regular factorizations of the characteristic function associate with these operators. We obtain criterions for joint similarity of n-tuples of operators to Cuntz row isometries. We consider a class of bounded linear operator on Hilbert space called n-hyper contractions which relates mutually to adjoint

shift operators on certain vector- valued weighed Bergman space on the unit disc. We generalized a characteristic operator function for a contraction operator which is an operator valued analytic function in the unit disc Hilbert spaces of defect type. The operator-valued analytic function is shown to act as a contractive multiplier from Hardy space into the associated standard weighted Bergman space.

The Content

Chapter 6