

Bitriangular Operators of Jordan form and Inverse Spectral Theory for Symmetric Operators on Joint Invariant Subspaces

المعتمد على رسالة الماجستير التي قدمتها
إلى جامعة السودان للعلوم والتكنولوجيا
في عام ٢٠٠٦

By

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***A thesis submitted in fulfillment for the degree of Ph.D in
Mathematics***

**Sudan University of Science & Technology
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July -2009

Dedication

To my parents, wife, sons

Acknowledgement

I wish to express my thanks to my supervisor Dr. Shawgy Hussein Abdalla for his extensive effort and unlimited support for carrying out this work. Also, I wish to extend my especial thanks to my family.

From the depth of my hart I wish to thank Sudan University of Science and Technology, Nyala University and their teaching staff. Especial thanks are due to Mr. Babiker Mohamed Zain and Mr. Sharaf Hassan Ahmed at Sudan Institute for Natural Sciences for their excellent typing.

Abstract

We show the sum rules and their applications of special form for Jacobi matrices, and the spectral properties of self-adjoint extensions of Weyl functions are considered. The representation and Jordan form of biquasitriangular operators are studied, and we determined the homogeneous shift with operators on Hilbert spaces and, also show the inverse spectral theory for symmetric operators with several gaps. We obtained the characteristic operator function of the class of n -hypercontractions on joint invariant subspaces.

الخلاصة

تم توضيح قوا عد الجمع وتطبيقاتها من النوع الخاص لمصفوفات الجاكوبيان واعتبار الخواص الطيفية لتمديدات المرافق الذاتي لدوال ويل، تم دراسة التمثيل وصيغة جوردان للمؤثرات الثنائية شبه المثلثية، وتحديد الأجزاء المتجانسة مع المؤثرات على فضاءات هيلبرت، ونظرية الطيف العكسي للمؤثرات المتماثلة طبقاً للمرات المتعددة، وإيضاح إعطاء دالة المؤثر المميز لعائلة المؤثرات الفوقية n - على الفضاءات الجزئية اللامتغيرة المقترنة.

Introduction

In the last two decades, there has been existing progress in multivariable dilation theory, in the attempt to extend the classical Nagy-Foias theory of contractions. Several years ago, P. R. Halmos introduced the remarkable class of quasitriangular operators on the complex separable infinite-dimensional Hilbert space, the study of this class was the spectral characterization of non-quasitriangular operators and Jordan form of a Bitriangular operator. Spectral properties of self-adjoint realizations of the Laplacian on a bounded domain have been investigated for a very long time. Originally these investigations had been motivated by the important role of such operators in a wide variety of models in acoustics, electromagnetism and thermodynamics. In the present century the interest gradually shifted to fascinating new research areas. In non-relativistic quantum mechanics realizations of the Laplacian on a bounded domain and in probability theory. The connection between the invariant subspaces of an operator and the corresponding characteristic function was first considered for certain particular classes of operators.

In chapter (1) we use some rules of a special form to study spectral properties of Jacobi matrices. We characterize the spectra of self-adjoint extensions of a symmetric operator with equal deficiency indices in terms of boundary values of their Weyl functions. We develop a dilation for row contractions subjected to constraints. A characteristic function associated with a row contraction is proved.

In chapter 2: are present a matrix representation for biquasitriangular operators and their existence and show the Jordan form of a bitriangular operator. We determine the homogeneous weighted shifts which they consist

of the unweighted bilateral shift and two one parameter families of unilateral shift.

In chapter 3, we study the homogenous operators and projective representations of the Möbius group. We construct a class of multiplication operators on reproducing Kernel Hilbert spaces which are homogenous with respect to the action of the Möbius group consisting of bi-holomorphic automorphism of the unit disc.

In chapter 4, we discuss the systematic application of sum rules for Jacobi matrices. We study the complete classification of the spectral measures of all Jacobi matrices and work with sum values of higher order.

In chapter 5, we show that if there exist a self-adjoint extension of the orthogonal sum of infinitely many pair wise unitarily equivalent symmetric operators with non-zero deficiency indices such that the open subset of a real line is contained in the resolvent set of the self-adjoint extension and the associated Weyl function of the pair of the orthogonal sum and the self-adjoint extension is monotone with respect to the open subset, then for any self-adjoint operator there exists a self-adjoint extension such that the spectral parts are unitarily equivalent, we investigate that the spectral properties of the self-adjoint extension of a symmetric operator in a separable Hilbert space can have inside the gap and provide prescribed spectral properties inside the gap.

In the last chapter we establish the existence of bijectivity between the joint invariant subspaces under n -tuple of operators on a Hilbert space and the regular factorizations of the characteristic function associate with these operators. We obtain criterions for joint similarity of n -tuples of operators to Cuntz row isometries. We consider a class of bounded linear operator on Hilbert space called n -hyper contractions which relates mutually to adjoint

shift operators on certain vector-valued weighted Bergman space on the unit disc. We generalized a characteristic operator function for a contraction operator which is an operator valued analytic function in the unit disc Hilbert spaces of defect type. The operator-valued analytic function is shown to act as a contractive multiplier from Hardy space into the associated standard weighted Bergman space.

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