Chapter one

Normal shock waves

:Introduction (1.1)

This section will develop relation for normal shock waves in .fluids with general equations of state

It will be specialized to calorically perfect ideal gases to illustrate the general features of the waves

:Assumption for this sectional

- one-dimensional flow
- steady flow
- no area change
- viscous effects and wall friction do not have time to influence
- flow

Heat conduction and wall heat transfer do not have time to flow

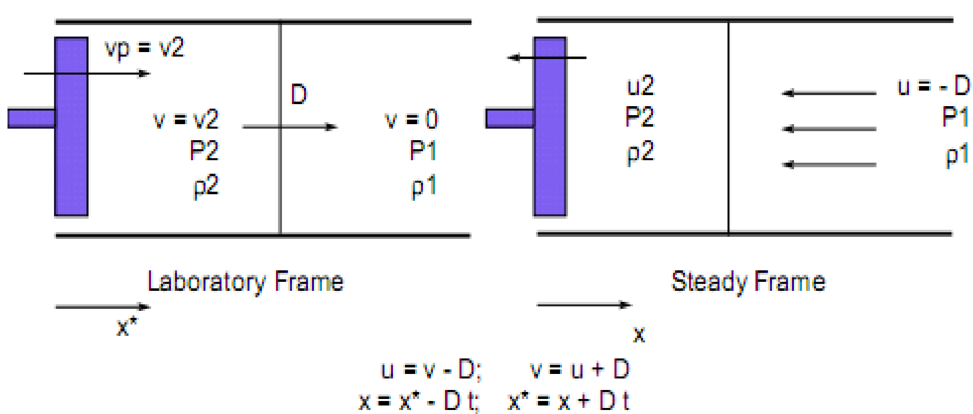


figure (1-1) Normal shock sketch

The piston problem as sketched in figure (1-1) will be .considered

:Physical problem

- Drive piston with known velocity v_p into fluid at rest $(v_1=0)$ with known properties, $p_1, \rho_1,$ in the x¿ laboratory frame
- Determine disturbance speed D
- Determine disturbance properties v_2, p_2, ρ_2 in this frame of reference unsteady problem

:Transformed problem

- Use Galilean transformation $x=x'-Dt, u=v-D$ to transform to the frame in which the wave is at rest, therefore the problem steady in this frame.

- Solve as though D is known to get downstream '2' conditions:

$$u_2(D), p_2(u_2)$$

- Invert to solve for D as function of u_2, the transformed piston velocity: $D(u_2)$

Back trans for to get all variables as function of v_2, the laboratory piston velocity: $D(v_2), p_2(v_2), \rho_2(v_2), \ldots$

(1-2) Governing equations:

Under these assumption the conservation principles in conservative form and equation of state are in the steady frame as follows:

$$\frac{d}{dx}(\rho u)=0$$

$$\frac{d}{dx}(\rho u^2+p)=0$$

$$\frac{d}{dx}\left(\rho u\left(h+\frac{u^2}{2}\right)\right)\Big|_{h=h(p,\rho)}=0$$

Upstream conditions are $\rho=\rho_1, p=p_1, u=-D,$ with knowledge of the equation of state, one stets $h=h_1$. Integrating the equations from upstream to state '2' gives

$$\rho_2 u_2=-\rho_1 D$$

$$\rho_2 u_2^2+p_2=\rho_1 D^2+p_1$$

$$h_2+\frac{u_2^2}{2}=h_1+\frac{D^2}{2}$$

$$h_2=h(p_2,\rho_2)$$

:Rayleigh line(1-3)

:Work on the momentum equation

$$p_2=p_1+\rho_1 D^2-\rho_2 u_2^2$$

$$p_2=p_1+\frac{\rho_1^2 D^2}{\rho_1}-\frac{\rho_2^2 u_2^2}{\rho_2}$$

Since Mass gives $\rho_2^2 u_2^2 = \rho_1^2 D^2$ one gets an equation for the :Rayleigh line, a line in $(p, \frac{1}{\rho})$ space

$$p_2 = p_1 + \rho_1^2 D^2 \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right) (1)$$

:Note

- Rayleigh line passes through ambient state
- Rayleigh line has negative slope
- Magnitude of slope proportional to square of wave speed
- Independent of state and energy equation

:Hugoniot curve(1-4)

Operate on the energy equation, using both mass .and momentum to eliminate velocity

:First eliminate v_2 via the mass equation

$$h_2 + \frac{u_2^2}{2} = h_1 + \frac{D^2}{2}$$

$$h_2+\frac{1}{2}\left(\frac{\rho_1 D}{\rho_2}\right)^2=h_1+\frac{D^2}{2}$$

$$h_2-h_1+\frac{D^2}{2}\left(\left(\frac{\rho_1}{\rho_2}\right)^2-1\right)=0$$

$$h_2-h_1+\frac{D^2}{2}\left(\frac{\rho_1^2-\rho_2^2}{\rho_2^2}\right)=0$$

$$h_2-h_1+\frac{D^2}{2}\left(\frac{(\rho_1-\rho_2)(\rho_1+\rho_2)}{\rho_2^2}\right)=0$$

Now use the Rayleigh line to eliminate D^2:

$$D^2=(p_2-p_1)\left(\frac{1}{\rho_1^2}\right)\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right)^{-1}$$

$$D^2=(p_2-p_1)\left(\frac{1}{\rho_1^2}\right)\left(\frac{\rho_2-\rho_1}{\rho_1\rho_2}\right)^{-1}$$

$$D^2=(p_2-p_1)\left(\frac{1}{\rho_1^2}\right)\left(\frac{\rho_2\rho_1}{\rho_2-\rho_1}\right)$$

So the energy equation becomes

$$h_2-h_1+\frac{1}{2}(p_2-p_1)\left(\frac{1}{\rho_1^2}\right)\left(\frac{\rho_2\rho_1}{\rho_2-\rho_1}\right)\left(\frac{(\rho_1-\rho_2)(\rho_1+\rho_2)}{\rho_2^2}\right)=0$$

$$h_2-h_1-\frac{1}{2}(p_2-p_1)\left(\frac{1}{\rho_1}\right)\left(\frac{(\rho_1+\rho_2)}{\rho_2}\right)=0$$

$$h_2 - h_1 - \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_2} + \frac{1}{\rho_1}\right) = 0$$

Solving finally for the enthalpy difference, on finds

$$h_2 - h_1 = (p_2 - p_1)\left(\frac{1}{2}\right)\left(\frac{1}{\rho_2} + \frac{1}{\rho_1}\right)(2)$$

.This equation is the Hugoniot equation

- enthalpy change equals pressure difference times mean volume
- u_2 independent of wave speed D and velocity
- .Independent of equation of state

Solution procedure for General (1-5) :equation of state

The shocked state can be determined by the following :procedure

- $h(p,\rho)$ Specify and equation of state
- Substitute the equation of state into the Hugoniot to ρ_2 get a second relation between p_2 and

- Use the Rayleigh line to eliminate p_2 in the Hugoniot
  ρ_2 to that the Hoguniot is a single equation in
- Solve for ρ_2 as functions of ‘I’ and D
- Back transform to laboratory frame to get D as function
  $v_2 = v_p$ of (1) state and piston velocity

Calorically perfect ideal gas solutions (1-6)

Follow this procedure for the special case of a calorically
.perfect ideal gas

$$h = c_p (T - T_0) + h_0$$

$$p = \rho R T$$

$$h = c_p \left(\frac{p}{R\rho} - \frac{p_0}{R\rho_0} \right) + h_0$$

$$h = \frac{c_p}{R} \left(\frac{p}{\rho} - \frac{p_0}{\rho_0} \right) + h_0$$

$$h = \frac{c_p}{c_p - c_v} \left(\frac{p}{\rho} - \frac{p_0}{\rho_0} \right) + h_0$$

$$h = \frac{\gamma}{\gamma - 1} \left(\frac{p}{\rho} - \frac{p_0}{\rho_0} \right) + h_0$$

:Evaluate at states 1 and 2 and substitute into Hoguniot

$$\left(\frac{\gamma}{\gamma-1}\left(\frac{p_2}{\rho_2}-\frac{p_0}{\rho_0}\right)+h_0-\frac{\gamma}{\gamma-1}\left(\frac{p_1}{\rho_1}-\frac{p_0}{\rho_0}\right)+h_0\right)$$

$$\text{¿}\left(p_2-p_1\right)\left(\frac{1}{2}\right)\left(\frac{1}{\rho_2}-\frac{1}{\rho_1}\right)$$

$$\frac{\gamma}{\gamma-1}\left(\frac{p_2}{\rho_2}-\frac{p_1}{\rho_1}\right)-\left(p_2-p_1\right)\left(\frac{1}{2}\right)\left(\frac{1}{\rho_2}+\frac{1}{\rho_1}\right)=0$$

$$p_2\left(\frac{\gamma}{\gamma-1}\frac{1}{\rho_2}-\frac{1}{2\rho_2}-\frac{1}{2\rho_1}\right)-p_1\left(\frac{\gamma}{\gamma-1}\frac{1}{\rho_1}-\frac{1}{2\rho_2}-\frac{1}{2\rho_1}\right)=0$$

$$p_2\left(\frac{\gamma+1}{2(\gamma-1)}\frac{1}{\rho_2}-\frac{1}{2\rho_1}\right)-p_1\left(\frac{\gamma+1}{2(\gamma-1)}\frac{1}{\rho_1}-\frac{1}{2\rho_2}\right)=0$$

$$p_2\left(\frac{\gamma+1}{\gamma-1}\frac{1}{\rho_2}-\frac{1}{\rho_1}\right)-p_1\left(\frac{\gamma+1}{\gamma-1}\frac{1}{\rho_1}-\frac{1}{\rho_2}\right)=0$$

$$p_2=\frac{\frac{\gamma+1}{\gamma-1}\frac{1}{\rho_1}-\frac{1}{\rho_2}}{\frac{\gamma+1}{\gamma-1}\frac{1}{\rho_2}-\frac{1}{\rho_1}}\,(3)$$

- a hyperbola in $(p,\frac{1}{\rho})$ space causes $p_2 \to \infty$, note γ =1.4, $\rho_2 \to 6$ for $\frac{1}{\rho_2} \to \frac{\gamma-1}{\gamma+1}\frac{1}{\rho_1}$ •

infinite pressure as $\frac{1}{\rho_2} \to \infty$, $p_2 \to -p_1\frac{\gamma-1}{\gamma+1}$ note

negative pressure, note physical here the Raylish line

(and Hugoniote curves are aketched in figure (1-1

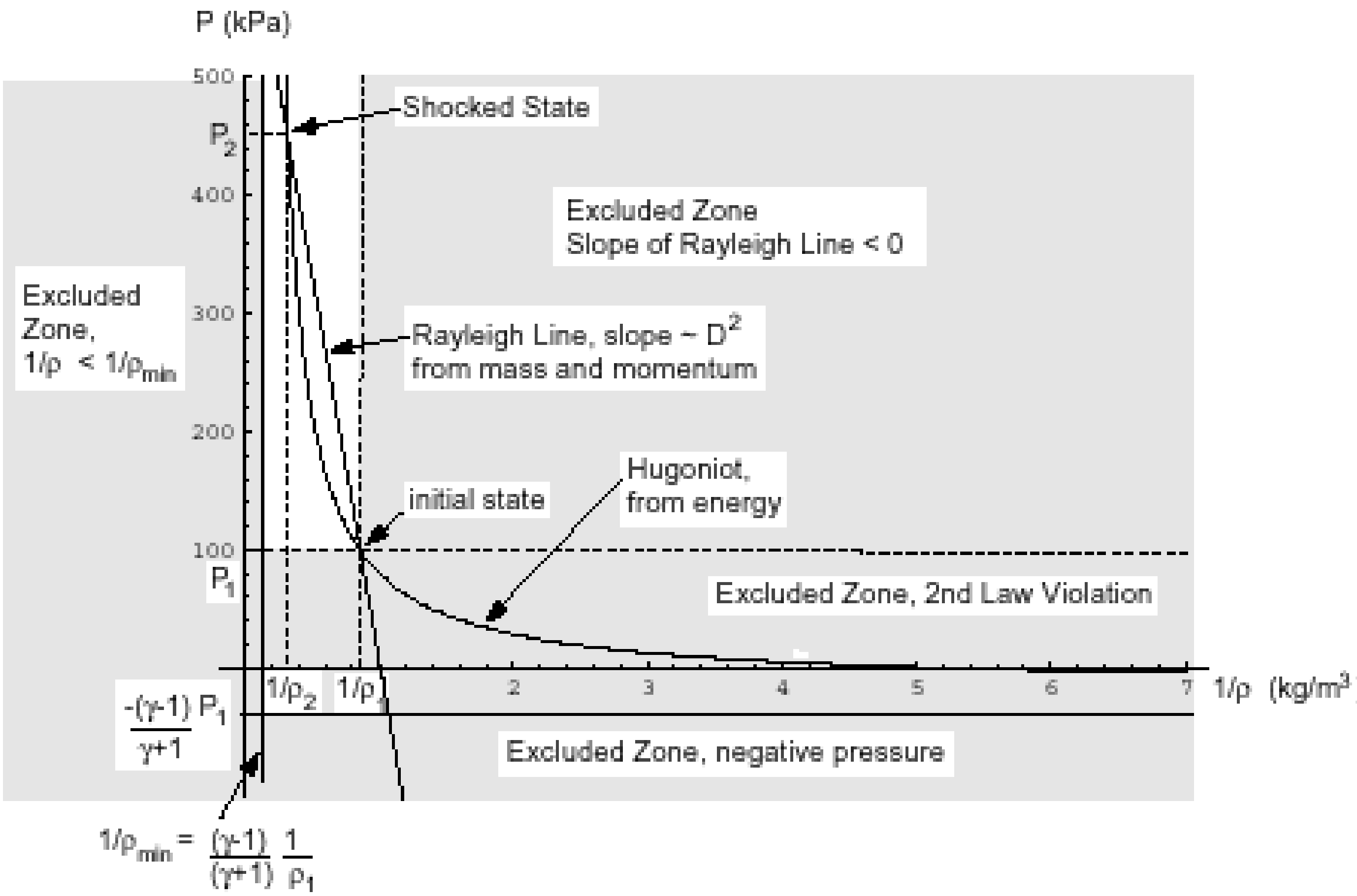


Figure (1-2) Rayleigh line and Hugoniot

Note

- Intersections of the two curves are solutions to the equations
- the ambient state "1" is one solution
- The other solution "2" is known as the shock solution
- shock solution has higher pressure and higher density
- higher wave speed implies higher pressure and higher density
- a minimum wave speed exist
  - occurs when Rayleigh line tangent to Hugoniot
  - occurs for every small pressure changes
  - corresponds to a sonic wave speed
  - disturbances are acoustic
- if pressure increases , can be shown entropy increases
- if pressure decreases (wave speed less than sonic), entropy decreases , this is nonphysical

Substitute Rayleigh line into Hugoniot to get single equation for γ_2

$$p_1 + ¿\, \rho_1^2 D^2 \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right) = p_1 \frac{\frac{\gamma+1}{\gamma-1}\frac{1}{\rho_1} - \frac{1}{\rho_2}}{\frac{\gamma+1}{\gamma-1}\frac{1}{\rho_2} - \frac{1}{\rho_1}}$$

¿

.This equation is quadratic in $\frac{1}{\rho_2}$ and factorizable

.Use computer algebra to solve and get tow solutions

:One ambient $\frac{1}{\rho_2} = \frac{1}{\rho_1}$ and one shocked solution

$$(\quad 1 + \frac{2\gamma}{(\gamma-1)D^2}\frac{p_1}{\rho_1}$$

$$\frac{1}{\rho_2} = \frac{1}{\rho_1}\frac{\gamma-1}{\gamma+1} ¿$$

The shocked density ρ_2 is plotted aginst wave speed D for

(CPIG air in figure(1-3

:Note

- $0 < D < \infty$ density solution allows all speeds
- $C_1 < D < \infty$ plot range , however , is
- $D \geq C_1$ Rayleigh line and Hugoniot show

- Solution for D=D(v_p¿, to be shown, rigorously shows

$D \geq C_1$

- $\rho_2 \to \frac{\gamma+1}{\gamma-1}$, $D^2 \to \infty$ Strong shock limit

$\rho_2 \to \frac{\gamma+1}{\gamma-1}$ $D^2 \to \infty$, :acoustic limit

non-physical limit

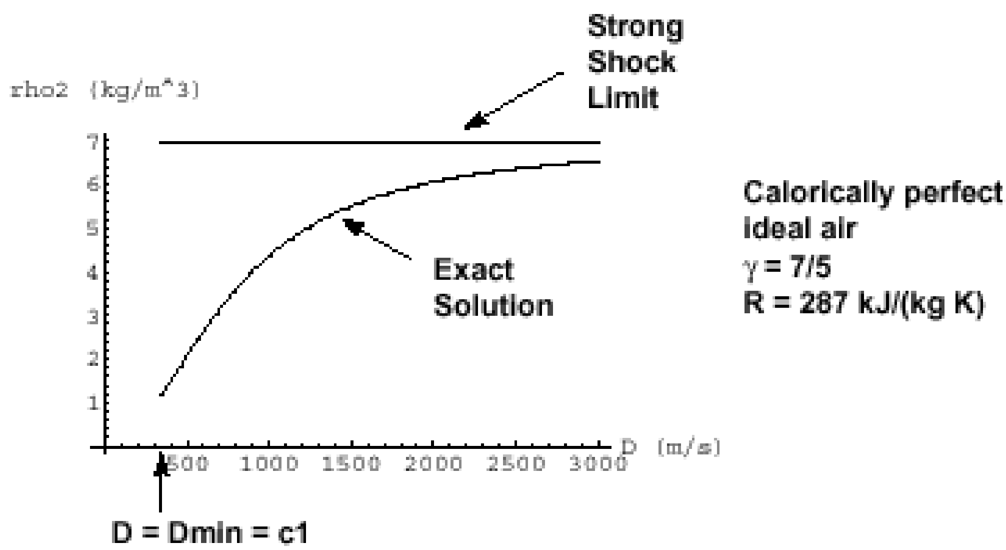


Figure (1-3) Shock density vs .shock wave speed for calorically perfect ideal air

Beak substitute into Rayleigh line and mass conservation to solve for the shocked pressure and the :fluid velocity in the shocked wave frame

$$p_2 = \frac{2}{\gamma+1}\rho_1 D^2 - \frac{\gamma-1}{\gamma+1} p_1$$

$$u_2 = -D\frac{\gamma-1}{\gamma+1}\left(1 + \frac{2\gamma}{(\gamma-1)D^2}\frac{p_1}{\rho_1}\right)$$

The shocked pressure p_2 is plotted against wave speed D for CPIG air in figure (1-4) including both the exact solution and the solution in the strong shock limit. Note for these parameters, the results are .indistinguishable

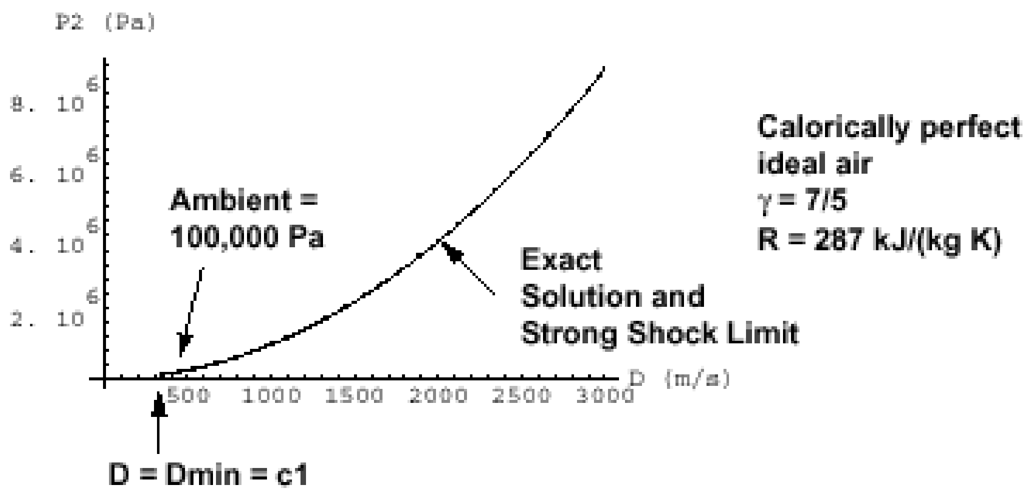


Figure (1-4): shock pressure vs. shock wave speed for calorically perfect ideal air

the shocked wave frame find particle velocity u_2 is plotted (against wave speed D for CPIG air in figure (1-5

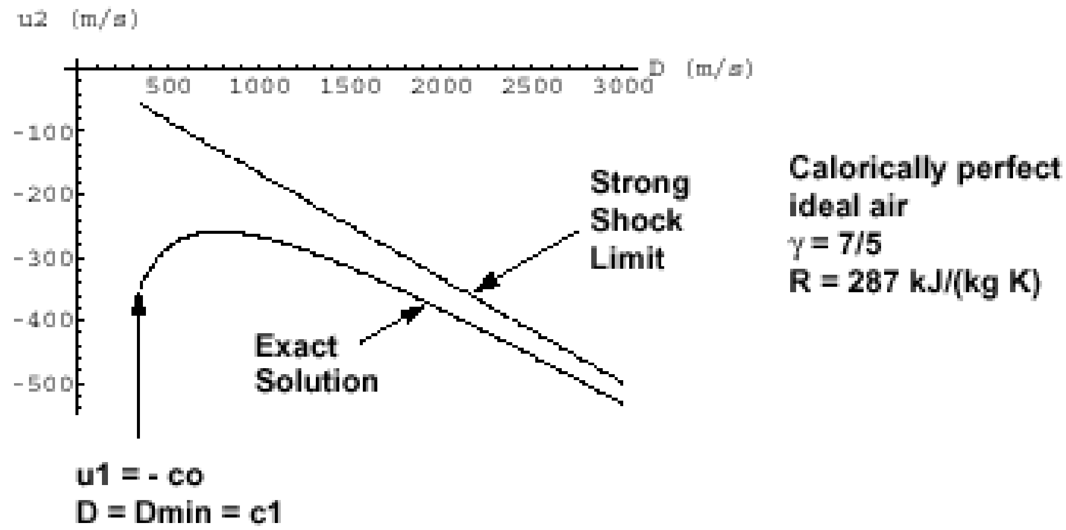


Figure (1-5) shock wave frame fluid particle velocity vs. shock wave speed calorically prefect ideal air

The shocked wave frame fluid particle velocity $M_2^2 = \frac{\rho_2 u_2^2}{\gamma p_2}$ is

(plotted against wave speed D for CPIG air in figure (1-6

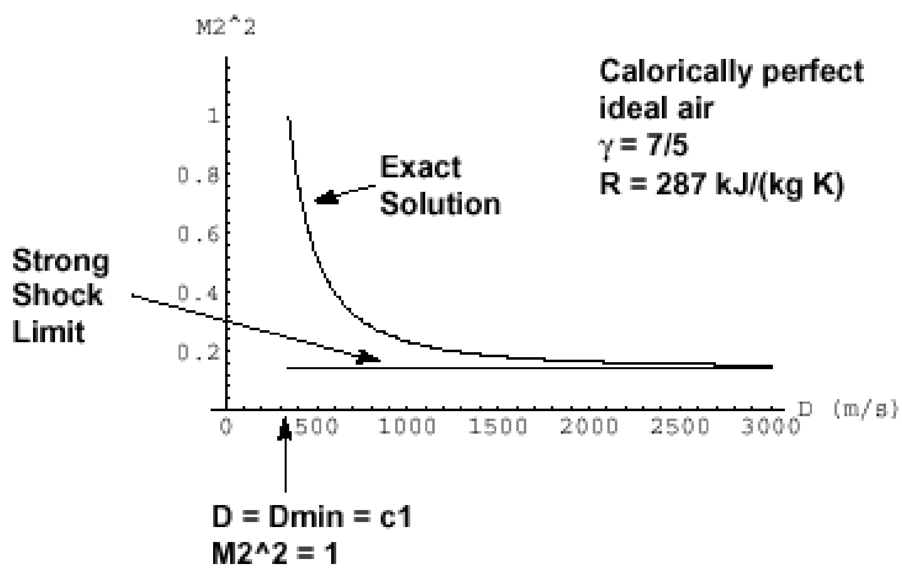


Figure (1-6) Mach number squared of shocked fluid particle vs. shock wave speed for calorically perfect ideal air

$u=v-D$: Transform back to the laboratory frame

$$v_2-D=-D\frac{\gamma-1}{\gamma+1}\left(1+\frac{2\gamma}{(\gamma-1)D^2}\frac{p_1}{\rho_1}\right)$$

$$v_2=D-D\frac{\gamma-1}{\gamma+1}\left(1+0\frac{2\gamma}{(\gamma-1)D^2}\frac{p_1}{\rho_1}\right)$$

Manipulate the above equation and solve the resulting quadratic equation for D and get

$$D=\frac{\gamma+1}{4}v_2\pm\sqrt{\frac{\gamma p_1}{\rho_1}+v_2^2\left(\frac{\gamma+1}{4}\right)^2}$$

Now if $v_2>0$ one expect $D>0$ so take positive root, also $v_2=v_p$ set velocity equal piston velocity

$$D=\frac{\gamma+1}{4}v_p\pm\sqrt{\frac{\gamma p_1}{\rho_1}+v_p^2\left(\frac{\gamma+1}{4}\right)^2}$$

:Note

- *acoustic limit*: as $v_p \to 0, D \to c_1$ the shock speed approaches the sound speed
- *Strong shock limit*: as $v_p \to \infty, D \to \frac{\gamma+1}{2} v_p$

The shock speed D is plotted against piston velocity v_p for CPIG air in figure (1-7) the exact solution and strong shock limit are shown.

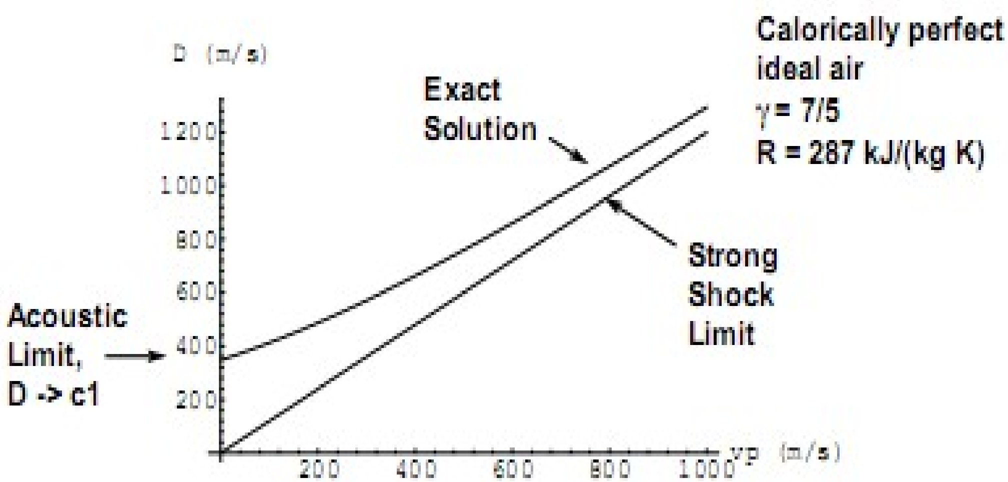


Figure (1-7) shock speed vs. piston velocity for calorically perfect ideal air

If the Mach number of the shock is defined as

$$M_s = \frac{D}{c_1}$$

One gets

$$M_s = \frac{\gamma+1}{4}\frac{v_p}{\sqrt{\gamma R T_1}} + \sqrt{1 + \frac{v_p^2}{\gamma R T_1}\left(\frac{\gamma+1}{4}\right)^2}$$

The shock Mach number M_s is plotted against piston velocity v_p for CPIG air figure (1-8) both the exact solution .and strong shock limit are shown

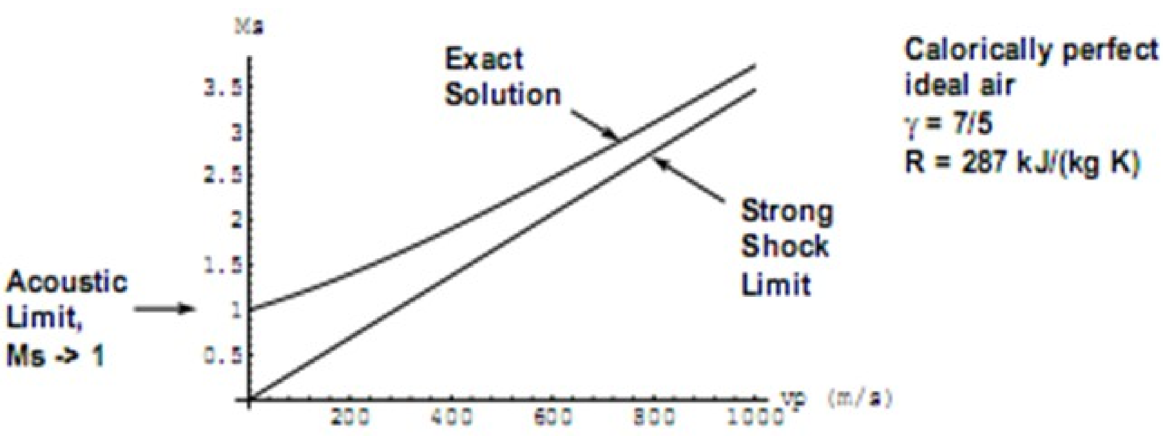


Figure (1-8) shock Mach number vs. piston velocity for calorically perfect ideal air

Pressure Ratio across normal shock (1-7)

Since normal shock is a compression process the pressure ratio across the shock is an indication of the stengh of the shock wave when the initial Mach number is unity .then the pressure ratio will also be unity

As the incident Mach number increases. The strength of the compression wave increases and hence pressure .downstream of the shock increases

The pressure ratio across the normal shock can be obtained from the simple form of the momentum equation applicable .to the control volume surrounding the normal shock

$$p_1+\rho_1 u_1^2=p_2+\rho_2 u_2^2=constant$$

For a perfect gas, ρu^2 can be written in terms of Mach number using the perfect gas equation and the definition of .Mach number

$$\rho u^2=\frac{p u^2}{RT}=u^2\frac{\gamma P}{\gamma RT}=\gamma P\frac{u^2}{c^2}=\gamma P M^2$$

Subtracting this in the momentum equation we obtain

$$p_1+\gamma p_1 M_1^2=p_2+\gamma p_2 M_2^2$$

$$p_1\left(1+\gamma M_1^2\right)=p_2\left(1+\gamma M_2^2\right)$$

$$\frac{p_2}{p_1}=\frac{1+\gamma M_1^2}{1+\gamma M_2^2}(4)$$

This equation gives the relation for the pressure ratio in terms of incident mach number M_1 is required it can be obtaind by substituting the relation between M_1 and M_2

$$M_2^2=\frac{\frac{2}{\gamma-1}+M_1^2}{\frac{2r}{\gamma-1}M_1^2-1}$$ from eq

The denominator of eq (4) can be simplified, after :substituting for M_2 as follows

$$1+\gamma M_2^2=1+\gamma\left(\frac{\frac{2}{\gamma-1}+M_1^2}{\frac{2\gamma}{\gamma-1}M_1^2-1}\right)=\frac{\left(\frac{2\gamma}{\gamma-1}M_1^2-1\right)+\gamma\left(\frac{2}{\gamma-1}+M_1^2\right)}{\frac{2\gamma}{\gamma-1}M_1^2-1}=\frac{M_1^2\left(\frac{2\gamma}{\gamma-1}+\gamma\right)+\left(\frac{2\gamma}{\gamma-1}-1\right)}{\frac{2\gamma}{\gamma-1}M_1^2-1}$$

$$¿\left(\frac{M_1^2\frac{(\gamma+1)\gamma}{\gamma-1}+\frac{\gamma+1}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2-1}\right)$$

$$¿\frac{\frac{\gamma+1}{\gamma-1}(1+\gamma M_1^2)}{\frac{2\gamma}{\gamma-1}M_1^2-1}(5)$$

Substituting eq (5) in the denominator of eq (4) we get

$$\frac{p_2}{p_1}=\frac{\left(\frac{2\gamma}{\gamma-1}M_1^2-1\right)(1+\gamma M_1^2)}{\frac{\gamma+1}{\gamma-1}(1+\gamma M_1^2)}$$

$$\frac{p_2}{p_1}=\frac{2\gamma}{\gamma+1}M_1^2-\frac{\gamma-1}{\gamma+1}$$

Temperature Ratio across normal (1-8) shock

Since the shock process is irreversible the kinetic energy of gas after crossing the shock is lower than that of isentropic comparison the equation of the temperature ratio across the shock be obtained from the isentropic stagnation temperature ratio at the upstream and downstream sides of . the shock

Since the stagnation temperature across the shock is constant

$\frac{T_2}{T_1}=\frac{T_2}{T_{01}}\times\frac{T_{01}}{T_1}$ we can write , $T_{01}=T_{02}=T_0¿$)

$$¿\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)(6)$$

The denominator of the equation contains M_2 .This can be eliminated by substituting equation: $M_2^2=\frac{\frac{2}{\gamma-1}+M_1^2}{\frac{2\gamma}{\gamma-1}M_1^2-1}$ in the denominator

.This the denominator becomes

$$1+\frac{\gamma-1}{2}M_2^2=1+\frac{\gamma-1}{2}\left(\frac{\frac{2}{\gamma-1}+M_1^2}{\frac{2\gamma}{\gamma-1}M_1^2-1}\right)$$

$$¿\frac{\frac{2\gamma}{\gamma-1}M_1^2-1+\frac{\gamma-1}{2}\left(\frac{2}{\gamma-1}+M_1^2\right)}{\frac{2\gamma}{\gamma-1}M_1^2-1}$$

$$¿\frac{M_1^2\left(\frac{2\gamma}{\gamma-1}+\frac{\gamma-1}{2}\right)-1+1}{\frac{2\gamma}{\gamma-1}M_1^2-1}$$

$$¿\frac{\frac{(\gamma+1)^2}{2(\gamma-1)}M_1^2}{\frac{2\gamma}{\gamma-1}M_1^2-1}(7)$$

Substituting the equation for denominator into eq(6) We get

$$\frac{T_2}{T_1}=\frac{(1+\frac{\gamma-1}{2}M_1^2)(\frac{2\gamma}{\gamma-1}M_1^2-1)}{\frac{(\gamma+1)^2}{2(\gamma-1)}M_1^2}(8)$$

The temperature ratio, similar to pressure ratio is a function of incident Mach number alone but for high temperature .flows and for M_1> 5

The temperature behind the shock is very high and γ no .longer remains constant

In solving such problems. Variation of γ with temperature .must also be take into account

:(Example(1-1

Air with a pressure and temperature of $100k$ p_a and 10^0c passes through a standing normal shock wave where flow Mach number is 1.6 drtermine the dencity and .temperature of the air after it has crossed the shock

-:Solution

Date $p=100\text{k}\,p_a, T=283\,k, M=1.6, \gamma=1.4$ in problem the Mach number and flow properties upstream of the normal shock are given and it is nesseserry to find out the flows properties .downstream of the shock wave

The problem can be solved using the relationship for pressure ratio and temperature ratio as given by eq (5) and eq (8) the numerical value of pressure ratio and temperature ratio across the shock can be directly read

$$\frac{p_2}{p_1}=\frac{2\gamma}{\gamma+1}M_1^2-\frac{\gamma-1}{\gamma+1}=\left(\frac{p_2}{p_1}\right)_{M_1=1.6}=2.82$$

Pressure downstream of the shock

$$p_2=100\times 2.82\,k\,p_a$$

And

$$\frac{T_2}{T_1}=\left(\frac{T_2}{T_1}\right)_{M_1=1.6}=1.388$$

Temperature downstream of the shock

$$T_2=283\times 1.388=392.8\,k$$

Density downstream of the shock

$$\rho_2=\frac{p_2}{RT_2}=\frac{282\times10^3}{287\times392.8}=2.5015\,kg/m^3$$

:(Example(1-2

Compression shock occurs in a divergent air flow passage on the upstream side of the shock, the velocity of air is 400 m/s and the pressure and temperature are 0.2 and $35^o c$

.respictively

:Determine

i) Mach number and air velocity on the downstream side of) the shock

ii) Change in the entropy per unit mass of air as result of) shock

:Solution

$p=0.2\,Mp_a, T=308\,k, u=400\,m/s$ date

The Mach number upstream of the shock is calculated from the given date

$$M_1=\frac{u_1}{c_1}=\frac{u_1}{\sqrt{\gamma RT_1}}=\frac{400}{\sqrt{1.4\times287\times308}}=1.14$$

The normal shock tables can be used for finding

The downstream Parameters from normal shock

$M=1.14$ Table's $\gamma=1.4$ corres panding to

$$M_2=0.882$$

$$\frac{p_2}{p_1}=1.3495$$

$$p_2=0.2\times 1.3495$$

$$p_2=0.269^o M\, p_a$$

$$\frac{T_2}{T_1}=1.0903$$

$$T_2=308\times 1.0903=335.8\,k$$

The velocity of air after the shock wave

$$u_2=M_2 c_2=M_2\sqrt{\gamma R T_2}$$

¿ $323.98\,m/s$ ¿ $0.882\times\sqrt{1.4\times 287\times 335.8}$

Also from shock tables

$$p_{01}/p_{02}=0.997$$

Champ in entropy across the shock

$$s_2-s_1=R\ln\frac{p_{01}}{p_{02}}=287\ln(\frac{1}{0.997})=0.862\,J/kg\,k$$

-:Acoustic limit (1-9)

Consider that state 2 is a small perturbation of state 1 so that

$$\rho_2=\rho_1+\Delta\rho$$

$$u_2=u_1+\Delta u$$

$$p_2=p_1+\Delta p$$

Substituting into the normal shock equation, one gets

$$(\rho_1+\Delta\rho)(u_1+\Delta u)=\rho_1 u_1$$

$$(\rho_1+\Delta\rho)(u_1+\Delta u)^2+(p_1+\Delta p)=\rho_1 u_1^2+p_1$$

$$\frac{\gamma}{\gamma-1}+\frac{p_1+\Delta p}{\rho_1+\Delta\rho}+\frac{1}{2}(u_1+\Delta u)^2=\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}+\frac{1}{2}u_1^2$$

Expending, one gets

$$\rho_1 u_1+\tilde{u}_1(\Delta\rho)+\rho_1(\Delta u)+(\Delta\rho)(\Delta u)=\rho_1 u_1$$

$$\left(\rho_1 u_1^2+2\rho_1 u_1(\Delta u)+u_1^2(\Delta\rho)\rho_1(\Delta u)^2+2u_1(\Delta u)(\Delta\rho)+(\Delta\rho)(\Delta u)^2\right)+(p_1+\Delta p)=\rho_1 u_1^2+p_1$$

$$\frac{\gamma}{\gamma-1}\left(\frac{p_1}{\rho_1}+\frac{1}{\rho_1}\Delta p-\frac{p_1}{\rho_2}\Delta\rho+\dots\right)+\frac{1}{2}\left(u_1^2+2u_1(\Delta u)+(\Delta u)^2\right)$$

$$¿\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}+\frac{1}{2}u_1^2$$

Subtracting the basic state and eliminating products of small quantities yields

$$u_1(\Delta\rho)+\rho_1(\Delta u)=0$$

$$2\rho_1 u_1(\Delta u)+u_1^2(\Delta\rho)+\Delta p=0$$

$$\frac{\gamma}{\gamma-1}\left(\frac{1}{\rho_1}\Delta p-\frac{p_1}{\rho_1^2}\Delta p\right)+u_1(\Delta u)=0$$

$$\begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}\Delta\rho\\\Delta u\\\Delta p\end{pmatrix} \begin{pmatrix}u_1 & \rho_1 & 0\\ u_1^2 & 2\rho_1 u_1 & 1\\ \frac{-\gamma}{\gamma-1}\frac{p_1}{\rho_1^2} & u_1 & \frac{\gamma}{\gamma-1}\frac{1}{\rho_1}\end{pmatrix}$$ In matrix form this is

As the right hand side is zero, the determinant must be zero .and there must be a linear dependency of the solution

First check the determinants

$$u_1\left(\frac{2\gamma}{\gamma-1}u_1-u_1\right)-\rho_1\left(\frac{\gamma}{\gamma-1}\frac{u_1^2}{\rho_1}+\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1^2}\right)=0$$

$$\gamma u_1^2+\gamma\frac{p_1}{\rho_2}=0$$

$$2\gamma-(\gamma-1)-\frac{1}{\gamma-1}¿$$

$$\frac{u_1^2}{\gamma-1}¿$$

$$u_1^2=\frac{\gamma p_1}{\rho_1}=c_1^2$$

So the velocity is necessarily sonic for a small disturbance

:Take Δu to be known and solve a resulting 2x2 system

$$\begin{pmatrix} u_1 & 0 \\ \frac{-\gamma}{\gamma-1}\frac{p_1}{\rho_1} & \frac{\gamma}{\gamma-1}\frac{1}{\rho_1}\end{pmatrix}\begin{pmatrix}\Delta\rho\\ \Delta p\end{pmatrix}=\begin{pmatrix}-\rho_1\Delta u\\ -u_1\Delta u\end{pmatrix}$$

Solving yields

$$\Delta\rho=\frac{-\rho_1\Delta u}{\sqrt{\gamma p_1/\rho_1}}$$

$$\Delta p=-\rho_1\sqrt{\gamma\frac{p_1}{\rho_1}}\Delta u$$

(1-10) Non-ideal Gas solution:

non-ideal effects are important

- near the critical point
- for strong shocks

Some other points:

- qualitative trends the same as for ideal gases
- analysis is much more algebraically complicated
- extraneous solution often arise which must be discarded

Example (1-3):

Shock in Van Der Waals gas

Given: shock wave $D=500\,m/s$ propagating into N_2 at rest at $T_1=125\,k,\ p_1=2\,M\,p_a$

Find: shocked state

Assume: Van Der Waals equation of state accurately models gas behavior, specific heat constant.

Analysis:

$p_1=2\,M\,p_a$ First, some data for N_2 are need. at

has abiling point of $115.5\,k$ so the material is in the M_2

.gas phase but very near vapor dome

$$R=296.8\frac{J}{kg\,k},c_v=744.8\frac{J}{kg\,k}$$

Since the material is near the vapor dome, the Van Der Weals equation may give a good first correction for non-.ideal effects

$$p=\frac{RT}{v-b}-\frac{a}{v^2}$$

$$p=\frac{RT}{\frac{1}{\rho}-b}-a\rho^2$$

$$p=\frac{\rho RT}{1-b\rho}-a\rho^2$$

as derived earlier, the corresponding caloric equation of state is

$$e(T,v)=e_0+\int_{T_0}^{T}c_v(\hat{T})\,d\hat{T}+a\left(\frac{1}{v_0}-\frac{1}{v}\right)$$

Taking c_v constant and exchanging v for ρ gives

$$e(T,\rho)=e_0+c_v(T-T_0)+a(\rho_0-\rho)$$

Eliminating T in favor of p then gives

$$e(p,\rho)=e_0+c_v\left(\frac{(p+a\rho^2)(1-b\rho)}{\rho R}-T_0\right)+a(\rho_0-\rho)$$

: $h=e+\frac{p}{\rho}$ And in terms of

$$h(p,\rho)=e_0+c_v\left(\frac{(p+a\rho^2)(1-b\rho)}{\rho R}-T_0\right)+a(\rho_0-\rho)+\frac{p}{\rho}$$

And h_2-h_1 allows cocellation of the "0" state so that

$$h_2-h_1=c_v\left(\frac{(p_2+a\rho_2^2)(1-b\rho_2)}{\rho_2 R}-\frac{(p_1+a\rho_1^2)(1-b\rho_1)}{\rho_1 R}\right)-a(\rho_2-\rho_1)+\frac{p_2}{\rho_2}-\frac{p_1}{\rho_1}$$

The constant a and b are fixed so that an isotherm passing through the critical point, $p=p_c, T=T_c,$ passes through with $\frac{\partial p}{\partial v} ¿_T=0$ and $\frac{\partial^2 p}{\partial v^2}¿_T=0$ A statndard analysis yields

$$a=\frac{27R^2T_c^2}{64p_c}$$

$Kg\,k$
$296.8\,J/¿$
$¿$
$¿^2$
$¿$
$¿$
$a=\frac{27}{64}¿$

kgk
$296.8\,J/¿$
$¿$
$(126.2\,k)$
$¿$
$3{,}390{,}000\,p_a$
$¿$
$¿$
$b=\frac{R\,T_c}{8\,p_c}=¿$

.Find the ambient density

kgk
$296.8\,J/¿$
$¿$
$(125\,k)$
$¿$
kg
$0.00138\,m^3/¿$
$¿$
ρ_1
$\rho_1¿$
$2{,}000{,}000\,p_a=¿$

:(Three solutions (from computer algebra

$$\rho_1=69.0926\,\frac{kg}{m^3}\,physical$$

$$\rho_1=(327.773+112.702\,i)\,\frac{kg}{m^3}\,non-physical$$

$$\rho_1=(327.773+112.702i)\frac{kg}{m^3} non-physical$$

$\rho_1=71.28\,kg/m^3$ Tabular data from experiments gives

Error ¿$(71.28-69.09)171.28=3$, so seems the first root is the .physical root

Not that the van der Waals prediction is a significant improvement over the ideal gas low which gives

$$\rho_1=\frac{p_1}{RT_1}=\frac{2{,}000{,}000}{296.8\,x\,125}=53.91\frac{kg}{m^3}$$

¿$(71.28-53.91)/71.28=21.4$ Error

Even with this improvement there are much better (and more complicated) equations of state for materials near the .vapor dome

Now use the Rayleigh line and Hugoniot equations to solve :for the shocked density

$$p_2=p_1+\rho_1^2D^2\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right)$$

$$\left(c_v\left(\frac{\left(p_2+a\rho_2^2\right)\left(1-b\rho_2\right)}{\rho_2 R}-\frac{\left(p_1+a\rho_1^2\right)\left(1-b\rho_1\right)}{\rho_1 R}\right)-a\left(\rho_2-\rho_1\right)+\frac{p_2}{\rho_2}-\frac{p_1}{\rho_1}\right)-\left(\frac{1}{2}\right)\left(p_2-p_1\right)\left(\frac{1}{\rho_2}+\frac{1}{\rho_1}\right)=0$$

Plugging in all the numbers into a computers algebra program yields the following solution for ρ_2:

$$\rho_2=195.309\,kg/m^3\;shocked\;solution$$

$$\rho_2=69.0926\,kg/m^3\;inert\;solution$$

$$\rho_2=\left(85.74+657.9\,i\right)\frac{kg}{m^3}\;non\;physical\;solution$$

$$\rho_2=\left(85.74-657.9\,i\right)\frac{kg}{m^3}\;non\;physical\;solution$$

The Rayleigh line then gives the pressure:

$$2{,}000{,}000\,p_a+\left(69.0926\,kg/m^3\right)$$
¿
¿
$$p_2=¿$$

$$p_2=13{,}162{,}593\,p_a=13.2\,M\,p_a$$

The state equation gives temperature

$$T_2=\frac{\left(p_2+a\rho_2^2\right)\left(1-b\rho_2\right)}{\rho_2 R}$$

$$13{,}162{,}593\,p_a + \left(174.6\,\frac{p_a m^6}{kg^2}\right)\left(195.3\,\frac{kg}{m^3}\right)^2$$

$$1-\left(0.00138\,\frac{m^3}{kg}\right)\left(195.3\,\frac{kg}{m^3}\right)$$

$$¿$$

$$T_2 = ¿¿$$

$$T_2 = 249.8\,k$$

Note the temperature is still quite low relative to standard atmospheric conditions, it is unlikely at these low temperatures that any effects due to vibration relaxation or .dissociation will be important

Our assumption of constant specific heat is probably pretty .good

:The mass equation gives the shock particle velocity

$$\rho_2 u_2 = \rho_1 u_1$$

$$2 = ¿\,\frac{\rho_1 u_1}{\rho_2}$$

$$u_¿$$

$$u_2 = \frac{\left(69.0926\,\frac{kg}{m^3}\right)(500\,m/s)}{195.3\,\frac{kg}{m^3}} = 176.89\,m/s$$

An ideal gas approximation ($\gamma N_2 = 1.4¿$ would have yielded

$$\frac{1}{\rho_2}=\frac{1}{\rho_1}\frac{\gamma-1}{\gamma+1}\left(1+\frac{2\gamma}{(\gamma-1)D^2}\frac{p_1}{\rho_1}\right)$$

$$\frac{1}{\rho_2}=\left(\frac{1}{53.91\,kg/m^3}\right)\frac{1.4-1}{1.4+1}\left(1+\frac{2(1.4)}{(1.4)(500\,kg/s)^2}\frac{2{,}000{,}000\,p_a}{53.91\,kg/m^3}\right)$$

$$\rho_2=158.65\,kg/m^3\,ideal\,gas\,approximation$$

$$¿\frac{195.3-158.65}{195.3}=18.8$$ relative error

The Rayleigh line then gives the pressure

$$p_2=2{,}000{,}000\,p_a+\left(53.91\frac{kg}{m^3}\right)^2(500\,m/s)^2\left(\frac{1}{53.91\,kg/m^3}-\frac{1}{158.65\frac{kg}{m^3}}\right)$$

$$p_2=10{,}897{,}783\,p_a=10.90\,M\,p_a$$

$$¿\frac{13.2-10.9}{13.2}=17.4$$ Relative error

Chapter two
Oblique Shock Waves

An oblique shock is a shock which is not normal to the incoming flow field. It can be shown that in the limiting case as the oblique shock strength goes to zero, the oblique shock wave becomes a Mach wave, as described in the previous chapter.

Oblique waves can be understood by considering the following problem.

Given:

- a straight wedge inclined at angle θ to the horizontal
- a freestream flow parallel to the horizontal with known velocity $v = u_1 i + 0j$
- known freestream pressure and density of P_1 and ρ_1
- steady flow of a calorically perfect ideal gas (this can be relaxed and one can still oblique shocks)

Find:

- angle of shock inclination β
- downstream pressure and density P_2, ρ_2

Similar to the piston problem, the oblique shock problem is easiest analyzed if we instead consider

- β as known
- θ as unknown

They are best modeled in a two-dimensional coordinate system with axes parallel and perpendicular to the shock, see Figure (2-1), so that

$$x = \acute{x} \sin\beta + \acute{y} \cos\beta$$

$$y = -\acute{x} \cos\beta + \acute{y} \sin\beta$$

$$u=\acute{u}\sin\beta+\acute{v}\cos\beta$$

$$v=-\acute{u}\cos\beta+\acute{v}\sin\beta$$

Consequently, in this coordinate system, the freestream is two-dimensional.

It is easily shown that the equations of motion are invariant under a rotation of axes, so that

$$\frac{\partial(\rho\acute{u})}{\partial\acute{x}}+\frac{\partial(\rho\acute{v})}{\partial\acute{y}}=0$$

$$\frac{\partial}{\partial\acute{x}}\left(\rho\acute{u}^2+p\right)+\frac{\partial}{\partial\acute{y}}\left(\rho\acute{u}\acute{v}\right)=0$$

$$\frac{\partial}{\partial\acute{x}}\left(\rho\acute{u}\acute{v}\right)+\frac{\partial}{\partial\acute{y}}\left(\rho\acute{v}^2+p\right)=0$$

$$\rho\acute{v}\left(e+\frac{1}{2}\left(\acute{u}^2+v^2\right)+\frac{p}{\rho}\right)=0$$

$$\rho\acute{u}\left(e+\frac{1}{2}\left(\acute{u}^2+\acute{v}^2\right)+\frac{p}{\rho}\right)+\frac{\partial}{\partial\acute{y}}¿$$

$$\frac{\partial}{\partial\acute{x}}¿$$

$$e=\frac{1}{\ }\frac{p}{\ }+e_0$$

To a ... al ... on

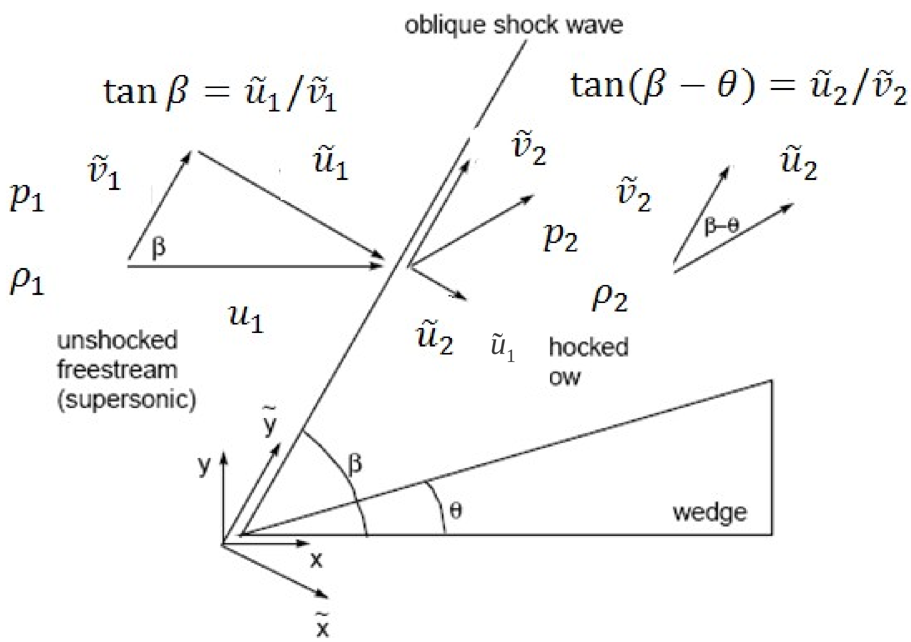


Fig: (2-1) oblique shock wave

- $\frac{\partial}{\partial y}=0$

Note however that, contrary to one-dimensional flow we will *not* enforce $\tilde{v}=0$, so

- $\tilde{v}\neq 0$

Consequently, all variables are a function of $\tilde{x}$ at most and $\frac{\partial}{\partial \tilde{x}}=\frac{d}{d\tilde{x}}$.

The governing equations reduce to

$$\frac{d}{d\tilde{x}}=(\rho \acute{u})=0$$

$$\frac{d}{d\tilde{x}}(\rho \acute{u}^2+\rho)=0$$

$$\frac{d}{d\tilde{x}}(\rho \acute{u}\acute{v})=0$$

$$\frac{d}{d\tilde{x}}\left(\rho \acute{u}\left(e+\frac{1}{2}(\acute{u}^2+v^2)+\frac{p}{\rho}\right)\right)=0$$

$$e=\frac{1}{\gamma-1}\frac{p}{\rho}+e_0$$

Integrate and apply freestream conditions

$$\rho_2 \tilde{u}_2 = \rho_1 \tilde{u}_1$$

$$\rho_2 \tilde{u}_2{}^2 + p_2 = \rho_1 \tilde{u}_1{}^2 + p_1$$

$$\rho_2 \tilde{u}_2 \tilde{v}_2 = \rho_1 \tilde{u}_1 \tilde{v}_1$$

$$e_1 + \frac{1}{2}\left(\tilde{u}_1{}^2 + \tilde{v}_1{}^2\right) + \frac{p_1}{\rho_1}$$

$$\rho_2 \tilde{u}_2 \left(e_2 + \frac{1}{2}\left(\tilde{u}_2{}^2 + \tilde{v}_2{}^2\right) + \frac{p_2}{\rho_2}\right) = \rho_1 \tilde{u}_1 ¿$$

$$e = \frac{1}{\gamma - 1}\frac{p}{\rho} + e_0$$

Now using the mass equation, then ˜y momentum equation reduces to

$$\tilde{v}_2 = \tilde{v}_1$$

Using this result and the mass a state equations gives

$$\rho_2 \tilde{u}_2 = \rho_1 \tilde{u}_1$$

$$\rho_2 \tilde{u}_2{}^2 + p_2 = \rho_1 \tilde{u}_1{}^2 + p_1$$

$$\frac{1}{\gamma - 1}\frac{p_2}{\rho_2} + \frac{1}{2}\tilde{u}_2{}^2 + \frac{p_2}{\rho_2} = \frac{1}{\gamma - 1}\frac{p_1}{\rho_1} + \frac{1}{2}\tilde{u}_1{}^2 + \frac{p_1}{\rho_1}$$

These are exactly the equations which describe a normal shock jump. All our old results applied in this coordinate system with the additional stipulation that the component of .velocity *tangent* to the shock is constant

Recall our solution for one-dimensional shocks in a :calorically perfect ideal gas

$$\frac{1}{\rho_2} = \frac{1}{\rho_1}\frac{\gamma - 1}{\gamma + 1}\left(1 + \frac{2\gamma}{(\gamma - 1)}\frac{p_1}{\rho_1}\right)$$

For this problem $D=\tilde{u}_1$ so

$$\frac{1}{\rho_2}=\frac{1}{\rho_1}\frac{\gamma-1}{\gamma+1}\left(\frac{2\gamma}{(\gamma-1)\tilde{u}_1^{\,2}}\frac{p_1}{\rho_1}\right)$$

With the freestream Mach number normal to the wave defined as

$$M_{1n}^2=\frac{\tilde{u}_1^2}{\gamma\frac{p_1}{\rho_1}}$$

we get

$$\frac{\rho_1}{\rho_2}=\frac{\gamma-1}{\gamma+1}\left(1+\frac{2}{(\gamma-1)M_{1n}^2}\right)$$

$\frac{\rho_1}{\rho_2}=\frac{\tilde{u}_2}{\tilde{u}_1}$ and since from mass

$$\frac{\tilde{u}_2}{\tilde{u}_1}=\frac{\gamma-1}{\gamma+1}\left(1+\frac{2}{(\gamma-1)M_{1n}^2}\right)$$

Now for our geometry

$$\tan\beta=\tilde{u}_1/\tilde{v}_1$$

$$\tan(\beta+\theta)=\frac{\tilde{u}_2}{\tilde{v}_2}=\frac{\tilde{u}_2}{\tilde{v}_1}$$

So

$$\frac{\tilde{u}_2}{\tilde{u}_1}=\frac{\tan(\beta-\theta)}{\tan\beta}$$

Thus

$$\frac{\tan(\beta-\theta)}{\tan\beta}=\frac{\gamma-1}{\gamma+1}\left(1+\frac{2}{(\gamma-1)M_{1n}^2}\right)$$

Now note that

$$M_{1n}^2=M_{1n}^2\sin^2\beta$$

$$\frac{\tan(\beta-\theta)}{\tan\beta}=\frac{\gamma-1}{\gamma+1}\left(1+\frac{2}{(\gamma-1)M_1^2\sin^2\beta}\right)$$

$$\frac{\tan(\beta-\theta)}{\tan\beta}=\frac{\gamma-1}{\gamma+1}\left(\frac{(\gamma-1)M_1^2\sin^2\beta+2}{(\gamma-1)M_1^2\sin^2\beta}\right)$$

$$\tan(\beta-\theta)=\tan\beta\,\frac{(\gamma-1)M_1^2\sin^2\beta+2}{(\gamma+1)M_1^2\sin^2\beta}$$

$$\frac{\tan\beta-\tan\theta}{1+\tan\theta\tan\beta}=\tan\beta\,\frac{(\gamma-1)M_1^2\sin^2\beta+2}{(\gamma+1)M_1^2\sin^2\beta}\equiv x$$

$$\tan\beta-\tan\theta=x+x\tan\theta\tan\beta$$

$$\tan\beta-x=\tan\theta(1+x\tan\beta)$$

$$\tan\theta=\frac{\tan\beta-x}{1+x\tan\beta}$$

With a little more algebra and trigonometry this reduces to

$$\tan\theta=2\cot\beta\,\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}$$

Given $M1,\gamma$ and β, this equation can be solved to find θ the wedge angle. It can be inverted to form an equation cubic in sin β to solve explicitly for β. Figure (2-2) gives a plot of oblique shock angle β versus wedge angle θ.

Note the following features:

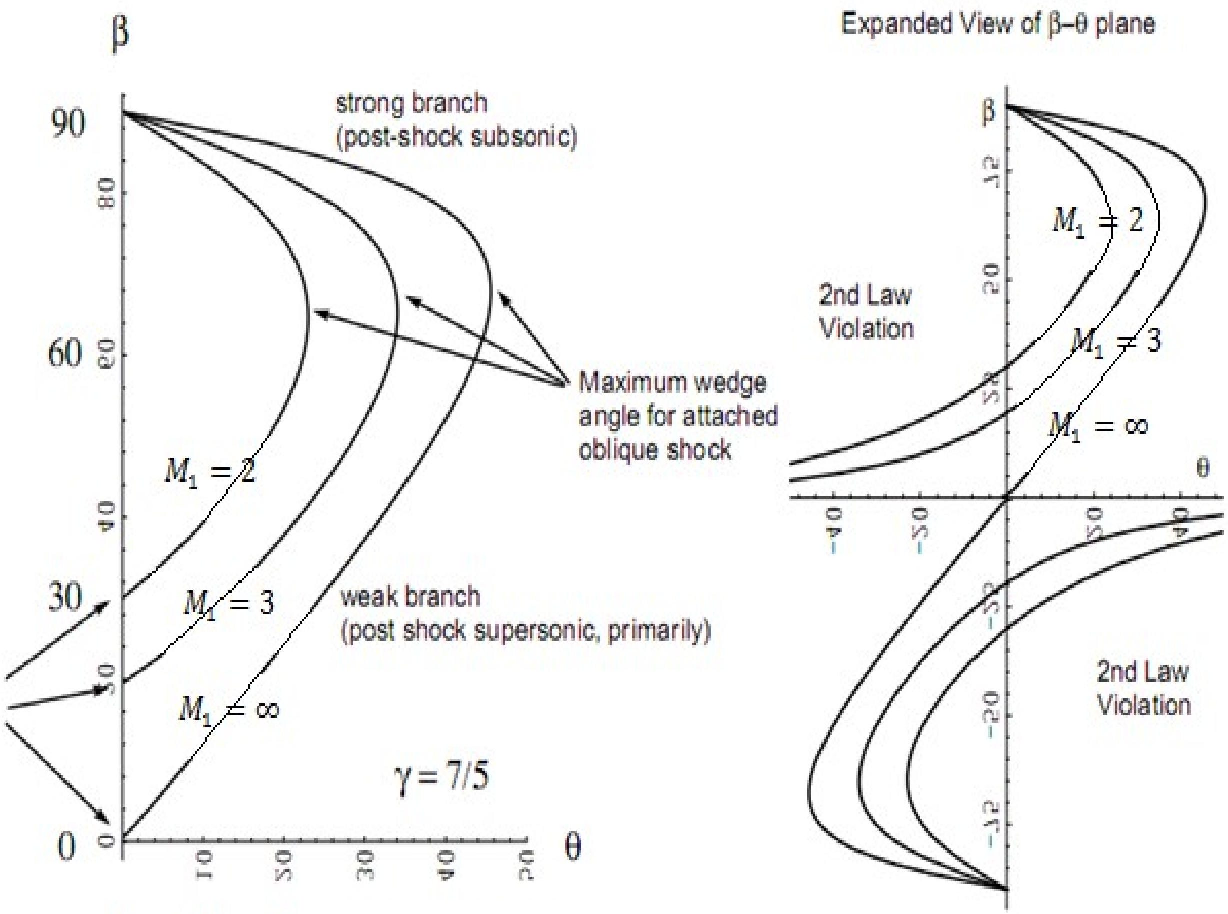


Figure 2.2: shock angle β versus

- for a given $\theta<\theta max$, there exist *two* β ’s
- lower β is *weak* solution -
- Mach waves M, a $\lim ¿_{\theta\to 0}\beta=\arcsin\frac{1}{M}$ ¿
- relevant branch for most external flows, matches in far-field to acoustic wave,can exist in internal flows total

Mach number primarily supersonic $M_2^2=\frac{\acute{u}^2+\acute{v}^2}{c_2^2}>1$ for $0<\theta<\theta max$ nearly all

$, M_{2n}^2=\frac{\tilde{u}_2}{c_2^2}<1$ normal Mach number subsonic •

for $\theta>\theta max$, no solution exists; shock becomes •

detached Consider fixed θ, increasing freestream $M1$ Mach number

subsonic incoming flow, no shocks , $0<M1<1$ -

continuous pressure variation

supersonic incoming flow, detached , $1<M1<M1a$ -

curved oblique shock

supersonic incoming flow, attached $M1a<M1<\infty$, -

straight oblique shock

$M1\rightarrow\infty, \beta\rightarrow\beta\infty$ as -

Consider fixed supersonic freestream Mach number •

($M1$, increasing θ see figure(2-4

and Mach wave, negligible disturbance, $\theta\sim0$ -

small θ , small β , small pressure and density rise -

medium θ, medium β, moderate pressure and density rise -

large θ, curved detached shock, large pressure and density rise -

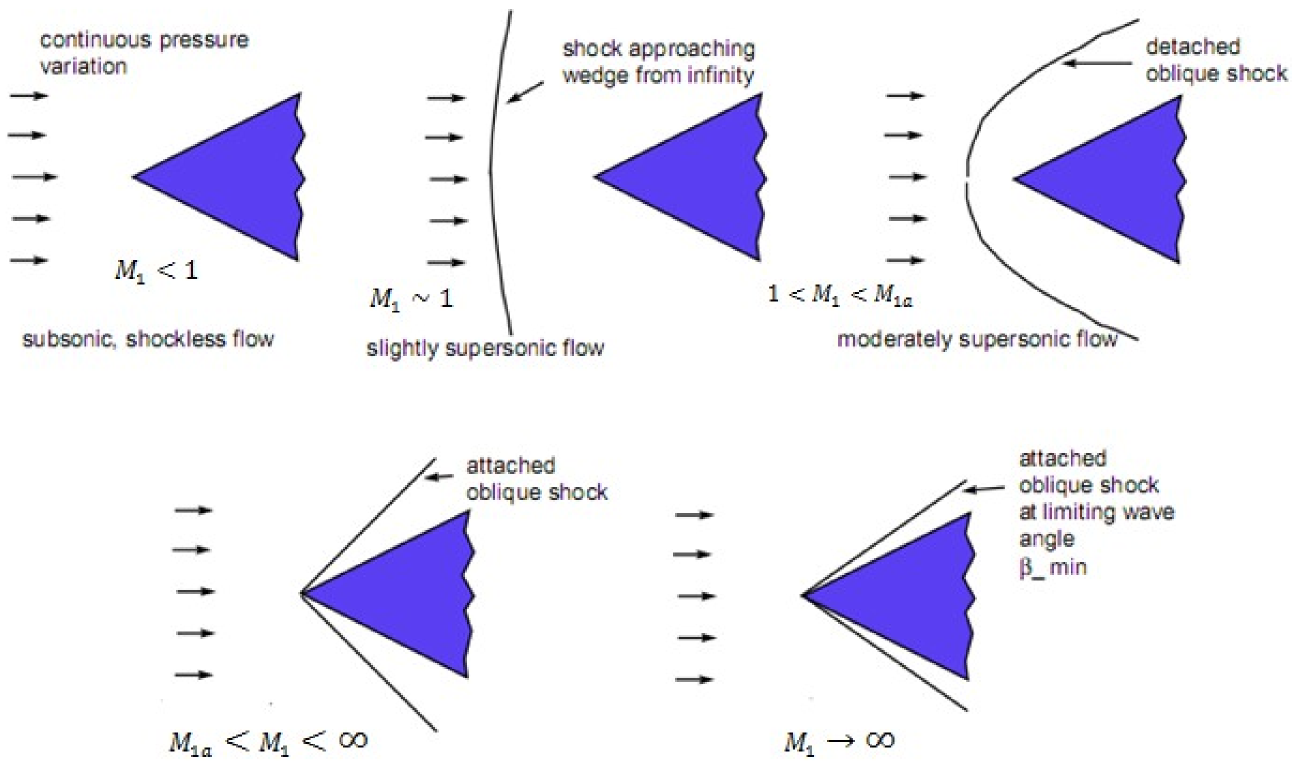


Figure (2-3) shock wave pattern as incoming Mach number varied

Example 2.1

Oblique Shock Example

,Given: Air flowing over a wedge $\theta = 20^{\circ}, P1 = 100\,kPa, T1 = 300\,K, M1 = 3.0$

Find: Shock angle β and downstream pressure and temperature $P2, T2.$

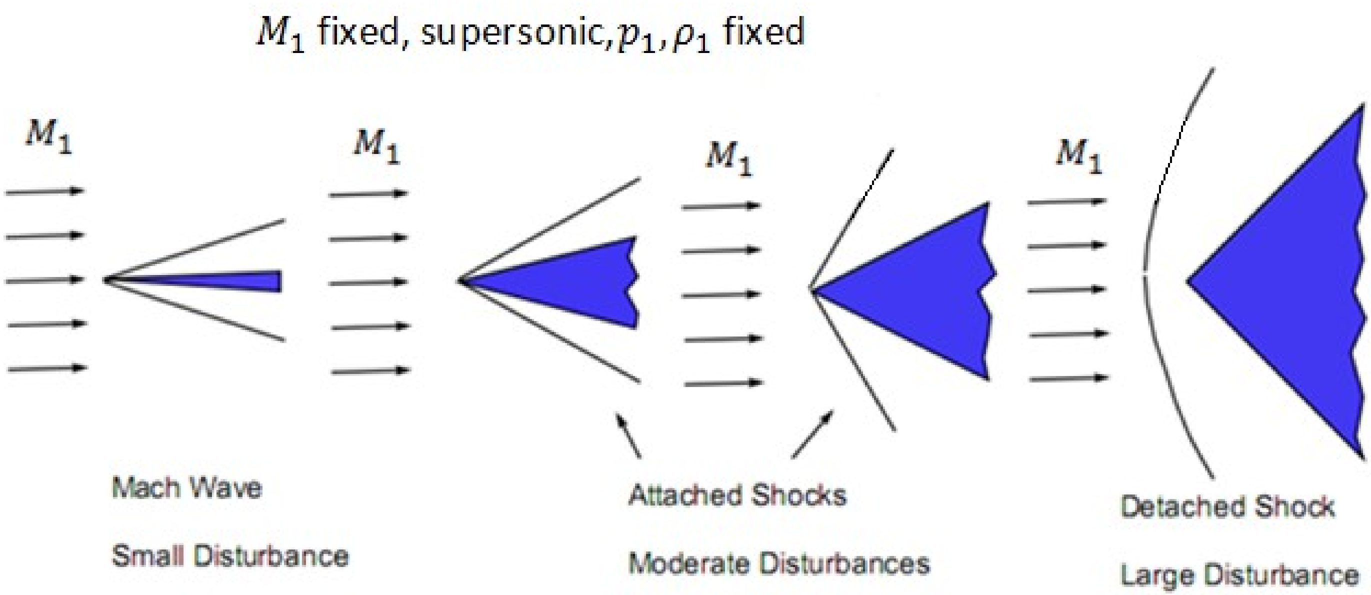


Figure (2-4) shock wave pattern as wedge angle varied

Assume: calorically perfect ideal gas

:**Analysis:** First some preliminaries

$$c_1=\sqrt{\gamma R T_1}=\sqrt{1.4\left(287\frac{J}{kgk}\right)(300\text{k})}=347.2\,m/s$$

$$u_1=M_1 c_1=(3.0)(347.2\,m/s)=1{,}041.6\,m/s$$

$$\rho_1=\frac{p_1}{RT_1}=\frac{100{,}000\,p_a}{\left(287\frac{J}{kgk}\right)(300\text{k})}=1.1614\,kg/m^3$$

:Now find the wave angle

$$\tan\theta=2\cot\beta\,\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}$$

$$\tan 20^o=2\cot\beta\,\frac{(3.0)^2\sin^2\beta-1}{(3.0)^2(1.4+\cos 2\beta)+2}$$

:Three solutions

- $\beta=37.76$ weak oblique shock; common
- $\beta=82.15$ strong oblique shock; rare
- $\beta=-9.91$ second law violating “rarefaction” shock

1- Weak Oblique Shock

$$\tilde{u}_1=u_1\sin\beta=(1{,}041.6\,m/s)$$

$$\tilde{v}_1 = u_1 \cos\beta = (1{,}041.6\, m/s)\cos 37.76^o = 823^o\, m/s$$

$$M_{1n} = \frac{\tilde{u}_1}{c_1} = \left(\frac{637.83}{347.2}\right) m/s = 1.837$$

$$\frac{\rho_1}{\rho_2} = \frac{\gamma-1}{\gamma+1}\left(1 + \frac{2}{(\gamma-1)\, M_{1n}^2}\right)$$

$$\frac{1.1614\, kg/m^3}{\rho_2} = \frac{1.4-1}{1.4+1}\left(1 + \frac{2}{(1.4-1)\, 1.837^2}\right) = 0.413594$$

$$\rho_2 = \frac{1.1614\, kg/m^3}{0.41359} = 2.8081\, kg/m^3$$

$$\rho_2 \tilde{u}_2 = \rho_1 \tilde{u}_1$$

$$\tilde{u}_2 = \rho_1 \tilde{u}_1 / \rho_2 = \frac{1.1614\, kg/m^3 (637.83^o\, m/s)}{2.8081\, kg/m^3} = 263.8\, m/s$$

$$\tilde{v}_2 = \tilde{v}_1 = 823.47\, m/s$$

$$u_2 = \tilde{u}_2 \sin\beta + \tilde{v}_2 \cos\beta$$

$$v_2 = -\tilde{u}_2 \cos\beta + \tilde{v}_2 \sin\beta$$

$$u_2 = (263.80\, m/s)\sin 37.76 + (823.47\, m/s)\cos 37.76^o = 812.56\, m/s$$

$$v_2 = -(263\, m/s)\cos 37.76^o + (823.47\, m/s)\sin 37.76 = 295.7\, m/s$$

$$\theta = \arctan\left(\frac{v_2}{u_2}\right) = \arctan\left(\frac{295.70\, m/s}{812.56\, m/s}\right) = 19.997^\circ$$

Check on wedge angle

$$p_2 = p_1 + \rho_1 u_1^2 - \rho_2 u_2^2$$

$$p_2 = 100{,}000\, p_a + \left(1.1614\, \frac{kg}{m^3}\right)\left(637.83\, \frac{m}{s}\right)^2 - \left(2.8081\, \frac{kg}{m^3}\right)\left(263.80\, \frac{m}{s}\right)^2$$

$$p_2 = 377{,}072\, p_a$$

$$T_2 = \frac{p_2}{\rho_2 R} = \frac{377{,}072\, p_2}{\left(2.8081\, \frac{kg}{m^3}\right)\left(287\, \frac{J}{kg\, K}\right)} = 467.88\text{K}$$

$$c_2 = \sqrt{\gamma R\, T_2} = \sqrt{(1.4)\left(287\, \frac{J}{kg\, K}\right)(467.88\text{K})} = 433.58\, m/s$$

$$M_{2\text{n}} = \frac{\tilde{u}_2}{c_2} = \frac{263.8\, m/s}{433.58\, m/s} = 0.608$$

$$M_2 = \frac{\sqrt{u_2^2 + v_2^2}}{c_2} = \frac{\sqrt{(812.56\, m/s)^2 + (295.7\, m/s)^2}}{433.58\, m/s} = 1.994$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$¿\left(1{,}0045\, \frac{J}{kg\, K}\right) \ln \frac{467.88\text{K}}{300\text{K}} - \left(287\, \frac{J}{kg\, K}\right) \ln \frac{377{,}072\, p_a}{100{,}000\, p_a}$$

$$s_2 - s_1 = 65.50\, \frac{J}{kg\, K}$$

strong oblique shock -2

$$\tilde{u}_1 = u_1 \sin\beta = (1{,}041.6\, m/s) \sin 82.15^o = 1{,}031.84\, m/s$$

$$\tilde{v}_1 = u_1 \cos\beta = (1{,}041\, m/s) \cos 82.15^o = 142.26\, m/s$$

$$M_{1\text{n}} = \left(\frac{\tilde{u}_1}{c_1}\right) = \left(\frac{1{,}031.84\, m/s}{347.2\, m/s}\right) = 2.972$$

$$\frac{\rho_1}{\rho_2} = \frac{\gamma - 1}{\gamma + 1}\left(1 + \frac{2}{(\gamma - 1)\, M_{1\text{n}}^2}\right)$$

$$\frac{1.1614\,\frac{kg}{m^3}}{\rho_2}=\frac{1.4-1}{1.4+1}\left(1+\frac{2}{(1.4-1)(2.972)^2}\right)=0.26102$$

$$\rho_2=\frac{1.1614\,\frac{kg}{m^3}}{0.26102}=4.4495\,\frac{kg}{m^3}$$

$$2=¿\,\rho_1\tilde{u}_1$$
$$\rho_2\tilde{u}¿$$

$$\tilde{u}_2=\frac{\rho_1\tilde{u}_1}{\rho_2}=\frac{\left(1.1614\,\frac{kg}{m^3}\right)\left(1{,}031.84\,\frac{m}{s}\right)}{4.4495\,\frac{kg}{m^3}}=269.33\,\frac{m}{s}$$

$$2=¿\,\tilde{u}_1=142.26\,\frac{m}{s}$$
$$\tilde{u}¿$$

$$2=¿\,\tilde{u}_2\,sin\beta+\tilde{v}_2\,cos\beta$$
$$u¿$$

$$2=¿-\tilde{u}_2\,cos\beta+\tilde{v}_2\,sin\beta$$
$$v¿$$

$$u_2=\left(269.33\,\frac{m}{s}\right)sin82.\,15^0+\left(142.26\,\frac{m}{s}\right)cos82.\,15^0=286.24\,\frac{m}{s}$$

$$v_2=-\left(269.33\,\frac{m}{s}\right)cos82.\,15^0+\left(142.26\,\frac{m}{s}\right)sin82.\,15^0=104.14\,\frac{m}{s}$$

$$\theta=\arctan\left(\frac{v_2}{u_2}\right)\qquad \check{o}n\ wedge\ angle$$

$$¿\arctan\left(\frac{104.14\,\frac{m}{s}}{286.24\,\frac{m}{s}}\right)=19.99^0$$

$$P_2 = P_1 + \rho_1 \tilde{u}_1{}^2 - \rho_2 \tilde{u}_2{}^2$$

$$269.33\,\frac{m}{s}¿^2$$

$$1{,}031.84\,\frac{m}{s}¿^2 - \left(4.4495\,\frac{kg}{m^3}\right)¿$$

$$P_2 = 100{,}000\,Pa + \left(1.1614\,\frac{kg}{m^3}\right)¿$$

$$P_2 = 1{,}013{,}775\,Pa$$

$$T_2 = \frac{P_2}{\rho_1 R} = \frac{1{,}013{,}775\,Pa}{\left(4.4495\,\frac{kg}{m^3}\right)\left(287\,\frac{J}{kg\,K}\right)} = 793.86\text{K}$$

$$c_2 = \sqrt{\gamma R\,T_2} = \sqrt{(1.4)\left(287\,\frac{J}{kg\,K}\right)(793.86\text{K})} = 564.78\,\frac{m}{s}$$

$$M_{2n} = \frac{\tilde{u}_2}{c_2} = \frac{269.33\,\frac{m}{s}}{564.78\,\frac{m}{s}} = 0.477$$

$$104.14\,\frac{m}{s}¿^2$$

$$¿$$

$$286.24\,\frac{m}{s}¿^2 + ¿$$

$$¿$$

$$\sqrt{¿}$$

$$M_2 = \frac{\sqrt{u_2^2 + v_2^2}}{c_2} = ¿$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$¿\left(1{,}004.5\,\frac{J}{kg\,K}\right)\ln\frac{793.86\,K}{300\,K} - \left(287\,\frac{J}{kg\,K}\right)\ln\frac{1{,}013{,}775\,p_a}{100{,}000\,p_a}$$

$$s_2 - s_1 = 312.86 \frac{J}{kg\, K}$$

rarefaction shock -3

$$\tilde{u}_1 = u_1 \sin\beta = \left(1{,}041.6 \frac{m}{s}\right) \sin\left(-9.91^o\right) = -179.26 \frac{m}{s}$$

$$\tilde{v}_1 = u_1 \cos\beta = \left(1{,}041.6 \frac{m}{s}\right) \cos\left(-9.91^o\right) = -1{,}026.06 \frac{m}{s}$$

$$M_{1n} = \left(\frac{\tilde{u}_1}{c_1}\right) = \left(\frac{-179.26 \frac{m}{s}}{347.2 \frac{m}{s}}\right) = -0.5163$$

$$\frac{\rho_1}{\rho_2} = \frac{\gamma - 1}{\gamma + 1}\left(1 + \frac{2}{(\gamma - 1) M_{1n}^2}\right)$$

$$\frac{1.1614 \frac{kg}{m^3}}{\rho_2} = \frac{1.4 - 1}{1.4 + 1}\left(1 + \frac{2}{(1.4 - 1)(-0.5163)^2}\right) = 3.2928$$

$$\rho_2 = \frac{1.1614 \frac{kg}{m^3}}{3.2928} = 0.3527 \frac{kg}{m^3}$$

$$\rho_2 \tilde{u}_2 = \rho_1 \tilde{u}_1$$

$$\tilde{u}_2 = \frac{\rho_1 \tilde{u}_1}{\rho_2} = \frac{\left(1.1614 \frac{kg}{m^3}\right)\left(-179.26 \frac{m}{s}\right)}{0.3527 \frac{kg}{m^3}} = -590.27 \frac{m}{s}$$

$$\tilde{v}_2 = \tilde{v}_1 = 1{,}026.06 \frac{m}{s}$$

$$u_2 = \tilde{u}_2 \sin\beta + \tilde{v}_2\, sin\beta$$

$$u_2 = -\tilde{u}_2 \cos\beta + \tilde{v}_2 \cos\beta$$

$$u_2 = \left(-590.27 \frac{m}{s}\right) \sin(-9.91^o) + \left(1{,}026.06 \frac{m}{s}\right) \cos(-9.91^o) = 1{,}112.34 \frac{m}{s}$$

$$v_2 = \left(-590.27 \frac{m}{s}\right) \cos(-9.91^o) + \left(1{,}026.06 \frac{m}{s}\right) \sin(-9.91^o) = 404.88 \frac{m}{s}$$

$\check{o}n$ *wedge angle*

$$\theta = \arctan\left(\frac{v_2}{u_2}\right) = \arctan\left(\frac{404.88 \frac{m}{s}}{1{,}112.34 \frac{m}{s}}\right) = 20.00^o$$

$$p_2 = p_1 + \rho_1 \tilde{u}_1^2 - \rho_2 \tilde{u}_2^2$$

$$p_2 = 100{,}000\, p_a + \left(1.1614 \frac{kg}{m^3}\right)\left(-179.26 \frac{kg}{m^3}\right)^2 - \left(0.3527 \frac{kg}{m^3}\right)\left(-590.27 \frac{m}{s}\right)^2$$

$$p_2 = 14{,}433\, p_a$$

$$T_2 = \frac{p_2}{\rho_2 R} = \frac{14{,}433\, p_a}{\left(0.3527 \frac{kg}{m^3}\right)\left(287 \frac{J}{kg\, k}\right)} = 142.59\, K$$

$$c_2 = \sqrt{\gamma R T_2} = \sqrt{(1.4)\left(287 \frac{J}{kg\, k}\right)(142.59\, K)} = 239.36 \frac{m}{s}$$

$$M_{2n} = \frac{\tilde{u}_2}{c_2} = \frac{-590.27 \frac{m}{s}}{239.36 \frac{m}{s}} = -2.47$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1}$$

$$s_2 - s_1 = \left(1{,}004.5\,\frac{J}{kg\,K}\right)\ln\frac{142.59\mathrm{K}}{300\mathrm{K}} - \left(287\,\frac{J}{kg\,k}\right)\ln\frac{14{,}433\,pa}{100{,}000\,pa}$$

$$s_2 - s_1 = -191.5\,\frac{J}{kg\,k}$$

(Chapter (3

Graphical solution of oblique shock wave

For the reason we now denote by u_1 and u_2 the normal . components of the velocity relative to shock

. q_2 With u_1 and u_2 writen in place of q_1 and

$$\rho_1 q_1 = \rho_2 q_2 (1)$$

For, in time δt a mass $\rho_1 q_1 \delta t$ with momentum $\rho_1 q_1^2 \delta t$ crosses unit area of the shock and becomes a mass $\rho_2 q_2 \delta t$

. $\rho_2 q_2^2 \delta t$ with momentum

Since force equals rate of chance of momentum we deduce that

$$p_1 - p_2 = \rho_2 q_2^2 - \rho_1 q_1^2 (2)$$

Or

$$p_1 + \rho_1 q_1^2 = p_2 + \rho_2 q_2^2 (3)$$

From (1) and (2) we conclude that

$$p_1 - p_2 = \rho_1 q_1 q_2 - \rho_2 q_1 q_2$$

hence

$$q_1 q_2 = \frac{p_1 - p_2}{\rho_1 - \rho_2} \quad (4)$$

Since the flow is isentropic both upstream and downstream of the shock , Bernoulli's

$$\frac{1}{2} q_1^2 + \frac{\gamma}{\gamma - 1} \frac{p_1}{\rho_1} = \frac{\gamma + 1}{\gamma - 1} \frac{c_1^2}{2}$$

$$\frac{1}{2} q_2^2 + \frac{\gamma}{\gamma - 1} \frac{p_2}{\rho_2} = \frac{\gamma + 1}{\gamma - 1} \frac{c_2^2}{2}$$

Since we assume that the flow is adiabatic, equation (5) .must hold along any stream line

The term in μ, however is assumed to be negligible at all .points not in the shock

We may therefore equation the left-hand sides of the last .tow equations

This establishes the fact that $c_1^{¿}=c_2^{¿}$ and we may express this by writing

$$\frac{1}{2}q_1^2+\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}=\frac{\gamma+1}{\gamma-1}c_1^{¿2}=\frac{1}{2}q_2^2+\frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2}(5)$$

hence

$$\frac{2\gamma}{\gamma-1}\left[\frac{p_1\rho_2-p_2\rho_1}{\rho_1\rho_2}\right]=q_2^2-q_1^2$$

But from eq (1) and (4) we have

$$\frac{q_1^2}{\rho_2^2}=\frac{q_2^2}{\rho_1^2}=\frac{q_1q_2}{\rho_1\rho_2}=\frac{p_1-p_2}{\rho_1-\rho_2}-\frac{1}{\rho_1\rho_2}$$

Thus

$$\frac{2\gamma}{\gamma-1}\left(\frac{p_1\rho_2-p_2\rho_1}{\rho_1\rho_2}\right)=\left(\rho_1^2-\rho_2^2\right)\left(\frac{p_1-p_2}{\rho_1-\rho_2}\right)\frac{1}{\rho_1\rho_2}$$

or

$$2\gamma\left(\frac{p_1\rho_2-p_2\rho_1}{\rho_1\rho_2}\right)=\left(\rho_1+\rho_2\right)\left(p_1-p_2\right)\left(\gamma-1\right)$$

Which simplifies to

$$\frac{p_1-p_2}{\rho_1-\rho_2}=\gamma\frac{p_1+p_2}{\rho_1+\rho_2}(6)$$

Further, the two equations of (5) can be written

$$(\gamma+1)\,c^{¿^2}\rho_1=(\gamma-1)\left(\rho_1 q_1^2+p_1\right)+(\gamma+1)\,p_1$$

$$(\gamma+1)\,c^{¿^2}\times\rho_2=(\gamma-1)\left(\rho_2 q_2^2+p_1\right)+(\gamma+1)\,p_2$$

Subtracting, and using eq (3) we deduce that

$$(\gamma+1)\,c^{¿}\left(\rho_1-\rho_2\right)=(\gamma+1)\left(p_1-p_2\right)$$

This equation combined with (4) and (6) yields

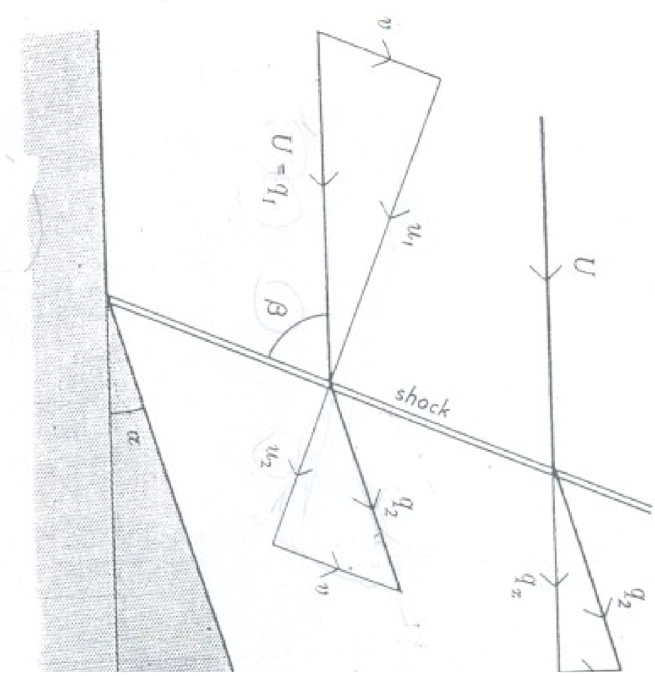
$$q_1 q_2=\frac{p_1-p_2}{\rho_1-\rho_2}=\gamma\frac{p_1+p_2}{\rho_1+\rho_2}=c^{¿}\,(7)$$

Further, these relation imply that

$$q_1 q_2=\frac{(\gamma+1)\,p_1+(\gamma-1)\,p_2}{2\rho_1}=\frac{(\gamma-1)\,p_1+(\gamma-1)\,p_2}{2\rho_2}\,(8)$$

The equation (1) to (8) must remain valid except in so far as

¿



(Fig: (3-1

In passing, we observe that if we superimpose a velocity v in the same direction as $u_1 \wedge u_2$ we have the case of an upstream uniform flow $q_1 = u_1 + v$ with a downstream uniform flow $q_2 = u_2 + v$ separeted by a plan shock wave moving downstream with speed v

We could even choose $v = -u_1$ so that $q_1 = 0$ and $q_2 = -(u_1 - u_2)$.

We would then have the case of shock wave rushing with .supersonic speed into still air followed by a uniform stream

If, however, we superimpose the uniform velocity v paralled to the shock it self we get the case of a stationary obligue shock illustrated in fig (3-1) we observe that the stream lines are tow refracted on passing through the shock and we conclude that a plane shock can occur when a supersonic flow is turned by a rectilinear boundary with a concave angle.

The flow behind the shock in this case is not necessary subsonic for it is possible that $q_2=\sqrt{u_2^2+v^2}>c_2$ although $u_2<c_2$, Bernoulli's equation now reads

$$\frac{1}{2}\left(u^2+v^2\right)+\frac{\gamma}{\gamma-1}\frac{p}{\rho}=\left(\frac{\gamma+1}{\gamma-1}\right)c_2^{¿2}$$

Or

$$\frac{1}{2}u^2+\frac{\gamma}{\gamma-1}\frac{p}{\rho}=\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)\left(c^{¿2}-\left(\frac{\gamma-1}{\gamma+1}\right)v^2\right)=\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)C^2$$

Where

$$C^2=c^{¿2}-\left(\frac{\gamma-1}{\gamma+1}\right)v^2$$

A moment's consideration will show that the formula (1)…(8) all remain valid for the oblique shock if we replace q_1 by u_1, q_2 by u_2 and $c^{¿}$ by C.

$$u_1u_2=c^{¿2}-\left(\frac{\gamma-1}{\gamma+1}\right)v^2\,(9)$$ In particular we have

In order to investigate the relationship between the angle β which the shock wave makes with the upstream flow and the angle α through which the flow is turned by the shock wave suppose that the upstream flow is of speed U in the x-direction and that we resolve the downstream velocity q_2 into components q_x, q_y parallel to, and pendicular to the upstream flow with this notation.

$$u_1 = U\sin\beta, v = U\cos\beta$$

And

$$q_x = v\cos\beta + u_2\sin\beta, q_y = v\sin\beta - u_2\cos\beta$$

Since, however

$$u_2 = \frac{1}{u_1}\left[c^{¿^2} - \left(\frac{\gamma-1}{\gamma+1}\right)v^2\right]$$

We find that

$$U q_x = U^2\cos^2\beta + \left[c^{¿^2} - \left(\frac{\gamma-1}{\gamma+1}\right)U^2\cos^2\beta\right]$$

$$¿ c^{¿^2} + \frac{2}{\gamma+1}U^2\cos^2\beta \quad (10)$$

And

$$U q_y = U^2\cos^2\beta\sin\beta - \left[c^{¿^2} - \left(\frac{\gamma-1}{\gamma+1}\right)U^2\cos^2\beta\right]\cot\beta$$

$$¿ (U^2 - U q_x)\cot\beta \quad (11)$$

We can eliminate β between these last two results in fact.

$$\left(\frac{U^2 - U q_x}{U q_y}\right)^2 = \tan^2\beta = \frac{1}{\cos^2\beta} - 1$$

Since

$$U q_x = c^{¿^2} + \frac{2}{\gamma+1} U^2 \cos^2\beta$$

Then

$$\cos^2\beta = \frac{\gamma+1}{2U^2}\left(U q_x - c^{¿^2}\right)$$

$$\left(\frac{U - U q_x}{U q_y}\right)^2 = \frac{2U^2}{(\gamma+1)\left(U q_x - c^{¿^2}\right)} - 1$$

$$¿\,\frac{2U^2 - (\gamma+1)\left(U q_x - c^{¿^2}\right)}{(\gamma+1)\left(U q_x - c^{¿^2}\right)}$$

$$q_y^2 = \frac{\left(U q_x - c^{¿^2}\right)(U - U q_x)^2}{\left(\frac{2U^2}{\gamma+1} - \left(U q_x - c^{¿^2}\right)\right)}$$

Now the angle α is siven by

$$\tan\alpha = \frac{q_y}{q_x} (12)$$

Hence, if αis a known angle, we have an equation

$$(U - q_x)^2\left(U q_x - c^{¿^2}\right) = \left(\frac{2U^2}{\gamma+1} - U q_x + c^{¿^2}\right) q_x^2 \tan^2\alpha$$

Which is a cubic equation for q_x in terms of the upstream .conditions

If this equation has a solution such that q_x is less than C, .(we can then determine β from the equation (11

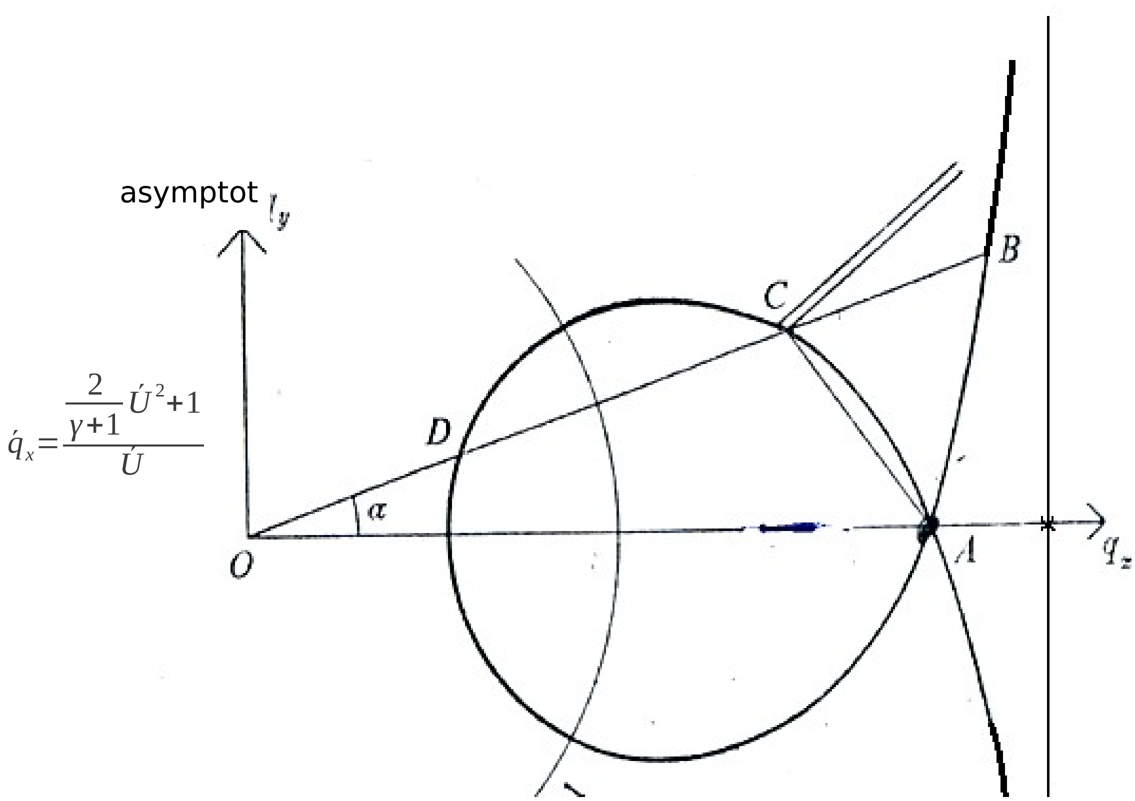
Conversely, if known, q_x and q_y are obtained from (12) .(and (11

.We then determine α from (12) graphically

If we write $q_x = c¿\acute{q}_x, q_y = c¿\acute{q}_y, U = c¿\acute{U}$, we have the relation

$$\acute{q}_y^2 = \frac{\left(\acute{U}-\acute{q}_x\right)^2\left(\acute{U}\acute{q}_x-1\right)}{\frac{2}{\gamma+1}\acute{U}^2-\acute{U}\acute{q}_x+1} (13)$$

.This equation is solved graphically



Wave number
surface

(Fig: (3-2

The plane of this curve is that of the coordinates q_x and q_y down on the scale for which $c^{¿}=1$, each paint point in the plane is associated with velocity vector $\acute{op}$.

The point A represents the upstream velocity since it is easily verified that $oA=\acute{u}$.

The points C and D both represent downstream velocities making an angle α with the incident stream $\acute{U}$ the downstream speed is clearly less for the point D than for C so that in general there are tow possibilities for a given angle α.

We may have a weak shock represented by point C in which there is a comparatively small diminution in the fluid velocity on passing through the shock.

Alternatively, we may have a strong shock, represented by D in which there is greater .reduction in speed

In actual experiments it is usually the weak shock .which accurse

The point is B physically irrelevant since it corresponds to a downstream speed which is greater than $\acute{U}$ if C by any point on the curve, then the gradient of CA is $\frac{\acute{U}-\acute{q}_x (¿) - \acute{q}_y}{¿}$,and according to eq(11)

. $-\cot\beta$ this is just

It is therefore apparent that the shock wave which gives rise to the downstream velocity $\acute{OC}$ is . CA perpendicular to

The cubic curve of our figure is called a shock polar or an oblique shock hodograph and several of these for varying values

.Of $\acute{U}$ may be drawn on the same diagram

From such a diagram the angle β of the shock wave as well as the downstream velocity can be read off graphically for a given angle α and a given $\acute{U}$ it will be observed that if for a given value of $\acute{U}$ the angle α exceeds a certain maximum value, the line

$$\acute{q}_y = \acute{q}_x \tan\alpha$$

.Will not intersect the loop of the shock polar at all

The physical meaning of this is that γ is too large it will not be passible to turn the flow through an .angle α by a plane shock wave

The formation of plane shock waves is well illustrated in plate which shows tow-dimensional flow past a sharp nosed aerofoil

In the flow illustrated α is less than the maximum for a plane shock and two straight shock waves (dark lines) are clearly seen attached to the nose of .the aerofoil

Plane shock waves are also to be observed radiating

.from the trailing edge

When the value of α exceeds the maximum an

.attached plane shock is impassible

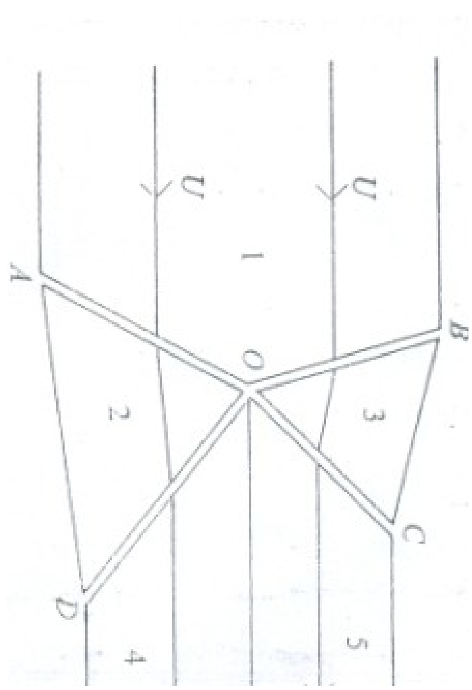
It is found, however, that there is now a curved (cylindrical) detached shock wave present and that the closer $\acute{U}$ is to unity the farther a way from the

.wedge is the detached shock wave

Plate shows a curved detached shock wave produced by a blunt nosed aerofoil travelling at

.supersonic speed

.Their verification is left to the reader



(Fig: (3-3

Intersections of oblique shock *(3-1)* waves

Space permits only a brief investigation of the flow pattern resulting when two plane shock waves intersect each other

fig(3-3) illustrates four plane shock waves OA, OB, OC, OD and a vortex sheet OS which separate the fluid into five regions which we denote .by athe suffixes 1,2,3,4,5

The regions corresponds to an undisturbed incident . U supersonic stream of known speed

The shock waves OA and OB may have originated from some disturbance on the boundary and we therefore suppose that their position as well .as the shock angles β_A and β_B are known

By the methods of the previous section it is .therefore

Possible to calculate the flow in regions 2 and 3

These flows will have different directions and so we cannot expect that whole of the downstream flow .belongs to one or other of the regions 2 and 3

In fact the point 0 of intersection of the shock waves OA and OB must it self behave as a disturbance and may give rise to shock waves OC and OD .whose positions have to be determined

If the flows in regions 4 and 5 are to be parallel we must have $p_4 = p_5$ though this dos not imply that $\rho_4 = \rho_5$ or that the mach numbers M_4 and M_5 are .equal

If M_4 and M_5 are in fact unequal then the line OS which separates regions 4 and 5 must .represent a vortex sheet

If it happens that $M_4 \wedge M_5$ are equal then it can be shown that $\rho_4=\rho_5$ and the conditions in regions 4 .and 5 are identical

The fact that the pressures are the same and that flows are parallel in regions 4 and 5 implies that

$$\frac{p_1}{p_2}\times\frac{p_2}{p_4}=\frac{p_1}{p_3}\times\frac{p_3}{p_5}$$

$\alpha_A+\alpha_D=\alpha_B+\alpha_C$ And that

Where α_P denotes the angle theough which the flow is turned by a shock wave OP and where all .these angles are measured in the same sense

The angles β_C and β_D in turn fix the positions of OC and OD, so that the flow in the regions 4 and .5 can eventually be evaluated

It may, however happen that the equations (1) and . β_D (2) have no real solution for β_C and

This would happen if the flow in region 2 were subsonic, in which case the shock wave OC would .be absent

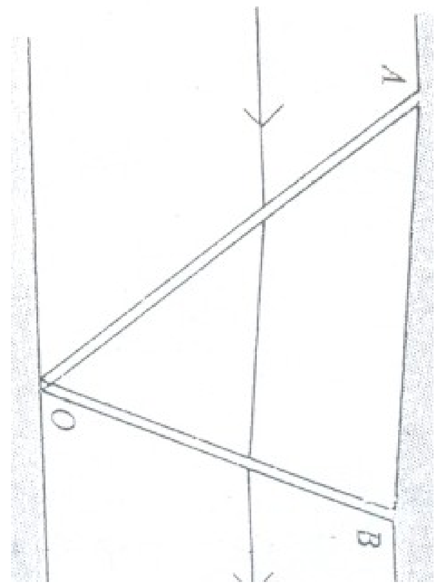
The equations to be solved in this case would be

$$\frac{p_1}{p_4}=\frac{p_1}{p_3}\times\frac{p_3}{p_5}$$

$\alpha_B=\alpha_A+\alpha_D$ And

In a configuration of three intersecting shock such as this three must necessarily be trailing vortex sheet . OS

if the flow were subsonic both in regions 2 and 3 there could be no further shock waves but we would not find uniform flow any where downstream of the OB shocks OA and



(Fig: (3-4

Even if the flow is supersonic in both the regions 2 and 3 there may be no solution for β_C and β_D for essentially the same reason that certain wedges can have no attached shock wave.

In this case the secondary shocks will be curved and may include a portion which is upstream of the .geometrical point 0

Fig (3-4) illustrates the reflection of a shock wave OP . at a plan boundary

The reflection wave OB, if this configuration is possible must be just sufficient to turn the flow .parallel to the wall again

Only one equation has to be solved, namely

$$\alpha_A + \alpha_B = 0$$

In this case also, circumstances may be such that no plane reflected wave is possible and a more .complicated system of carved waves may result

it is now possible to see how shock waves may a rise when a supersonic stream is turned by a curved concave boundary for we can approximate to such a boundary by a rectilinear one such as is exhibited in fig (3-5) the uniform stream with the initial direction p_0, p_1 is turned in succession by the shock waves originating at the corners $p_1, p_2, \ldots$ if the shocks from p_1 and p_2 intersect in A then the flow in the region $p_1 A p_2$ will be uniform while beyond A the tow shocks combine to yield a shock AB and also a wave reflected from A towards the .boundary

This reflective wave is generally much weaker than .the others and we shall disregard it

The angle through which the stream is turned in passing through AB is then equal to the sum of the angles through which the flow is turned in crossing $p_1 A$ and $p_2 A$, so that the shock AB is stronger .than either of these

The next shock from p_3 meets AB in B and the flow is parallel to $p_2 p_3$ in the region $p_2 AB p_3$ it is uniform in each of the region $p_2 AS p_3$ and were AS (not shown in the figure) is the vortext sheet trailing from A .

The shock p_3 B and AB unite to form a still .stronger shock be and so on

In the limiting case when the boundary p_0, p_1, p_2, p_3 , ... become a smooth curve the finite shocks a rising at $p_1, p_2, p_3, \ldots$ are replaced by an infinite number of infinitesimal shocks whose envelope replaces $ABC\ldots$ the gradient of an infinitesimal shock relative to the flow just in front of it, is that of the normal to the shock polar at the point A in fig(3-.(2

It may be obtained graphically from a shock polar

$$\left(\frac{-d\acute{q}_x}{d\acute{q}_y}\right)_{\acute{q}_x=\acute{U}}$$ diagram, or from (13) by calculating

After some calculation it may be verified, as we might have guessed, that an infinitesimal shock makes the mach angle μ with the flow just in front .of it

.Indeed an infinitesimal shock is just a mach line

The envelope of these mach lines drawn from the .boundary is found to have a cusp

The infinitesimal shocks build up in strength to form a shock wave lying between the tow branches of the envelope and starting with zero strength at the .cusp

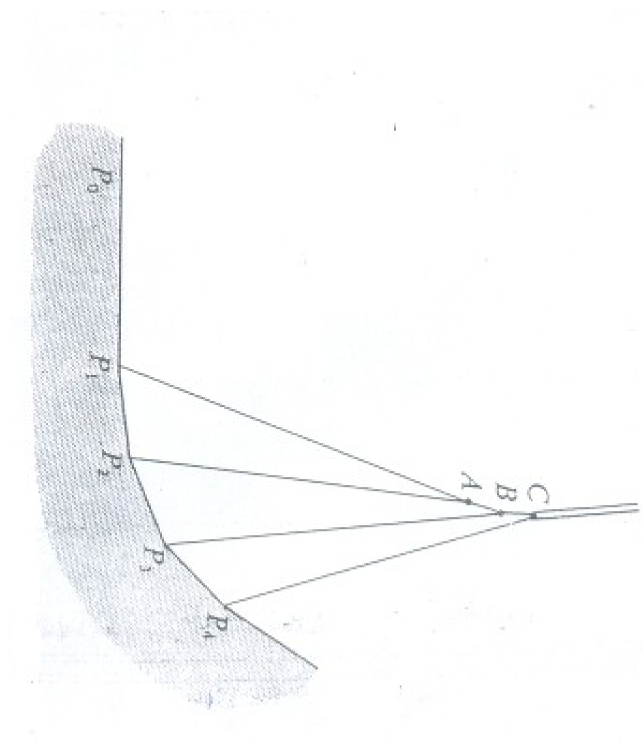


Fig:4

Chapter Four

Conclusion &Recommendations (4-1)

Thanks, my Goodness, I wrote about this subject, because it is so important, in our life and I want to reach up and I want this destination to create a new innovation so as to benefit .the Human beings and to extend my knowledge

I came to a conclusion that this normal and oblique shock waves in Gas subject can be studied more and more with .more information and knowledge

In case of oblique shock wave it's necessary to solve the

$$\acute{q}_y^2=\frac{\left(\acute{U}-\acute{q}_x\right)^2\left(\acute{U}\,\acute{q}_x-1\right)}{\frac{2}{\gamma+1}\acute{U}^2-\acute{U}\,\acute{q}_x+1}$$

equation graphically

It is seen that the graph possesses an asymptote at

$$\acute{q}_x=\frac{\frac{2}{\gamma+1}\acute{U}^2+1}{\acute{U}}$$

This asymptote called the wave number surface it advance the whole wave which is always behind it

Reference (4-2)

A. Joseph M. Powers, Lecture Notes On Gas Dynamics Department of Aerospace and Mechanical Engineering University of Notre Dame Notre Dame, Indiana 46556-5637 USA updated 1 July 23, 2010.

B. Teng Hong-Hui, Zhao Wei, Jiang Zong, Anovel Oblique Detonation Structer and it's Stability, 22 March 2007.

C. Dr. Narayanan Komerath Professor, AE 6020 Compressible Flow, Spring 2008

D. Reno, Calculations For Study Propagation Of a Generic Ram Accelerator Configuration, received. 8 Dec 1993 present as paper 94-0550 at the alaa 32nd aerocpace sciences meeting and Exhibit, Jan 10-13-1994.

E. Jean-Christophe Robinte, Critical Interaction Of a Shock Wave With an Acoustic Wave, received 28 Apr 2000, accepted 8 Jan 2001

F. Yungester, Numerical Study Of Shock Wave Boundary Layer Interaction in Premixed Combustible Gases, 10 Nov 1992.

G. Krishnan Mahesh, Amodel For The Onset of Breakdown In an Ax Symmetric Compressible Vortex, 28 May 1996, accepted 5 Sep 1996.

H. D.E.Rutherfood, Fluid Dynamics ,publisher: Oliverod Boyd 1966.

Shunsuke Usami and Ohsawa, Evolution of Relativsticicions Incessantly as Accelerated by an Oblique Shock Wave, 7 .July 2003, accepted 20 Nov 2003 .I

Tetuya Ukon,Joshiyuki Aoki, Taish Sakai and Kazuyasu Matsou, Visual Study of Supersonic Plasma Flow in .Constant Area MHD Channel, received 31 Aug 1994 .J

A.H. Shapiro, Compressible Fluid Flow, Vols. 1& 2. Ronald .Press, 1953 .K

.M.A. Saad, Compressible Fluid Flow. Prentice-Hall, 1993 .L

P.A. Thompson, Compressible Fluid Dynamics. McGraw-Hill, .1972 .M

Ya.B. Zel’dovich and Yu.P. Raizer, Physics of Shock Waves and HighTemperature Hydrodynamic Phenomena. Dover Publications, 2002 (originally in two volumes from .(Academic Press, 1967 .N