Sudan University of Science and Technology

College of Graduate Studies

Derivation of Lorentz Field Dependent Transformation using Lorentz Electromagnetic Force Equation

اشتقاق تحويل لورنس المعتمد على المجال باستخدام معادلة قوة لورنس الكهرومغنطيسية

A thesis submitted for fulfilment requirement of the degree of Doctor in Physics

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الآية

قال تعالي:

(يَرْفَعُ اللهُ الذينَ امَنُوا مِنْكُمْ وَالذِينَ أُوْتُوا العِلْمَ دَرَجَات وَاللهُ

بِمَا تَعْمَلُونَ خَبِيرٌ)

صدق الله العظيم

سوره المجادله ,ايه رقم 11

Dedication

To the soul of my parents …

My dear brother **Jaafrar Abdelrahman …**

My dear husband **Alfatih Talab…**

Acknowledgement

I would like to express my deep gratitude to **Sudan University** of science and technology, Graduate College, Faculty of science and physics department for their moral support and facilities to do this research. Thanks extends **to prof. Mubarak Dirar abd-allah** for faithful suggestions and king warm help. I am also deeply indebted to **Mr. Mohammed Sharaf Aldeen** and **Mr. Ahmed Alhaj** for their kind help in writing this manuscript.

Abstract

Lorentz transformation is one of the cornerstone of special relativity. It is concerned only with inertial frames within the framework of special relativity, but doing nothing for non-inertial frames. Attempts were made to account for this effect by using space-time language, without any link with electromagnetic theory.

The aim of this work is to cure this defect. The research methodology is based on the mathematical analytical framework of electromagnetic Lorentz force. Then the result obtained is compared with previous studies and observations. The Lorentz electromagnetic expression for the force, beside Maxwell relation between electric and magnetic field intensity are used to find Lorentz force in terms of velocity and electric field intensity. This relation of magnetic and electric field intensity was found by using two approaches, in one of them the relation between the curl of electric intensity with the time variation of the magnetic one was used, in the second approach the relation between curl of magnetic intensity with the displacement current was used also. The velocity in accelerated frame and curved space-time is incorporated in this expression to make Lorentz sensitive to field's potentials. The field potential was incorporated first by replacing the acceleration with the potential in Newton's equations of motion, then replacing the average velocity with

final velocity and potential. In another approach, the expression for the interval was also used to relate the average velocity to the final velocity and potential. A third approach used the interval in a curved space to incorporate the potential through space-time Lorentz transformation. Fortunately, this transformations reduces to that of SR in the absence of fields, thus share with it all its success and compatibility with observations. It also conforms with generalized special relativity, thus shares with it also all its successes. More importantly it link electromagnetic theory with generalized Lorentz transformations.

المستخلص

تعتبر تحويلات لورنس واحدة من أحد الركائز الأساسية للنسبية الخاصة. وهي تختص بمحاور الإسناد القصورية في إطار النسبية الخاصة ولكن لا شأن لها بالمحاور اللاقصورية، وقد جرت محاولات عديدة لتلافي هذا القصور باستخدام لغة الزمكان ولكن بدون أي حلقة وصل بالنظرية الكهرمغنطيسية.

يهدف هذا البحث لمعالجة هذا الخلل ويعتمد منهج البحث على الإطار التحليلي الرياضي لقوة لورنتز. ثم مقارنة النتائج المتحصل عليها مع الدراسات السابقة والمشاهدات. وقد استخدمت صيغة لورنس الكهرومغنطيسية للقوة، بالإضافة لعلاقات ماكسويل بين شدتي المجال الكهربي والمغنطيسي لإيجاد صيغة لورنس بدلالة السرعة وشدة المجال الكهربي. وقد تم إيجاد علاقة شدتي المجال الكهربي والمغناطيس بطريقتين، في إحداهما تم استخدام علاقة التواء شدة المجال الكهربي بتغير شدة المجال المغناطيسي مع الزمن، وفي الثانية تم استخدام علاقة التواء المجال المغناطيسي مع تيار الإزاحة أيضا. وقد ضمنت صيغة السرعة في الإسناد المعجل والزمكان المنحني لجعل تحويل لورنس حساسا لجهود المجالات. وقد ضمن الجهد أولا باستبدال العجلة بالجهد في معادلات نيوتن للحركة، ثم استبدال السرعة المتوسطة بالسرعة النهائية والجهد. وفي طريقة أخرى استخدمت صيغة الفاصل أيضا لإيجاد علاقة السرعة المتوسطة بالسرعة النهائية والجهد. وفي طريقة ثالثة استخدمت صيغة الفاصل في الفراغ المحدب لتضمين الجهد عبر تحويلات لورنتس الزمكانية. ولحسن الحظ أن هذا التحويل يؤول للنسبية الخاصة في غياب المجالات، لذا فهي تشاركه في كل نجاحاته وتوافقه مع المشاهدات. كما أنها اتسقت مع النسبية الخاصة المعممة، وبالتالي شاركته كل نجاحاته. والأهم من ذلك هو أنها أصبحت حلقة وصل بين النظرية الكهرمغنطيسية وتحويلات لورنس المعممة.

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Chapter One

Introduction

1.1 Electromagnetic Theory

Atoms are the building blocks of matter. They consist of negatively charged electrons and positively charged protons. These charges generat electric field. When they move they generate magnetic field(1,2,3).Later on it was discovered that the time changing magnetic field can generate electric field and vice versa. This fact is formulated mathematically by Maxwell, and know as Maxwells equations. Maxwell's equations can successfully describe a wide variety of physical phenomena. For instance it can describe the generation of electromagnetic waves, beside the description of the electric and magnetic properties of matter (4,5,6).These equations were widely used in telecommunications, computers, and electronic devices (7,8,9).

Maxwell’s equations not can fit with experimental data of charges and currents, by considering the charges and currents produced in materials. They are used to describe the fields produced for example, by all the important technologically useful classes of material. They also include a prohibition on the creation of net charge that is consistent with experimentation to date. However this prohibition does open the question of how the scientific community would consider Maxwell’s equations if an experiment created net charge. Recently Maxwell’s equations are generalized such that allow processes such as net charge creation. This is done if one

consider the conditions under which laws, that in their original form contain no time-dependence, are used to derive time-dependent differential equations. One can then extend Coulomb's law and the BiotSavart law to the temporal domain under relevant time-varying conditions (10, 11, 12)

1-2 Special Relativity Theory(SR)

The theory of relativity resulted from an analysis of the physical consequences implied by the absence of a universal frame of reference. The special theory of relativity developed by Albert Einstein in 1905, treats problems involving inertial frames of reference, which are frames of reference moving at constant velocity with respect to one another (13, 14, 15). The general theory of relativity, proposed by Einstein a decade later, treats problems involving frames of reference accelerated with respect to one another. An observer in an isolated laboratory can detect accelerations. Anybody who has been in an elevator or on a merry-ground can verify this statement from his own experience. The special theory has had a profound influence on all physics, and we shall concentrate on it with only a brief glance at the general theory (16, 17, 18).

The special theory of relativity is based upon two postulates. The first states that the laws of physics may be expressed in equations having the same form in all frames of reference moving at constant velocity with respect to one another (19,20,21) This postulate expresses the absence of a universal frame of reference. If the laws of physics had different forms for different observers in relative motion, it could be determined from this difference which objects are "stationary" in space and which are "moving". But because there is no universal frame of reference, this distinction does no iexist in nature, hence the above postulate.

The second postulate of special relativity states that the speed of light in free space has the same value for all observers, regardless of their state of motion.

This postulate follows directly from the results of the Michelson-Morley experiment and many others (22,23,24,25)

1.3 Research Problems

Special relativity (SR) study the effect of velocity on space, time and mass. Unfortunately SR does not account for the effect of fields on them.The research problem is related to the need of using simple transformation to account for the effect of fields on electric and magnetic fields

1.4 Literature Review

Different attempts were made to account for the effect of fields (26,27,28, 29, 30). In some of them the effect of fields is embedded through the mean average velocity(31,32,33),while in others the curvature of space is used (34,35,36) .This model is known as generalized special relativity (GSR). It successfully explains awide variety of physical phenomena, like effective mass of electrons in Crystals, time dilation and photon gravitational red shift (37,38,39,40).

1.5 Aim of the Work

The aim of this work is to get a new transformation that accounts for the effect of fields on magnetic field and electric field

1.6 Thesis Layouts

The thesis consists of 5 Chapters Chapter 1 is the introduction. Electromagnetic theory and special relativity are exhibited in chapters 2 and 3. Chapters 4 and 5 are devoted for Literature review and contribution.

Chapter two

Special Relativity

2.1 Introduction

Einstein SR is one of the big achievements that change the classical concept of absolute space and time coordinate.

2.2 The Special Theory of Relativity

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The special theory of relativity is based upon two postulates. The first states that the laws of physics may be expressed in equations having the same form in all frames of reference moving at constant velocity with respect to one another. This postulate expresses the absence of a universal frame of reference. If the laws of physics had different forms for different observers in relative motion, it could be determined from this difference which objects are “stationary” in space and which are “moving”. But because there is no universal frame of reference, this distinction does no iexist in nature, hence the above postulate.

The second postulate of special relativity states that the speed of light in free space has the same value for all observers, regardless of their state of motion.

This postulate follows directly from the results of the Michelson-Morley experiment and many others. (43, 44, 45)

At first sight these postulates hardly seem radical. Actually they subvert almost all the intuitive concepts of time and space we form on the basis of our daily experience. A simple example will illustrate this statement. If we have two boats A and B once more, with boat A at rest in the water while boat B drifts at the constant velocity v. there is a low-lying fog present, and so on neither boat does the observer have any idea which is the moving one. At the instant that B is abreast of A, a flare is fired. The light from the flare travels uniformly in all directions, according to the second postulate of special relativity. An observer on either boat must find a sphere of light expanding with himself at its center, according to the first postulate of special relativity, even though one of them is changing his position with respect to the point where the flare went off. The observers cannot detect which of them is undergoing such a change in position since the fog eliminates any frame of reference other than each boat itself, and so since the speed of light is the same for both of them, they must both see the identical phenomenon.

Why the situation is unusual. Let us consider a more familiar analog. The boats are at sea on a clear day and somebody on one of them drops a stone into the water when they are abreast of each other. A circular pattern of ripples spreads out, as at the bottom of fig (1.7), which appears different to observers on each boat. Merely by observing whether or not he is at the center of the pattern of ripples, each observer can tell whether he is moving relative to the water or not. Water is in itself a frame of reference, and an observer on a boat moving through it measures ripple speeds with respect to himself that are different in different directions, in contrast to the uniform to ripple speed measured by an observer on a stationary boat. It is important to recognize that motion and waves in water are entirely different from motion and waves in space; water is in itself a frame of reference while space is not, and wave speeds in water vary with the observer's motion while wave speeds of light in space do not.

The only way of interpreting the fact that observers in the two boats in our example perceive identical expanding spheres of light is to regard the coordinate system of each observer, from the point of view of the other, as being affected by their relative motion. When this idea is developed using only accepted laws of physics and Einstein's postulates, we shall see that many peculiar effects are predicted. One of the triumphs of modern physics is the experimental confirmation of these effects.

2.3 Time Dilation

We shall first use the postulates of special relativity to investigate how relative motion affects measurements of time intervals and lengths.

A clock moving with respect to an observer appears to tick less rapidly than it does when at rest with respect to him. That is, if someone in a spacecraft finds that the time interval between two events in the spacecraft is t_0, we on the ground would find that the same interval has the longer dilation t_0. The quantity t_0, which is determined by events that occur at the same place in an observer's frame of reference, is called the proper time of the interval between the events. When witnessed from the ground, the events that mark the beginning and end of the time interval occur at different places, and in consequence the duration of the interval appears longer than the proper time. This effect is called time dilation.

To see how time dilation comes about, let us examine the operation of the particularly simple clock shown in fig (1.8) and inquire how relative motion affects what we measure. This clock consists of a stick L_0long with a mirror at each end. A pulse of light is reflected up and down between the mirrors and an appropriate device is attached to one of the mirrors to give a "tick" of some kind each time the pulse of light strikes it. (such a device might be a photosensitive surface on the mirror which can be arranged to give an electric signal when the light pulse arrives). The proper time t_0 between ticks is

$$t_0 = \frac{2L_0}{c} \tag{2.3.1}$$

Now how time elapse between two ticks. Each tick involves light passage with speed c from the lower mirror to the upper one and back. Which means that the pulse of light, as seen from the ground, actually follows a zigzag path fig (1.9). on its way from the lower mirror to the upper one in the time $\frac{t}{2}$, the pulse of light travels a horizontal distance of $\frac{vt}{2}$ and a total distance of $\frac{ct}{2}$. Since $L_0$is the vertical distance between the mirrors.

$$\left(\frac{ct}{2}\right)^2 = L_0^2 + \left(\frac{vt}{2}\right)^2 \tag{2.3.2}$$

$$\frac{t^2}{4}(c^2 - v^2) = L_0^2 \tag{2.3.3}$$

$$t^2 = \frac{4L_0^2}{c^2 - v^2} = \frac{(2L_0)^2}{c^2\left(1 - \frac{v^2}{c^2}\right)} \tag{2.3.4}$$

And

$$t = \frac{\frac{2L_0}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2.3.5)$$

But $\frac{2L_0}{c}$ is the time interval t_0 between ticks on the clock on the ground, as in equation (1.4) and so

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2.3.6)$$

The moving clock in the spaceship appears to tick at s rate than the stationary one the ground, as seen by an observer on the ground.

Exactly the same analysis holds for measurements of the clock on the ground by the pilot of the spaces rip. To him the light pulse of the ground clock follows a zigzag path which requires a total time t per round trip, while his own clock at rest in the spaceship ticks at intervals of t_0. He too finds that

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2.3.7)$$

So the effect is reciprocal every observer finds that clocks in motion relative to him tick more slowly Chan when they are at rest.

Our discussion has been based on a somewhat unusual clock that employs light pulse bouncing back and between two mirrors. Do the same conclusions apply to more conventional clocks that use machinery-spring-controlled escapements, tuning forks, or whatever-to produce ticks at constant time intervals. The answer mast be yes since if a mirror clock and a conventional clock in the spaceship agree with each other on the ground but not when in flight, the disagreement between them could be used to determine the speed of the spaceship without reference to any other object which contradicts the principle that all motion is relative. Detailed calculations of what happens to conventional clocks in motion as see from the ground confirm this answer for example, as we shall learn in Sec. 1.10, the mass of an object is greater when it is in motion, so that the period of an oscillating object must be greater in the moving spaceship. Therefore all clocks at rest relative to one another behave the same to all observers, regardless of any motion at constant velocity of either the group of clocks or the observers.

The relative character of time has many implications. For example, events that seem to take place simu tenuously to one observer may not be simultaneous to anther observer in relative motion, and vice versa. Who is right the question is, of course meaningless: both observers are "right" since each simply measures what he sees.

Because simultaneity is a relative motion and not an absolute one, physical theories which require somral taneity in events at different locations must be discarded. The principle of conservation of energy in its elementary form states that the total energy content of the universe is constant, but it does not rule out a process in which a certain amount of energy ΔE vanishes at one point while an equal amount of energy ΔE spontaneously comes into being somewhere else with no actual transport of energy from one place to the other. Because simultaneity is relative, some observers of the process will find energy not being conserved. To rescue conservation of energy in the light of special relativity, then it is necessary to say that when energy disappears somewhere and appears elsewhere, it has actually flowed from the first location to the second. (there are many ways in which a flow of energy can occur, of course). Thus energy is conserved loyally in any arbitrary region of space at any time, not merely when the universe as whole is considered a much stronger statement of this principle.

Although time is a relative quantity, not all the notions of time formed by everyday experience are incorrect. Time dose not run backward to any observer for instance a sequence of events that occur somewhere at $t_1, t_2, t_3, ...$will appear in the same order to all observers $t_2 - t_1, t_3 - t_2, ...$ between each pair of events. Similarly, on distant observer regardless of his state of motion can see an event before it happens more precisely before a nearby observer sees it since the speed of light infinite and signals require the minimum period of time L/c to travel a distance L. There is no way to peer into the future; although temporal (and as we shall see spatial) perspectives of past events may appear different to different observers.

2.4 Length Contraction

Measurements of lengths as well as of time intervals are affected by relative motion. The length L of an object motion with respect to an observer always appears to the observer to be shorter than its length L_0 when it is at rest with respect to him, a phenomenon known as the Lorentz Fitz Gerald contraction. This contraction occurs only in the direction of the relative motion. The length L_0 of an object in its rest frame is called its proper length (46, 47, 48).

We can use the light clock of the previous section to investigate the Lorentz contraction. For this purpose we imagine the clock oriented so that the light pulse travel back and forth parallel to the direction in which the clock is moving relative to the observer. At $t = 0$ the light pulse starts from the rear mirror, and reach the front mirror where from the diagram

$$ct_1 = L + ct_1$$

Hence

$$t_1 = \frac{L}{c - v} \tag{2.4.1}$$

Where L is the distance between the mirrors as measured by the observer at rest?

The pulse is then reflected by the front mirror and returns to the rear mirror at t after traveling the distance

$$c(t - t_1) = L - c(t - t_1) \tag{2.4.2}$$

Hence the entire time interval$t$, as determined from the ground is

$$t = \frac{L}{c + v} + t_1 \tag{2.4.3}$$

We eliminate $t_1$with the help of equation (2.4.3) to find that

$$t = \frac{L}{c + v} + \frac{L}{c - v}$$

$$= \frac{2Lc}{(c + v)(c - v)}$$

$$= \frac{2Lc}{c^2 - v^2}$$

$$= \frac{2Lc}{1 - v^2/c^2} \tag{2.4.4}$$

Equation (2.4.4) gives the time interval t between ticks of the moving clock as measured by an observer on the ground.

We earlier found another expression for t.

$$t = \frac{2L_0/c}{\sqrt{1 - v^2/c^2}} \tag{2.4.5}$$

Which is in terms of L_0, the proper distance between the mirrors, instead of in terms of L, the distance as measured by an observer motion. The two formulas must be equivalent and hence we have.

$$\frac{2L/c}{1 - v^2/c^2} = \frac{2L_0/c}{\sqrt{1 - v^2/c^2}}$$

$$L = L_0\sqrt{1 - v^2/c^2} \qquad (2.4.6)$$

Because the relative velocity appears only as v^2 in equation (2.4.6), the Lorentz contraction is a reciprocal effect. To man in a spacecraft objects on the earth appear shorter than they did when he was on the ground by the same factor $\sqrt{1 - v^2/c^2}$, that the spacecraft appears shorter to somebody at rest. The proper length of an object is the maximum length any observer will find.

The relativistic length contraction is negligible for ordinary speeds but it is an important effect at speeds close to the speed of light. A speed of 1.000mi/s seems enormous to us, and yet it results in a shortening in the direction of motion to only

$$\frac{L}{\mathrm{L_O}} = \sqrt{1 - v^2/c^2}$$

$$= \sqrt{1 - \frac{(1.000mi/s)^2}{(186.00mi/s)^2}}$$

$$= 0.999985$$

$$= 99.9985percent$$

Of the length at rest. On the other hand a body traveling at 0.9 the speed of light is shortened to

$$\frac{L}{\mathrm{L_O}} = \sqrt{1 - \frac{(0.9c)^2}{c^2}}$$

$$= 0.436$$

$$= 43.6\, percent$$

Of the length at rest a significant change.

The ratio between L and $\mathrm{L_O}$ in equation (2.4.4) is the same as that in equation (2.4.5) when it is applied to the times of travel of the two light beams, so that we might be tempted to consider the Michelson-Morley result solely as evidence for the contraction of the length of their apparatus in the direction of the earth's motion.

This interpretation was tested by Kennedy and ThorndiKe in a similar experhnent using an interferometer with arms of unequal length. They also found no fringe shift, which means that these experiments must be considered evidence for the absence of ether with all this implies and only for contractions $c^4$the apparatus.

An actual photograph of an object in very rapid relative motion would reveal a somewhat different distortion, depending upon the direction from which the object is viewed and the rationv/c. The reason for this effect is that light reaching the camera (or eye for that matter) from the more distant parts of the object was emitted earlier than that coming from the nearer parts; the camera "sees'' a picture that is actually a composite, since the object was at different locations when the various elements of the single image that reaches the film left it. This effect supplements the Lorentz contraction by extending the apparent length of a moving object in the direction of motion. As a result a three-dimensional body such as a cube may be seen as rotated in orientation as well as changed in shape again depending upon the position of the observer and the value ofv/c. This result must be distinguished from the Lorentz contraction itself which is a physical phenomenon. If there were no Lorentz contraction, the appearance of a moving body would be also different from what it is at rest. But in another way.

It is interesting to note that the above approach to the visual appearance of rapidly moving object was not made until 1959, 54 years after the publication of the special theory of relativity.

2.5 The Lorentz Transformation

Let us suppose that we are in a frame of reference S and find that the coordinates of some event that occurs at the time t are $,y,z$. an observer with respect to S at the constant velocity v will find that the same event occurs at the time t' and has the coordinates x',y',z'. (in order to simplify our work, e shall assume that v is in the $+x$ direction, haw are the measurements x,y,z,t related to x',y',z',t'.

If we are unaware of special relativity, the answer seems obvious enough. If time in both systems is measured from the instant when the origins of S and S' coincided measurements in the x direction made in S will exceed those made in $S'$by the amount vt, which represents the distance that S' has moved in the x direction. That is (49,50,51)

$$x' = x - vt \qquad (2.5.1)$$

There is no relative motion in the $y$and z directions and so

$$y' = y$$

$$z' = z$$

In the absence of any indication to the contrary in our everyday experience we further assume that

$$t' = t$$

The set of equation (1.11) to (1.14) is known as the Galilean transformation. To convert velocity components measured in the S frame to their equivalents in the S'frame according to the Galilean transformation, we simply differentiate $x', y'$and $z'$with respect to time:

$$v'_x = \frac{dx'}{dt'} = v_x - v$$

$$v'_y = \frac{dy'}{dt'} = v_y$$

$$v'_z = \frac{dz'}{dt'} = v_z \qquad (2.5.2)$$

While the Galilean transformation and the velocity transformation it leads to are both in accord with our intuitive expectations, they violate both of the postulates of special relativity. The first postulate calls for identical equations of physics in both the $S$and S'frames of reference, but the fundamental equations of electricity and magnetism assume very different forms when the Galilean transformation is used to convert quantities measured in one frame into their equivalents in the other. The second postulate calls for the same value of the speed of light c whether determined in $S\ or\ S'$. If we measure the speed of light in the x direction in the S system to be c, however in S' system it will be

$$c' = c - v$$

According to equation (2.4.5) clearly a different transformation is required if the postulates of special relativity are to be satisfied. We would expect both time dilation and length contraction to follow naturally from this new transformation.

A reasonable guess as to the nature of the correct relationship between x and x' is

$$x' = k(x - vt) \tag{2.5.3}$$

Where k is a factor of proportionality that does not depend upon either $x \ or \ t$ but may be a function of$v$. The choice of equation (2.5.3) follows from several considerations:

1- It is linear in x and x', so that a single event in frame S corresponds to a single event in frame S', as it must.

2- It is simple, and a simple solution to a problem should always be explored first.

3- It has the possibility of reducing to equation (2.4.6) which we know to be correct in ordinary mechanics.

Because the equations of physics must have the same form in both S and S'we need only change the sign of v (in order take into account the difference in the direction of relative motion) to write the corresponding equation for x and solving for x, in terms of $x'$and t':

$$x = k(x' + vt') \tag{2.5.4}$$

The factor k must be the same in both frames of reference since there is no difference between S and S' other than in the sign of v.

As in the case of the Galilean transformation, there is nothing to indicate that there might be difference between the corresponding coordinates y, y' and z, z' which are normal to the direction of v. hence we again take

$$y' = y$$

$$z' = z$$

The time coordinates t *and* t', however are not equal. We can see this substituting the value of x' given by equation (2.5.3) in to equation (2.4.6). We obtain

$$x = k^2(x - vt) + kvt' \quad (2.5.5)$$

From which we find that

$$t' = kt + \left(\frac{1 - k^2}{kv}\right)x \quad (2.5.6)$$

Equations (2.5.3) and (2.5.6) constitute a coordinate transformation that satisfies the first postulate of special relativity.

The second postulate of relativity enables to evaluate k. at instant $t = 0$ the origins of the two frames of reference S *and* S' are in the same place according to our initial conditions, and $t' = 0$ then also. Suppose that a flare is set off at the common origin of S *and* S'at $t = t' = 0$, and the observer in each system proceed to measure the speed with which the light from it spreads out. Both observers must find the same speed$c$, which means that in the S frame

$$x = ct \quad (2.5.7)$$

While in the S' frame

$$x' = ct' \quad (2.5.8)$$

Substituting for x' and t' in equation (2.5.7) with the help of equations (2.5.4) and (2.5.6)

$$k(x - vt) = ckt + \left(\frac{1 - k^2}{kv}\right)cx \quad (2.5.9)$$

And solving for x

$$x = \frac{ckt + vkt}{k - \left(\frac{1 - k^2}{kv}\right)c} \tag{2.5.10}$$

$$= ct\left[\frac{k + \frac{v}{c}k}{k - \left(\frac{1 - k^2}{kv}\right)c}\right]$$

$$= ct\left[\frac{1 + \frac{v}{c}}{1 - \left(\frac{1}{k^2} - 1\right)\frac{c}{v}}\right] \tag{2.5.11}$$

This expression for x will be the same as that given by equation (2.5.11), namely$x = ct$, provided that quantity in the brackets equals1. Therefore

$$\frac{1 + \frac{v}{c}}{1 - \left(\frac{1}{k^2} - 1\right)\frac{c}{v}} = 1$$

$$k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Inserting the above value of k in equations. (2.5.4) and (2.5.6) we have for the complete transformation of measurements of an event made in S to the corresponding measurements made inS', the equations

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2.5.12)$$

These equations comprise the Lorentz transformation. They were first obtained by the Dutch physicist H.A. Lorentz who showed that the basic formulas of electromagnetism are the same in all frames of reference I uniform relative motion only when these transformation equations are used. It was not until a number of years later that Einstein discovered their full significance. it is obvious a that the Lorentz transformation reduces to the Galilean transformation when the relative velocity v is small compared with the velocity of light c.

The relativistic length contraction follows directly from the Lorentz transformation. Let us consider a rod lying along the x' axis in the moving frame S'. An observer in this frame determines the coordinates of its ends to be x_1' and x_2' and so the proper length of the rod is

$$L_0 = x_2' - x_1' \qquad (2.5.13)$$

In order to find $L = x_2 - x_1$ the length of the rod as measured in the stationary frame S at the time t, we make use of equation (2.5.11). we have

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2.5.14)$$

And so

$$L = x_2 - x_1$$

$$= x_2' - x_1' \sqrt{1 - \frac{v^2}{c^2}}$$

$$= L_0 \sqrt{1 - \frac{v^2}{c^2}} \qquad (2.5.15)$$

Which is the same as equation (2.4.6)

So to him the duration of the interval t is

As we found earlier with the help of a light-pulse clock

2.6 velocity addition

One of the postulates of special relativity states that the speed of light c in free space has the same value for all observer, regardless of their relative motion but “common sense” tells us that if we throw a ball forward at 50ft/s from car moving at 80ft/s, the ball’s speed relative to the ground is 130ft/s the similarly sum of the two speeds. Hence we would expect that a ray of light emitted in a frame of reference S' in the direction of its motion at velocity v relative to another frame S will have a speed of $c + v$ as measured in S, contradicting the above postulate. “common sense” is no more reliable as a guide as a guide in science that it is elsewhere, and we must turn to the Lorentz transformation equations for the correct scheme of velocity addition.

Let us consider something moving relative to both S and S'. An observer in S measures its three velocity components to be

$$V_x = \frac{dx}{dt} \quad V_y = \frac{dy}{dt} \quad V_z = \frac{dz}{dt} \qquad (2.6.1)$$

While to an observer in S' they are

$$V_x' = \frac{dx'}{dt} \quad V_y' = \frac{dy'}{dt} \quad V_z' = \frac{dz'}{dt} \tag{2.6.2}$$

By differentiating the inverse Lorentz transformation equation for x, y, z and t we obtain

$$dx = \frac{dx' + vdt'}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2.6.3}$$

$$dy = dy'$$

$$dz = dz'$$

$$dt' = \frac{dt' + \frac{vdt'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2.6.4}$$

$$V_x = \frac{dx}{dt}$$

$$= \frac{dx' + vdt'}{dt' + \frac{vdx'}{c^2}}$$

$$\frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$

$$\frac{V_x' + v}{1 + \frac{vV_x'}{c'}}$$

$$V_y = \frac{V_y'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{vV_x'}{c'}} \tag{2.6.5}$$

$$V_z = \frac{V_z'\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{vV_x'}{c'}} \tag{2.6.6}$$

If $V_x' = c$, that is if a ray of light is emitted in the moving reference frame S' its direction of motion relative to S. An observer in frame S will measure the velocity

$$\frac{V_x' + v}{1+\frac{vV_x'}{c'}} \tag{2.6.7}$$

$$= \frac{c+v}{1+\frac{vc}{c^2}}$$

$$= \frac{c(c+v)}{c+v}$$

$$= c$$

Both observer determine the same value for the speed of light, as they must.

The relativistic velocity transformation has other peculiar consequences. For instance we might imagine wishing to pass a space ship whose speed with respect to the earth in 0.9c at a relative speed of 0.5c. according to conventional mechanics our required speed relative to the earth would have to be 1.4c, more than the velocity of light. According to equation 1.34, however, with $V_x' = 0.5c$ and $v = 0.9c$, the necessary speed is only

$$V_x = \frac{V_x' + v}{1+\frac{vV_x'}{c^2}}$$

$$= \frac{0.5c + 0.9c}{1+\frac{(0.9c)(0.5c)}{c^2}}$$

$$= 0.9655c$$

Which is less that c. We need go less than 10 percent faster than a space ship traveling at 0.9c in order to pass it a relative speed of 0.5c.

2.7 The relativity of Mass

Until now we have been considering only the purely kinematical aspects of special relativity. The dynamical consequences of relativity are at least as remarkable, including as they do the variation of mass with velocity and the equivalence of mass and energy(52,53,54,55)

We begin by considering an elastic collision (that is a collision in which kinetic energy is conserved) between two particles A and B, as witnessed by observers in the reference frames S and S'which are in uniform relative motion. The properties of A and B are identical when determined in reference frames in which they are at rest. The frames S and S' are oriented. With S' moving in the $+x$ direction with respect to S at the velocity v.

Before the collision particle A had been at rest in frame S and particle B in frame S'. Then at the same instant, A was thrown in the $+y$ direction at the speed V_A while B was thrown in the $-y$ direction at the speed V'_B where

$$V_A = V'_B \tag{2.7.1}$$

Hence the behavior of A as seen from S is exactly the same as the behavior of B as seen from S'. When the two particle collide, A rebounds in the $-y$ direction at the speed V_A, while B rebounds in the $+y$ direction at the speed V'_B. If particles are thrown from positions γ apart an observer in S finds that the collision occurs at $y = \frac{1}{2}\gamma$ and one in S' finds that it occurs at $y' = \frac{1}{2}\gamma$

.

The round-trip time T_0 for A as measured in frame S is therefore

$$T_0 = \frac{\gamma}{V_A} \tag{2.7.2}$$

And it is the same for B in S'

$$T_0 = \frac{\gamma}{V_B'} \tag{2.7.3}$$

If momentum is conserved in the S frame, it must be true be that

$$m_A V_A = m_B V_B \tag{2.7.4}$$

Where m_A and m_B the masses of A and B, and V_A and V_B their velocities as measured in the S frame. In S the speed V_B is found from

$$V_B = \frac{\gamma}{T} \tag{2.7.5}$$

Where T is the time required for B to make its round trip as measured in S. in S'however B is trip requires the time T_0, where

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2.7.6}$$

According to our previous results. Although observer in both frame see the same event, they disagree as to the length of time the particle thrown from the other frame requires to make the collision and return.

Replacing T in equation (2.7.5) with is equivalent in terms of T_0 we have

$$V_B = \frac{\gamma\sqrt{1 - \frac{v^2}{c^2}}}{T_0}$$

From equation (2.7.1)

$$V_A = \frac{\gamma}{T_0} \tag{2.7.7}$$

Inserting these expressions for $V_A$and V_B in equation (2.7.5) we see momentum is conserved provided that

$$m_A = m_B \sqrt{1 - \frac{v^2}{c^2}} \qquad (2.7.8)$$

Our original hypothesis was that A and B are identical when at with respect to an observer; the difference between m_A and $m_B$therefore means that measure mints of mass, like those of space and time, depend upon the relative speed between an observer and whatever he is observing.

In the above example both A and B are moving in S. In order to obtain a formula giving the mass m of a body measured while in motion terms of its mass m_0 when measured at rest, we need only consider a similar example in which $V_A$and $V_B'$are very small. In this case an observer in S will see B approach A with velocity v, make a glancing collision(since $V_B' \ll v$). And then continue on. In S

$$m_A = m_0$$

$$m_A = m$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2.7.9)$$

The mass of a body moving at the speed v relative to an observer is larger than its mass when at rest relative to the observer by the factor $\frac{1}{\sqrt{1-v^2/c^2}}$. This mass increase is reciprocal; to an observer in S'

$$m_A = m \qquad (2.7.10)$$

$$m_B = m_0 \qquad (2.7.11)$$

Measured from the earth, a rocket ship in fight is shorter than its twin still on the ground and its mass is greater. To somebody on the rocket ship in flight the ship on the ground also appears shorter and to have a greater mass. (the effect is of course unobservable small for actual rocket speeds). Equation (2.7.9) is plotted

Provided that momentum is defined as

$$mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Conservation of momentum is valid in special relativity just as in classical physics. However Newton's second law of motion is only in the form

$$F = \frac{d}{dt}(mv) \qquad (2.7.12)$$

$$= \frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

This is not equivalent to saying that

$$F = ma$$

$$= m\frac{dv}{dt}$$

Even with m given by equation (1.43) because

$$\frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt} \qquad (2.7.13)$$

And $\frac{dm}{dt}$ dose not vanish if the speed of the body varies with time. The resultant force on a body is always equal to the time rate of change of its momentum.

Relativistic mass increases are increases are significant only at speeds approaching that of light. At a speed one-tenth that of light the mass increase amounts to only 00.5 percent, but this increase is over 100 percent at a speed nine-tenths that of light. Only atomic particles such as electrons, protons, mesons and soon have sufficiently high speeds for relativistic effects to be measurable, and in dealing with these particles the “ordinary” laws of physics cannot be used. Historically the first confirmation of equation (2.7.9) was the discovery by Bucherer in 1908 that the ratio e/m of the electron like the others of special relativity has been verified by so many experiments that it is now recognized as one of the basic formulas of physics.

2.8 Mass and Energy

The most famous relationship Einstein obtained from the postulates of special relativity concerns mass and energy. This relationship can be derived directly from the definition of the kinetic energy T of a moving body as the work done in bringing it from rest to its state motion. That is

$$T = \int_0^3 F ds \qquad (2.8.1)$$

Where F is the component of the applied force in the direction of the displacement ds and s is the distance over which the force acts. Using the relativistic form of the second law of motion

$$F = \frac{d(mv)}{dt} \tag{2.8.2}$$

The expression for kinetic energy becomes

$$T = \int_0^s \frac{d(mv)}{dt} ds$$

$$= \int_0^{mv} vd(mv)$$

$$= \int_0^v vd\left(\frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}\right) \tag{2.8.3}$$

Integrating by parts($\int xdy = xy - \int ydx$)

$$T = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 \int_0^v \frac{vdv}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2.8.4}$$

$$= \frac{m_0 v^2}{\sqrt{1 - \frac{v^2}{c^2}}} + m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}}\Bigg|_0^v$$

$$\frac{m_0 v^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2$$

$$mc^2 - m_0 c^2 \tag{2.8.5}$$

Equation (2.8.5) states that the kinetic energy of a body is equal to the increase in its mass consequent upon its relative motion multiplied by the square of the speed of light.

$$mc^2 = T + m_0 c^2 \tag{2.8.6}$$

Equation (2.8.5) may be rewritten

$$mc^2 = T + m_0c^2 \tag{2.8.7}$$

If we interpret mc^2as the total energy E of the body, it follows that when the body is at rest$T = 0$, it nevertheless possesses the energy m_0c^2. Accordingly m_0c^2 is called the rest energy E_0 of a body whose mass at rest ism_0. Equation (2.8.6) therefore becomes

$$E = E_0 + T \tag{2.8.8}$$

Where

$$E_0 = m_0c^2 \tag{2.8.9}$$

In addition to its kinetic, potential electromagnetic thermal and other familiar guises then energy can manifest as mass. The conversion factor between the unit of mass (kg) and the unit of energy (J) is c^2, so $1kg$ of matter has an energy content of $9 \times 10^2 J$. Even a minute bit of matter represent a vast amount of energy and in fact the conversion of matter into energy it the source of the power liberated in all the exothermic reactions of physics and chemistry.

Since mass and energy are not independent entities the separate conservation principles of energy and mass are properly a single one the principle of conservation of mass energy. Mass can be created or destroyed but when the happens an equivalent amount of energy simultaneously vanishes or comes into being and vice verse. Mass and enegy are different aspects of the same thing.

When the relative speed v is small compared with c, the formula for kinetic energy must reduce to the familiar $\frac{1}{2}m_0v^2$, which has been verified by experiment at low speeds. Let us see whether this is true. The binomial theorem of theorem of algebra tells us that if some quantity x is much smaller than.

$$(1 \pm x) \approx 1 \pm nx \qquad (2.8.10)$$

The relativistic formula for kinetic energy is

$$T = mc^2 - m_0c^2$$

$$= \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0c^2$$

Expanding the first term of this formula with the help of the binomial theorem with $\frac{v^2}{c^2} \ll 1$ since v is much less than c.

$$T = \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)m_0 - m_0c^2$$

$$= \frac{1}{2}m_0v^2 \qquad (2.8.11)$$

Hence at how speeds the relativistic expression for the kinetic energy of a moving particle reduces to the classical one. The total energy of such a particle is

$$E = m_0c^2 + \frac{1}{2}m_0v^2 \qquad (2.8.12)$$

In the foregoing calculation relativity has once again met an important test; has yielded exactly the same results as those of ordinary mechanics at low speeds, where we know by experience that the latter are perfectly valid. It is nevertheless important to keep in mind that, so far as is known the correct formulation of mechanics has its basis in relativity, with classical mechanics no more than an approximation correct only under certain circumstances.

It is often convenient to express several of the relativistic formulas obtained above in forms somewhat different from their original ones. The new equations are so easy to derive that we shall simply state them without proof:

$$E = \sqrt{m_0^2 c^4 + p^2 c^2} \qquad (2.8.13)$$

$$p = m_0 c^2 \sqrt{\frac{1}{1 - \frac{v^2}{c^2}} - 1}$$

$$T = m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left[1 + \left(\frac{T}{m_0 c^2}\right)\right]^2}}$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \sqrt{1 - \frac{p^2}{m_0^2 c^2}}$$

$$= 1 + \frac{T}{m_0^2 c^2} \qquad (2.8.14)$$

The symbol p is used for the magnitude of the linear momentum mv.

These formulas are particularly useful in nuclear and elementary-particle physics where the kinetic energies of moving particle are customarily specified rather than their velocities. Equation (2.8.14) for instance permits us to find v/c directly from T/m_0c^2, the ratio between the kinetic and rest energies of a particle.

Chapter Three

Maxwell Equation

3.1 Introduction

Maxwell's equations describe the relation between electric and magnetic fields. It also describes how they are generated. This chapter is concerned with how they are each other generated and related.

3.2 Gausses Law and First Maxwell's Equation

The magnetic flux ∅ can be written in farms' of electric flux density D in the form (56,57,58)

$$\phi = \int D.ds \qquad (3.2.1)$$

Where ds is the area element also the change ϕ can be expressed in terms of charge density ρ according to the relation

$$\phi = \int \rho dv \qquad (3.2.2)$$

Where dv is the volume element but according to vector algebra

$$\int D.ds = \int \nabla.dv \qquad (3.2.3)$$

Hence

$$\phi = \int \nabla.Ddv \qquad (3.2.4)$$

But Gauss Law states that

$$\phi = Q$$

$$\int \nabla . D dv = \int \rho dv \quad (3.2.5)$$

Thus

$$\nabla . D = \rho \quad (3.2.6)$$

3.3 Amperes Law and Second Maxwell Equation

According to amperes Law the work done by unit magnetic (charge) due to the effect of magnetic flux density B is related to the current density according to the relation (59, 60, 61)

$$\int \mathrm{B}. dL = \mu \int J. ds \quad (3.3.1)$$

Where dL is the Length element, μ the magnetic permeability?

According to vector algebra B satisfies

$$\int \mathrm{B}. dL = \int (\nabla \times B). ds \quad (3.3.2)$$

Thus inserting (3.3.2) in (3.3.1) yields

$$\int (\nabla \times B). ds = \mu \int J. ds$$

Hence a direct comparison of both sides yields

$$\nabla \times \underline{B} = \mu \underline{J}$$

3.4 Faraday Law and Third Maxwell Equation

Faraday Law states that the electromotive force or potential v is related to the magnetic flux ϕ entering the electric circuit, where (62,63,64)

$$V = -\frac{d\phi}{dt} \quad (3.4.1)$$

But V is related to E according to the relation:

$$V = \int E.dL \qquad (3.4.2)$$

Using equations (3.4.1) and (3.4.2) one gets

$$\int E.dL = -\frac{d}{dt}\int B.ds \qquad (3.4.3)$$

Where:

$$\phi = \int B.ds \qquad (3.4.4)$$

Bat from vector algebra

$$\int E.dL = \int (\nabla \times E).ds \qquad (3.4.5)$$

Hence (3.4.5) and (3.4.3) yields

$$\int (\vec{\nabla} \times \underline{B}).ds = -\int \frac{\partial B}{\partial t}.ds \qquad (3.4.6)$$

There fore

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

3.5 Magnetic Flux and Forth Maxwell Equation

The magnetic field is known to form close Loop. Thus the total magnetic flux that enters or Leave any closed Loop vanishes. Hence (65)

$$\phi = \int B.ds = 0 \qquad (3.5.1)$$

But from vector algebra:

$$\int B.ds = \int \nabla.Bdv$$

Hence

$$\int \nabla.Bdv = 0$$

$$\nabla.B = 0$$

Chapter four

Literature Review

4.1 introduction

The SR theory is one of the biggest achievements in physics. It changes our view of space and time coordinates. The SR theory succeeded in explaining a wide variety of physical phenomena but it failed in explaining the situations in which the fields are involved. Many attempts were made to account for the effect of fields (66).The most popular one is known as GSR (67, 68, 69)

4.2 The Special Relativity in the Presence of Gravitational and other Fields

The gravitational field system properties was discussed in many standard texts [70] .n these text the equation of motion of matter in gravitational and the matter energy momentum tensor are treated separately. The equation of motion of matter is obtained either by expressing the equation of motion of straight line in curvilinear coordinate system [4] or by minimizing the proper time [2] or even by using Euler-Lagrange equations [3].The energy momentum tensor of matter was found by generalizing its special relativistic form in a curved space [4]. This situation is not inconformity with the classical field theories, where the equation of motion and the expression for the energy momentum tensor stem from only one action and from the same Lagrangian [70]

On the other hand the physical properties of matter; like time, length, and mass; in special relativity (SR) are incomplete for not recognizing the effect of fields on them [6].

Many attempts were made to modify SR to include the effect of gravity and other fields [7, 8, 9]. The attempts concentrate on the notion of mass and energy without accounting the influence of both fields and motion on time and length. Using the ordinary classical Euler-Lagrange equations [5] a full expression for the equation of motion of matter in an arbitrary gravitational field and the energy momentum tensor are obtained from the same Lagrangian in section (2). Stemming from General Relativity (GR) the effect of gravitation and other fields on time, length and mass are obtained in section (3).

The Equation of motion and the Energy-Momentum Tensor for Matter:

Using the action principle a useful expression for the energy-momentum tensor of matter in form of a perfect fluid as well as the equation of motion of matter, in particle form, in the gravitational field can be obtained .By variation of the matter action the equation of motion and the energy-momentum tensor can be derived.

Taking the field variables to be x^k and assuming the Lagrangian to depend only on x^k and its first derivate it follows that (71)

$$\mathcal{L} = \mathcal{L}(x^k, U^\mu) \tag{4.2.1}$$

With

$$U^\mu = \frac{dx^\mu}{dt}$$

Being the four-velocity, and t the proper time.

To obtain the energy-momentum tensor of a perfect fluid we choose the Lagrangian of matter to have the form:

$$\mathcal{L} = A_1 + A_2 g_{\mu\nu} U^\mu U^\nu \tag{4.2.2}$$

Where the parameters A_1 and A_2 are independent of the metric $g_{\mu\nu}$ and the velocity U^{μ} .

The energy-momentum tensor of matter is given to be

$$T_{\rho\sigma} = g_{\rho\sigma}\mathcal{L} - g_{\lambda\sigma} \frac{\partial \mathcal{L}}{\partial \partial_{\lambda} x^{k}} \partial_{\rho} x^{k}$$

$$= g_{\rho\sigma}\mathcal{L} - g_{\lambda\sigma} \frac{\partial \mathcal{L}}{\delta_{\lambda}^{\mu} \partial U^{k}} \delta_{\rho}^{\mu} U^{k}$$

$$= g_{\rho\sigma}\mathcal{L} - g_{\lambda\sigma} \frac{\partial \mathcal{L}}{\partial U^{\rho}} U^{\lambda} \qquad (4.2.3)$$

According to formula

$$g_{\mu\nu} U^{\mu} U^{\nu} = -1$$

Then the Lagrangian becomes

$$\mathcal{L} = A_1 - A_2 \qquad (4.2.4)$$

Using this equation and inserting (4) in equation (3) yields.

$$T_{\rho\sigma} = g_{\rho\sigma}(A_1 - A_2) - 2A_2 U_{\rho} U_{\sigma} \qquad (4.2.5)$$

If we set

$$A_1 - A_2 \equiv p \,, A_1 + A_2 = -\rho \qquad (4.2.6)$$

Then

$$T_{\rho\sigma} = g_{\rho\sigma} p + (\rho + p) U_{\rho} U_{\sigma} \qquad (4.2.7)$$

Which is the expression for the energy-momentum tensor of matter in a perfect fluid form [1].

The equation of motion can be obtained by using Euler-Lagrange equation, where

$$\frac{\partial \mathcal{L}}{\partial x^k} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial U^k}\right) = 0 \qquad (4.2.8)$$

Using equations (6) and (2) the various terms in the equation of motion are given by,

$$\frac{\partial \mathcal{L}}{\partial x^k} = \frac{\partial A_1}{\partial x^k} + g_{\mu\nu} U^\mu U^\nu \frac{\partial A_2}{\partial x^k} + A_2 U^\mu U^\nu \frac{\partial g_{\mu\nu}}{\partial x^k}$$

$$= \partial \frac{(A_1 - A_2)}{\partial x^k} + A_2 U^\mu U^\nu \frac{\partial g_{\mu\nu}}{\partial x^k}$$

$$= \frac{\partial p}{\partial x^k} + A_2 U^\mu U^\nu \frac{\partial g_{\mu\nu}}{\partial x^k} \qquad (4.2.9)$$

And

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial U^k}\right) = 2\frac{d}{dt}\left[A_2 g_{\mu k} U^\mu\right]$$

$$= 2U^\lambda U_k \frac{dA_2}{dx^\lambda} + 2A_2 \frac{\partial g_{\mu k}}{\partial x^\lambda} U^\mu U^\lambda + 2A_2 g_{\mu k} \frac{dU^\mu}{dt}$$

$$\lambda \rightarrow \nu, \lambda \rightarrow \mu \ and \ \mu \rightarrow \nu$$

We get

$$U^\mu U^\lambda \frac{\partial g_{k\mu}}{\partial x^\lambda} = \frac{1}{2}\left[U^\mu U^\nu \frac{\partial g_{k\mu}}{\partial x^\nu} + U^\mu U^\nu \frac{\partial g_{k\nu}}{\partial x^\mu}\right]$$

Hence

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial U^k}\right) = A_2 U^\mu U^\nu \left[\frac{\partial g_{k\mu}}{\partial x^\nu} + \frac{\partial g_{k\nu}}{\partial x^\mu}\right] + 2A_2 g_{\mu k} \frac{dU^\mu}{dt} + 2U^\lambda U_k \frac{dA_2}{dx^\lambda} \qquad (4.2.10)$$

The equation of motion is then given by substituting equation (4.2.9) and (4.2.10) in equation (4.2.8) and by multiplying both sides by $g^{k\lambda}$ to get

$$-2A_2 U^\mu U^\nu \frac{g^{k\lambda}}{2}\left[\frac{\partial g_{k\mu}}{\partial x^\nu} + \frac{\partial g_{k\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^k}\right]$$

$$-2A_2 \delta^\lambda_\mu \frac{dU^\mu}{dt} - 2g^{k\lambda} U_\lambda U_k \frac{dA_2}{dx^\lambda} + g^{k\lambda} \frac{dp}{dx^k} = 0$$

When we consider the motion of a point test particle of small mass, the pressure p vanishes and the density variation is negligible. Therefore by equation (6) we get

$$P = 0, \rho = constant, A_1 = A_2 = \frac{-\rho}{2} \quad (4.2.11)$$

The equation of motion of matter in a gravitational field is then given by [15, 16, and 17].

$$\Gamma^{\lambda}_{\mu\nu}\frac{dx^{\mu}}{dt}\frac{dx^{\nu}}{dt} + \frac{d^2x^{\lambda}}{dt^2} = 0 \quad (4.2.12)$$

It is very interesting to note that this expression obtained from the matter action represents an alternative derivation of the geodesic equation.

3. Special Relativity in the presence of Gravitation:

In SR the time, length and mass can be obtained in any moving frame by either multiplying or dividing their values in the rest frame by a factor γ

$$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

To see how gravity effect these quantities it is convenient to re express γ in terms of the proper time (4)

$$c^2 d\tau^2 - g_{\mu\nu}dx^{\mu}dx^{\nu} \quad (4.2.13)$$

Which is a common language to both SR and GR. We know that in SR (4.2.13) reduces to [10]

$$c^2 d\tau^2 = c^2 dt^2 - dx^i dx^i, xx^0 = ct$$

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{1}{c^2}\frac{dx^i}{dt}\frac{dx^i}{dt}} = \sqrt{1 - \frac{v^2}{c^2}} = \gamma \qquad (4.2.14)$$

Thus we can easily generalized γ to include the effect of gravitation by using (4.2.13) and adopting the weak field approximation where [11]

$$g_{11} = g_{22} = g_{33} = -1, g_{00} = 1 + \frac{2\Phi}{c^2} \qquad (4.2.15)$$

$$\gamma = \frac{d\tau}{dt} = \sqrt{g_{00} - \frac{1}{c^2}\frac{dx^i}{dt}\frac{dx^i}{dt}} = \sqrt{g_{00} - \frac{v^2}{c^2}} \qquad (4.2.16)$$

When the effect of motion only is considered, the expression for time in SR take the form [4]

$$dt = \frac{dt_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (4.2.17)$$

Where the subscript 0 stands for quality measured in a rest frame. While if gravity only affect time, its expression is given by [11].

$$dt = \frac{dt_0}{\sqrt{g_{00}}} \qquad (4.2.18)$$

In view of equation (4.2.17), (4.2.18) and (4.2.16) the expression

$$dt = \frac{dt_0}{\gamma} \qquad (4.2.19)$$

Can be generalized to recognize the effect of motion as well as gravity on time, to get

$$dt = \frac{dt_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \qquad (4.2.20)$$

$$V = V_0\sqrt{1 - \frac{v^2}{c^2}} \qquad (4.2.21)$$

$$V = \sqrt{g}V_0 = \sqrt{g_{00}}V_0 \qquad (4.2.22)$$

The generalization can be done by utilizing (4.2.14) and (4.2.16) to find that

$$V = \gamma V_0 = \sqrt{g_{00} - \frac{v^2}{c^2}}V_0 \qquad (4.2.23)$$

To generalized the concept of mass to include the effect of gravitation we use the express for the Hmiltonian in GR, i.e. [13].

$$H = \rho c^2 = g_{00}T^{00} = g_{00}\rho_0\left(\frac{dx^0}{dt}\right)^2 = g_{00}\frac{\rho_0 c^2}{\gamma^2}$$

$$g_{00}\frac{m_0 c^2}{v_0\gamma^2} \qquad (4.2.24)$$

Using equation (4.2.23) and (4.2.24) yields

$$\rho c^2 = \frac{mc^2}{v} = \frac{g_{00}m_0}{\gamma v} \qquad (4.2.25)$$

Therefore

$$m = \frac{g_{00}m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \qquad (4.2.26)$$

Which is the expression of mass in the presence of gravitation.

Using equations (4.2.15) and (4.2.26) when the field is weak and the speed is small, the energy E is given by

$$E = mc^2 = m_0 g_{00}\left(g_{00} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \qquad (4.2.27)$$

In the weak field

$$E = m_0\left(1+\frac{2\phi}{c^2}\right)\left(1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} C^2$$

$$\approx m_0(1)\left(1-\frac{2\phi}{c^2}+\frac{1}{2}\frac{v^2}{c^2}\right)c^2$$

$$E = m_0c^2 + \frac{1}{2}m_0v^2 - m_0\phi$$

$$E = m_0c^2 + T + V \qquad (4.2.28)$$

Unlike SR which doesn't include the potential energy, equation (28) shows that the energy is reduced to the classical expressions which include potential energy.

$$V = -m_0\phi$$

According to general relativity (GR) and standard model (SM) the effect of the field on physical quantities manifests itself via the space. The space deformation in our model manifests itself through the ϕ which can be given with the aid of equation (13) and (16) to be.

$$\phi = \frac{dT}{dt} = \sqrt{\frac{g_{\mu\nu}}{c^2}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}} = \sqrt{\frac{g_{00}c^2}{c^2}\frac{dt^2}{dt^2} - \frac{1}{c^2}g_{\alpha}\beta v^\alpha v^\beta}$$

$$\gamma = \sqrt{g_{00} - \frac{g\alpha\beta}{c^2} v^\alpha v^\beta} \qquad (4.2.29)$$

Where $\alpha = 1,2,3$

The effect of the field on gamma is incorporated in the deformation parameters $g_{\alpha\beta}$ andg_{00}. According to SM [14] the presence of the gauge fields W_μ and B_μ deform the space by changing the ordinary derivative ∂_μ to the covariant derivative D_μ.i.e

$$D_\mu = \partial_\mu + igl.W_\mu + i\left(\frac{g}{2}\right)YB_\mu \qquad (4.2.30)$$

Where the factors g, g, I and Y are parameters determining the nature of interaction. On the other hand the covariant derivative in GR [1] is given by

$$D_\mu = \partial_\mu - \Gamma_{\mu\nu}^{\lambda} \qquad (4.2.31)$$

Where

$$\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2} g\left[\partial_\mu g_{k\nu} + \partial_\nu g_{k\mu}\ \right] \qquad (4.2.32)$$

The relation between the metric $g_{\mu\nu}$ and field can be obtained from relation (4.2.30) and (4.2.31) with the aid of the relation

$$\partial_\lambda g_{\mu\nu} - \Gamma_{\mu\nu}^{\rho} g_{\rho\mu} = g_{\mu\nu;\lambda} \qquad (4.2.33)$$

Where (4.2.30) and (4.2.31) gives:

$$\Gamma_{\mu\nu}^{\lambda} = -igl.W_\mu - i\left(\frac{g}{2}\right)\lambda B_\mu \qquad (4.2.34)$$

According to these relations the genenralized expression for the time volume, mass, and energy is given according to equaions (4.2.29) (4.2.19), (4.2.23) with the aid of the relation

$$dt = \frac{dt_0}{\gamma} \tag{4.2.35}$$

$$m = \frac{g_{00}\, m_{00}}{\gamma} \tag{4.2.36}$$

4.3 New Lorentz Field Dependent Lorentz Transformation Due to Photon Direction Change

Lorentz transformation is one of the most beautiful mathematical framework that changed radically the concept of space time and mass. The SR one is suitable for inertial frames but it doesn't not account for the effect of fields. To take care of the effect of fields consider the Lorentz transformation [72]

$$x = \gamma\left(\grave{x} + v\grave{t} - \frac{a\grave{t}^2}{2}\right) \tag{4.3.1}$$

$$\grave{x} = \gamma \quad x - vt + \frac{at^2}{2} \tag{4.3.2}$$

Consider the two frames $(x.\ t)$ and $(\grave{x}.\ \grave{t})$ have their origin coincide at = $\grave{t}$ = 0. If a pulse of light is received from a source S then its position in the two frames becomes at t and $t^{`}$ respectively

$$x = ct \qquad (4.3.3 \cdot a)$$

$$x = c\grave{t} \qquad (4.3.3 \cdot b)$$

Substitute (4.3.3.a) & (4.3.3.b) in (4.3.1) yields

$$ct = \gamma\left(ct' + vt' - \frac{at^2}{2}\right) = \gamma\left((c+v)t' - \frac{at'^2}{2}\right)$$

$$t = \gamma\left(\left(1 + \frac{v}{c}\right)t' - \frac{at^2}{2c}\right)$$

$$t = c_1 t' + C_2 t'^2 \quad (4.3.4)$$

Where

$$C_1 = \gamma\left(1 + \frac{v}{c}\right)(5 \cdot a)$$

$$C_2 = -\frac{\gamma a}{}(5 \cdot b)$$

$$C_1 = \gamma\left(1 + \frac{v}{c}\right) \quad (4.3.5\ a)$$

$$C_2 = -\frac{\gamma a}{c} \quad (4.3.5\ b)$$

In (4.3.2) gives

$$c\grave{t} = \gamma \quad ct - vt + \frac{at}{2}$$

$$\grave{t} = \gamma\left(\left(1 - \frac{v}{c}\right)t + \frac{at^2}{2c}\right)$$

2

$$\acute{t} = C_3 t + C_4 t^2 \quad (4.3.6)$$

$$C_3 = \gamma\left(1 - \frac{v}{c}\right) \quad (4.3.7.a)$$

$$C_4 = \frac{\gamma a}{2C} \quad (4.3.7.b)$$

Substitute (4.3.6) in (4.3.4) to get

$$t = C_1(C_3 t + C_4 t^2) + C_2(C_3 t + C_4 t^2)^2 \quad (4.3.8)$$

$$t = C_1 C_2 t + C_1 C_4 t^2 + C_2 C_3 2t^2 + 2C_2 C_3 C_4 t^3 + C_2 C_4 2t^4 \quad (4.3.9)$$

Comparing the coefficients of t. t^2. $t^3$and t^4 on both sides gives

$$C_1 C_3 = 1 \quad (4.3.10)$$

$$C_1 C_4 = -C_2 C_3^2 \quad (4.3.11)$$

$$2C_2 C_3 C_4 = 0 \quad (4.3.12)$$

$$C_2 C_4^2 = 0 \quad (4.3.13)$$

From (4.3.5.a) and (4.3.7. a), (4.3.10) becomes

$$\gamma^2\left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right)=1$$

$$\therefore \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (4.3.14)$$

$$\gamma^2\left(1+\frac{v}{\ }\right)\frac{a}{\ } = \frac{\gamma\ \ a}{\ }\left(1-\frac{v}{\ }\right) \qquad (4.3.15)$$

From (4.3.5.b), (4.3.7.a) and (4.3.7.b), (4.3.12) becomes

$$-\frac{2\gamma^3 a^2}{4C^2}\left(1-\frac{v}{c}\right) \qquad (4.316)$$

From (4.3.5.b) and (4.3.7.b), (4.3.13) becomes

$$-\frac{\gamma a}{8c^3}=0 \qquad (4.3.17)$$

In view of equation (4.3.14) γ take the same special relativity form. However, equations (4.3.15), (4.3.16) and (4.3.17) shows that the Lorentz transformation (4.3.1) and (4.3.2) gives consistent results only when (a = 0). This requires trying another transformation to take care of effect of fields. One can assume that the light is accelerated due to the effect of field on photon trajectory. It is well known in mechanics that any particle can be accelerated if its magnitude of velocity v is constant when it change its direction. This happens for particles having constant speed v and moving in a circular orbit, thus changing its direction regularly and possessing an acceleration

$$a=\frac{v^2}{r} \qquad (4.3.18)$$

Towards the Centre of a circular orbit. According to general relativity (GR) the photon move in a curved trajectory in a gravitational field, although the magnitude of photon speed c is constant, but it is accelerated due to the change of photon direction, since the change of photon direction decreases its speed in the original direction. For example if the photon change its direction by $\Delta\emptyset$ during time intervalΔt, its acceleration becomes

$$a = \frac{\Delta c}{\Delta t} = \frac{c - csin\Delta\emptyset}{\Delta t} \approx \frac{c(1-\Delta\emptyset)}{\Delta t} \qquad (4.3.19)$$

This means that SR and GR are not in conflict with each other, this shows how beauty is Einstein relativity compared to Newton's laws. The photon acceleration can be found by using the relation between work done and energy change according to gravity red shift. The change in photon energy is given by

$$\Delta E = hf - hf = V \qquad (4.3.20)$$

Where V is the field potential. Here one assume that V is potential of any field; not gravity field only. The change of energy is equal to the work done, again assuming constant mass and constant acceleration, one gets

$$F \cdot x = max = V \qquad (4.3.21)$$

The photon displacement can be found by using the expression for photon interval in a curved space, to get

$$0 = c^2 d\tau^2 = g_{00} c^2 dt^2 - g_{xx} dx^2 \qquad (4.3.22)$$

Assuming that the photon obeys static isotropic constraints $g_{00} = g_{xx}$, one gets

$$dx^2 = g_{00}^2 c^2 dt^2 = \left(1 + \frac{2\varphi}{c^2}\right)^2 c^2 dt^2 \qquad (4.3.23)$$

$$dx = \left(1 + \frac{2\varphi}{c^2}\right) cdt$$

Thus integrating both sides yields

$$x = \left(1 + \frac{2\varphi}{c^2}\right) ct \qquad (4.3.24)$$

Similar relation can be obtained by finding the photon acceleration by assuming$x = ct$ to get

$$a = \frac{V}{mx} = \frac{\varphi}{x} = \frac{\varphi}{ct} \tag{4.3.25}$$

In view of equation (4.3.3.a) and (4.3.25) the position is given by

$$x = ct - \frac{at}{2} = ct - \frac{\varphi t}{2ct} = ct - \frac{\varphi}{2c}t \tag{4.3.26}$$

Similarly, equation (4.3.3.b) and (4.3.25) gives

$$\grave{x} = c\grave{t} + \frac{at}{2} = c\grave{t} + \frac{\varphi t}{2\grave{x}} = c\grave{t} + \frac{\varphi}{2c}\grave{t} \tag{4.3.27}$$

Consider the Lorentz transformation

$$x = \gamma\left(\grave{x} + v\grave{t} - \frac{a\grave{t}^2}{2}\right) \tag{4.3.28}$$

Where the average velocity v_m is given by

$$v_m = \frac{v + v_0}{2} = \frac{v + v - a\grave{t}}{2} = v - \frac{a\grave{t}}{2} \tag{4.3.30}$$

Thus

$$\grave{l} = v\grave{t}^2 - \frac{a\grave{t}^2}{2} = \left(v - \frac{a\grave{t}}{2}\right)\grave{t} = v_m\grave{t} \tag{4.3.31}$$

$$x = \gamma(x' + v_m t') \tag{4.3.31}$$

Similarly

$$\grave{x} = (x - v_m t) \tag{4.3.32}$$

For static source in a frame S the photon is not accelerated, thus

$$x = ct \tag{4.3.33}$$

But the observer in $\grave{S}$ sees the source S is accelerated and the photon moves in curved space, thus (see equation (4.3.14))

$$\grave{x} = c\grave{t} + \frac{\varphi}{2c}\grave{t} \tag{4.3.34}$$

By substituting (4.3.33) and (4.3.34) in (31) yields

$$ct = \gamma\left(c + \frac{\varphi}{2c} + v_m\right)\grave{t} \tag{4.3.35}$$

Similarly if the source is at rest in frame $\grave{S}$ the photon position is given by

$$\grave{x} = c\grave{t} \tag{4.3.36}$$

Since $\dot{S}$ is accelerated with respect to S due to the field effect, therefore the photon move in a curved space , thus it is accelerated, hence

$$x = ct - \frac{\varphi}{2c}t \tag{4.3.37}$$

Substitute (4.3.36), (4.3.37) in (4.3.32) to get

$$c\grave{t} = \gamma\left(c - \frac{\varphi}{2c} - v_m\right)t \tag{4.3.38}$$

From (4.3.34) and (4.3.37)

$$t' = \frac{t'}{c}\gamma^2\left(c + \frac{\varphi}{2c} + v_m\right)\left(c - \frac{\varphi}{2c} - v_m\right) \tag{4.3.39}$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{\varphi}{2c} + \frac{v_m}{c}\right)^2}} \tag{4.3.40}$$

Thus the generalized special relativistic energy is given by

$$E = \gamma m_0 c^2 = \frac{m_0 c^2}{1 - \quad \frac{\varphi}{2} + \frac{v_m}{c} \quad ^2} \tag{4.3.41}$$

Neglect the term consisting of c^2, yields

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_m^2}{c^2}}} \tag{4.3.42}$$

Where

$$v_m = v + \frac{v_0}{2} \tag{4.3.43}$$

But when the particle moves against the field

$$v^2 = v_0^2 - 2ax = v_0^2 - 2\varphi \therefore v_0^2 = v^2 + 2\varphi \tag{4.3.44}$$

By Assuming that $v$and v_0 represent the average values that related to maximum values v_{max} and v_{0max} according to relations $v = \frac{v_{max}}{\sqrt{2}}$ and

$$v_0 = \frac{v_{0max}}{\sqrt{2}}$$

$$\therefore v_{0max}^2 = v_{max}^2 + 4ax \tag{4.3.45}$$

Then

$$v_m^2 = \left(\frac{v_{max} + v_{0max}}{2}\right)^2 = \frac{v_{max}^2 + 2v_{max}v_{0max} + v_{0max}^2}{4}$$

$$= \frac{v_{max}^2 + 2v_{max}\sqrt{v_{max}^2 + 4ax} + v_{max}^2 + 4ax}{4}$$

$$v_m^2 = \frac{2v_{max}^2 + 2v_{max}^2\sqrt{1 + \frac{4ax}{v_{max}^2}} + 4ax}{4}$$

$$\approx \frac{2v_{max}^2 + 2v_{max}^2\left(1 + \frac{2ax}{v_{max}^2}\right) + 4ax}{4} \qquad (4.3.46)$$

$$\therefore v_m^2 = \frac{4v_{max}^2 + 8ax}{4} = v_{max}^2 + 2ax = v_{max}^2 + 2\varphi \qquad (4.3.47)$$

But from equation (4.3.38) for $\frac{v_m^2}{c^2} < 1$ then

$$\gamma = 1 + \frac{v_m^2}{2c^2} = 1 + \frac{v_{max}^2 + 2\varphi}{2c^2} \qquad (4.3.48)$$

$$\therefore \gamma = 1 + \frac{1}{C^2}\left(\frac{v_{max}^2}{2} + \varphi\right) \qquad (4.3.49)$$

Thus equation (4.3.41) and (4.3.38) gives

$$E = \gamma m_0 c^2 = m_0 c^2 + \frac{1}{2} m_0 v_{max}^2 + m_0 \varphi = m_0 c^2 + T + V \qquad (4.3.50)$$

Thus the generalized special relativity energy relation satisfies the Newtonian limit. This is since the energy include kinetic beside potential energy term.

The gravitational red shift of photons can also be explained by using GSR. Assuming photon in free space so its potential energy $V = 0$, by using (4.3.50) and plank hypothesis, one can get

$$hf = m_0 c^2 + T \qquad (4.3.51)$$

If the photon enters gravitational field its frequency (4.3.51) changes also To $\grave{f}$. Thus equation (4.3.50) gives

$$h\grave{f} = m_0 c^2 + T + V = hf + V \qquad (4.3.52)$$

Thus fortunately equation (48) explains the gravitational red shift.

4.4 Lorentz Transformation Einstein Derivation simplified

Those who have studied Einstein's special relativity theory know that everything there is the result of his two postulates and of the distant clock synchronization procedure that he

(x, y=0) located at that point reads t. In order to be operational the different clocks of that frame, located along the OX axis should display the same running time. Einstein satisfied that condition proposing the synchronization procedure shown in the figure below [75]

Clock $C_0(0,0)$ located at the origin O is ticking and when it reads a zero time the source of light S(0,0) located in front of it emits short light signals in the positive and in the negative directions of the OX axis. Clocks $C_+(0,0)$ and $C_-(0,0)$ are initially stopped and fixed to display a time t=x/c. The light signals arriving at the corresponding clocks start them and from that very moment the clocks display the same running time. The events associated with the synchronization of clocks C_0, C_+ and C_- are $E_0(0,0,t)$, $E_+(x,0,t)$ and $E_-(-x,0,t)$ respectively. It is obvious that their space-time coordinates are related by

$$x = \pm ct \quad (t > 0) \qquad (4.4.1)$$

Or by

$$x^2 - c_2 t^2 = 0 \qquad (4.4.2)$$

Special relativity becomes involved when we consider a second inertial reference frame K' (X'O'Y') in the standard arrangement with the K (XOY) reference frame, K' moving with constant velocity V in the positive direction of the overlapped OX (O'X') axes. The events associated with the synchronization of the clocks in K' are $E_0'(0,0,t)$, $E_+'(x',0,t')$ and $E_-'(-x',0,t')$.

The clocks $C_0'(0,0)$, $C_+'(x',0)$ and $C_-'(0,0)$ of that frame are synchronized following the same procedure as in K and we have obviously

$$x'^2 - c'^2 t'^2 = 0. \tag{4.4.3}$$

Equating (4.4.2) and (4.4.3) we obtain

$$x^2 - c^2 t^2 = x'^2 - c^2 t'^2. \tag{4.4.4}$$

Because at the origin of time the origins of K and K' are located at the same point in space we can consider that Δx=x-0, Δt=t-0, Δx'=x'-0 and Δt'=t'-0 presenting (4.4.4) as

$$(\Delta x)^2 - c^2(\Delta t)^2 = (\Delta x')^2 - c^2(\Delta t')^2 \tag{4.4.5}$$

Equation (4.4.5) is a starting point in Einstein's derivation of the Lorentz transformations[1] which establish a relationship between the space-time coordinates of events E(x,0.t) and E'(x',0,t').

Relativists consider that one event E(x,0,t) detected from the K frame and an event E'(x',0,t') detected from the K' frame represent the **same** event if they take place at the same point in space when the clocks C(x,0) and C'(x',t') located at that point read t and t' respectively. The Lorentz transformations establish a relationship between the coordinates of events E(x,0,t) and $E'(x',0,t)$ defined above and considered to represent the same event. We derive them in two steps. Figure 2 presents the relative position of the reference frames K and K' as detected from the K frame when its clocks read t.

When the clock $C_0'(0,0)$ is reading t' it is located in front of a clock

C_1(x=Vt,0) reading t. The problem is to establish a relationship between Δt and Δt'. The clock $C_0'(0,0)$ being in a state of rest in K' we have in its case Δx'=0. The position of clock $C_0'(0,0)$ is defined in K by Δx=VΔt , the change in the reading of clock C_1(x=Vt,0) being Δt . The events involved are E(x=Vt,t) in K and (x'=0,t') in K'. Imposing the condition (4.4.4) that relates correctly their space-time coordinates we obtain

$$t = \frac{t'}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4.4.6}$$

Which relates the readings of the two clocks when they are located at the same point in space, equation

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4.4.7}$$

Relating the changes in theirs readings.[2] It is of essential importance to make a net distinction between the ways in which the time intervals Δt and Δt' are measured.

The time interval Δt is measured as a difference between the reading t of clock C(x=V_t,0) and the reading t=0 for clock C_0(0,0) when the moving clock C_0'(0,0) passes in front of them respectively. Relativists call a time interval measured under such conditions **coordinate time interval.** The time interval Δt' is measured as a difference between the readings of the same clock C_0'(0,0) when it passes in front of clock C_1(x=V_t,0), (t') and when it passes in front of clock C_0(0,0) (t'=0). A time interval measured under such conditions is called **proper time interval**. As we see (4.4.7) relates a coordinate time interval measured in the reference frame K and a proper time interval measured in K. Because Δt>Δt' relativists say that a **time dilation effect** takes place. If we consider the same experiment from the inertial reference frame K' then we see that observers of that reference frame measure a coordinate time interval whereas observers from K measure a proper time interval related by (4.4.7). Figure 3 presents the relative positions of the reference frames K and K' when all the clocks of the first frame read t.

Encouraged by Galileo's transformation equations

$$x = x' + Vt' \qquad 4.4.(8)$$

$$x' = x - Vt \qquad (4.4.9)$$

$$t = t' \qquad (4.4.10)$$

we guess that in Einstein's special relativity theory, one of the transformation equations should have the shape

$$x = ax' + cbt' \qquad (4.4.11)$$

where a and b represent factors which, due to the linear character of a transformation equation, could depend on the relative velocity V but not on the space-time coordinates of the involved events. In order to find them we impose the condition that it should correctly relate the space-time coordinates of events E(x=Vt,0,t) and E'(x'=0,0,t') and of events E'(x'=-Vt',0,t'), E(0,0,t) we have defined deriving the formula which accounts for the time dilation effect. In the case of the first pair of events (4.4.11) Works as

$$Vt = bct' = bct\sqrt{1 - \frac{v^2}{c^2}} \qquad (4.4.12)$$

Where from we obtain

$$b = \frac{\frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} = \beta c^{-1}\gamma(V) \qquad (4.4.13)$$

In the case of the second pair of events (4.4.11) works as

$$0 = -aVt' + cbt' \qquad (4.4.14)$$

Resulting that

$$a = \gamma(V) \qquad (4.4.15)$$

(4.4.11) becoming

$$x = \gamma(V)(x' + Vt') . \qquad (4.4.16)$$

Dividing both sides of (4.4.16) by c and taking into account that all the involved clocks are synchronized à la Einstein (t=x/c,t'=x'/c) we obtain

$$t = \gamma(V)(t' + \beta c^{-1} x') \quad (4.4.17)$$

Combing (4.4.16) and (4.4.17) we obtain with some algebra

$$x' = \gamma(V)(x - Vt) \quad (4.4.18)$$

$$t' = \gamma(V)(t - \beta c^{-1} x). \quad (4.4.19)$$

Equations (4.4.16) and (4.4.17) are known as the **inverse Lorentz transformations** whereas equations (4.4.18) and (4.4.19) are known as the **direct Lorentz transformations.**

Compared with Einstein's derivation and with other derivations we found in the literature of this subject, our derivation presents the advantage that it is shorter, revealing the fact that the Lorentz transformations are a direct consequence of the two relativistic postulates and of the clock synchronization procedure proposed by Einstein.

The Lorentz transformations become more transparent if we present them as a function of changes in the space-time coordinates of the same event. Equations (4.4.16) and (4.4.17) become

$$\Delta x = \gamma(V)(\Delta x' + V\Delta t') \quad (4.4.20)$$

And

$$\Delta t = \gamma(V)(\Delta t' + Vc^{-2}\Delta x') . \quad (4.4.21)$$

The way in which the transformation equations were derived ensures the fact that they account for the time dilation effect. They account in a transparent way for the addition law of relativistic velocities. Consider a particle that starts to move at t=t'=0 from the common origin of K and K' with speed u_x relative to K and with speed u'_x relative to K'. After a time of motion t the particle generates the event E(x=u_xt,0,t) as detected from K and E'($x' = u'_x t',0,t'$) when detected from K'. In accordance with the Lorentz transformations we have

$$u'x \ + V$$

$$\Delta x = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} \tag{4.4.22}$$

4.5 Evolution of Stars by Kinetic Theory and Quantum Physics on the Basis Generalized Special Relativity

Let us now discuss ideal gases from a purely quantum mechanical standpoint. It turns out that this approach is necessary to deal with either low temperature or high density gases. Furthermore, it also allows us to investigate completely nonclassical"gases", such as photons. From the kinetic theory and quantum physics; we can get an equation of star evolution by the pressure force and the force of gravity. For stars one have tow forces, pressure force which counter balance the gravity force, thus [76]:

$$P = \frac{1}{3}nmv^2 \quad , \quad mv^2 = 3KT \tag{4.5.1}$$

The number density can be assumed to satisfy Maxwell's distribution

$$n = n_0 e^{-\beta} \tag{4.5.2}$$

We first consider an ideal gas consisting of a single type of non-relativistic particles. The ideal-gas law for the gas contained in a volume V is commonly written as

$$P = \frac{1}{3}\frac{N}{V}(3KT) = nKT \tag{4.5.3}$$

Where: $n = N/V(3KT)$ is the number of particles per unit volume). Thus the pressure force is given by

$$F_P = PA = (4\pi r^2) = 4\pi nKTr^2 = c_1 r^2 \tag{4.5.4}$$

The gravity force is given by

$$F_g = \int \left(\frac{4\pi}{3}\rho r^3\right)(4\pi r^2 \rho)\, dr$$

For constant density

$$F_g = \frac{(4\pi)^2}{3}\rho^2 \int_0^r r^5 dr = \frac{1}{6}\frac{(4\pi)^2}{3}\rho^2 r^6$$

Thus

$$F_g = \frac{8}{9}\pi^2\rho^2 r^6 = c_2 r^6 \qquad (4.5.5)$$

Equation of hydrostatic equilibrium requires

$$F_P = F \qquad (4.5.6)$$

Thus from equation (4.5.4), (4.5.5) and (4.5.6) one gets

$$c_1 r^2 = c_2 r^6 \qquad \Rightarrow \qquad r = \left(\frac{c_1}{c_2}\right)^{1/4}$$

The critical radius is thus given by

$$r_c = \left(\frac{9nKT}{2\pi\rho^2}\right)^{1/4} \qquad (4.5.7)$$

Expansion takes place

$$F_P > F \qquad (4.5.8)$$

While contraction is observed when

$$F_P < F \qquad (4.5.9)$$

But according to the laws of quantum mechanics for particle in box the energy is given by

$$E = c_0 V^{-2/3} \qquad (4.5.10)$$

At $T = 0$ all quantum states whose energy is less than the Fermi energy E_F are filled. The Fermi energy corresponds to a Fermi momentum $p_F = \hbar k_F$ is thus given by

$$E_F = \frac{p_F^2}{2m} = \frac{\hbar^2 k_F^2}{2m} \qquad (4.5.11)$$

The above expression can be rearranged to give

$$k_F = (3\pi^2 n)^{1/3} = \frac{\Lambda}{\hbar}\left(\frac{N}{V}\right)^{1/3}$$

$$\Lambda = (3\pi^2)^{1/3}\hbar$$

Hence

$$\lambda_F = \frac{2\pi}{K_F} = \frac{2\pi}{(3\pi^2 n)^{1/3}} = \frac{2\pi\hbar}{A}\left(\frac{V}{N}\right)^{1/3}$$

Which implies that the De-Broglie wavelength λ_F corresponding to the Fermi energy is of order the mean separation between particles $(V/N)^{1/3}$.All quantum states with De-Broglie wavelengths $\lambda > \lambda_F$ are occupied at $T = 0$, whereas all those with $\lambda < \lambda_F$ are empty.

According to equation (4.5.11), the Fermi energy at $T = 0$ takes the form

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} = \frac{A}{2m}\left(\frac{N}{V}\right)^{2/3} = C_0 V^{-\frac{2}{3}} \qquad (4.5.12)$$

$$\frac{dE_F}{dV} = -\frac{2}{3}c_0 V^{-\frac{5}{3}} \qquad (4.5.13)$$

But for spherical body

$$V = \frac{4\pi}{3}r^3$$

Thus

$$\frac{dE_F}{dV} = -\frac{2}{3}c_0\left(\frac{4\pi}{3}r^3\right)^{-5/3} = -\frac{2}{3}\left(\frac{4\pi}{3}\right)^{-5/3} c_0 r^{-5} \qquad (4.5.14)$$

But according to canonical Gibbs's distribution

$$P = n\frac{dE_F}{dV} \qquad (4.5.15)$$

Hence the pressure takes the form

$$P = -\frac{2}{3}\left(\frac{4\pi}{3}\right)^{-5/3} n c_0 r^{-5} = c_1 r^{-5} \qquad (4.5.116)$$

Thus the pressure force is given by

$$F_P = P(4\pi r^2)$$

$$F_P = -\frac{2}{3}\left(\frac{4\pi}{3}\right)^{-5/3} n c_0 r^{-5}(4\pi r^2) = 4\pi c_1 r^{-3}$$

$$F_P = c_2 r^{-3} \tag{4.5.17}$$

But gravity force is given by

$$F_g = \frac{GmM}{r^2}$$

Where we assume that the density is constant within the star. The mass at distance r from the star center is

$$M(r) = \frac{4\pi}{3}\rho r^3$$

$$F_g = \frac{4\pi r^3 \rho Gm}{3r^2}$$

Thus

$$F_g = \frac{4}{3}\pi\rho Gmr = c_3 r \tag{4.5.18}$$

Equation of hydrostatic equilibrium requires

$$FP = c_2 r^{-3} = c_3 r \tag{4.5.19}$$

The critical radius r_c is thus given by

$$r_c^4 = \frac{c_2}{c_3}$$

$$r_c = \left(\frac{c_2}{c_3}\right)^{1/4} = \left(\frac{-(4\pi)^{-5/3}\, nc_0(4\pi)}{2\pi\rho mG\ (3)^{-5/3}}\right)^{1/4} \tag{4.5.20}$$

Expansion takes place

$$F_P > \tag{4.5.21}$$

While contraction happens when

$$F_P < \tag{4.5.22}$$

The conditions of star evolution can be started by adopting classical limit, of generalized special relativity (GSR) energy relation where

$$E = m_0 c^2 \left(1 + \frac{2\varphi}{c^2}\right)\left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{-1\ 2} \tag{4.5.23}$$

Considering Newtonian potential and thermal motion

$$\varphi = -\frac{G}{M}, \frac{1}{2}mv^2 = \frac{3}{2}KT \Rightarrow v^2 = \frac{3KT}{m_0} \tag{4.5.24}$$

$$E = mc^2 = m_0c^2\left(1-\frac{2GM}{Rc^2}\right)\left(1-\frac{2GM}{Rc^2}-\frac{3KT}{m_0c^2}\right)^{-1/2} \quad (4.5.25)$$

If the gravitational potential and thermal energy are everywhere small, so

$$\frac{2GM}{Rc^2} \ll 1 \quad , \quad \frac{3KT}{m_0c^2} \ll 1 \quad (4.5.26)$$

Thus (25) reduces to

$$E = m_0c^2\left(1-\frac{2GM}{Rc^2}\right)\left(1+\frac{GM}{Rc^2}+\frac{3KT}{2m_0c^2}\right) \quad (4.5.27)$$

Neglecting higher order terms, yields

$$E = m_0c^2\left(1+\frac{GM}{Rc^2}+\frac{3KT}{2m_0c^2}-\frac{2GM}{Rc^2}-\frac{2G^2M^2}{R^2c^4}-\frac{3GMKT}{Rm_0c^4}\right)$$

Thus the energy E become

$$E = m_0c^2 + \frac{3}{2}KT - \frac{GMm_0}{R} \quad (4.5.28)$$

Assuming the kinetic energy is due to thermal motion

$$K.E = \frac{3}{2}KT \quad (4.5.29)$$

Assuming also the potential energy of mass m_0 to be

$$V = -\frac{GMm_0}{R} \quad (4.5.30)$$

Thus equation (4.5.28) gives

$$E = m_0c^2 + K.E + V$$

Thus the expression of energy includes the total kinetic energy of the degenerate electrons (the kinetic energy of the ion is negligible), the rest energym_0c^2 and the gravitational potential energy$V$. Let us assume, for the sake of simplicity, that the density of the star is its uniform. The total energy of a star is its gravitational potential energy, its internal energy and its kinetic energy (due to bulk motions of gas inside the star, not the thermal motions of the gas particles).

Using the hypothesis of universe expansion, the star explodes and expands when the energy E is positive

$$E = m_0 c^2 + \frac{3}{2}KT - \frac{Gm_0 M}{R} > 0 \qquad (4.5.31)$$

i.e.

$$m_0 c^2 + \frac{3}{2}KT > \frac{Gm_0 M}{R} \qquad (4.5.32)$$

This is quite obvious from the point of view of common sense because this equation indicates that expansion happen when thermal and rest mass energies exceeds attractive gravity energy. However it collapse and contract when the energy E is negative, this requires

$$m_0 c^2 + \frac{3}{2}KT < \frac{Gm_0 M}{R} \qquad (4.5.33)$$

Thus collapse takes place when gravity energy exceeds thermal one.

Can be obtained the critical radius, using the following energy for generalized special relativity

$$E = m_0 c^2 \left(1 - \frac{2GM}{rc^2}\right)\left(1 - \frac{2GM}{rc^2} - \frac{3KT}{m_0 c^2}\right)^{-1/2}$$

$$E = m_0 c^2 \left(1 + \frac{2c_1}{rc^2}\right)\left(1 + \frac{2c_1}{rc^2} - \frac{3KT}{m_0 c^2}\right)^{-1/2}$$

Where

$$c_1 = -GM$$

$$E = m_0 c^2 (1 + c_2 r^{-1})\left(1 + c_2 r^{-1} - \frac{3KT}{m_0 c^2}\right)^{-1/2} \qquad (4.5.34)$$

Where

$$c_2 = \frac{c_1}{c^2}$$

The critical radius of the star requires minimizing the total energy E and can be found by using the conditions for minimum value, i.e.

$$\frac{dE}{dr}=\frac{-m_0c^2c_2\,r^{-2}}{\left(1+c_2r^{-1}-\frac{3KT}{m_0c^2}\right)^{1/2}}+\frac{\frac{1}{2}m_0(1+c_2r^{-1})(c^2c_2r^{-2})}{\left(1+c_2r^{-1}-\frac{3KT}{m_0c^2}\right)^{3/2}}$$

$$\frac{dE}{dr}=\frac{-m_0c^2c_2\,r^{-2}\left(1+c_2r^{-1}-\frac{3KT}{m_0c^2}\right)+\frac{1}{2}m_0(1+c_2r^{-1})(c^2c_2r^{-2})}{\left(1+c_2r^{-1}-\frac{3KT}{m_0c^2}\right)^{3/2}}=0$$

$$-m_0c^2c_2\,r^{-2}\left(1+c_2r^{-1}-\frac{3KT}{m_0c^2}\right)+\frac{1}{2}m_0(1+c_2r^{-1})(c^2c_2r^{-2})=0$$

$$m_0(1+c_2r^{-1})(c^2c_2r^{-2})=2m_0c^2c_2\,r^{-2}\left(1+c_2r^{-1}-\frac{3KT}{m_0c^2}\right)$$

$$1+c_2r^{-1}=2+2c_2r^{-1}-\frac{6KT}{m_0c^2}$$

$$2c_2r^{-1}-c_2r^{-1}=\frac{6KT}{m_0c^2}-1$$

$$c_2r^{-1}=\frac{6KT-m_0c^2}{m\ \ c^2}$$

When temperature is neglected, i.e.

when

$$T=0$$

One gets

$$c_2r^{-1}=-1$$

$$r=-c_2=-\frac{2c_1}{c^2}=\frac{2MG}{c^2}$$

The critical radius is thus given by

$$r_c=\frac{2GM}{c^2} \qquad (4.5.35)$$

(This is the black hole radius)

Using the generalized special relativity energy relation

$$' = m_0c^2\left(1-\frac{2GM}{rc^2}\right)\left(1-\frac{2GM}{rc^2}-\frac{3KT}{m_0c^2}\right)^{-1/2} \qquad (4.5.36)$$

For star having spherical shape:

$$-GM=-G\left(\frac{4\pi}{3}\rho r^3\right)=-\frac{4\pi}{3}G\rho r^3=c_3r^3 \qquad (4.5.37)$$

$$E = m_0 c^2 \left(1 + \frac{2c_3 r^2}{c^2}\right)\left(1 + \frac{2c_3 r^2}{c^2} - \frac{3KT}{m_0 c^2}\right)^{-1/2}$$

$$E = m_0 c^2 (1 + c_4 r^2)\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) \qquad (4.5.38)$$

Where

$$c_4 = \frac{2c_3}{c^2}$$

The radius of the star r that dimension which reduces the total energy E and his can be found by using the minimum energy condition that has to be less energy as soon as possible, i.e.

$$\frac{dE}{dr} = 0 \qquad (4.5.39)$$

$$\frac{dE}{dr} = \frac{m_0 c^2 (2c_4 r)}{\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{1/2}} - \frac{m_0 c^2 (1 + c_4 r^2)\left(\frac{1}{2}\right)(2c_4 r)}{\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{3/2}} = 0$$

$$\frac{2m_0 c^2 c_4 r\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) - m_0 c^2 c_4 r(1 + c_4 r^2)}{\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{3/2}} = 0$$

$$2m_0 c^2 c_4 r\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) - m_0 c^2 c_4 r(1 + c_4 r^2) = 0$$

$$2m_0 c^2 c_4 r\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) = m_0 c^2 c_4 r(1 + c_4 r^2)$$

$$2\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) = 1 + c_4 r^2$$

$$2 + 2c_4 r^2 - \frac{6KT}{m_0 c^2} = 1 + c_4 r^2$$

$$c_4 r^2 = \frac{6KT}{m_0 c^2} - 1$$

$$r^2 = \frac{6KT - m_0 c^2}{m_0 c^2 c_4} = \frac{6KT - m_0 c^2}{2m_0 c_3}$$

The minimum radius

$$r = \left(\frac{6KT - m_0c^2}{2m_0c_3}\right)^{1/2}$$

$$6KT > m_0c^2 \tag{4.5.40}$$

$$m_0 < \frac{6KT}{2}$$

c

Thus the critical mass is given by:

$$m_{0_c} = \frac{6KT}{c^2}$$

Hence for equilibrium

$$m_0 < m_0c$$

Using equation (4.5.37)

$$c_3 = \frac{4\pi}{3}G\rho$$

The critical radius is thus given by

$$r_c = \frac{6KT - m_0c^2}{\frac{8\pi}{3}m_0G\rho}^{1/2} \tag{4.5.41}$$

When

$$\frac{d\ \ E}{dr^2}=\frac{2m_0c\ \ c_4}{\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{1/2}}-\frac{2m_0c\ \ c_4r}{\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{3/2}}-\frac{m_0c\ \ c_4(1+3c_4r\ \ )}{\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{3/2}}+\frac{3m_0c^2c_4^2r^2(1+c_4r^2)}{\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{5/2}}$$

$$\frac{d^2E}{dr^2}=\frac{2m_0c^2c_4\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)}{\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{3/2}}-\frac{[2m_0c^2c_4^2r^2+m_0c^2c_4(1+3c_4r^2)]}{\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{3/2}}+\frac{3m_0c^2c_4^2r^2(1+c_4r^2)\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{-1}}{\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{3/2}}$$

$$=\frac{m_0c^2c_4-6c_4KT-3m_0c^2c_4^2r^2+3m_0c^2c_4^2r^2(1+c_4r^2)\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{-1}}{\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{3/2}}$$

For maximum values

$$\frac{d^2E}{dr^2}<0 \qquad (4.5.42)$$

$$\frac{m_0c^2c_4-6c_4KT-3m_0c^2c_4^2r^2+3m_0c^2c_4^2r^2(1+c_4r^2)\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{-1}}{\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{3/2}}<0$$

$$m_0c^2c_4-6c_4KT-3m_0c^2c_4^2r^2+3m_0c^2c_4^2r^2(1+c_4r^2)\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{-1}<0$$

$$\left[3m_0c^2c_4^2r^2(1+c_4r^2)\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{-1}\right]<[6c_4KT+3m_0c^2c_4^2r^2-m_0c^2c_4]$$

$$\left[3m_0c^2c_4^2r^2(1+c_4r^2)\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{-1}\right]<\left[m_0c^2c_4\left(\frac{6KT}{m_0c^2}+3c_4r^2-1\right)\right]$$

$$\left[c_4r^2(1+c_4r^2)\left(1+c_4r^2-\frac{3KT}{m_0c^2}\right)^{-1}\right]<\left(\frac{2KT}{m_0c^2}+c_4r^2-\frac{1}{3}\right)$$

$$c_4r^2(1+c_4r^2)<\left(\frac{2KT}{m\ \ c^2}+c_4r^2-\frac{1}{3}\right)\left(1+c_4r^2-\frac{3KT}{m\ \ c^2}\right)$$

When temperature is neglected, i.e. when

$$T = 0$$

$$c_4 r^2(1 + c_4 r^2) < \left(c_4 r^2 - \frac{1}{3}\right)(1 + c_4 r^2)$$

$$c_4 r^2 + c_4^2 r^4 < c_4 r^2 + c_4^2 r^4 - \frac{1}{3} - \frac{1}{3} c_4 r^2$$

$$\frac{1}{3} c_4 r^2 + \frac{1}{3} < 0$$

$$c\ \ r^2 + 1 < 0$$

$$r^2 < -\frac{1}{c_4}$$

$$r < \left(\frac{1}{c_4}\right)^{1/2}$$

Where

$$c_4 = \frac{2c_3}{c^2}, \qquad c_3 = \frac{4\pi G_\rho}{3}$$

$$c_4 = \frac{8\pi G_\rho}{3c^2} \qquad (4.5.43)$$

$$r < c\left(\frac{3}{8\pi G_\rho}\right)^{1/2}$$

While contraction takes place when

$$r < \sqrt{3}\, c(8\pi G\rho)^{-1/2} \qquad (4.5.44)$$

For minimum values

$$\frac{d^2E}{dr^2} > 0 \qquad (4.5.45)$$

Thus explosion is expected when

$$r > \sqrt{3}\ c(8\pi G\rho)^{-1/2} \qquad (4.5.46)$$

Thus the critical radius is given by

$$r_c = \sqrt{3}\, c(8\pi G\rho)^{-1/2} \qquad (4.5.47)$$

4.6 Generation of Elementary Particles inside Black Holes at Planck Time

Generalized special relativistic energy (GSR) expression, beside ordinary Newtonian gravity potential are given by (78)

$$E = m_0 c^2 \left(1+\frac{2\varphi}{c^2}\right)\left(1+\frac{2\varphi}{c^2}-\frac{v^2}{c^2}\right)^{-1/2} \qquad (4.6.1)$$

Where the Newtonian potential takes the form

$$\varphi = -\frac{MG}{R} \qquad (4.6.2)$$

$$E = m_0 c^2 \left(1-\frac{2MG}{Rc^2}\right)\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)^{-1/2} \qquad (4.6.3)$$

Minimizing E w.r.t M yields

$$\frac{dE}{dM} = m_0 c^2 \left[\frac{-\frac{2G}{Rc^2}}{\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)^{1/2}} + \frac{\left(1-\frac{2MG}{Rc^2}\right)\left(-\frac{1}{2}\right)\left(\frac{-2G}{Rc^2}\right)}{\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)^{3/2}}\right] = 0$$

Thus

$$\frac{-\frac{2G}{Rc^2}\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)+\frac{G}{Rc^2}\left(1-\frac{2MG}{Rc^2}\right)}{\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)^{3/2}} = 0$$

If one consider

$$v^2 \ll c^2$$

$$-\frac{2G}{Rc^2}\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)+\frac{G}{Rc^2}\left(1-\frac{2MG}{Rc^2}\right) = 0$$

$$-\frac{G}{Rc^2}\left(1-\frac{2MG}{Rc^2}\right) = 0$$

This requires

$$\frac{2MG}{Rc^2} = 1$$

$$2MG = Rc^2 \qquad (4.6.4)$$

Thus the mass which makes E minimum is

$$M = \frac{Rc^2}{2G} \qquad \textbf{(4.6.5)}$$

Consider also the generalized special relativity energy E equilibrium condition by minimizing $E$with respect to radius r from equation (4.6.3), when the star particles speed are small compared to speed of light

$$\frac{v^2}{c^2} \ll 1$$

Thus

$$E = m_0 c^2 \left(1 - \frac{2MG}{rc^2}\right)^{1/2} \qquad (4.6.6)$$

$$\frac{dE_r}{dr} = m_0 c^2 \left(\frac{2MG}{r^2c^2}\right)\left(\frac{1}{2}\right)\left(1 - \frac{2MG}{rc^2}\right)^{-1/2}$$

$$\frac{dE_r}{dr} = \frac{m_0 c^2 \left(\frac{MG}{r^2c^2}\right)\left(1 - \frac{2MG}{rc^2}\right)}{\left(1 - \frac{2MG}{rc^2}\right)^{3/2}} = 0$$

Thus the radius which makes E minimum is given by

$$1 - \frac{2MG}{rc^2} = 0$$

The critical radius is thus given by

$$r_c = \frac{2MG}{2} \qquad (4.6.7)$$

(This is the black hole radius r_c)

But the critical mass is given by equation (4.6.7), i.e.

$$M = mc = \frac{c^2 r_c}{2G} \qquad (4.6.8)$$

Hence from (4.6.8)

$$2m_c G \ = \ r_c c^2 \qquad (4.6.9)$$

The condition governing the equilibrium of the universe, from (4.6.9) and (4.6.4) we get

$$\frac{m_c R}{M r_c} = 1 \qquad (4.6.10)$$

Where M and R are the mass and radius of the universe respectively. The mass of the universe ($M = 2.2 \times 10^{56} g$) and the radius ($R = 1.6 \times 10^{28} cm$). According to generalized general relativity (GGR) there is a short range repulsive gravitational force beside long range attractive gravity force given by [1]:

$$\varphi_s = \frac{c_1}{r} e^{-\frac{r}{r_c}} (11)$$

$$\varphi_L = -\frac{GM}{r} (12)$$

$$\varphi = \varphi_s + \varphi_L = \frac{c_1}{r} e^{-\frac{r}{r_c}} - \frac{GM}{r}$$

$$\varphi = \frac{1}{r}\left[c_1 e^{-\frac{r}{r_c}} - GM\right] \qquad (4.6.13)$$

For small radius r or strictly speaking small $\frac{r}{r_c}$:

$$e^{-\frac{r}{r_c}} = 1 - \frac{r}{r_c} \qquad (4.6.14)$$

Hence

$$\varphi = \frac{1}{r}\left[c_1 - c_1 \frac{r}{r} - GM\right] \qquad (4.6.15)$$

To secure finite self-energy φ at small r_c, one requires

$$c_1 = GM \qquad (4.6.16)$$

Thus the star self-energy is given by

$$\varphi = -\frac{c_1}{r_c} = -\frac{GM}{r_c} \qquad (4.6.17)$$

Since the star is a particle at rest thus the minimization of E requires (see equation (4.6.2), (4.6.4) and (4.6.17))

$$\varphi = -\frac{c_1}{r_c} = -\frac{c^2}{2} \qquad (4.6.18)$$

For photon ($v = c$) thus one gets

$$\varphi = \frac{c^2}{2} \qquad (4.6.19)$$

From equation (4.6.17) and (4.6.18)

$$\varphi = -\frac{GM}{r_c} = -\frac{c^2}{2} \qquad (4.6.20)$$

Thus the critical radius is given by

$$r_c = \frac{2GM}{c^2} \qquad (4.6.21)$$

(This is the black hole radius)

Since r_c should be small as shown by equation (4.6.14), thus requires

$$r_c < 1 \quad , \quad \frac{2GM}{c^2} < 1$$

$$M < \frac{c^2}{2G} \qquad (4.6.22)$$

Thus there is a critical mass

$$M_c = \frac{c^2}{2G} \qquad (4.6.23)$$

Above it the particle rest mass energy cannot be formed form potential.

We see from equation (4) that the present radius of the universe should be

$$R_0 = \frac{2GM_0}{c^2} \sim 10^{28} cm \qquad (4.6.24)$$

Which conforms to observations.Consider a star as consisting of photons gas, such that the critical radius is related to the wave number according to the relation

$$p = m_0 c = \hbar k = \frac{\hbar}{r_c} \quad , \quad k = \frac{1}{r_c} \qquad (4.6.25)$$

For oscillating string the energy takes the form

$$E_{r_c} = m_0 c^2 = \frac{\hbar c}{r_c} \qquad (4.6.26)$$

Hence

$$r_c = \frac{\hbar}{m_0 c} \qquad (4.6.27)$$

The photon which obeys quantum laws equations (4.6.19) and (4.6.1) gives

$$E = \frac{2m_0c^2}{\sqrt{2-1}} = 2m_0c^2 \tag{4.6.28}$$

This conforms to the fact that photons can produce particle pairs.

Newton's law of potential gives

$$E_{r_c} = U(r) = -G\frac{m_1m_2}{r} \tag{4.6.29}$$

Gravity force is also given by

$$F = -G\frac{m_1m_2}{r^2}.\frac{\underline{r}}{r} \tag{4.6.30}$$

If

$$m_1 = m_2 = m_c$$

Thus (4.6.26) and (4.6.29) given

$$E_{r_c} = \frac{G\ m_c^2}{r_c} = \frac{\hbar c}{r_c} \tag{4.6.31}$$

Therefore

$$\hbar c = Gm_c^2 \tag{4.6.32}$$

Hence

$$m_c = \left(\frac{\hbar c}{G}\right)^{1/2} \tag{4.6.33}$$

$$m_c = \left(\frac{\hbar c}{G}\right) \quad \sim 2.2 \times 10^{-5}\text{g} \tag{4.6.34}$$

(Equivalent Planck's mass)

Which matches the proposed value. The same equation applies to Planck's length, namely

$$R_P = \frac{G_PM_P}{c^2} \sim 10^{-33}cm \tag{4.6.35}$$

(Planck's length)At distances smaller than this scale the gravitational interaction should be stronger than the quantum effects [2].Also the critical distance r_c is equal

$$r_c = \frac{\hbar}{m\ c} = \left(\frac{G\hbar}{c^3}\right)^{1/2} \sim 1.6 \times 10^{-33} cm \quad (4.6.36)$$

One can calculate the critical density σ_c of the material when the particles are considered as a hollow sphere surrounded by thin layer or membrane. In this case the surface density is given by

$$\sigma = \frac{m_c}{A}, \quad m_c = \frac{\hbar}{r_c c}, \quad A = 4\pi r_c^2 \quad (4.6.37)$$

$$\sigma = \left(\frac{\hbar}{r_c c}\right)\left(\frac{1}{4\pi r_c^2}\right) = \frac{\hbar}{4\pi r_c^3} \quad (4.6.38)$$

$$m_c = \frac{\hbar}{r_c c} \quad (4.6.39)$$

Where

$$\sigma = \frac{m_c}{4\pi r_c^2} \sim 6.7 \times 10^{59} \text{g}.\, cm^{-2} \quad (4.6.40)$$

Thus the critical density satisfies

$$\sigma_c = \frac{m_c}{r_c^2} = \left(\frac{c^7}{G^3\hbar}\right)^{1/2}$$

Where

$$\sigma_c = 4\pi\sigma \sim 8.4 \times 10^{60} \text{g}.\, cm^{-2} \quad (4.6.41)$$

According to this model the universe began at a time and specific place, at the critical point (r_c , t_c), where all fundamental forces are unified into a single force.

The Planck time is thus given by

$$t_c = \frac{r_c}{c} = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}}\left(\frac{1}{c}\right) = \left(\frac{G\hbar}{c^5}\right)^{1/2} \sim 5.4 \times 10^{-44}\ s \quad (4.6.42)$$

(Equivalent Planck’s time)

The value speed of light c at the critical point(r_c, t_c).

$$c = \frac{r_c}{t_c} \sim 3 \times 10^{10} cm.\, s^{-1} \quad (4.6.43)$$

Began creation of the universe at the critical point (r_c , t_c), and show the fundamental constants such as ($\hbar$, c, G) known values, since that time and keep as it is without any change, the structure of the our universe is sensitive to precise degree to less change in these fundamental constants. The status of the universe at different stages is shown to be described in terms of the constants ($\hbar$, c, G) only. This masterly organization of the universe is the result for precise tuning arbitrator. The acceleration was great, which is equal to [3]:

$$a_c = R_c = \frac{c}{t_c} \qquad (4.6.44)$$

Where R_c critical curvature (the maximal acceleration occurred at Planck's time). From a purely dimensional argument one can constant a quantum acceleration from the set of fundamental constants ($\hbar$, c, G) to be valid at Planck's time, and according to our hypothesis, an analogous acceleration of the form

$$a_c = \frac{c}{t_c} = \frac{r_c}{t_c^2} = \left(\frac{c^7}{G\hbar}\right)^{1/2} \sim 5.7 \times 10^{53} cm.s^{-2} \qquad (4.6.45)$$

Getting limited value to a larger curvature or maximal acceleration in the relation (4.6.43) resolved the problem singular behavior. And the matching bending dimensions to pry acceleration are consistent with the principles of general relativity. Conform to the critical value of the acceleration a_c in this relation with the researches results [4].This acceleration on unwavering$c$ constants, and associated critical point (t_c). The existence of this greatest acceleration confirms the occurrence of stretch accelerator of the universe at the beginning of time [5]. The acceleration declining at critical value a_c generates the force to attract at the beginning of time, when the universe takes its way to expansion, and this explains why the presence of the cosmic force of the overall attraction.

The critical force F_c as follows

$$F_c = m_c a_c = \frac{c^4}{} \sim 1.25 \times 10^{49}\ dyne \qquad (4.6.46)$$

$$E_c = m_c c^2 = \left(\frac{\hbar c^5}{G}\right) \quad \sim 10^{19}\ GeV \qquad (4.6.47)$$

4.7 Equilibrium of Stars within the Framework of Generalized Special Relativity Theory

Consider first the Generalized Special Relativity GSR energy E equilibrium condition by minimizing E w.r.t. [79]

$$E = m_0 c^2 \left(1 + \frac{2\varphi}{c^2}\right)\left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{-1/2} \qquad (4.7.1)$$

$$\varphi = -\frac{GM}{r}, m_0 = M \qquad (4.7.2)$$

$$\frac{v^2}{c^2} = \frac{m^2 v^2}{m^2 c^2} = \frac{p^2}{m^2 c^2} = \frac{p^2}{M^2 c^2} \qquad (4.7.3)$$

For simplicity consider the average momentum p is equal to the maximum momentum p, where

$$p = \frac{p_F}{\sqrt{2}}$$

Thus

$$p = p_F = \Lambda \left(\frac{N}{V}\right)^{\frac{1}{3}} = \Lambda n_0$$

Where

Therefor, with the aid of equation (4.7.2), (4.7.4), equation (4.7.1) reads

$$3\pi^2 \frac{1}{3\bar{h}}$$

$$E = E_f = Mc^2\left(1-\frac{2MG}{r}\right)\left(1-\frac{2MG}{r}-\frac{p_f^2}{M^2C^2}\right)^{1/2} \qquad (4.7.5)$$

The radius r which makes the energy E minimum is given when

$$\frac{dE_r}{dr} = \frac{Mc^2\left(\frac{2MG}{r^2}\right)}{\left(1-\frac{2MG}{r}-\frac{p_F{}^2}{M^2c^2}\right)^{1/2}} + \frac{Mc^2\left(1-\frac{2MG}{r}\right)\left(-\frac{1}{2}\right)\left(\frac{2MG}{r^2}\right)}{\left(1-\frac{2MG}{r}-\frac{p_F{}^2}{M^2c^2}\right)^{3/2}} = 0$$

$$\frac{Mc^2\left[\left(\frac{2MG}{r^2}\right)\left(1-\frac{2MG}{r}-\frac{p_F{}^2}{M^2c^2}\right)-\left(\frac{MG}{r^2}\right)\left(1-\frac{2MG}{r}\right)\right]}{\left(1-\frac{2MG}{r}-\frac{p_F{}^2}{M^2c^2}\right)^{3/2}} = 0$$

$$\frac{M^2c^2G}{r^2}\left(-1+2+\frac{4MG}{r}-\frac{p_F{}^2}{M^2c^2}\right) = 0 \qquad (4.7.6)$$

This is satisfied when

$$\frac{4MG}{r} = \frac{p_f^2}{M^2C^2} - 1 \qquad (4.7.7)$$

Thus the minimum radius is given by

$$r = \frac{4M^3C^2G}{p_f^2 - M^2C^2} \qquad (4.7.8)$$

Where

$$p_f = (3\pi^3)^{1/3}\,\hbar n^{1/3} = A\left(\frac{N}{V}\right)^{1/3} = \left(\frac{9\pi}{4}\right)^{1/3}\frac{N^{1/3}}{r_f}\hbar \qquad (4.7.9)$$

The equilibrium takes place when r is non negative, i.e when

$$pF2 > M2c2$$

$$p_F > Mc \qquad (4.7.10)$$

The critical mass is given by

$$M_c = \frac{p_f}{c} \qquad (4.7.11)$$

Thus for star to be at equilibrium one requires

$$\frac{p_f}{c} > M$$

$$M_c > M \tag{4.7.12}$$

$$M < M_c$$

Thus the maximum mass for stable star is

$$M_c = \frac{p_f}{c} = \frac{(3\pi^2)^{\frac{1}{3\hbar}}}{c}\left(\frac{N}{V}\right)^{1/3} \tag{4.7.13}$$

This condition resembles Chandrasekhar limit for stable white dwarf. I.e. the star mass need to be less than the critical value in equation (4.7.11).

The equilibrium condition can also be found by using generalized special relativity energy momentum relation

$$g_{00}\, E^2 - p^2c^2 = m_0^2c^4(g_{00})^2$$

$$E^2 = (g_{00})^{-1}p^2c^2 + g_{00}\, m_0^2c^4 \tag{4.7.14}$$

One can rewrite equation (4.7.14) to be

$$E = (a_1 - a_2p^2)^{1/2} \tag{4.7.15}$$

Where

$$a_1 = g_{00}m_0^2c^4 = \left(1 - \frac{2MG}{rc^2}\right)m_0^2c^4 \quad , \quad a_2 = (g_{00})^{-1}c^2$$

$$a_2p^2 = a_1cos^2\theta \tag{4.7.16}$$

$$E = \int_0^{p_F}(a_1 - a_1cos^2\theta)^{1/2}dp\ (17)$$

Where

$$-dp = \sqrt{\frac{a_1}{a_2}}\ sin\theta d\theta \tag{4.7.18}$$

$$E = \sqrt{a_1}\int(1 - cos^2\theta)^{1/2}\left(-\sqrt{\frac{a_1}{a_2}}\right)sin\,\theta\, d\theta \tag{4.7.19}$$

$$= \sqrt{a_1}\left(-\sqrt{\frac{a_1}{a_2}}\right)\int \sin^2\theta\, d\theta \tag{4.7.20}$$

$$cos2\theta = cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

(4.7.21)

$$E = \frac{\sqrt{a_1}}{2}\left(-\sqrt{\frac{a_1}{a_2}}\right)\left(\theta - \frac{sin2\theta}{2}\right)$$

$$sin2\theta = 2sin\theta cos\theta \quad , \quad cos\theta = \sqrt{\frac{a_2}{a_1}}\ p$$

$$sin\theta = (1 - \cos^2\theta)^{1/2} = \left(1 - \frac{a_2}{a_1}p^2\right)^{1/2}$$

$$E = \sqrt{a_1}\sqrt{\frac{a_1}{a_2}}\sqrt{\frac{a_2}{a_1}}p_F(1 - a_3p_F^2)^{1/2} + cos^{-1}\sqrt{\frac{a_2}{a_1}}p_F - \frac{\pi}{2}$$

$$= \sqrt{a_1p_f}(1 - a_3p_F^2)^{1/2} + \cos^{-1}\sqrt{\frac{a_2}{a^1}}p_F - \frac{\pi}{2} \qquad (4.7.22)$$

Where

$$a_3 = \frac{a_2}{a_1} = \frac{g_{00}^{-1}c^2}{g_{00}m_0^2c^4} = \frac{g_{00}^{-2}}{m_0^2c^2}$$

$$E = \left(1 - \frac{2MG}{rc^2}\right)^{\frac{1}{2}} p_F\left[1 - \frac{p_F^2\ c^2}{m_0^2c^4\left(1 - \frac{2MG}{rc^2}\right)^2}\right]^{1/2} + cos^{-1}\left(\frac{p_Fc}{m_0c^2\left(1 - \frac{2MG}{rc^2}\right)}\right) - \frac{\pi}{2} \qquad (4.7.23)$$

It is clear from equation (4.7.23) that stability requires E to be real. This can be satisfied when

$$1 - \frac{2MG}{rc^2} > \quad 0$$

$$rc^2 > \ 2MG$$

$$r > \frac{2MG}{c^2}$$

The critical radius is given by

$$r_c = \frac{2MG}{c^2} \tag{4.7.24}$$

Thus the radius should be greater than the black hole radius. Also

$$1 - \frac{p_F^2\, c^2}{m_0^2 c^4 \left(1 - \frac{2MG}{rc^2}\right)^2} > \ 0$$

$$m_0^2 c^4 \left(1 - \frac{2MG}{rc^2}\right)^2 > p_F^2\, c^2$$

Thus

$$\left(1 - \frac{2MG}{rc^2}\right) > \pm \frac{p_F c}{m_0 c^2}$$

$$rc^2 - 2MG > \pm \left(\frac{p_F c}{m_0 c^2}\right) rc^2 \tag{4.7.25}$$

$$\left(1 \pm \frac{p_F c}{m_0 c^2}\right) rc^2 > 2MG$$

$$r > \frac{2MGm_0}{(m_0 c^2 \pm p_F c)} \tag{4.7.26}$$

Thus the critical radius is given by

$$r_c = \frac{2Mm_0 G}{(m_0 c^2 \pm p_F c)} \tag{4.7.27}$$

The equilibrium mass also satisfies

$$2MG \ > \ -rc^2 \pm \left(\frac{p_F c}{m_0 c^2}\right) rc^2$$

$$M \ < \frac{rc^2}{2G} \pm \frac{p_F rc}{m_0} \tag{4.7.28}$$

Hence the critical maximum mass is given by

$$M_c = \frac{rc^2}{2G} \pm \frac{p_F rc}{m_0} \tag{4.7.29}$$

The equilibrium condition can also be found by minimizing$E$, where

$$E = mc^2 = m_0 c^2 \left(1 + \frac{2\varphi}{c^2}\right)\left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{-1/2} \tag{4.7.30}$$

Assuming the mass to be equal to the rest mass, and the potential to be the Newtonian, one gets

$$m_0 = M \qquad , \qquad \varphi = -\frac{GM}{R} \tag{4.7.31}$$

Therefore

$$E = Mc^2 \left(1 - \frac{2GM}{Rc^2}\right)\left(1 - \frac{2GM}{Rc^2} - \frac{v^2}{c^2}\right)^{-1/2} \tag{4.7.32}$$

For small φ and velocity v compared to speed of light c, i.e

$$\frac{GM}{R} < \ 1 \quad , \quad \frac{v^2}{c^2} < \ 1$$

One gets

$$E = Mc^2 \left(1 - \frac{2GM}{Rc^2}\right)\left(1 + \frac{GM}{Rc^2} + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$E = \left(Mc^2 - \frac{2GM^2}{R}\right)\left(1 + \frac{GM}{Rc^2} + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$E = Mc^2 + \frac{GM^2}{R} + \frac{1}{2}Mv^2 - \frac{2GM^2}{R} - \frac{2G^2M^3}{R^2c^2} - \frac{GM^2v^2}{Rc^2} \tag{4.7.33}$$

The mass which make the energy minimum for constant radius is given by

$$\frac{dE}{dM} = \frac{c^2\left(1 - \frac{2GM}{Rc^2}\right)}{\sqrt{1 - \frac{2GM}{Rc^2} - \frac{v^2}{c^2}}} + \frac{Mc^2\left(\frac{-2G}{Rc^2}\right)}{\sqrt{1 - \frac{2GM}{Rc^2} - \frac{v^2}{c^2}}} + \frac{\frac{1}{2}Mc^2\left(1 - \frac{2GM}{Rc^2}\right)\left(\frac{2G}{Rc^2}\right)}{\left(1 - \frac{2GM}{Rc^2} - \frac{v^2}{c^2}\right)^{3/2}} \tag{4.7.34}$$

Neglecting the kinetic term yields

$$\frac{dE}{dM} = \frac{\left(c^2 - \frac{4MG}{R}\right)\left(1 - \frac{2MG}{Rc^2}\right) + \frac{MG}{R} - \frac{2M^2G^2}{R^2c^2}}{\left(1 - \frac{GM}{Rc^2} + \frac{1}{2}\frac{v^2}{c^2}\right)^{3/2}} = 0 \tag{4.7.35}$$

This requires

$$c^2 - \frac{2MG}{R} - \frac{4MG}{R} + \frac{8M^2G^2}{R^2c^2} + \frac{MG}{R} - \frac{2M^2G^2}{R^2c^2} = 0$$

$$c^2 - \frac{5MG}{R} + \frac{6M^2G^2}{R^2c^2} = 0$$

$$\frac{6G^2}{R^2c^2}M^2 - \frac{5G}{R}M + c^2 = 0$$

$$ax^2 + bx + c = 0 \ , \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$M = \frac{\frac{5G}{R} \pm \sqrt{\left(\frac{5G}{R}\right)^2 - \frac{24G^2c^2}{R^2c^2}}}{\frac{12G^2}{R^2c^2}} = \frac{R^2c^2}{12G^2}\left(\frac{5G}{R} \pm \sqrt{\frac{G^2}{R^2}}\right)$$

$$M = \frac{R^2c^2}{12G^2}\left(\frac{G}{R}\right)(5 \pm 1) = \frac{Rc^2}{12G}(5 \pm 1)$$

$$M = \frac{1}{2}\frac{Rc^2}{G}, \ \ \frac{1}{3}\frac{Rc^2}{G} \tag{4.7.36}$$

For stars one have two forces, pressure force which counter balance the gravity force, thus

$$P = \frac{NKT}{V} = \frac{1}{3}\frac{mv^2}{V} \tag{4.7.37}$$

Thus the pressure force is given by

$$F_P = PA = \frac{\frac{1}{3}mv^2(4\pi r^2)}{\frac{4\pi}{3}r^3} = \frac{mv^2}{r} \tag{4.7.38}$$

The gravity force is given by

$$F_g = \frac{GmM}{r^2} \tag{4.7.39}$$

At equilibrium the two forces counter balances themselves thus

$$F_p = F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r}, \qquad mv^2 = \frac{GmM}{r} \tag{4.7.40}$$

If particles are considered as strings with v representing max speed. Thus the average value is given by

$$v_a = \frac{v_m}{\sqrt{2}} \qquad , \quad mv_a^2 = \frac{mv_m^2}{2} \tag{4.7.41}$$

Thus

$$mv_a^2 = \frac{mv_m^2}{2} = \frac{1}{2}mv^2 \tag{4.7.42}$$

One thus gets

$$\frac{1}{2}mv^2 = \frac{GmM}{r} = m\varphi \tag{4.7.43}$$

Hence

$$v^2 = 2\varphi \tag{4.7.44}$$

Hence

$$E = \frac{m_0c^2\left(1 + \frac{2\varphi}{c^2}\right)}{\left(1 + \frac{2\varphi - v^2}{c^2}\right)^{1/2}} = m_0c^2\left(1 + \frac{2\varphi}{c^2}\right) \tag{4.7.45}$$

But

$$m_0 = M \quad , \quad \varphi = -\frac{GM}{R} \tag{4.7.46}$$

For attractive force

$$E = M\left(c^2 - \frac{2GM}{R}\right) \tag{4.7.47}$$

$$\frac{dE}{dM} = \left(c^2 - \frac{2GM}{R}\right) + M\left(\frac{-2G}{R}\right) = 0$$

$$-\frac{4GM}{R} + c^2 = 0$$

$$\frac{4GM}{R} = c^2 \tag{4.7.48}$$

$$= \frac{Rc^2}{4G} \tag{4.7.49}$$

$$M = \frac{R}{2G}c_a^2 = \frac{R}{2G}\left(\frac{c_m}{\sqrt{2}}\right)^2 = \frac{R}{2G}c^2 \tag{4.7.50}$$

In the works done by many authors the incorporation of the effect of fields on physical quantities is proved to be in agreement with many experimental observations that cannot be explained within the frame work of special relativity (80,81,82,84,85). These attempts ,specially the so called GSR, reduced to SR in the absence of fields, thus share with SR all its successes (86,87,88,89,90).Despite these remarkable successes , these models suffers from beying weakly linked with the electromagnetic theory (91,92,93,94,95).This needs a transformation that accounts for electromagnetic theory. This is quite natural as far as the fact that the Lorentz transformation originated from the electromagnetic theory. (96, 97, 98, 99,100)

Chapter five

Lorentz Transformation on the Basis of Maxwell's Equation

5.1 Introduction:

Lorentz transformation (LT) is of the corner stones of SR. usually LT is used on space time relations. It ere one tries to return back to ME to derive GSR relations.

5.2 Lorentz Transformation and electromagnetic Filed:

The Lorentz force in frame S is given by in the frame

$$F = e\left(E + v \times \underline{B}\right) \tag{5.2.1}$$

In the frame S' it is given by

$$F' = e(E' + v \times B') \tag{5.2.2}$$

Assume that e is constant and the electromagnetic force transforms from frame S to frame S' as

$$eE' = e\gamma(E + v \times B) \tag{5.2.3}$$

If one assumes that the charge is at rest in frame S', thus no magnetic field is exerted therefore the force in S' is given by (assume the electric field in the Z direction)

$$E_z' = \gamma\left(E_z + vB_y\right) \tag{5.2.4}$$

If in contrary, the charge is at rest in S, hence:

$$F = eE_z$$

And

$$F' = e\left(E_z' - vB_y'\right) \tag{5.2.5}$$

Using Maxwell's equations

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{5.2.6}$$

$$\nabla \times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y}E_z - \frac{\partial}{\partial z}E_y\right)\hat{\imath} - \left(\frac{\partial}{\partial x}E_z - \frac{\partial}{\partial z}E_x\right)\hat{\jmath} + \left(\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x\right)\hat{k}$$

$$= -\frac{\partial B_x}{\partial t}\hat{\imath} - \frac{\partial B_y}{\partial t}\hat{\jmath} - \frac{\partial B_z}{\partial t}\hat{k} \tag{5.2.7}$$

The $\hat{\jmath}$ component is given by

$$-\frac{\partial E_z}{\partial y} + \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} \tag{5.2.8}$$

Let

$$E_z = E_0 e^{i(k.r - \omega t)}$$

$$B_y = B_0 e^{i(k.r - \omega t)} \tag{5.2.9}$$

$$k.r = k_x x + k_y y + k_z z$$

$$k_x = k_y = k_z = k$$

$$\frac{\partial E_z}{\partial x} = ik.E_z \tag{5.2.10}$$

$$\frac{\partial B_y}{\partial t} = -i\omega B_y \tag{5.2.11}$$

Sub (5.2.10) and (5.2.11) in (5.2.8) yields

$$-ikE_z = +i\omega B_y$$

$$B_y = -\frac{k}{\omega}E_z = -\frac{2\pi E_z}{\lambda(2\pi f)} = -\frac{E_z}{\lambda f} = -\frac{E_z}{c}$$

$$B_y = -\frac{E_z}{c}$$

$$B'_y = -\frac{E'_z}{c} \tag{5.2.12}$$

Sub (5.2.12) in (5.2.4) and (5.2.5) thus

$$E'_z = \gamma\left(E_z - \frac{v}{c}E_z\right) = \gamma\left(1 - \frac{v}{c}\right)E_z \tag{5.2.13}$$

$$E_z = \gamma\left(E'_z + \frac{v}{c}E'_z\right) = \gamma\left(1 + \frac{v}{c}\right)E'_z \tag{5.2.14}$$

$$E_z = \gamma\left(1 + \frac{v}{c}\right)E'_z$$

$$\gamma^2\left(1 + \frac{v}{c}\right)\left(1 - \frac{v}{c}\right) = 1$$

$$\left(1 - \frac{v^2}{c^2}\right)\gamma^2 = 1$$

Therefore

$$\gamma = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \tag{5.2.15}$$

$$= \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

5.3 Generalized special relativity Faraday Electromagnetic Lorentz Transformation:

Consider a particle moving with acceleration a, the velocity is thus given by

$$v = v_0 + at = v_0 \frac{axt}{x} = \frac{\phi t}{x} + v_0 \quad (5.3.1)$$

Where

$$V = potential = Fx = max$$

$$\phi = potential\ per\ unit\ mass \quad (5.3.2)$$

$$= \frac{V}{m} = ax \quad (5.3.3)$$

Sub in (5.2.15) and assuming the relation hold for all physical system

$$\gamma = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} \quad (5.3.4)$$

$$\gamma = \frac{1}{\left(1 - \left(\frac{\phi t}{xc} + \frac{v_0}{c}\right)^2\right)^{\frac{1}{2}}} \quad (5.3.4)$$

For photon

$$x = ct \quad (5.3.5)$$

$$\gamma = \frac{1}{\left(1 - \left(\frac{\phi t}{c^2 t} + \frac{v_0}{c}\right)^2\right)^{\frac{1}{2}}} \quad (5.3.6)$$

$$\gamma = \frac{1}{\left(1 - \left(\frac{\phi}{c^2} + \frac{v_0}{c}\right)^2\right)^{\frac{1}{2}}} \quad (5.3.7)$$

For no potential

$$\phi = O$$

$$\gamma = \frac{1}{\left(1 - \frac{v_0^2}{c^2}\right)^{\frac{1}{2}}} \quad (5.3.8)$$

Which is the ordinary Lorentz transformation coefficient

Assume again that the relation

$$\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-\frac{1}{2}} \quad (5.3.9)$$

For particle in a field causing constant acceleration a

$$v = v_0 - at \quad (5.3.10)$$

$$x = v_0 t - \frac{1}{2} a t^2 \quad (5.3.11)$$

But

$$\left(\frac{v_0 + v}{2}\right) t = v_m t \quad (5.3.12)$$

Where

v_m is the mean velocity which is given by

$$v_m = \frac{v_0 + v}{2} \quad (5.3.13)$$

Replacing v by v_m in (5.3.11) one gets

$$\gamma = \left[1 - \frac{v_m^2}{c^2}\right]^{-\frac{1}{2}} \quad (5.3.14)$$

Using the relation

$$v^2 = v_0^2 - 2ax = v_0^2 - 2\phi \quad (5.3.15)$$

Thus

$$v_0^2 = v^2 + 2\phi$$

$$v_0 = \sqrt{v^2 + 2\phi} \quad (5.3.16)$$

Incorporating (5.3.16) in (5.3.14) and (5.3.15) one gets

$$\gamma = \left[1 - \left(\frac{v + \sqrt{v^2 + 2\phi}}{2c^2}\right)^2\right]^{-\frac{1}{2}} \quad (5.3.17)$$

When no filed exists

$$\phi = 0$$

Thus

$$\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-\frac{1}{2}} \quad (5.3.18)$$

Which is again the ordinary Lorentz transformation coefficient

From (5.3.11)

$$v = v_0 - at = v_0 - \frac{ax}{x}t$$

$$v = v_0 - \frac{\phi}{x}t \tag{5.3.19}$$

Thus

$$v_0 = \left(v + \frac{\phi}{x}t\right) \tag{5.3.20}$$

Therefore equation (5.3.16) rends

$$v_m = \frac{v_0 + v}{2} = v + \frac{\phi}{2x}t \tag{5.3.21}$$

Using (5.3.21) in equation (5.3.17) given

$$\gamma = \left[1 - \left(v + \frac{\phi}{2xc}t\right)^2\right]^{-\frac{1}{2}}$$

Assuming this relation is general. For pulse of light

$$x = ct$$

$$\gamma = \left[1 - \left(\frac{v + \frac{\phi}{2ct}t}{c}\right)^2\right]^{-\frac{1}{2}}$$

$$\gamma = \left[1 - \left(\frac{v + \frac{\phi}{2c}}{c}\right)^2\right]^{-\frac{1}{2}}$$

5.4 Displacement Current Lorentz transformations:

Consider the magnetic field generated by displacement current

$$\nabla \times H = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} \qquad (5.4.1)$$

But

$$\underline{B} = \mu \underline{H} \qquad (5.4.2)$$

$$\frac{1}{\mu} \nabla \times B = \varepsilon \frac{\partial E}{\partial t} \qquad (5.2.3)$$

$$\nabla \times B = \mu\varepsilon \frac{\partial E}{\partial t}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \mu\varepsilon \frac{\partial E}{\partial t} \qquad (5.2.4)$$

$$[\frac{\partial}{\partial x} B_y - \frac{\partial}{\partial y} B_x]\hat{k} \ \mu\varepsilon \frac{\partial E_z}{\partial t} \hat{k}$$

$$B_y - B_0 e^{i(k.r - \omega t)}$$

$$E_z - E_0 e^{i(k.r - \omega t)}$$

$$\frac{\partial B_y}{\partial x} = ikB_y \qquad (5.2.5)$$

$$\frac{\partial E_z}{\partial t} - i\omega E_z$$

$$\mu\varepsilon = \frac{1}{c^2}$$

$$ikB_y = -i\mu\varepsilon\omega E_z \qquad (5.2.6)$$

$$B_y = -\mu\varepsilon \frac{\omega}{k}\, E_z = -\frac{1}{c^2}\left(\frac{2\pi f\lambda}{2\pi}\right) \tag{5.2.7}$$

$$= -\frac{1}{c^2}(f\lambda)E_z = -\frac{c}{c^2}E_z$$

$$B_y = -\frac{E_z}{c}$$

$$B'_y = -\frac{E'_z}{c} \tag{5.2.8}$$

Sub these relations in (5.2.4) and (5.2.5) given

$$E'_z = \gamma\left(E_z - \frac{v}{c}E_z\right) = \gamma\left(1 - \frac{v}{c}\right)E_z \tag{5.2.9}$$

$$E_z = \gamma\left(E'_z + \frac{v}{c}E'_z\right) = \gamma\left(1 + \frac{v}{c}\right)E'_z \tag{5.2.10}$$

The term γ can be found by using relations (5.2.10) and (5.2.9) to get

$$E_z = \gamma^2\left(1 - \frac{v}{c}\right) = \gamma\left(1 + \frac{v}{c}\right)E_z$$

Hence

$$\gamma^{-2} = \left(1 - \frac{v^2}{c^2}\right)$$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{5.4.11}$$

Which is the ordinary SR transformation coefficient when the space is permeated with field the space is deformed [ ]. Thus

$$c^2 d\tau^2 = c^2 g_{00}(2)dt^2 - g_{xx}(2)dx^2$$

$$= c^2 g_{00}(1)dt^2 - g_{xx}(1)dx^2$$

Thus

$$c^2\big(g_{00}(2) - g_{00}(1)\big)dt^2 = [g_{xx}(2) - g_{xx}(1)]dx^2$$

$$[c^2 dg_{00}]dt^2 = [dg_{xx}]dx^2 \qquad (5.4.12)$$

But

$$g_{xx} = -g_{00}^{-1}$$

Thus

$$dg_{xx} = g_{00}^{-2} dg_{00} \qquad (5.4.13)$$

Inserting (5.4.13) in (5.4.12) gives

$$c^2 g_{00}^2 dt^2 = dx^2 \qquad (5.4.14)$$

Hence the velocity is gives by

$$v = \frac{dx}{dt} = cg_{00} = c\left(1 + \frac{2Q}{c^2}\right) \qquad (5.4.15)$$

Consider a particle having initial velocity v_0 initial potentialϕ_0, final velocity v and final potentialϕ_1.

According to equation (5.4.15), one gets

$$v_0 = c\left(1 + \frac{2\phi_0}{c^2}\right)$$

$$v = c\left(1 + \frac{2\phi_1}{c^2}\right) \qquad (5.4.16)$$

Hence

$$v - v_0 = \frac{2}{c}[\phi_1 - \phi_0] = \frac{2\phi}{C}$$

$$v = v_0 + \frac{2\phi}{c} \tag{5.4.17}$$

Thus the average mean velocity is given by

$$v_m = \frac{v + v_0}{2} = \frac{v + v - \frac{2\phi}{c}}{2}$$

$$v_m = v - \frac{\phi}{c} \tag{5.4.18}$$

Replacing v by the mean velocity $v_m$in equation (5.4.11), one gets

$$\gamma = \left(1 - \frac{{v_m}^2}{c^2}\right)^{-\frac{1}{2}} \tag{5.4.19}$$

Inserting equation (5.4.18) in (5.4.19) gives

$$\gamma = \left[1 - \frac{\left(v - \frac{\phi}{c}\right)^2}{c^2}\right]^{-\frac{1}{2}} \tag{5.4.20}$$

It is very interesting to note that when no field exist equation (5.4.20) becomes

$$\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-\frac{1}{2}} \tag{5.4.21}$$

Which is the ordinary SR transformation coefficient.

5.5 Lorentz Transformation:

In a curved space time according to Einstein hypothesis the space-time interval is invariant i.e.

$$c^2 d\tau^2 = c^2 g_{00} dt^2 - g_{xx} dx^2 \tag{5.5.1}$$

With

$$x_0 = ict \quad x_1 = x$$

In a Newtonian Limit for static isotropic metric, the Schwarzschild solution suggests that

$$g_{xx}(\phi) = -g_{00}^{-1} \tag{5.5.2}$$

Consider now two arbitrary points 1 and 2 in 4-dimensional space-time, such that

$$dt_1 = dt_2 = dt \quad dx_1 = dx_2 = dx$$

Thus according to equation (5.5.1)

$$c^2[g_{00}(2) - g_{00}(1)]dt^2 = [g_{xx}(2) - g_{xx}(1)]dx^2$$

$$c^2(dg_{00})dt^2 = (dg_{xx})dx^2 \tag{5.5.3}$$

But from equation (5.4.13)

$$dg_{xx} = g_{00}^{-2} dg_{00}$$

Therefore equation (5.5.3) becomes

$$c^2 g_{00}^2 dt^2 = dx^2$$

Hence

$$dx = c g_{00} dt \tag{5.5.4}$$

Thus the velocity is gives by

$$v = \frac{dx}{dt} = cg_{00} \tag{5.5.5}$$

But from equation (5.5.4) ϕ to be independent of t

$$x = cg_{00} \int dt = cg_{00}t \tag{5.5.6}$$

A direct substitution of (5.5.5) in (5.5.6) gives

$$x = vt \tag{5.5.7}$$

It is well known that in a weak field Limit

$$g_{00} = \left(1 + \frac{2\phi}{C^2}\right) \tag{5.5.8}$$

Thus according to equation (5.5.8) and (5.5.5)

$$v_2 - v_1 = c[g_{00}(2) - g_{00}(1)]$$

$$= c\left[\frac{2\phi_2}{c^2} - \frac{2\phi_1}{c^2}\right] \tag{5.5.9}$$

By choosing v_1 to stand for the initial velocity $v_0$and $v$to stand for the final velocity$v$, and defining

$$\phi = \phi_2 - \phi_1 \tag{5.5.10}$$

$$v = v_0 + \frac{2\phi}{c} \tag{5.5.11}$$

A direct insertion of (5.5.11) in (5.5.7) yields

$$x = \left(v_0 + \frac{2\phi}{c}\right)t \tag{5.5.12}$$

Thus for a frame S' moving with initial velocity v_0 in a field ϕ, the displacement l is given by

$$L = \left(v_0 + \frac{2\phi}{c}\right)t \tag{5.5.13}$$

When one assumes that the origin of the frames $S\ and\ S'$ coincide at

$$t = t' = 0$$

Thus one gets the following Lorentz transformation

$$x = \gamma(x' + L') = \gamma\left(x' + \left(v_0 + \frac{2\phi}{c}\right)t'\right) \tag{5.5.14}$$

$$x' = \gamma(x - L) = \gamma\left(x + \left(v_0 + \frac{2\phi}{c}\right)t\right) \tag{5.5.15}$$

If a source of light emit a photon at $(t = t' = 0)$ when the two origins of $S\ and\ S'$ coincide. Then after a time t in S and t' in S'

$$x = ct \qquad x' = ct' \tag{5.5.16}$$

A direct substitution of (5.5.16) in (5.5.14) and (5.5.15) gives

$$ct = \gamma\left[c + \left(v_0 + \frac{2\phi}{c}\right)\right]t'$$

$$t = \gamma\left[1 + \left(\frac{v_0}{c} + \frac{2\phi}{c^2}\right)\right]t' \tag{5.5.17}$$

$$ct' = \gamma\left[c - \left(v_0 + \frac{2\phi}{c^2}\right)\right]t$$

$$t' = \gamma\left[1 - \left(\frac{v_0}{c} + \frac{2\phi}{c^2}\right)\right]t \tag{5.5.18}$$

Inserting (5.5.18) in (5.5.17) yields

$$t = \gamma^2 \left[1 + \left(\frac{v_0}{c} + \frac{2\phi}{c^2}\right)\right] \left[1 - \left(\frac{v_0}{c} + \frac{2\phi}{c^2}\right)\right] t$$

This indicates that

$$\gamma = \frac{1}{\sqrt{\left(1 - \left(\frac{v_0}{c} + \frac{2\phi}{c^2}\right)^2\right)}} \tag{5.5.19}$$

To express γ in terms of the instantaneous velocity v, one uses the relation

$$v^2 = v_0^2 + 2\phi$$

Thus

$$v_0^2 = v^2 - 2\phi \tag{5.520}$$

Therefore

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{\sqrt{v^2 - 2\phi}}{c} + \frac{2\phi}{c^2}\right)^2}} \tag{5.5.21}$$

When no field exists

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Which is the ordinary SR transformation coefficient.

5.6 Discussion:

The SR Lorentz transformation can be found by using the electromagnetic force relation for a charged electron moving in an electromagnetic field, as shown by equations (5.2.2), (5.2.3), (5.2.4), and (5.2.5).

Using Maxwell equations, concerning generation of electric field by variable magnetic field, one gets a relation between Ycomponent of the magnetic field and Z component of electric field in equation (5.2.12). Using all above relations the Einstein SR coefficient γ is shown to be typical to that of SR.

Coefficient γ for particles moving in a field is found by using ordinary relation between velocity, acceleration and potential per unit mass [see equations (5.3.1 to 5.3.8). fortunately this relation reduces to that of SR in the absence of afield, as equation (5.3.9) indicate replacing the velocity v with the average velocity v_m in SR Einstein coefficient in (5.3.10), one gets γ in terms of v_m. Again using the relations between velocity and potential per unit mass, one gets two different expressions for γ depending on the time and time free relation of v and v_0[see equations (5.3.19), (5.3.26)] . fortunately the two expressions reduces to that of SR, as shown by equations (5.3.20) and (5.3.27).

Section (5.4) deals with another Lorentz transformation based on Maxwell equation which shows how time varying electric field generates magnetic field as shown by equation (5.4.1).

According to equations (5.4.5) the electromagnetic wave is a travelling wave in the x-direction, with a magnetic field vibrating in the y-direction and the electric field is vibrating in the Z-direction.

According to this version by and E_z are related according to equation (5.4.7). using the electromagnetic force relations (5.2.4) and (5.2.5), one gets relations in equations (5.4.9) and (5.4.10) which relates E_z to E'_z. These two relations are used to derive Einstein coefficientγ, which is strikingly the same as that of SR.

To find γ for any field, one uses the invariance of the interval to get an expression which relates the initial velocity v_0 to the final velocity v[see equation (5.4.17)]. This equation is the Einstein counter part of the Newton one which is given by

$$v = v_0 - at \tag{5.6.1}$$

But (5.4.17) which reflects space curvature which is written as

$$v = v_0 - \frac{2\phi}{c} \tag{5.6.2}$$

But

$$g_{00} = \left(1 + \frac{2\phi}{c}\right)$$

Thus the velocity time evolution is described by time metric as

$$v = v_0 + g_{00} - 1 \tag{5.6.3}$$

One can find (5.6.1) a doping an approximation which assumes that

$$g_{xx} = 1 \qquad g_{00} = \left(1 + \frac{2\phi}{c^2}\right) \tag{5.6.4}$$

By assuming $d\tau$to be very small equation (5.5.1) gives

$$dx = c\left(1 + \frac{2\phi_1}{c^2}\right) = c + \frac{\phi_1}{c} \tag{5.6.5}$$

When

$$v = v_0 \quad \phi = \phi_0 \tag{5.6.6}$$

Thus

$$v_0 = c + \frac{\phi_0}{c} \tag{5.6.7}$$

Hence

$$v = v_0 + \left(\frac{\phi_1 - \phi_0}{c}\right) = v_0 + \frac{\phi}{c} \tag{5.6.8}$$

But

$$\phi = ax \tag{5.6.9}$$

Thus (5.6.8) becomes

$$v = v_0 + \frac{ax}{c} \tag{5.6.10}$$

For a photon

$$x = ct \tag{5.6.11}$$

Thus

$$v = v_0 + at \tag{5.6.12}$$

Which is the ordinary Newton second Low.

By replacing v by the mean velocity v_m in (5.6.12). then using equations (5.4.18) and (5.4.19), one gets γ for any field ϕ.

To incorporate the effect of the field one can also use a relation between field potential per unit mass and the time metric g_{00} to derive anew Lorentz transformation coefficientγ. According to the expression for interval (5.5.1) a useful relation for $v \; and \; L$ in terms of ϕ were found in equations

(5.5.11) and (5.5.13). These were used to express x in terms of x' and vice versa [see (5.5.14) and (5.5.15). then a field dependent relation for γ are found in (5.5.21) this expression fortunately reduced to that of SR in the absence of field.

5.7 Conclusion:

The expression of the electric and magnetic force on the electrons beside the expression of the displacement current is used to derive special relativistic and generalized special relativistic Lorentz transformation which can successfully describe a wide variety of physical phenomena in the presence and absence of fields.

5.8 outlook:

This research can be extended to be applied for quantum field theory, it cn also be applied to drive new relativistic quantum equations.

The experimental verification can also be done by analyzing the information about the space and astronomical object observed by electromagnetic waves or laser beam.

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