CHAPTER THREE

MATHEMATICAL MODEL

3.1 Introduction

An accurate knowledge of the electrostatic field distribution and the onset voltage of corona from the discharge wires is of fundamental importance in developing good models of the different processes that take place in precipitators .This chapter shows all equations that developed to calculation electric fields, corona onset voltage and collection efficiency.

3.2 Electric Field Calculation

The well-known, charge-simulation technique [6] is used to calculate the electric field for wire-duct ESP with m wires (m is odd). Figure 3.1 shows a one-quadrant of the cross section of the ESP in the X-Y plane. For simplicity y, the precipitator is assumed infinitely long the Z-direction. The surface charge on each wire is simulated by ($N1$) line charges located at radius r_f, where $r_f = f \times r_c$, where f is a fraction, chosen 0.5 in the present work and r_c is the radius of discharge wire. The surface charge on each plate of the ESP is simulated by a number ($N2$) line charges located outside the plate at a distance from the plate (a) equal to the distance between two adjacent simulation charges (b) as shown in Figure 3.1. Thus, the total number of unknown simulation charges is ($mN1 + 2N2$). There is a symmetry around both x and y axes as shown in Figure 3.1, reduces the number of unknowns to $n = ((mN1 + 2N2)/4)$.

To evaluate the unknown simulation charges Qj, $j = 1, 2, 3, \dots , n$, a set of boundary points equal to the simulation charges is chosen on the surface of discharge wires and collecting plates as shown in Figure(3.1) to satisfy the boundary conditions: $\Phi = V$ at discharge wires, $\Phi = 0$ at collecting plates.

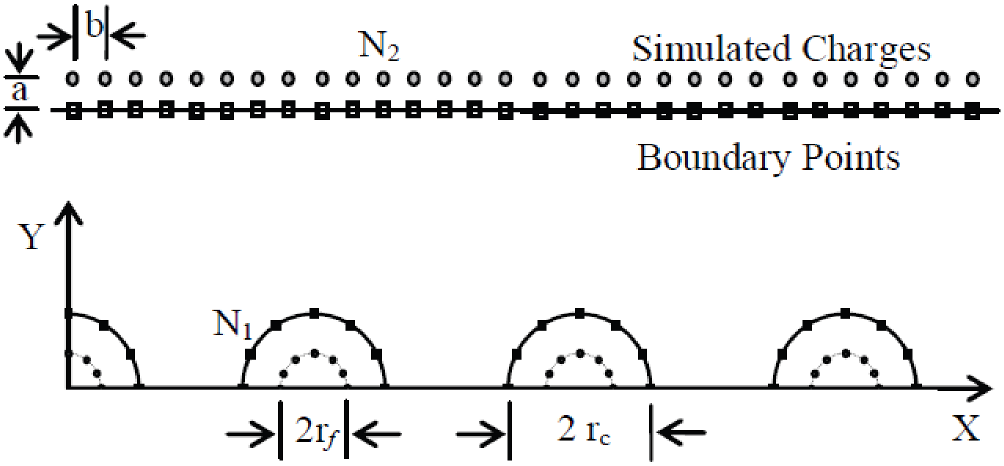

Figure 3.1: Arrangement of simulation charges and boundary points for one quarter of wire duct ESP

The potential Φ*i* is at the *i*th boundary point of coordinates (*xi*, *yi*) is the sum of potential contributions due to all simulation charges.

$$\Phi_i = \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{P_{ij}}\ \mathrm{Q_j} \tag{3.1}$$

Where, P_{ij} is the potential coefficient expressed as:

$$\mathrm{P_{ij}} = \ln\left({}^{1}\!/_{R_1\ R_2\ R_3\ R_4} \right) \tag{3.2}$$

$i\ =\ 1,2,3,4,\ldots,n$ and $j\ =\ 1,2,3,4,\ldots,n$

Where,

$$R_1^2 = (x_i - x_j)^2 + (y_i - y_j)^2 \tag{3.3}$$

$$R_2^2 = (x_i + x_j)^2 + (y_i - y_j)^2 \tag{3.4}$$

$$R_3^2 = (x_i + x_j)^2 + (y_i + y_j)^2 \tag{3.5}$$

$$R_4^2 = (x_i - x_j)^2 + (y_i + y_j)^2 \tag{3.6}$$

x_j and y_j are the coordinates of the jth simulation charge. Satisfaction of the boundary conditions at the boundary points formulates a set of equations relating the values of simulation charges to the potential values at the boundary points. This is expressed in a matrix form as:

$$[\mathrm{P}][\mathrm{Q}] = [\mathrm{V_b}] \tag{3.7}$$

Where $[P]$ is the potential coefficient matrix (with dimension $n\times n$), $[Q]$ are unknown simulation charges (with dimension $n\times 1$), and $[Vb]$ are the potential values of the boundary points (with dimension $n\times 1$). Solution of the set of Equation (3.7) determines the unknown simulation charges Qj. To check the accuracy of the solution, a set of check points is chosen (each check point is located between two successive boundary points). The potential value is to be checked against the applied value (V at the wire surface and zero at the collecting plates). Once the accuracy is checked and the simulation charges are known, the electric field intensity at any point p (x_p, y_p) can be determined:

$$E_x = \sum_{j=1}^{n} Q_j((x_p - x_j)\left(\frac{1}{R_1^2} + \frac{1}{R_4^2}\right) + (x_p + x_j)\left(\frac{1}{R_2^2} + \frac{1}{R_3^2}\right)) \tag{3.8}$$

$$E_y = \sum_{j=1}^{n} Q_j((y_p - y_j)\left(\frac{1}{R_1^2} + \frac{1}{R_4^2}\right) + (y_p + y_j)\left(\frac{1}{R_2^2} + \frac{1}{R_3^2}\right)) \tag{3.9}$$

The magnitude of the electric field intensity at point p is calculated as:

$$E_p = \sqrt{(E_x^2 + E_y^2)} \quad \frac{\mathrm{V}}{\mathrm{m}} \tag{3.10}$$

3.2 Corona Onset Voltage Calculation

With the wires of a wire-duct precipitator stressed by negative HV supply, the electric field near a wire reaches the threshold value for ionization of gas molecules by electron collision. A primary electron avalanche starts to develop along the direction of maximum field away from the wire. The avalanche grows through the so-called “ionization zone” whose boundary defines the zone where the ionization coefficient exceeds the coefficient of electron attachment as shown in Figure 3.2.

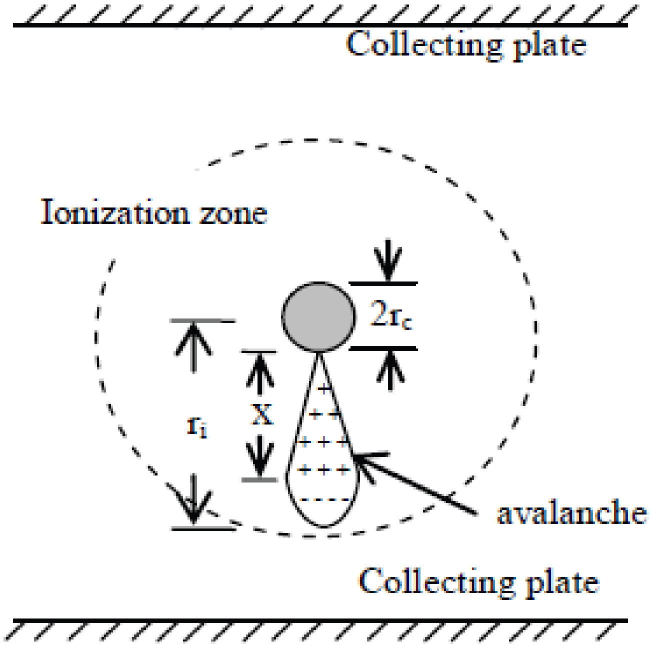

Figure 3.2: the development of avalanche

In order to simplify the calculations, the following assumptions are made:

1) The ionizing-zone extends from the wire surface to the point where almost all (i.e. 99.99%) of the electrons get attached to neutral gas molecules and from negative ions.

2) The electron avalanche grows under the influence of its own space-charge field besides the applied electrostatic field.

3) The space-charge-field of the avalanche is the same as if all its positive ions were concentrated at distance $1/\alpha$ from its tip. This assumption has been found satisfactory in similar work on uniform and no uniform fields [7-10]. At the onset voltage, the avalanche should somehow provide an initiating electron

at the wire surface to start a successor avalanche, possibly by photoemission, positive ion impact, meta stable action or field emission. Field emission is possible only at field strengths exceeding 5×10^{7}V/m [9]. Electron emission by positive ion impact is more than two orders of magnitude less probable than photoemission. Meta stables have been reported to have an effect approximately equal to that of positive ion impact [5]. Therefore, only the first mechanism (electron emission by photons) was considered in determining the onset voltage. The condition for a new (successor) avalanche to develop is

$$\gamma_{ph}\left[\int_{0}^{(r_f-r_c)} \alpha(x) \quad e^{\int_0^x(\alpha-\eta)d_x}\, g(x)\ e^{-\mu x}\ d_x\right] \geq 1 \tag{3.11}$$

Where:

$\boldsymbol{\alpha}$: Townsend's first ionization coefficient.

γ_{ph}: Coefficient of electron emission by the action of photons.

μ: Electron attachment coefficient.

g (*x*): Geometry factor to account for the factor that.

Some photons will not be received by the discharge wires of ESP, [7-10]. The corona onset voltage V_0 does not appear explicitly in last relation, however, the applied voltage affects the values of $\boldsymbol{\alpha}$, η, ... The onset voltage V_0 is the critical value which fulfills Equation (3.11). figure 3.3 shows the configuration of wire duct ESP

3.3 Precipitator Collection Efficiency

Precipitator collection can be determined using theoretical as well as practical methods. Both the methods are discussed as follows:

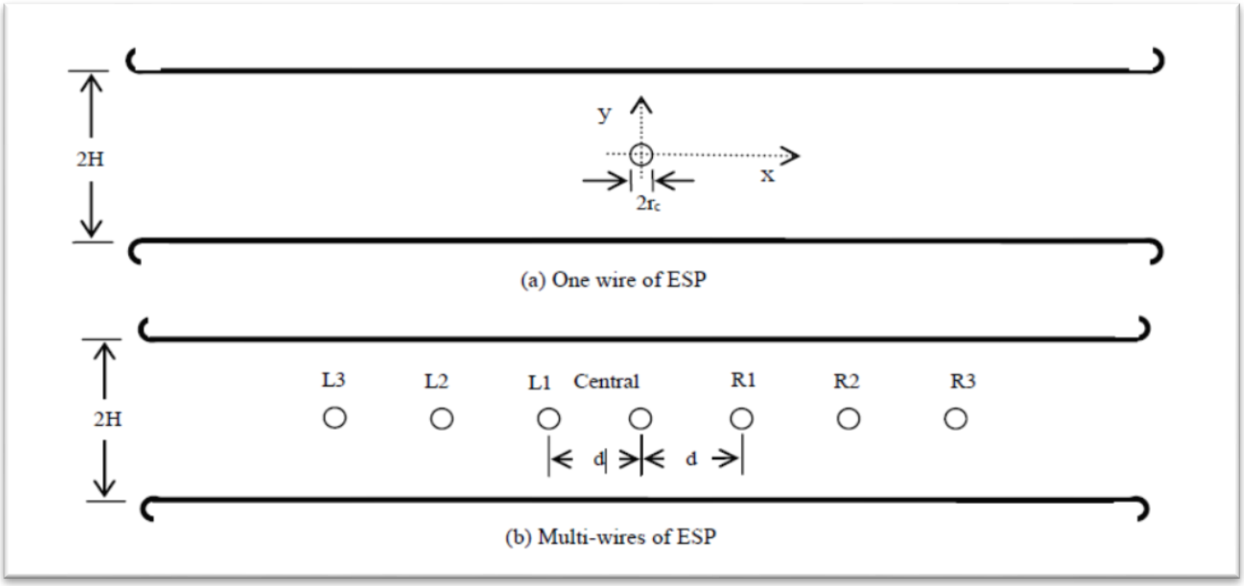

Figure 3.3: configuration of wire duct ESP

3.3.1 Theoretical collection efficiency

For an ESP with a known collection are A and a known volumetric flow Q, the Theoretical collection efficiency can be found out by calculating the electrical migration velocity, ν_p as follows. Electrical migration velocity, ν_p is the speed at which a particle is attracted to the collecting electrode.

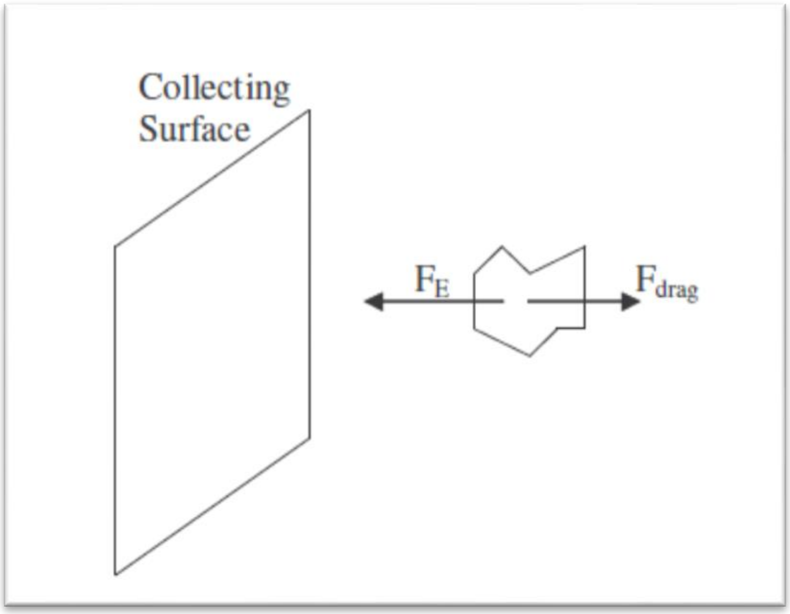

Figure 3.4: Forces acting on particle

Figure 3.4 shows force balance based on electrostatic force and the drag force in steady state condition. The theoretical collection efficiency can be derived as follows:

$$\sum F_x = F_E - F_{drag} = 0 \tag{3.12}$$

$$F_{drag} = C_d * A_p * \frac{\rho_{gas} * \nu_p}{2} * \frac{1}{C_c} \tag{3.13}$$

$$F_{drag} = \frac{C_d}{2C_c}\, \pi r_d^2\, \rho_{gas} {\nu_p}^2 \tag{3.14}$$

$$F_E = q * E \tag{3.15}$$

Where:

$$q = \left(\frac{3K}{K+2}\right) \pi \varepsilon_0 E {D_p}^2$$

K= dielectric constant of fly ash.

ε_0 = Ppermittivity of free space ($8.85*10^{-12}$ F/m).

E= The electric field strength (V/m).

C_c is the Cunningham correction factor given by

$$C_c = 1 + Kn\, \left(1.257 + o.4\, exp(\tfrac{-1.1}{Kn})\right) \tag{3.16}$$

Kn= The Knudsen number given by,

$$Kn = -\frac{\mu_g}{0.499\, P_g r_p (\frac{8MW}{\pi RT})^{0.5}} \tag{3.17}$$

C_d is the darg coefficient given by:

$$C_d = \frac{24}{Re}X \tag{3.18}$$

Where :

X=1 when Re<0.1

X=0 when Re>0.1

Reynolds number, $Re = \frac{\rho_{gas} * \nu_p * d_p}{\mu_{gas}}$ (3.19)

Substituting in equation 1 we get,

$$\nu_p = \frac{(\frac{K}{K+2})E^2 \varepsilon_0 D_P C_c}{(1+X)\mu_f} \tag{3.20}$$

As shown in Figure 3.5 and to find the theoretical efficiency, an ESP is considered with:

H= Height of collecting electrode.

W= Width of collecting electrode.

Δx = Distance between collection and discharge electrodes.

If the particle travels H in y direction, before it travels L in x direction, we will say it is collected > let us assume,

ν_p = Velocity in x- direction.

U = Velocity in y- direction.

In a time interval dt, fraction collected in $dx = \frac{\nu_p \, dt}{H}$. Particle concentration (c) will decrease as moving in x-direction

$$dC = -C \,.\, fraction\ collected\ in\ dx \tag{3.21}$$

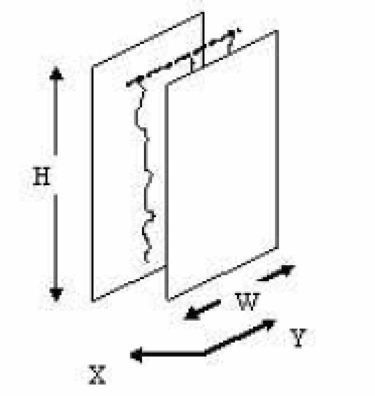

Figure 3.5: Collection Surface Dimensions

$$dC = -\mathrm{C}\frac{\mathrm{v_p\,dt}}{\mathrm{H}} \qquad (3.22)$$

$$\mathrm{dC} = -\mathrm{C}\frac{\mathrm{v_p}}{\mathrm{H}}\frac{\mathrm{dx}}{\mathrm{U}} \quad ,\text{since } \mathrm{dt} = \frac{\mathrm{dx}}{\mathrm{U}} \qquad (3.23)$$

Integrating Equation (3.23) , then get :

$$\int \frac{\mathrm{dC}}{\mathrm{C}} = -\int \frac{\mathrm{v_p}}{\mathrm{H\,U}}\mathrm{dx} \qquad (3.24)$$

Take Ln to both side at Equation (3.24) ; then get :

$$\ln\left(\frac{\mathrm{C_{outlet}}}{\mathrm{C_{inlet}}}\right) = -\frac{\mathrm{v_p}}{\mathrm{H}}\ \mathrm{L} \qquad (3.25)$$

At x=0 , $C=C_{inlet}$, x=L and $C=C_{outlet}$ also:

$$\eta_{(\mathrm{D_p})} = \frac{\mathrm{C_{inlet}} - \mathrm{C_{outlet}}}{\mathrm{C_{inlet}}} \qquad (3.26)$$

From Equation (3.25) and (3.26) the $\eta_{(\mathrm{D_p})}$ equal to :

$$\eta_{(D_p)} = 1 - \exp\left(-\frac{A\,v_p}{Q}\right) \tag{3.27}$$

Where :

$\eta_{(Dp)}$ = theoretical efficiency .

ν_p = the electrical migration velocity in m/s

Q= the volumetric flow rate in m^3.

A = the collection area in m^2.

In 1922, Deutsch derived this equation using a differential equation method and in 1919 was empirically derived by Anderson based on field experiments [9]. Equation 3.27 will be used to determine the theoretical efficiency of the wet metal ESP in capturing fly ash and iron oxide particulates.

3.3.2 Actual collection efficiency

The actual collection efficiency is calculated by doing a mass balance of the particulates captured before and after the ESP. Iso-kinetic sampling is done at the inlet and outlet using ESP method five. The concentration for calculating efficiency is found by dividing the differential filter weight by the volume of gas sampled by the gas meter. The following formulae will be used to find the actual collection efficiency of the wet metal ESP in capturing fly ash and iron oxide particulates .

$$V_{std} = \frac{K_1 * V_m * Y * \left(P_{bar} + \frac{\Delta H}{13.6}\right)}{T_m} \tag{3.28}$$

The concentration at the inlet is calculation as :

$$\mathrm{Conc_{inlet}} = \frac{\mathrm{m_n}}{\mathrm{V_{std}}} \qquad (3.29)$$

The actual collection efficiency found by :

$$\eta = \frac{\mathrm{Conc_{inlet}} - \mathrm{Conc_{outlet}}}{\mathrm{Conc_{inlet}}} \qquad (3.30)$$

Where :

η = Actual collection efficiency.

V_{std} = Volume of the gas sample measured by the dry gas meter , corrected to standard conditions

V_m = Volume of gas sample measured by dry gas meter.

ΔH = Average pressure differential across the orifice meter in mm of H_2O.

T_m = Absolute average DGM temperature.

P_{bar} = Barometric pressure at the sampling site in mm of Hg.

m_n = Differential filter weight.

$Conc_{inlet}$ = Concentration at inlet.

$Conc_{outlet}$ = Concentration outlet.