

Dedication

To my Father and mother

And to these

..... Whom I love

Acknowledgment

My praise and thanks to Allah , who gave me the strength to conduct such work . I am greatly indebted to my supervisor **Dr: Emad Eldeen Abdallah Abdel Rahim. .** Thanks

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Abstract

In this research we consider the recent exploring problem of existence (or non existence) of warped product submanifold in known spaces, for this reason first we have discussed a necessary and sufficient condition in terms of two canonical structures on a Cauchy Riemannian CR-submanifold of a nearly Kähler manifold under which the submanifold reduces to a locally CR-warped product manifold, More over an estimate for the second fundamental form is investigated. Also we have illustrated the proof of an

inequality for squared norm of second
fundamental form

الخلاصة

في هذا البحث إعتبرنا المسألة الحالية لإستكشاف وجود (أو عدم وجود) متعددات طيات الضرب الإلتوائي في الفضاءات المعروفة ، لهذا السبب أولاً ناقشنا الشروط الضرورية والكافية في نطاق بناءين قانونين على متعدد طيات كوشي- ريمان لمتعدد طيات كهلر

التقريبي بحيث يكون متعدد الطيات الجزئي ممهد لمتعدد طيات
كوشي- ريمان للضرب الإلتوائي ، كذلك تم التحقيق في تقدير
للصيغة الأساسية الثانية .أيضا وضحنا إثبات متباينة التنظيم
التربيعي للصيغة الأساسية الثانية.

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Introduction

A warped product manifolds provide an excellent setting model space time near black holes or bodies with large gravitational field , the study of these manifolds assumes significance in general . B.Y.chen initiated the study of CR-warped product submanifolds in a kähler manifolds . He obtained a characterization for CR-submanifold to be locally a CR-warped product and an estimate for the squared norm of the second fundamental form of CR-warped products in a complex space form .this work investigate some methods to obtained this complex space and discuss the extend of some results of chen

: to a more general setting , and it is organized as follows

Firstly we begin with a brief introduction to Riemannian manifolds ,and we introduce the notion of a connection ,and the important notion of the curvature of connection . As applications of these notions , we present the Levi-civita connection theorem ,and Bianchi identity proposition .Also we discuss the cartan theorem of the connection forms of the .Levi-civita connection

In chapter 2 we present the contact CR-submanifold of a sasakian manifold ,and provide some insights into the structure of the space of geometries , which plays an important role in determine every contact CR-product whether is an invariant submanifold or anti-invariant submanifolds. Also we illustrate the presentation of some properties of warped product contact CR-submanifold in sasakian manifolds and some of their .applications

In chapter 3 we investigate a more general structure on an almost Hermitian manifold with an almost complex structure and a Hermitian metric ,namely kaehler structure , and then sufficient condition for them to be kaehler manifold ,Also we obtained necessary and sufficient condition forcing a CR-submanifold to be locally a CR-warped product manifold ,and we discuss some applications on kaehler manifold an CR-warped products in generalized . Complex space form

Finally we give some applications of Riemannian metric on sasakian space , also we obtained necessary and sufficient condition for which a semi-invariant submanifold is a semi-invariant warped product submanifold in the setting of almost contact metric manifold , and then we discuss some .applications