SUDAN UNIVERSITY OF SCIENCE AND TECHNOLOGY

COLLEGE OF SCIENCE

Similarity of quasinilpotent operators and Norm estimates on Hardy spaces with interpolation of applications to controllability

تشابه المؤثرات شبه متلاشية القوى وتقديرات النظيم على فضاءات هاردي طبقاً لاستكمال التطبيقات لإمكانية التحكم

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﴿ اقْرَأْ بِاسْمِ رَبِّكَ الَّذِي خَلَقَ ، خَلَقَ الْإِنسَانَ مِنْ عَلَقٍ ، اقْرَأْ وَرَبُّكَ الْأَكْرَمُ ، الَّذِي عَلَّمَ بِالْقَلَمِ ،عَلَّمَ الْإِنسَانَ مَا لَمْ يَعْلَمْ ﴾

صدق الله العظيم

سورة العلق الآية (١-٥)

Dedication

To my family

Acknowledgments

I wish to express my thanks to my supervisor Dr. Shawgy Hussein AbdAlla for his help and guidance throughout this work. Special thanks to my family for their help encouragement and support during this research. And full thanks to Sudan University of Science and Technology. And thanks to My friend Khalid Osman for his help. And thanks to Amir & Albrraa Dawash for typing this research.

Abstract

We Show the uniqueness of the norm on the Lebesgue space of the compact group. We give some applications of the property of Kazhdan to the method of automatic continuity. We determine the similarity of quasinilpotent operators. The symmetric Meixner- Pollaczek polynomials and a system of orthogonal polynomials with Hardy spaces for the strip are considered. We investigate the behaviour of the Lebesgue space of the integral means of the analytic functions and the vector- valued BMOA and multipliers. We determine the norm estimates for operators from the Hardy space to the Hilbert space and the interpolation by the vector- valued analytic function that have applications to controllability. We discuss and show new abstract Hardy spaces.

الخلاصة

تم توضيح وحدانية النظيم على فضاء لبيق للزمرة المتراصة. أعطينا بعض التطبيقات لخاصية كازدان لطريقة الاستمرارية ذاتية الفعل. تم تحديد تشابه المؤثرات شبه متلاشية القوى. تم اعتبار تماثل كثيرات حدود مسنر– بولازيك ونظام كثيرات الحدود الناظمة طبقاً لفضاءات هاردي للشريط. تم تقصي سلوك فضاء لبيق لمتوسطات التكامل للدوال التحليلية وقيم متجه BMOA والمضاريب. تم تحديد تقديرات النظيم للمؤثرات من فضاء هاردي إلى فضاء هلبرت والاستكمال بواسطة الدالة التحليلية قيمة المتجه والتي لها تطبيقات لإمكانية التحكم. تم مناقشة وتوضيح فضاءات هاردي الجديدة.

Introduction

We show that every complete norm$|\cdot|$on$L^p\ (G)$ that makes translation from $\left(L^p\ (G), |\cdot|\right)$ into itself continuous and that makes the map $t \to L_t$ from $G\ in\ L(L^p\ (G, |\cdot|))$bounded is equivalent to $\|\cdot\|_p$. We characterize the continuity of all the linear operators from $L^p\ (\mu)$ which commute with translations and the property of $\|\cdot\|_p$. Being the unique norm (up to equivalence) on $L^p\ (\mu)$ with respect to which all translations are continuous in terms of the finite dimensionality of the sub space of all invariant functions.

A sufficient condition for an injective bounded linear operator on a Banach space to determine its topology is obtained . From this condition it follows , for instance , that the Volterra operator acting on the Hardy space H^p of the unit disk determines the topology of H^p for any$p \in [1, \infty]$.

Moreover, there are some simple operators that connect the systems with each other. We have designated the special case of the symmtric Meixner - Pollaczek polynomial systems by $\tilde{\tau}_n$ and the latter system on the strip by $\tilde{\sigma}_n$ and we were able to show that this system is the limiting case of the symmetric Meixner Pollaczek polynomial systems, $p_n^{(\lambda)}\left(\frac{x}{2}, \frac{\pi}{2}\right)$ as $\lambda \to 0$ this we define anon-standard inner product with respect to which the polynomials , $p_n^{(\lambda)}\left(\frac{x}{2}, \frac{\pi}{2}\right)$, for as $\lambda \leq 0$, become orthogonal polynomials, it examines the major properties of the polynomials, $p_n^{(\lambda)}\left(\frac{x}{2}, \frac{\pi}{2}\right)$ for as $\lambda > 0$ which are also shared by the polynomials , $p_n^{(\lambda)}\left(\frac{x}{2}, \frac{\pi}{2}\right)$ for $\lambda \leq 0$. We also present an orthogonal in $H^2(S)$ basis of polynomials.

We establish new connections between some classical inequalities concerning Hardy spaces. We study space of multipliers between , H^p and BMOA in the vector – valued setting . This leads us to the consideration of some geometric properties depending upon the validity of certain inequalities due to Littlewood and Paley on the g, function for vector valued functions. We give upper and lower estimates of the norm of a bounded linear operator from the Hardy space H^p to l^q in terms of the norm of the rows and the columns of its associated matrix in certain vector - valued sequence spaces.

Applications are given to the controllability properties of linear semi group systems with a Riesz basis of eigenvectors. We also obtain some results on weighted norm inequalities. Finally we present partial results in order to understand a characterization of the duals of Hardy spaces.

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