

Dedication:

*To my family and friends who have kept me going during this
Arduous task...I thank you.*

*To my mother and father...I cannot express how grateful I am
for having
this opportunity; I could not have done it without you. Your
Unconditional love and support has allowed me to accomplish
something I never dreamed possible.*

To my grandmother's soul...God's mercy to you

To the soul of AL_SHaheed

Eng. Abdulrahim Saleh Hamza... allah bless his soul

To my brothers, sisters and colleagues....thank you.

Abstract:

The goal of this research is to design a minimum length supersonic nozzle (MLN) and develops a computer code which uses the Method of Characteristics to design supersonic nozzle contours for isentropic, inviscid, irrotational supersonic flows of any working fluid for any user-defined exit Mach number. The contours were compared to theoretical isentropic exit area ratios and exit area ratios for GA10 nozzle, the percentage of error does not exceed 2% for all exit area ratios, and percentage of error does not exceed 5% for all mass flow rates. Also the contour was compared to Britton's code, where the difference does not exceed 10% of Britton's code results.

The accuracy of the nozzle to produce the desired exit Mach number was also checked by using the commercial Computational Fluid Dynamics (CFD) code FLUENT. FLUENT simulation was used for three nozzles (three exits Mach number), results gives percentage error for desired exit Mach number does not exceed 3% for each case, and there is *no* shock wave.

According to results obtained, we find that the program gives acceptable design to the minimum length supersonic nozzle contour.

مستخلص البحث:

يهدف هذا البحث إلى تصميم منفث فوق صوتي ذو طول قصير (MLN) وإنشاء برنامج ماتلاب لحساب شكل المنفث باستخدام طريقة الخواص (MOC) وذلك لجريان فوق صوتي، ايسنتروبي، غير لزج، غير دوراني لأي وسيط تشغيل مثالي واي رقم ماخ عند الخروج. بعد ذلك تمت مقارنة نسبة المساحات عند المخرج مع نظيراتها للجريان الايسنتروبي وايضاً مقارنة نسب المساحات عند المخرج مع منفث فوق صوتي لنفق هوائي (GA10) وكانت نسبة الخطأ لا تتجاوز 2% ومقارنة معدل سريان الكتله وكانت نسبة الخطأ لا تتجاوز 5%. ايضاً عقدت مقارنة بين شكل المنفث ونظيره المنتج من برنامج اخر (Britton's Code) وكانت نسبة الخطأ لا تتجاوز 10%. بحيث ان هذه النتائج في الثلاث مقارنات لنسب المساحة جيده ومرضيه جداً.

اخيراً اجرينا تحليلاً للمنفث بواسطة برنامج (FLUENT) وذلك للتأكد من ان رقم ماخ عند المخرج يكون بنفس القيمة التصميمية ومعرفة ما إذا تكونت صدمه داخل المنفث. ومن خلال نتائج المحاكاة اتضح ان رقم ماخ عند المخرج يكون قريب جداً من القيمة التصميمية بحيث لا يتجاوز الخطأ 3% وانه لا توجد صدمة داخل المنفث.

وفقاً للنتائج المتحصل عليها من الطرق اعلاه نجد ان البرنامج يعطي تصميمًا مقبولاً لشكل المنفث فوق الصوتي ذو الطول القصير (MLN).

Acknowledgements:

I would like to thank my supervisor, Dr. Eimad Eldin Elhadi, for his guidance and continued support throughout my research.

I would also like to thank Dr. Tagelsir Hassan and Eng. Sabir Abushosha Ahmed for their advice and support.

My special thanks are due to faculties of engineering in both Omdurman Islamic and Sudan University of Science and Technology.

I would also like to thank all of my family, especially my mother, who supported me in my studies.

My thanks also go to my Lecturers Department for providing me a teaching assistantship during my graduate studies and colleagues at the department of mechanical engineering at the Faculty of Engineering, Omdurman Islamic University for creating an enjoyable working environment.

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Nomenclatures

Symbols:

A	Area	v	Prandtl-Meyer function
ρ	density	V	Velocity
M	Mach Number	μ	Mach angle
γ	Ratio of specific heats	α	Left-running Characteristic Line angle
P	Pressure	β	Right-running Characteristic Line angle
v	ydirection component velocity	num	Number of Characteristic Lines
θ	Flow direction angle	fix	MATLAB command “Round towards zero”
u	x direction component velocity	Φ	velocity potential
a	speed of sound	d θ	Increment of left-running characteristics angles

Subscripts:

char	for characteristics	i	variable is known
C_-	For left-running characteristics	i +1	variable is unknown
C_+	For right-running characteristics	o	variable is at stagnation condition
K $_-$	Along C_- characteristics	exit	variable is an exit condition
K $_+$	Along C_+ characteristics	max	Maximum Value

Prefixes:

Δ - Incremental increase in the variable
f () - Function of the internal variables

Abbreviations:

CFD	Computational fluid dynamics
PDEs	Partial differential equations
ODEs	Ordinary differential equations
2-D	Two-Dimensional
DBA	Design by Analysis
MOC	Method of Characteristics
MLN	Minimum Length Nozzle