Verification of Program Results

5.1 Introduction:

The finite element formulations described in chapter three were implemented in the computer program written using MATLAB as presented in chapter four. The program was tested for linear analysis using different element numbers. Two numerical examples of large deformation problems were examined to demonstrate the degree of accuracy that can be obtained by using the incremental non-linear formulations for thin beams based on Green strain. The results of displacements, total internal forces, residual and stress values of the different elements were obtained. The following presents the two selected examples.

5.2 Linear Program Test:

5.2.1 Cantilever under point load at free end:

The cantilever is of dimensions L=2.5m as shown in Figure (5.1). The numerical values of material property parameters are; Young's modulus, E = 2 x 10^8 kN/m^2, the second moment of area I= 8.33 x 10^{-5} m^4, P = 13328 kN and area of beam A = 0.1 m^2.

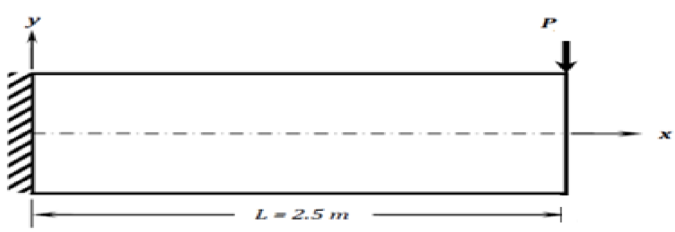

Figure 5.1: Cantilever with vertical load at free end.

This example demonstrate the efficiency of the linear analysis for different element numbers (1, 2, 4, 8, 16, 32), the results are shown in Tables (5.1) to (5.18). Graphical of results is also presented in Fig (5.2) to (5.19). Almost the same results were obtained for all cases. Tables (5.19) to (5.21) show that the displacements at nodes for sixteen elements and (5, 10, 20) increments and gave accurate results.

Table 5.1: Displacements at nodes for one element

Node No.	u (m)	v (m)	θ (rad)
1	0	0	0
2	0	-4.1667	-2.500

Table 5.2: Stresses at nodes for one element

Node No.	F (kN)	M=10^4*(kN.m)
1	0	-3.3320
2	0	0.0000

Table 5.3: Reactions at nodes for one element

Node No	Fr = x 10^4(kn)		
1	0	1.3328	3.3320
2	0	-1.3328	0.0000

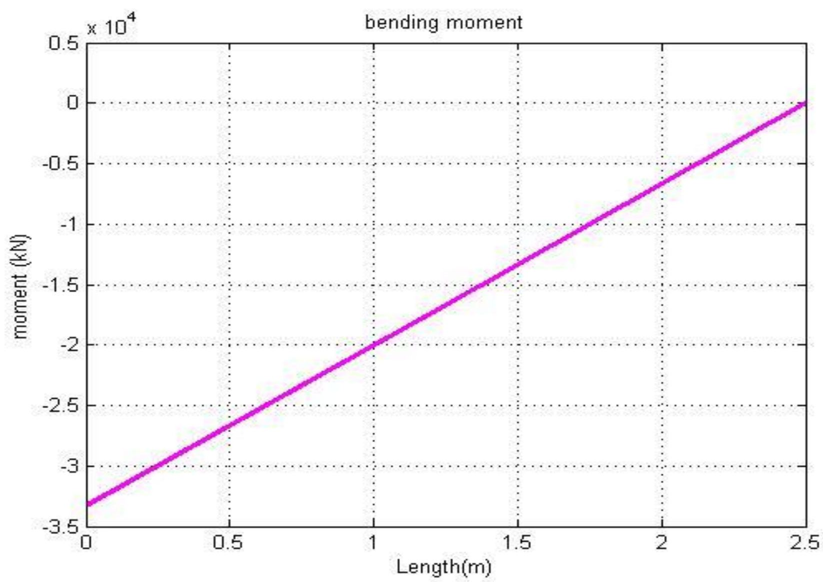

Figure 5.2: bending moment at nodes for one element

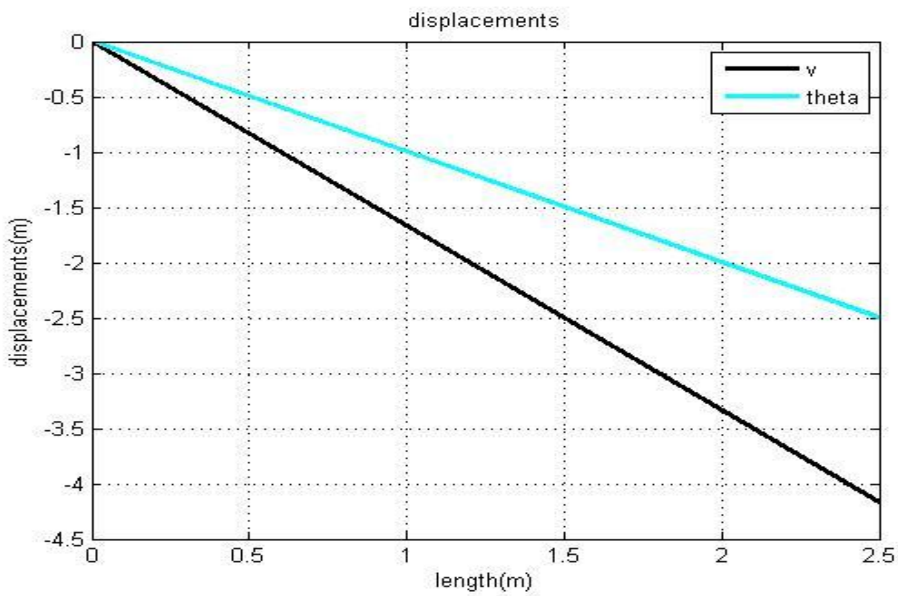

Figure 5.3: displacements at nodes for one element.

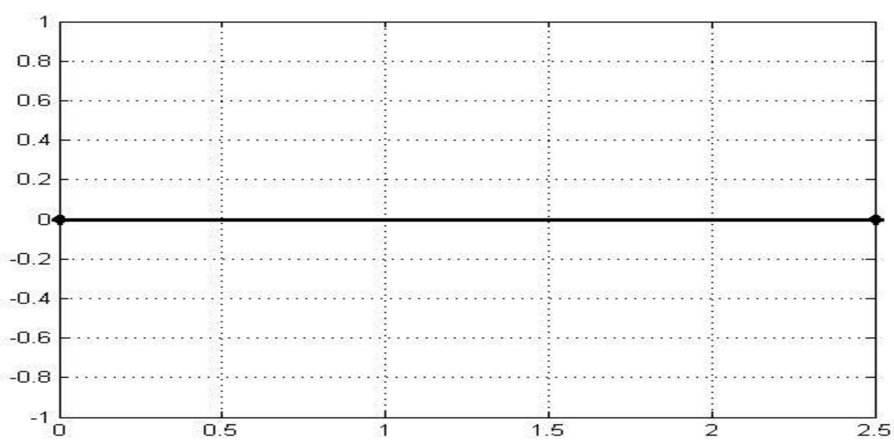

Figure 5.4: Mesh for one element.

Table 5.4: Displacements at nodes for two elements

Node No.	u (m)	v (m)	θ (rad)
1	0	0	0
2	0	-1.3021	-1.8750
3	0	-4.1667	-2.5000

Table 5.5: Stresses at nodes for two elements

Node No.	F (kN)	M = x 10^4 (kN.m)
1	0	-3.3330
2	0	-1.6660
3	0	0.0000

Table 5.6: Reactions at nodes for two elements

Node No.	Fr = x10^4 (kN)		
1	0	1.3328	3.3320
2	0	0.0000	0.0000
3	0	-1.3328	0.0000

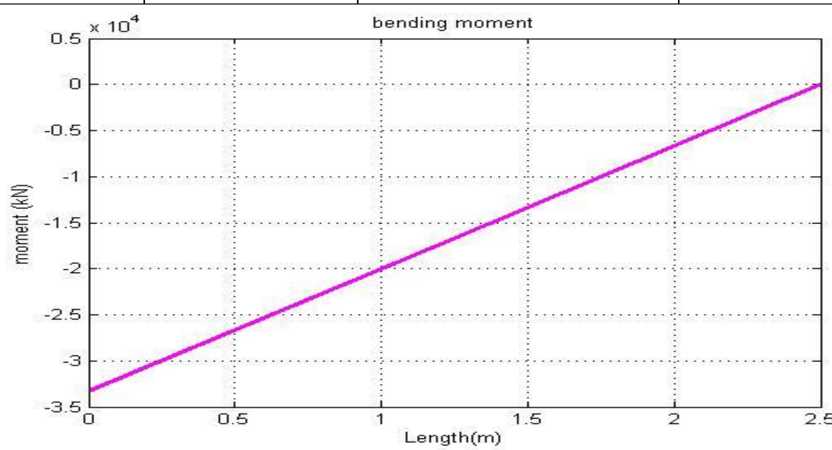

Figure 5.5: bending moment at nodes for two elements.

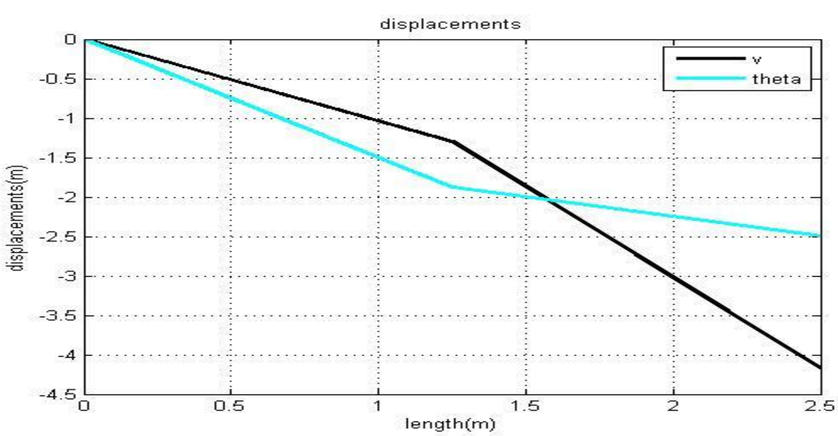

Figure 5.6: displacements at nodes for two elements.

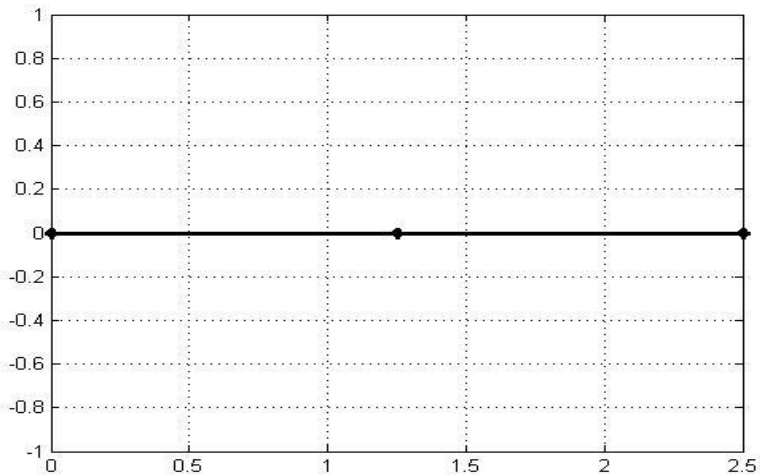

Figure 5.7: Mesh for two elements.

Table 5.7: Displacements at nodes for four elements

Node No.	u (m)	v (m)	θ (rad)
1	0	0	0
2	0	-0.3581	-1.0938
3	0	-1.3021	-1.8750
4	0	-2.6367	-2.3438
5	0	-4.1667	-2.5000

Table 5.8: Stresses at nodes for four elements

Node No.	F (kN)	M = x10^4 (kN.m)
1	0	-3.3332
2	0	-2.4990
3	0	-1.6666
4	0	-0.8330
5	0	0.0000

Table 5.9: Reactions at nodes for four elements

Node No.	Fr = 10^4* (kN)		
1	0	1.3328	3.3320
2	0	0	0.0000
3	0	0	-0.0000
4	0	0.0000	0.0000
5	0	-1.3328	0.0000

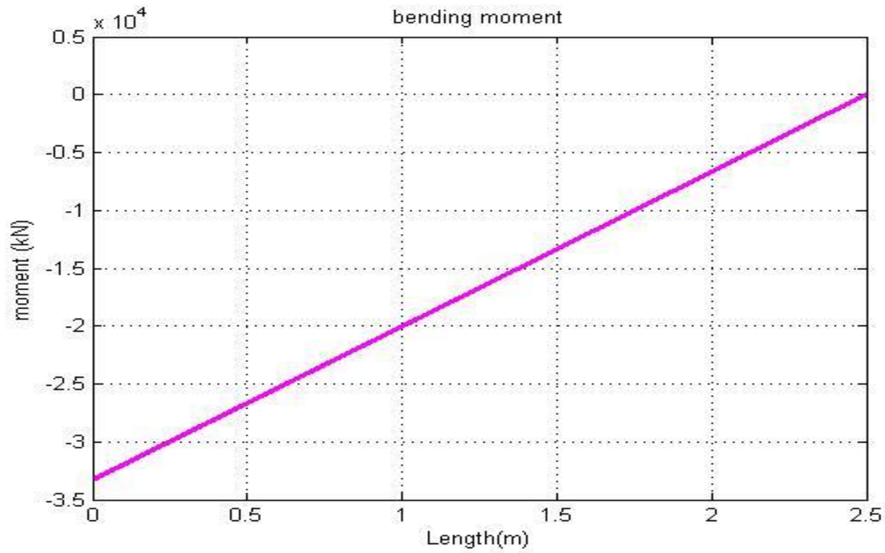

Figure 5.8: Bending moment at nodes for four elements.

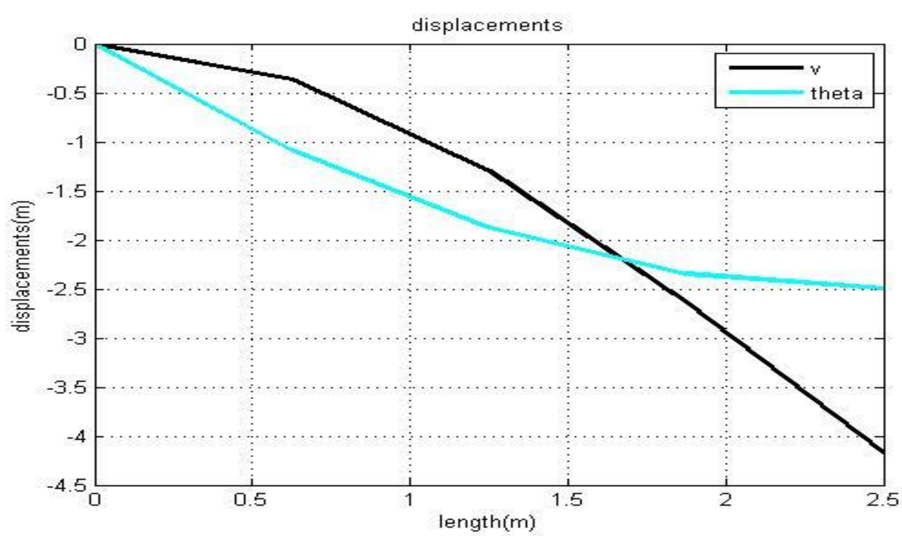

Figure 5.9: displacements at nodes for four elements.

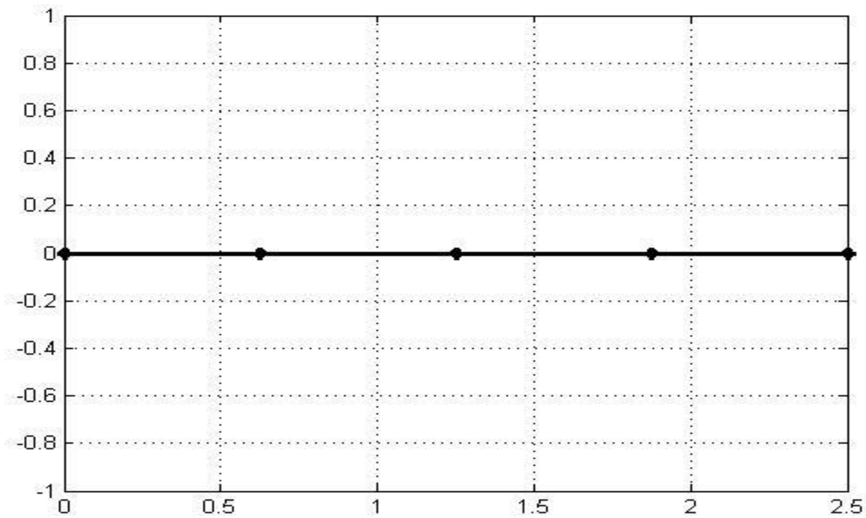

Figure 5.10: Mesh for four elements.

Table 5.10: Displacements at nodes for eight elements

Node No	u (m)	v (m)	θ (rad)
1	0	0	0
2	0	-0.0936	-0.5859
3	0	-0.3581	-1.0938
4	0	-0.7690	-1.5234
5	0	-1.3021	-1.8750
6	0	-1.9328	-2.1484
7	0	-2.6367	-2.3438
8	0	-3.3895	-2.4609
9	0	-4.1667	-2.5000

Table (5.11) Stresses at nodes for eight elements

Node No.	F (kN)	M = x 10^4 (kN)
1	0	-3.3320
2	0	-2.9155
3	0	-2.4990
4	0	-2.0825
5	0	-1.6666
6	0	-1.2495
7	0	-0.8330
8	0	-0.4165
9	0	0.0000

Table 5.12: Reactions at nodes for eight elements

Node No.	Fr = x10^4 (kN)		
1	0	1.3328	3.3320
2	0	0.0000	-0.0000
3	0	0.0000	-0.0000
4	0	-0.0000	-0.0000
5	0	0.0000	-0.0000
6	0	0	-0.0000
7	0	0.0000	0.0000
8	0	-0.0000	0.0000
9	0	-1.3328	0.0000

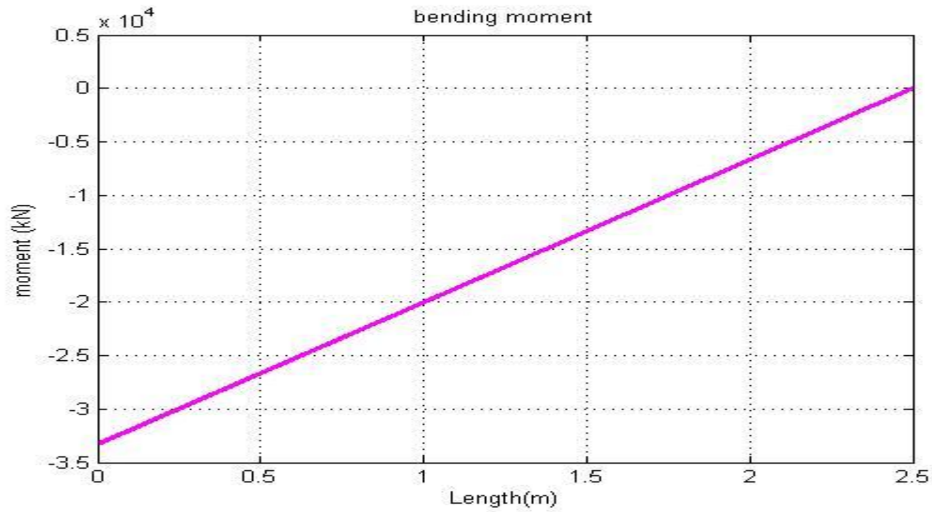

Figure 5.11: Bending moment at nodes for eight elements.

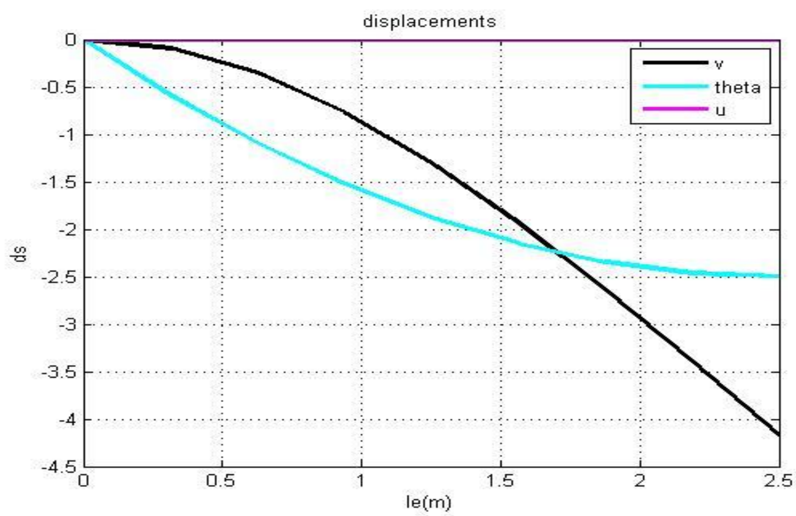

Figure 5.12: displacements at nodes for eight elements.

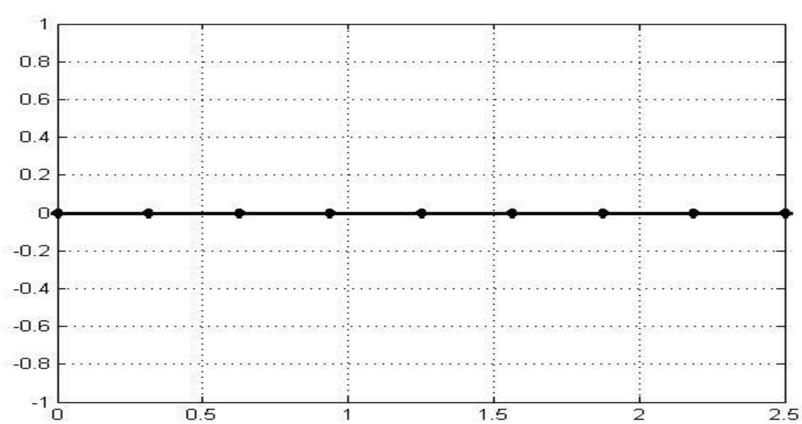

Figure 5.13: Mesh for eight elements.

Table (5.13) Displacements at nodes for sixteen elements

Node No.	u (m)	v (m)	θ (rad)
1	0	0	0
2	0	-0.0239	-0.3027
3	0	-0.0936	-0.5859
4	0	-0.2060	-0.8496
5	0	-0.3581	-1.0938
6	0	-0.5468	-1.3184
7	0	-0.7690	-1.5234
8	0	-1.0218	-1.7090
9	0	-1.3021	-1.8750
10	0	-1.6068	-2.0215
11	0	-1.9328	-2.1484
12	0	-2.2771	-2.2559
13	0	-2.6367	-2.3438
14	0	-3.0085	-2.4121
15	0	-3.3895	-2.4609
16	0	-3.7766	-2.4902
17	0	-4.1667	-2.5000

Table 5.14: Stresses at nodes for sixteen elements

Node No.	F (kN)	M=10^4*(kN.m)
1	0	-3.3320
2	0	-3.1238
3	0	-2.9155
4	0	-2.7073
5	0	-2.4990
6	0	-2.2908
7	0	-2.0825
8	0	-1.8743
9	0	-1.6660
10	0	-1.4578
11	0	-1.2495
12	0	-1.0413
13	0	-0.8330
14	0	-0.6248
15	0	-0.4165
16	0	-0.2083
17	0	0.0000

Table 5.15: Reactions at nodes for sixteen elements

Node No.	Fr = x 10^4 (kN)		
1	0	1.3328	3.3320
2	0	0	-0.0000
3	0	-0.0000	-0.0000
4	0	0.0000	-0.0000
5	0	-0.0000	-0.0000
6	0	-0.0000	-0.0000
7	0	-0.0000	0.0000
8	0	-0.0000	-0.0000
9	0	0.0000	-0.0000
10	0	-0.0000	-0.0000
11	0	0.0000	-0.0000
12	0	-0.0000	-0.0000
13	0	-0.0000	-0.0000
14	0	0.0000	-0.0000
15	0	-0.0000	0.0000
16	0	0.0000	0.0000
17	0	-1.3328	0.0000

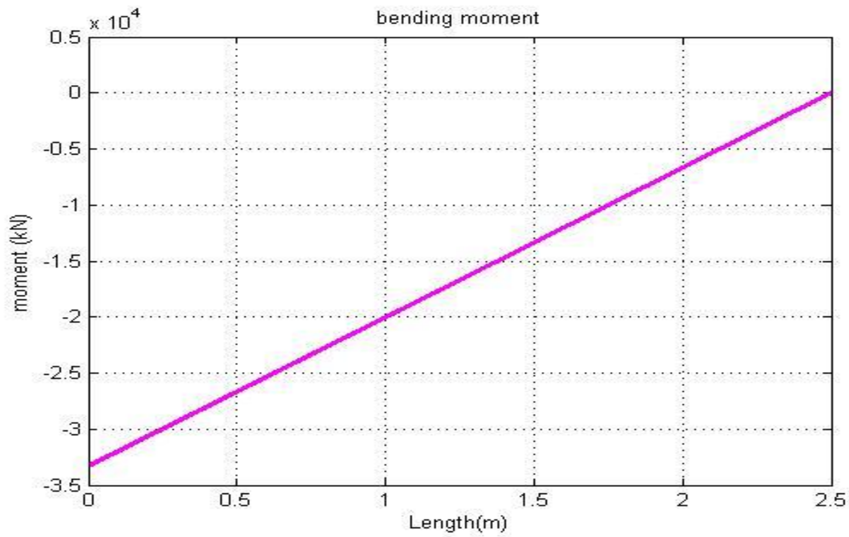

Figure 5.14: Bending moment at nodes for sixteen elements.

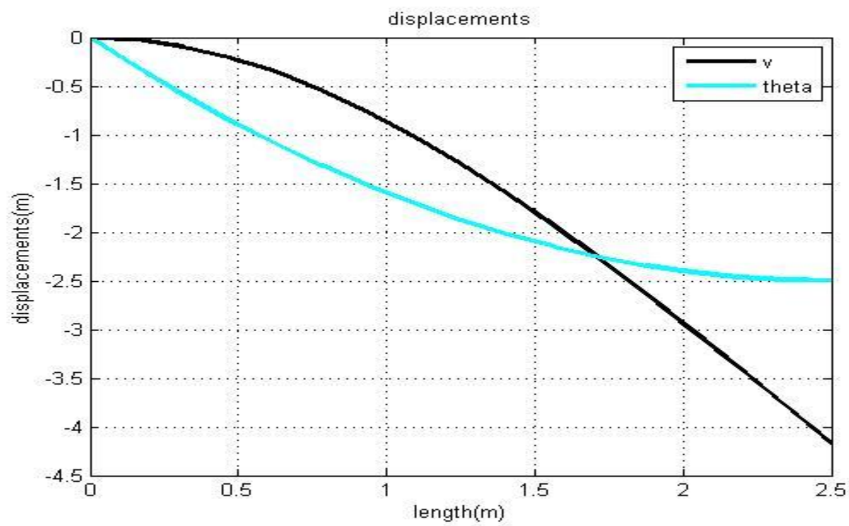

Figure 5.15: displacement at nodes for sixteen elements.

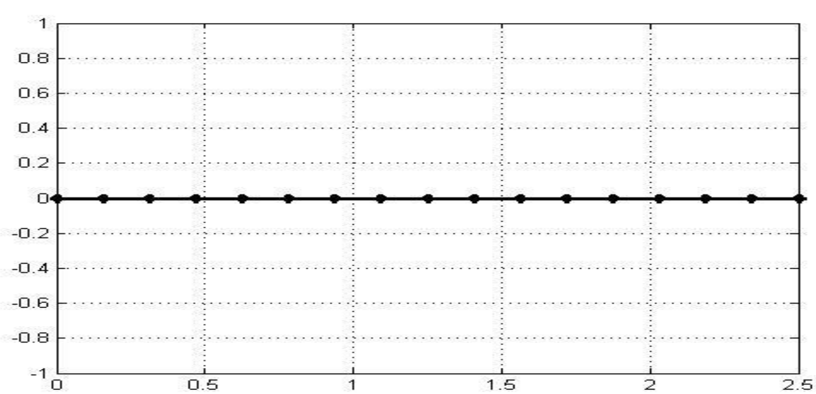

Figure 5.16: Mesh for sixteen elements.

Table 5.16: Displacements at nodes for thirty-two elements.

Node No	u (m)	v (m)	θ (rad)
1	0	0	0
2	0	-0.0060	-0.1538
3	0	-0.0239	-0.3027
4	0	-0.0532	-0.4468
5	0	-0.0936	-0.5859
6	0	-0.1446	-0.7202
7	0	-0.2060	-0.8496
8	0	-0.2773	-0.9741
9	0	-0.3581	-1.0937
10	0	-0.4480	-1.2085
11	0	-0.5468	-1.3184
12	0	-0.6539	-1.4233
13	0	-0.7690	-1.5234
14	0	-0.8918	-1.6187
15	0	-1.0218	-1.7090
16	0	-1.1587	-1.7944
17	0	-1.3021	-1.8750
18	0	-1.4516	-1.9507
19	0	-1.6068	-2.0215
20	0	-1.7673	-2.0874
21	0	-1.9328	-2.1484
22	0	-2.1029	-2.2046
23	0	-2.2771	-2.2559
24	0	-2.4552	-2.3022
25	0	-2.6367	-2.3437

26	0	-2.8213	-2.3804
27	0	-3.0085	-2.4121
28	0	-3.1981	-2.4390
29	0	-3.3895	-2.4609
30	0	-3.5824	-2.4780
31	0	-3.7766	-2.4902
32	0	-3.9714	-2.4976
33	0	-4.1667	-2.5000

Table 5.17: Stresses at nodes for thirty-two elements

Node No.	F(kN)	M = x 10^4(kN.m)	Node No	F(kN)	M = x 10^4(kN.m)
1	0	-3.3320	14	0	-1.9784
2	0	-3.2279	15	0	-1.8742
3	0	-3.1237	16	0	-1.7701
4	0	-3.0196	17	0	-1.6660
5	0	-2.9155	18	0	-1.5619
6	0	-2.8114	19	0	-1.4577
7	0	-2.7072	20	0	-1.3536
8	0	-2.6031	21	0	-1.2495
9	0	-2.4990	22	0	-1.1454
10	0	-2.3949	23	0	-1.0412
11	0	-2.2907	24	0	-0.9371
12	0	-2.1866	25	0	-0.8330
13	0	-2.0825	26	0	-0.7289
27	0	-0.6247	31	0	-0.2082
28	0	-0.5206	32	0	-0.1041
29	0	-0.4165	33	0	0.0000
30	0	-0.3124			

Table 5.18: Reactions at nodes for thirty-two elements

Node No.	Fr = x 10^4 (kN)			Node No	Fr = x 10^4 (kN)		
1	0	1.3328	3.3320	18	0	0.0000	0.0000
2	0	0	-0.0000	19	0	0.0000	0.0000
3	0	-0.0000	-0.0000	20	0	-0.0000	0.0000
4	0	0.0000	-0.0000	21	0	0.0000	0.0000
5	0	0.0000	-0.0000	22	0	-0.0000	0.0000
6	0	-0.0000	-0.0000	23	0	-0.0000	-0.0000
7	0	0.0000	-0.0000	24	0	0.0000	0.0000
8	0	-0.0000	-0.0000	25	0	-0.0000	-0.0000
9	0	0.0000	-0.0000	26	0	0.0000	-0.0000
10	0	-0.0000	0.0000	27	0	-0.0000	0.0000
11	0	0.0000	-0.0000	28	0	0.0000	0.0000
12	0	-0.0000	0.0000	29	0	-0.0000	-0.0000
13	0	-0.0000	0.0000	30	0	0.0000	-0.0000
14	0	-0.0000	-0.0000	31	0	0.0000	-0.0000
15	0	0.0000	0.0000	32	0	0.0000	0.0000
16	0	-0.0000	0.0000	33	0	-1.3328	0.0000
17	0	0.0000	0.0000				

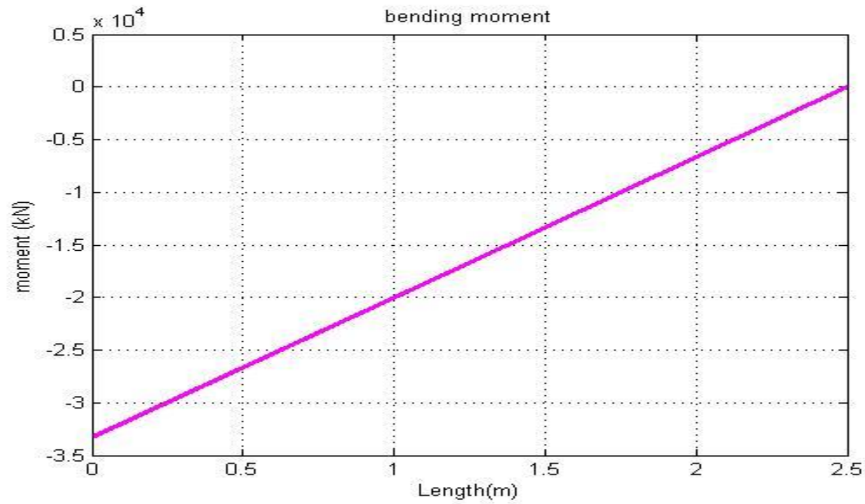

Figure 5.17: Bending moment at nodes for 32 elements.

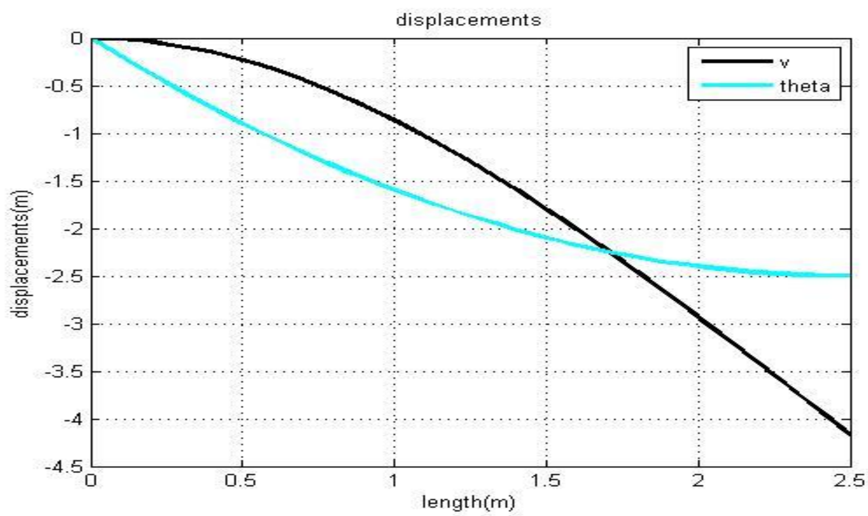

Figure 5.18: displacements at nodes for 32 elements.

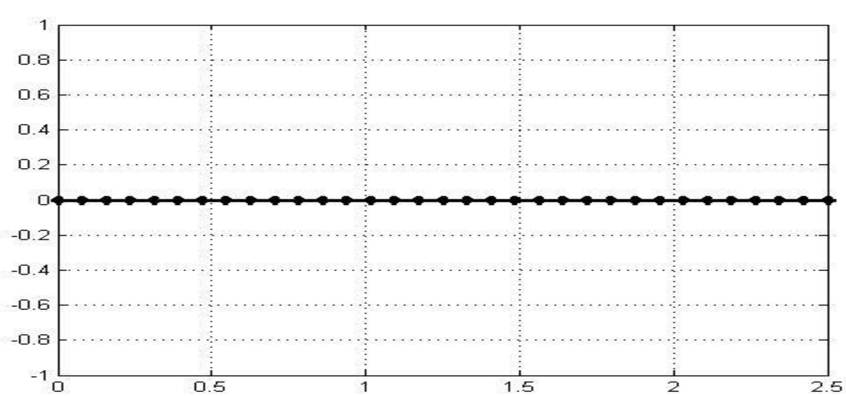

Figure 5.19: Mesh for 32 elements

Table 5.19: Displacements at nodes for sixteen elements (5INC)

$\frac{PL^2}{EI}$ (kN)	u (m)	v (m)	θ (rad)
0	0	0	0
1	0	-0.8333	-0.5000
2	0	-1.6667	-1.0000
3	0	-2.5000	-1.5000
4	0	-3.3333	-2.0000
5	0	-4.1667	-2.5000

Table 5.20: Displacements at nodes for sixteen elements (10INC)

$\frac{PL^2}{EI}$ (kN)	u (m)	v (m)	θ (rad)
1	0	-0.4167	-0.2500
2	0	-0.8333	-0.5000
3	0	-1.2500	-0.7500
4	0	-1.6667	-1.0000
5	0	-2.0833	-1.2500
6	0	-2.5000	-1.500
7	0	-2.9167	-1.7500
8	0	-3.3333	-2.0000
9	0	-3.7500	-2.2500
10	0	-4.1667	-2.5000

Table (5.21) Displacements at nodes for sixteen elements (20INC)

$\frac{PL^2}{EI}$ (kN)	u (m)	v (m)	θ (rad)	$\frac{PL^2}{EI}$ (kN)	u (m)	v (m)	θ (rad)
1	0	-0.2083	-0.1250	11	0	-2.2917	-1.3750
2	0	-0.4167	-0.2500	12	0	-2.5000	-1.5000
3	0	-0.6250	-0.3750	13	0	-2.7083	-1.6250
4	0	-0.8333	-0.5000	14	0	-2.9167	-1.7500
5	0	-1.0417	-0.6250	15	0	-3.1250	-1.8750
6	0	-1.2500	-0.7500	16	0	-3.3333	-2.0000
7	0	-1.4583	-0.8750	17	0	-3.5417	-2.1250
8	0	-1.6667	-1.0000	18	0	-3.7500	-2.2500
9	0	-1.8750	-1.1250	19	0	-3.9583	2.3750
10	0	-2.0833	-1.2500	20	0	-4.1667	-2.5000

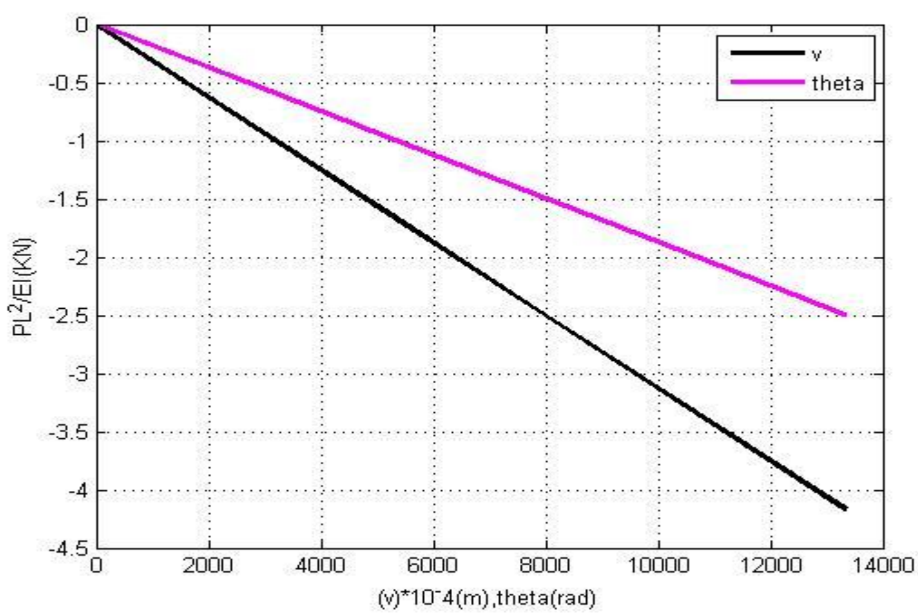

Figure 5.20: Displacements at nodes for sixteen elements (any INC).

5.2.2 Cantilever under pure bending at free end:

A cantilever subjected to pure moment is considered. . The cantilever is of dimensions L=10 m as shown in Figure (5.2). The numerical values of material property parameters are Young's modulus, E = 1x10^6 kN/m^2, the second moment of area I= 10^-4 m4,M=24KN.m, And area of beam A=1 m^2.

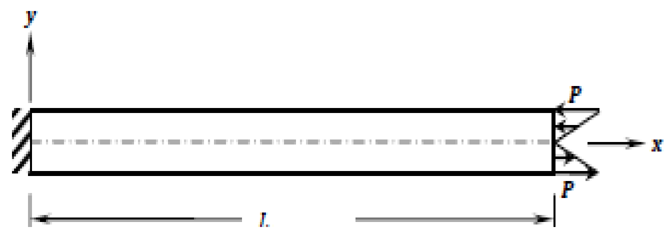

Figure 5.21: Cantilever under pure bending at free end.

This example demonstrate the efficiency of the linear analysis for different element numbers (1, 2, 4, 8, 16, 32), the results are shown in Tables (5.22) to (5.39). Graphical of results is also presented in Fig (5.22) to (5.34). Almost the same results were obtained for all cases. Tables (5.40) to (5.42) show that the displacements at nodes for sixteen elements and (5, 10, 20) increments and gave accurate results.

Table 5.22: Displacements at nodes for one element

Node No.	u(m)	v(m)	θ(rad)
1	0	0	0
2	0	12	2.4

Table 5.23: Stresses at nodes for one element

Node No	F(kN)	M (kN.m)
1	0	24
2	0	24

Table 5.24: Reactions at nodes for one element

Node No	Fr (kN)		
1	0	0	-24
2	0	0	24

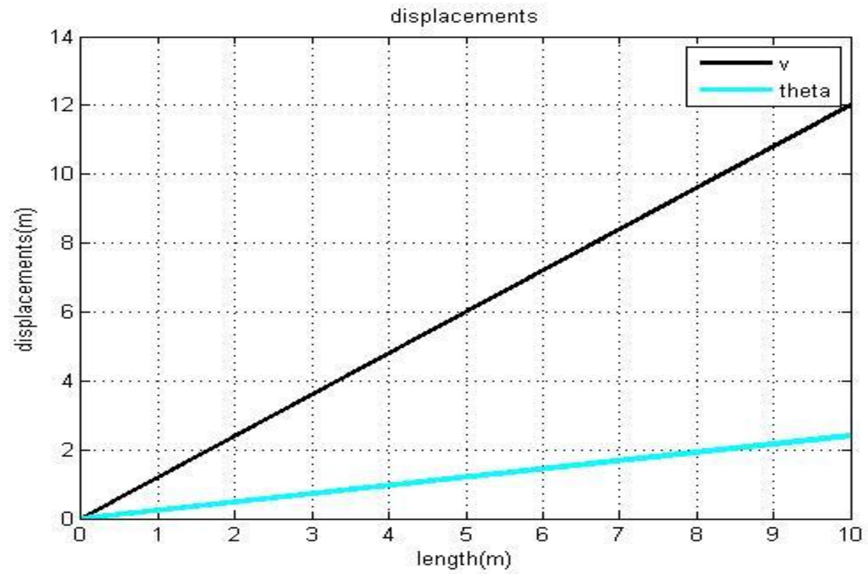

Figure 5.22: Displacements at nodes for one element.

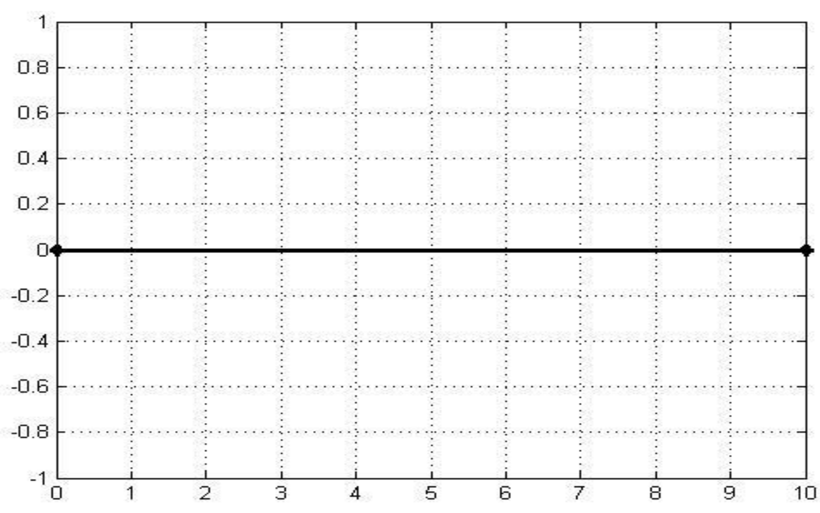

Figure 5.23: Mesh for one element.

Table 5.25: Displacements at nodes for two elements

Node No.	u(m)	v(m)	θ(rad)
1	0	0	0
2	0	3	1.2
3	0	12	2.4

Table 5.26: Stresses at nodes for two elements

Node No.	F (kN)	M (kN)
1	0	24
2	0	24
3	0	24

Table 5.27: Reactions at nodes for two elements

Node No.	Fr (kN)		
1	0	-0.0000	-24
2	0	0.0000	0
3	0	-0.0000	24

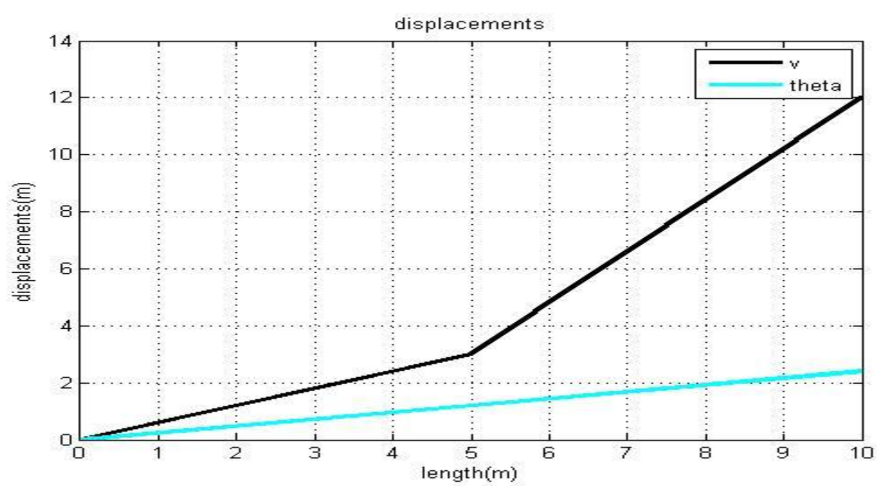

Figure 5.24: Displacements at nodes for two elements

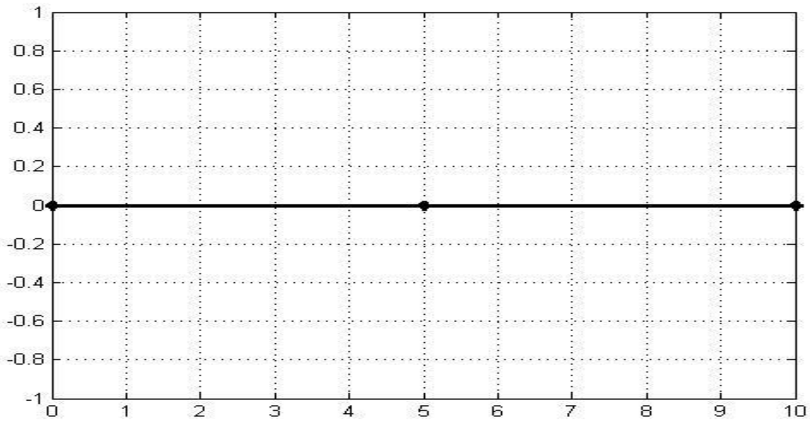

Figure 5.25: Mesh for two elements

Table 5.28: Displacements at nodes for four elements

Node No	u(m)	v(m)	θ(rad)
1	0	0	0
2	0	0.75	0.6
3	0	3	1.2
4	0	6.75	1.8
5	0	12	2.4

Table 5.29: Stresses at nodes for four elements

Node No	F (kN)	M (kN.m)
1	0	24
2	0	24
3	0	24
4	0	24
5	0	24

Table 5.30: Reactions at nodes for four elements

Node No	Fr (kN)		
1	0	-0.0000	-24
2	0	0.0000	0.0000
3	0	0	-0.0000
4	0	0	-0.0000
5	0	0	24

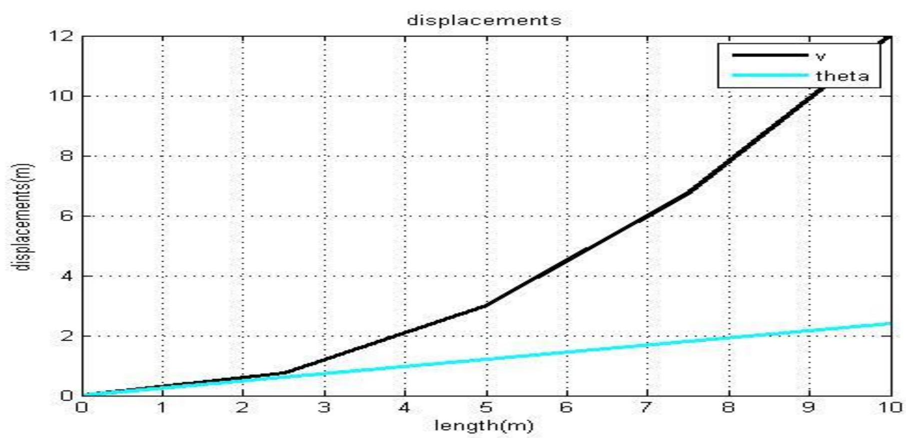

Figure 5.26: Displacements at nodes for four elements.

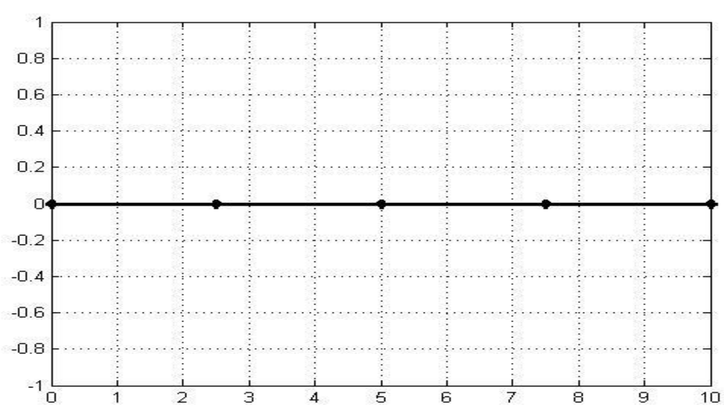

Figure 5.27: Mesh for four elements.

Table 5.31: Displacements at nodes for eight elements

Node No	u(m)	v(m)	θ(rad)
1	0	0	0
2	0	0.1875	0.3
3	0	0.75	0.6
4	0	1.6875	0.9
5	0	3	1.2
6	0	4.6875	1.5
7	0	6.75	1.8
8	0	9.875	2.1
9	0	12	2.4

Table 5.32: Stresses at nodes for eight elements

Node No.	F (kN)	M (kN.m)
1	0	24
2	0	24
3	0	24
4	0	24
5	0	24
6	0	24
7	0	24
8	0	24
9	0	24

Table 5.33: Reactions at nodes for eight elements

Node No	Fr(kN)		
1	0	-0.0000	24
2	0	0.0000	0.0000
3	0	0.0000	0
4	0	-0.0000	-0.0000
5	0	0.0000	-0.0000
6	0	-0.0000	-0.0000
7	0	0.0000	-0.0000
8	0	0.0000	0.0000
9	0	-0.0000	24

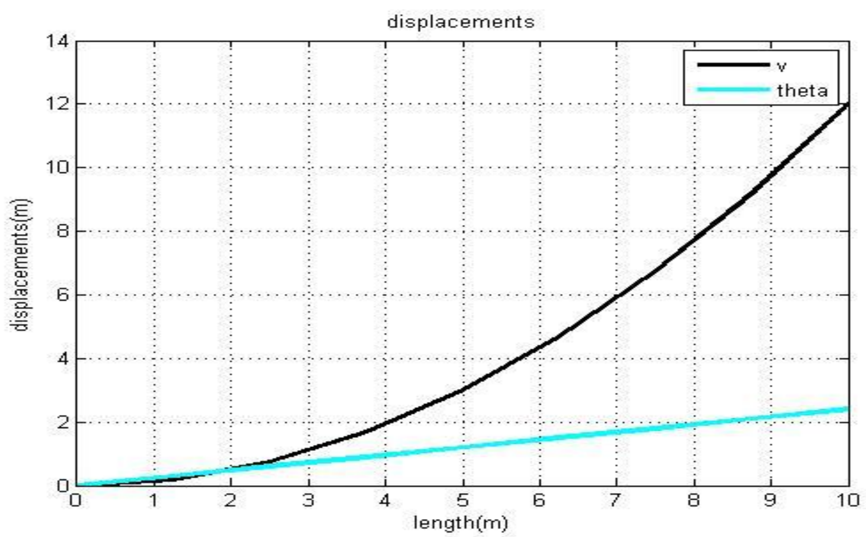

Figure 5.28: Displacements at nodes for eight elements

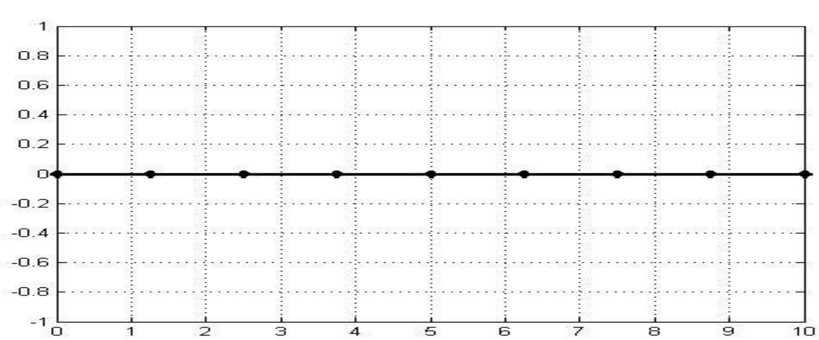

Figure 5.29: Mesh for eight elements

Table 5.34: Displacements at nodes for sixteen elements

Node No.	u(m)	v(m)	θ(rad)
1	0	0	0
2	0	0.0469	0.15
3	0	0.1875	0.30
4	0	0.4219	0.45
5	0	0.7500	0.60
6	0	1.1719	0.75
7	0	1.6875	0.90
8	0	2.2969	1.05
9	0	3	1.20
10	0	3.77969	1.35
11	0	4.6875	1.50
12	0	5.6719	1.65
13	0	6.7500	1.80
14	0	7.9219	1.95
15	0	9.1875	2.10
16	0	10.5469	2.25
17	0	12	2.40

Table 5.35: Stresses at nodes for sixteen elements

Node No.	F(kN)	M (kN.m)
1	0	24
2	0	24
3	0	24
4	0	24
5	0	24
6	0	24
7	0	24
8	0	24
9	0	24
10	0	24
11	0	24
12	0	24
13	0	24
14	0	24
15	0	24
16	0	24
17	0	24

Table 5.36: Reactions at nodes for sixteen elements

Node No.	Fr(kN)		
1	0	0.0000	-24
2	0	-0.0000	0.0000
3	0	0.0000	0.0000
4	0	-0.0000	-0.0000
5	0	0	-0.0000
6	0	0.0000	0.0000
7	0	-0.0000	0.0000
8	0	0.0000	0.0000
9	0	-0.0000	-0.0000
10	0	-0.0000	-0.0000
11	0	0	0.0000
12	0	0.0000	-0.0000
13	0	-0.0000	0.0000
14	0	0.0000	0.0000
15	0	-0.0000	-0.0000
16	0	0.0000	0.0000
17	0	0.0000	24

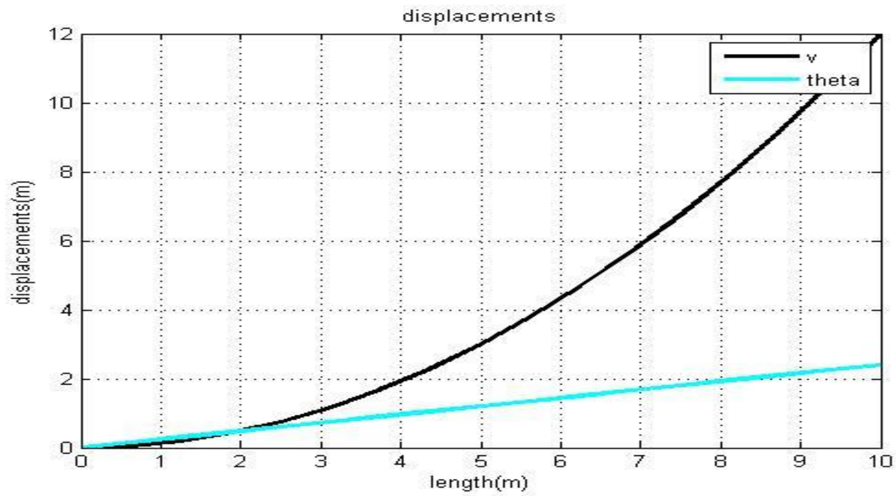

Figure 5.30: Displacements at nodes for sixteen elements

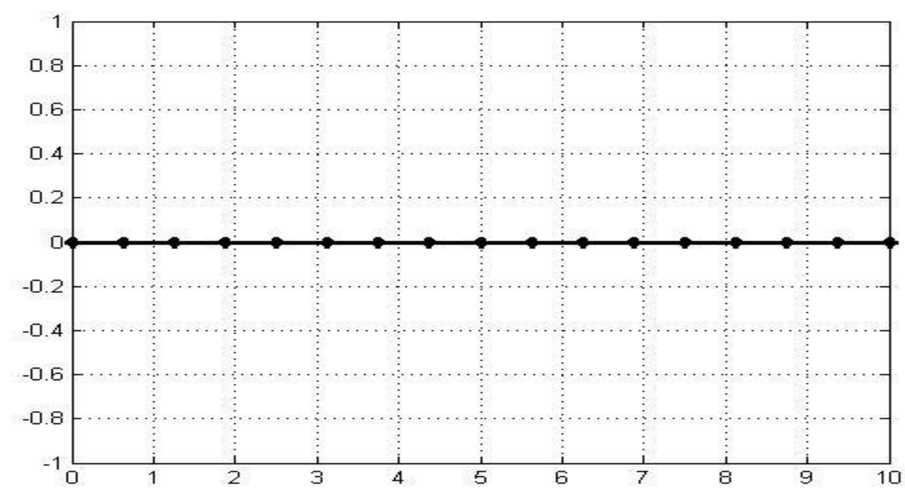

Figure 5.31: Mesh for sixteen elements

Table 5.37: Displacements at nodes for thirty-two elements

Node No.	u(m)	v(m)	θ(rad)
1	0	0	0
2	0	0.0117	0.0750
3	0	0.0469	0.1500
4	0	0.1055	0.2250
5	0	0.1875	0.3000
6	0	0.2930	0.3750

7	0	0.4219	0.4500
8	0	0.5742	0.5250
9	0	0.7500	0.6000
10	0	0.9492	0.6750
11	0	1.1719	0.7500
12	0	1.4180	0.8250
13	0	1.9875	0.9000
14	0	1.9805	0.9750
15	0	2.2969	1.0500
16	0	2.6367	1.1250
17	0	3	1.2
18	0	3.3867	1.2750
19	0	3.7969	1.35
20	0	4.2305	1.4250
21	0	4.6875	1.500
22	0	5.1680	1.5750
23	0	5.6719	1.65
24	0	6.1992	1.725
25	0	6.75	1.8
26	0	7.3242	1.875
27	0	7.9219	1.95
28	0	8.5430	2.0250
29	0	9.1875	2.1
30	0	9.8555	2.1750
31	0	10.5469	2.25
32	0	11.2617	2.3250
33	0	12	2.4

Table 5.38: Stresses at nodes for thirty-two elements

Node No	F(kN)	M(kN.m)	Node No	F(kN)	M(kN.m)
1	0	24	18	0	24
2	0	24	19	0	24
3	0	24	20	0	24
4	0	24	21	0	24
5	0	24	22	0	24
6	0	24	23	0	24
7	0	24	24	0	24
8	0	24	25	0	24
9	0	24	26	0	24
10	0	24	27	0	24
11	0	24	28	0	24
12	0	24	29	0	24
13	0	24	30	0	24
14	0	24	31	0	24
15	0	24	32	0	24
16	0	24	33	0	24
17	0	24			

Table 5.39: Reactions at nodes for thirty-two elements

Node No	Fr (kN)			Node No	Fr(kN)		
1	0	1.3328	3.3320	18	0	0.0000	0.0000
2	0	0	-0.0000	19	0	0.0000	0.0000
3	0	-0.0000	-0.0000	20	0	-0.0000	0.0000
4	0	0.0000	-0.0000	21	0	0.0000	0.0000
5	0	0.0000	-0.0000	22	0	-0.0000	0.0000
6	0	-0.0000	-0.0000	23	0	-0.0000	-0.0000
7	0	0.0000	-0.0000	24	0	0.0000	0.0000
8	0	-0.0000	-0.0000	25	0	-0.0000	-0.0000
9	0	0.0000	-0.0000	26	0	0.0000	-0.0000
10	0	-0.0000	0.0000	27	0	-0.0000	0.0000
11	0	0.0000	-0.0000	28	0	0.0000	0.0000
12	0	-0.0000	0.0000	29	0	-0.0000	-0.0000
13	0	-0.0000	0.0000	30	0	0.0000	-0.0000
14	0	-0.0000	-0.0000	31	0	0.0000	-0.0000
15	0	0.0000	0.0000	32	0	0.0000	0.0000
16	0	-0.0000	0.0000	33	0	-1.3328	0.0000
17	0	0.0000	0.0000				

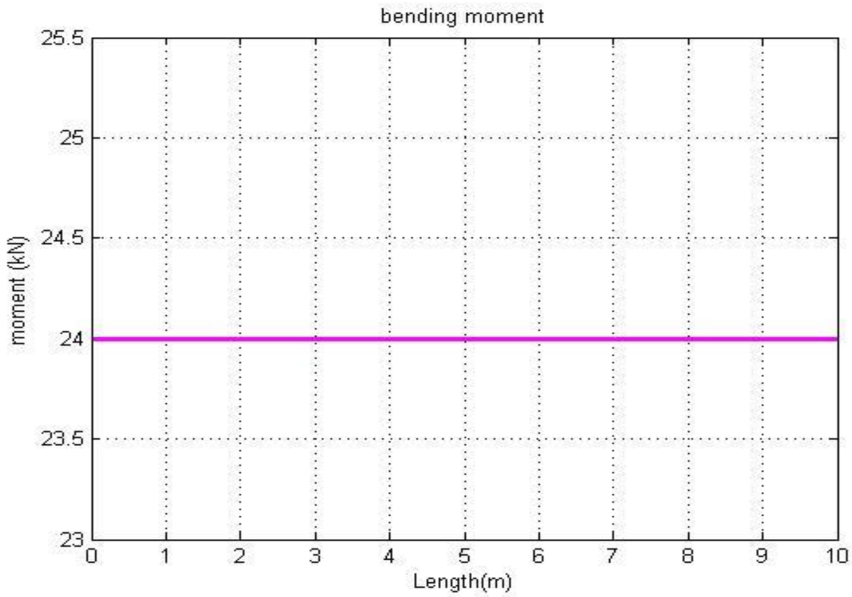

Figure 5.32: Bending moment at nodes for any elements.

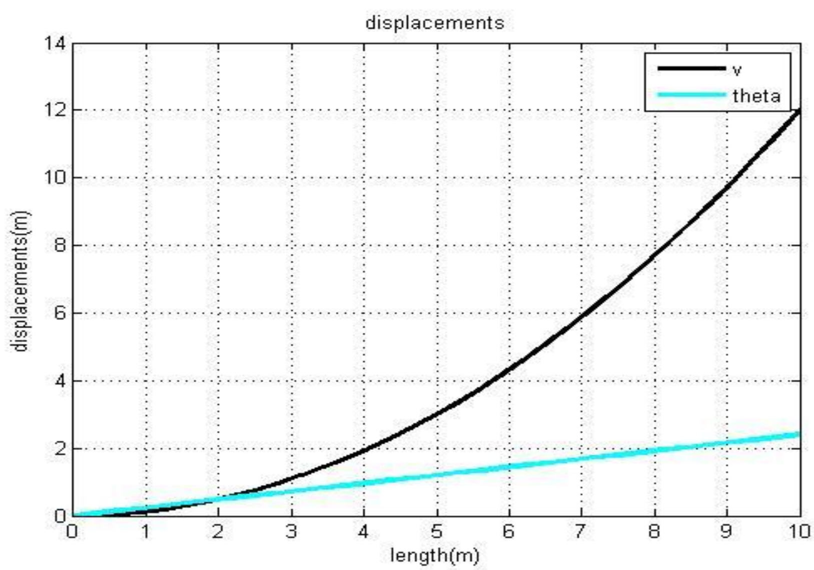

Figure 5.33: Displacements at nodes for 32 elements.

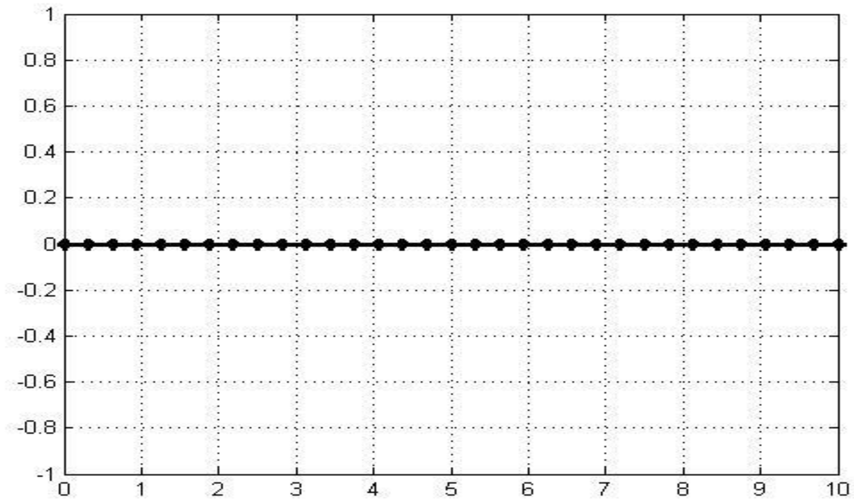

Figure 5.34: Mesh for 32 elements.

Table 5.40: Displacements at nodes for six elements (5INC)

$\frac{PL^2}{EI}$(kN)	u(m)	v(m)	θ(rad)
0	0	0	0
1	0	2.4	0.4800
2	0	4.8	0.9600
3	0	7.2	1.4400
4	0	9.6	1.9200
5	0	12	2.400

Table 5.41: Displacements at nodes for six elements (10INC)

$\frac{PL^2}{EI}$(kN)	u(m)	v(m)	θ(rad)
1	0	1.2	0.24
2	0	2.4	0.48
3	0	3.6	0.72
4	0	4.8	0.96
5	0	6	1.2
6	0	7.2	1.44

7	0	8.4	1.68
8	0	9.6	1.92
9	0	10.8	2.16
10	0	12	2.40

Table 5.42: Displacements at nodes for six elements (20INC)

$\frac{PL^2}{EI}$(kN)	u(m)	v(m)	θ(rad)	$\frac{PL^2}{EI}$(kN)	u(m)	v(m)	θ(rad)
1	0	0.6	0.12	11	0	6.6	1.20
2	0	1.2	0.24	12	0	7.2	1.32
3	0	1.8	0.36	13	0	7.8	1.56
4	0	2.4	0.48	14	0	8.4	1.68
5	0	3	0.60	15	0	9	1.80
6	0	3.6	0.72	16	0	9.6	1.92
7	0	4.2	0.84	17	0	10.2	2.04
8	0	4.8	0.96	18	0	10.8	2.16
9	0	5.4	1.08	19	0	11.4	2.28
10	0	6	0.24	20	0	12	2.4

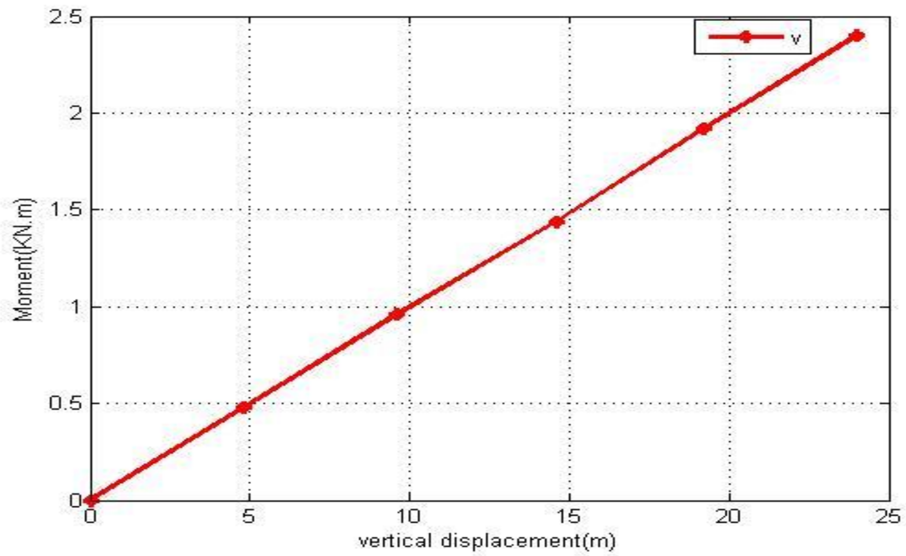

Figure 5.35: Displacements at nodes for six elements (any INC).

5.3 Non Linear Program:

5.3.1 Point load:

The results are shown in Tables (5.43) to (5.45). Table (5.43) shows that 5 increments load were applied at the cantilever and using (8, 16, and 32) elements. By comparing this results with Ref. (9) results. Table (5.44) shows percentage error for displacements. Table (5.45) shows cantilever under point load at free end-residuals for using (8, 16, and 32) elements. Tables (5.46) to (5.50) shows that stress resultants 5 increments load and (9) iteration were applied at the cantilever and using (16) elements.

Fig (5.36) to (5.39) shows the displacements result for cantilever under vertical load at free end for (8,16, 32) elements and also displacements for linear. Fig (5.40) percent’s error for displacements.

Table (5.43) Cantilever under point load at free end-displacements result

$\frac{PL^2}{EI}$	TLG		8 elements		16 elements		32 elements	
	V/L	1-u/L	V/L	1-u/L	V/L	1-u/L	V/L	1-u/L
0	0	0	0	0	0	0	0	0
1	0.3019	0.9435	0.3398	0.9268	0.3398	0.9268	0.3398	0.9268
2	0.4939	0.8390	0.4681	0.8595	0.4698	0.8580	0.4699	0.8580
3	0.6040	0.7448	0.5641	0.7854	0.5643	0.7843	0.5640	0.7844
4	0.6710	0.6700	0.6551	0.6813	0.6530	0.6815	0.6523	0.6819
5	0.7150	0.6110	0.7241	0.5373	0.7180	0.5406	0.7166	0.5421

Table (5.44) percentage error for displacements

L	% Error					
	8 elements		16 elements		32 elements	
	V/L	1-u/L	V/L	1-u/L	V/L	1-u/L
0	0	0	0	0	0	0
0.5	12.6	1.77	12.6	1.77	12.6	1.77
1	5.22	2.44	4.88	2.26	4.86	2.26
1.5	6.6	5.45	6.57	5.30	6.62	5.32
2	2.37	1.69	2.68	1.72	2.78	1.77
2.5	1.27	12.06	0.42	11.52	0.22	11.28

Table (5.45) Cantilever under point load at free end-residuals result

$\frac{PL^2}{EI}$	8elements		16 elements		32 elements	
	TRINT *10^3	Residuals *10^3	TRINT *10^3	Residuals *10^3	TRINT *10^3	Residuals *10^3
0	0	0	0	0	0	0
1	-4	1.3	-4	1.3	-4	1.3
2	-2.05	-3.28	-1.75	-3.58	-1.6	-3.73
3	-2.54	-5.46	-2.51	-5.49	-2.54	-5.46
4	-2.63	-8.03	-2.36	-8.31	-2.30	-8.36
5	-2.33	-11.00	-2.29	-11.04	-2.28	-11.04

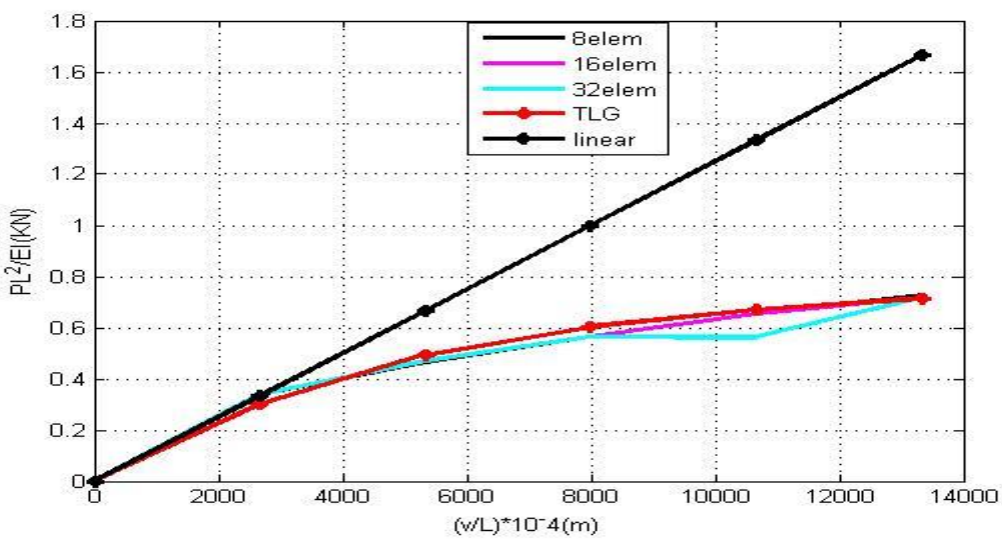

Figure 5.36: Cantilever under Vertical Load at free end vertical displacements result (V/L)

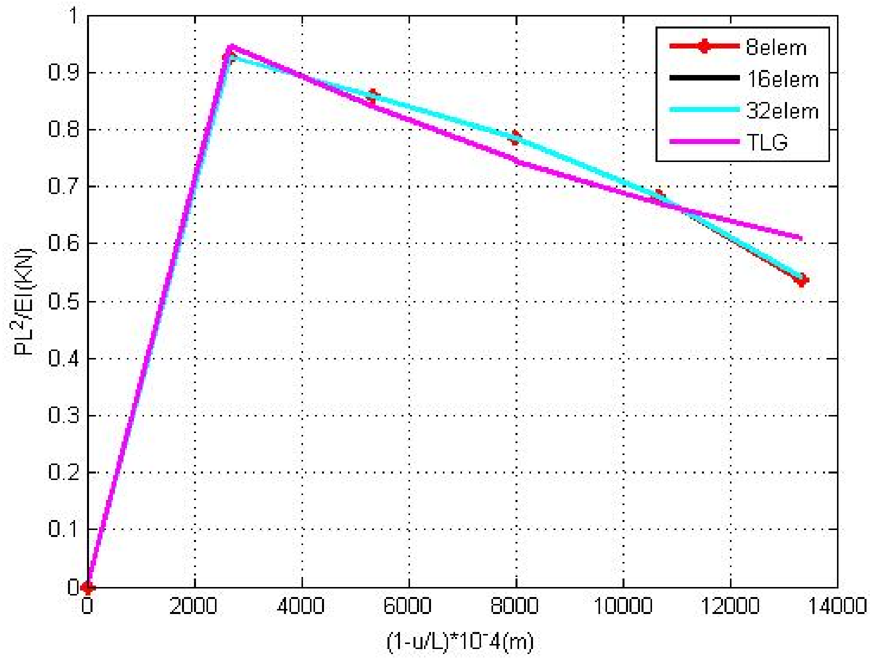

Figure 5.37: Cantilever under Vertical Load at free end horizontal displacements result(1-u/L)

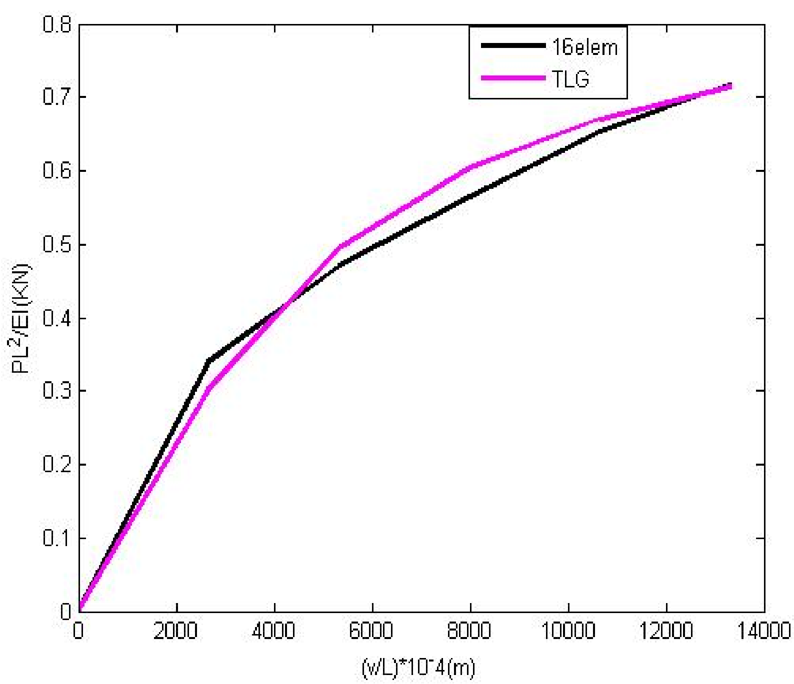

Figure 5.38: Cantilever under Vertical Load at free end vertical displacements (v/l) result for16elements.

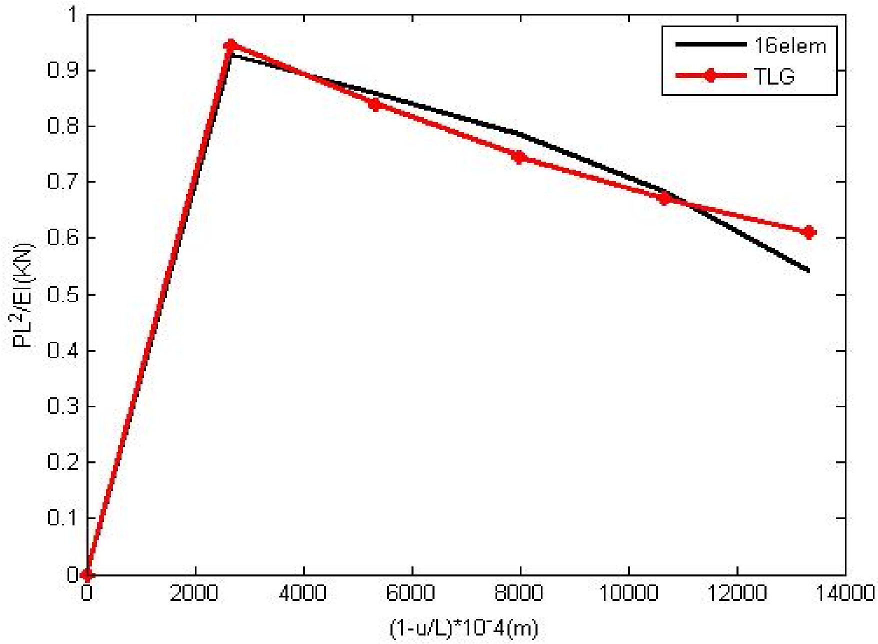

Figure 5.39: Cantilever under Vertical Load at free end horizontal displacements (1-u/l) result for16elements.

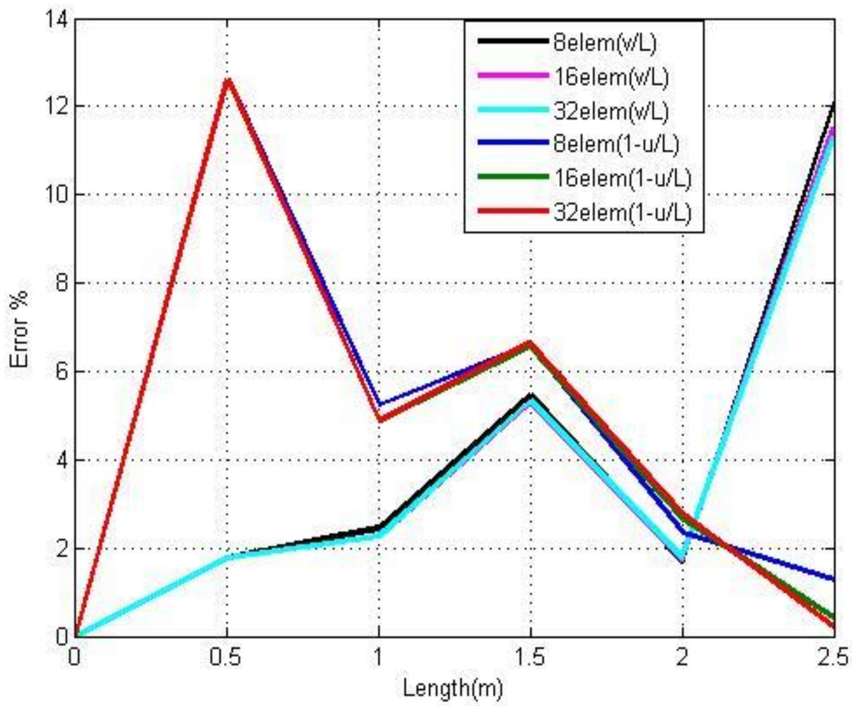

Figure 5.40: Percentage error of displacements.

Table (5.46) Stress resultants for sixteen elements (Inc=1,itr=9)

Node No	F =*10^4 (kN)	M =*10^3(kN.m)
1	-1.7920	-6.5307
2	-1.5556	-6.1833
3	-1.2041	-5.7674
4	-0.8499	-5.3520
5	-0.4870	-4.9370
6	-0.1117	-4.5225
7	0.2776	-4.1085
8	0.6807	-3.6951
9	1.0949	-3.2821
10	1.5148	-2.8697
11	1.9314	-2.4580
12	2.3319	-2.0468
13	2.6999	-1.6362
14	3.0161	-1.2262
15	3.2606	-0.8168
16	3.4161	-0.4077
17	3.4633	0.0668

Table (5.47) Stress resultants for sixteen elements (Inc=2,itr=9)

Node No	F=10^5* (kN)	M=10^3*(kN.m)
1	-0.3746	-9.5801
2	-0.2979	-8.8878
3	-0.1746	-8.0959
4	-0.0353	-7.3362
5	0.1226	-6.6068
6	0.3005	-5.9080
7	0.4985	-5.2398
8	0.7153	-4.6022
9	0.9481	-3.9945
10	1.1920	-3.4160
11	1.4400	-2.8653
12	1.6825	-2.3404
13	1.9079	-1.8390
14	2.1029	-1.3576
15	2.2540	-0.8922
16	2.3495	-0.4381
17	2.3724	-0.0639

Table (5.48) Stress resultants for sixteen elements (Inc=3,itr=9)

Node No	F =10^5* (kN)	M =10^4*(kN.m)
1	-0.5733	-1.1922
2	-0.4378	-1.0886
3	-0.2302	-0.9733
4	-0.0036	-0.8657
5	0.2491	-0.7652
6	0.5312	-0.6715
7	0.8444	-0.5846
8	1.1888	-0.5041
9	1.5631	-0.4297
10	1.9642	-0.3609
11	2.3856	-0.2974
12	2.8156	-0.2387
13	3.2352	-0.1845
14	3.6165	-0.1341
15	3.9251	-0.0869
16	4.1255	-0.0420
17	4.1707	-0.0056

Table (5.49) Stress resultants for sixteen elements(Inc=4,itr=9)

Node No	F=10^5* (kN)	M=10^4*(kN.m)
1	-0.7959	-1.4065
2	-0.6008	-1.2685
3	-0.3251	-1.1174
4	-0.0444	-0.9783
5	0.2567	-0.8501
6	0.5881	-0.7326
7	0.9586	-0.6252
8	1.3780	-0.5271
9	1.8588	-0.4379
10	2.4167	-0.3569
11	3.0686	-0.2837
12	3.8260	-0.2182
13	4.6806	-0.1604
14	5.5818	-0.1104
15	6.4154	-0.0677
16	7.0121	-0.0311
17	7.1717	-0.0034

Table (5.50) Stress resultants for sixteen elements (Inc=5,itr=9)

Node No	F =10^6*(kN)	M =10^4* (kN.m)
1	-0.0996	-1.5736
2	-0.0750	-1.4013
3	-0.0441	-1.2166
4	-0.0167	-1.0498
5	0.0097	-0.8986
6	0.0369	-0.7614
7	0.0666	-0.6368
8	0.1007	-0.5233
9	0.1426	-0.4198
10	0.1975	0.3253
11	0.2753	-0.2393
12	0.3941	-0.1616
13	0.5859	-0.0929
14	0.8944	-0.0364
15	1.3303	0.0005
16	1.7674	0.101
17	1.9407	0.0002

5.3.2 Concentrated End moment:

The results shown in Table (5.51) and (5.52). Table (5.51**)** shows 5 increments moment were applied at the cantilever and using 6 elements. By comparing this results with Ref. (9), Table (5.52**)** shows Comparative results for cantilever under pure moment. Tables (5.53) to (5.57) shows that stress resultants 5 increments load and (3) iteration were applied at the cantilever and using (6) elements.

Fig (5.41) shows the displacements result for cantilever under pure moment at free end for 6 elements.

Table 5.51: displacement Results for Cantilever under pure moment (six elements)

Increment	V (m)						
1	0	0.0667	0.2667	0.60	1.0667	1.6667	2.4
2	0	0.0667	0.2667	0.60	1.0667	1.6667	2.4001
3	0	0.0667	0.2667	0.60	1.0667	1.6667	2.3999
4	0	0.0661	0.2657	0.5996	1.0679	1.6703	2.4060
5	0	0.0683	0.2681	0.6038	1.0750	1.6813	2.4215

Table 5.52: Comparative results for cantilever under pure moment

Increment	V (m)			
	6 elements	8 elements	16 elements	32 elements
0	0	0	0	0
1	2.4	2.4	2.4	2.4
2	2.4001	2.4001	2.4001	2.4001
3	2.3996	2.3999	2.4004	2.4007
4	2.4060	2.3908	2.4010	2.4033
5	2.4215	2.3936	2.4012	2.4062

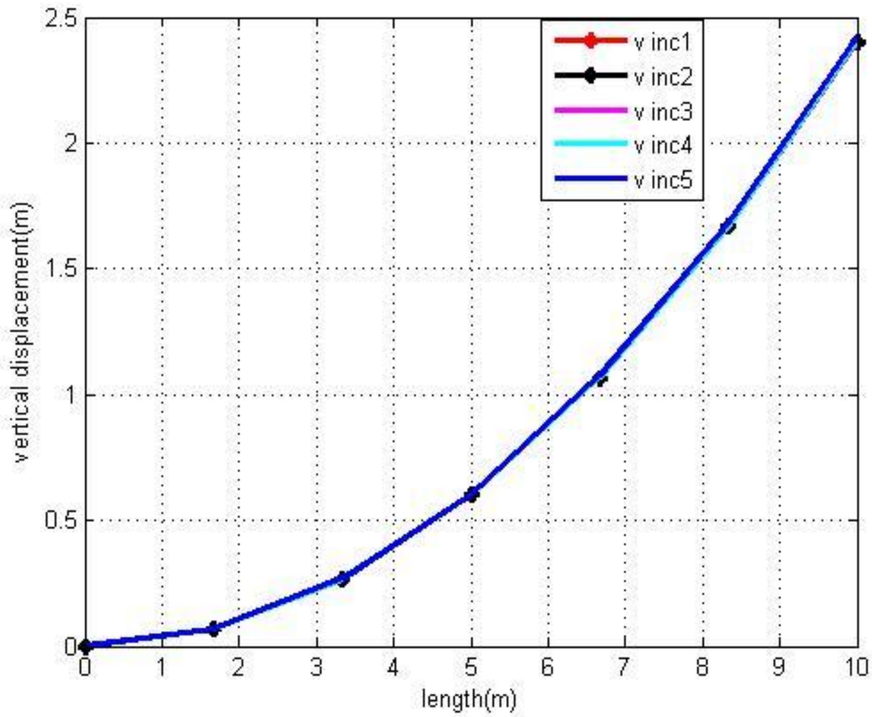

Figure 5.41: displacement Results for Cantilever under pure moment (six elements)

Table (5.53) Stress resultants for six elements (Inc=1,itr=3)

Node No	F=10^4* (kN)	M (kN.m)
1	-0.1600	4.7882
2	-0.1591	4.7797
3	-0.1563	4.7304
4	-0.1465	4.6518
5	-0.1212	4.5414
6	-0.0652	4.3850
7	1.8998	4.3463

Table (5.54) Stress resultants for six elements (Inc=2,itr=3)

Node No	F=10^4* (kN)	M (kN.m)
1	-0.1639	4.8252
2	-0.1608	4.6213
3	-0.1556	4.7835
4	-0.1470	4.7189
5	-0.1372	4.6136
6	-0.1310	4.4518
7	1.7942	4.2981

Table (5.55) Stress resultants for six elements (Inc=3,itr=3)

Node No	F =10^4* (kN)	M(kN.m)
1	-0.1696	4.9122
2	-0.1640	4.3941
3	-0.1507	4.9032
4	-0.1306	4.8520
5	-0.1105	4.7408
6	-0.1010	4.5553
7	1.8135	4.0664

Table (5.56) Stress resultants for six elements (Inc=4,itr=3)

Node No	F = 10^4* (kN)	M (kN.m)
1	-0.1798	5.0096
2	-0.1694	3.9954
3	-0.1356	5.1662
4	-0.0909	5.1460
5	-0.0548	5.0087
6	-0.0511	4.7458
7	1.7904	3.1402

Table (5.57) Stress resultants for six elements (Inc=5,itr=3)

Node No	F =10^4* (kN)	M(kN.m)
1	-0.2026	5.0559
2	-0.1789	3.2171
3	-0.0892	5.7775
4	0.0164	5.8234
5	0.0793	5.5839
6	0.0451	5.0813
7	1.6446	0.4980

5.4 Discussion of Results:

In this research, a Total Lagrangian formulation for geometric nonlinear beam problems based on Green strains was developed, implemented and tested. The program was first tested for linear analysis. Geometric nonlinear Total Lagrangian formulations applied on beam elements using different numbers finite elements were used. The formulations were implemented by using MATLAB software. The nonlinear equilibrium equations were solved by incremental method. Two numerical examples were presented here. The first example demonstrate the efficiency of the linear analysis for different element numbers (1, 2, 4, 8, 16, 32), the results are shown in Tables (5.1) to (5.18). Graphical of results is also presented in Fig (5.2) to (5.19). Almost the same results were obtained for all cases. Tables (5.19) to (5.21) show that the displacements at nodes for sixteen elements and (5, 10, 20) increments and gave accurate results. Fig (5.20) shows the displacements at nodes for sixteen elements and for any number of increments.

The second example demonstrate the efficiency of the linear analysis for different element numbers (1, 2, 4, 8, 16, 32), the results are shown in Tables (5.22) to (5.39). Graphical of results is also presented in Fig (5.22) to (5.34). Almost the same results were obtained for all cases. Tables (5.40) to (5.42) show that the displacements at nodes for six elements and (5, 10, 20) increments and gave accurate results. Fig (5.35) shows the displacements at nodes and for six elements for any number of increments.

For nonlinear analysis the results of first example are shown in tables (5.43) to (5.45). Table (5.43) shows that 5 increments load were applied at the cantilever and using (8, 16, and 32) elements. By comparing this results with Ref. (9) results, increments (2), (3) and (4) gave nearly equal results, increment (1) do not closely agree with the reference results with a percentage error of (12.6%). Increment (5) with sixteen elements gave results closely equal to reference results. Table (5.44) shows percentage error for displacements. Table (5.45) shows cantilever under point load at free end-residuals for using (8, 16, and 32) elements.

Fig (5.36) shows the vertical displacements result for cantilever under vertical load at free end for (8, 16, 32) elements and also displacements for linear. Fig (5.37) shows the horizontal displacements result for cantilever under vertical load at free end for (8, 16, 32) elements. Fig (5.38) to (5.39) shows the Cantilever under Vertical Load at free vertical and horizontal displacements result for 16elements. Fig (5.40) presents error for displacements. Tables (5.46) to (5.50) shows that stress resultants 5 increments load and (9) iteration were applied at the cantilever and using (16) elements. For nonlinear analysis the results of second example shown in Table (5.51) and (5.52).Table (5.51**)** shows 5 increments moment were applied at the cantilever and using 6 elements. By comparing this results with Ref. (9) results, increments (1, 2, 3, 4), and (5) gave nearly equal results. Table (5.52) shows Comparative results for cantilever under pure moment at free end-displacements for using (6, 8, 16, and 32) elements.

Fig (5.41) shows the displacements result for cantilever under pure moment at free end for 6 elements and 5 increments. Tables (5.53) to (5.57) shows that stress resultants 5 increments load and (3) iteration were applied at the cantilever and using (6) elements.

Results using iterations within each increment, despite being implemented in program, are not presented. This was due to the inconsistency of the results. To look into the reasons of this, the component matrices for geometrical stiffness and element residuals developed in chapter three were compared with those in Appendix(C) These are in very close an agreement. Thus the inconsistency of the results may be mainly due to calculating stresses at the nodes and/or due to programming errors. One of the reasons may be the linear interpolation of the in-plane displacement (u), which affects the iteration between axial force and moment.

The linear displacement stiffness matrix K_L was also neglected as a result of the inconsistency in calculation of displacements. K_L is usually neglected for beams as stated in ref (9).