CHAPTER THREE

MATHMATICAL MODEL

3.1 Introduction

A renaissance in the high performance of induction motors has been brought by the demonstration that the induction motor can be controlled as a separately excited DC motor. The field oriented control has become very popular, as it guarantees high dynamic and static performance [26].

FOC is based on the idea of decoupling torque and flux via nonlinear coordinate transformation and controlling these variables by acting on the direct and quadrature current vector components by means of unit vector (sinθe and cosθe). FOC has two methods: (a) direct or feedback vector control and (b) indirect or feed-forward vector control. In direct FOC, unit vectors are generated by stator voltages and currents (Voltage model estimator) or by stator currents and rotor speed (Current model estimator) [26].

The drawbacks of this method are: (a) dependency on the rotor resistance value and (b) computation requirements and time delay due to the use of current control loops and axes transformation [26].

As Field Orientated Control is simply based on projections the control structure handles instantaneous electrical quantities. This makes the control accurate in every working operation [26].

This chapter presents the mathematical model of three phase induction motor, indirect vector control and a relatively simple fuzzy logic controller that is efficient in the speed tracking, rejecting the disturbances and the variation in the parameters without any need of the complex control technique which require mathematical models. This is achieved by carefully designing the linguistic rule base [26].

3.2Mathmatical model of induction motor

The mathematical modeling of induction motor is established using a rotating (d,q) field reference ,before the implementation of any control mode, it was necessary to define the function equations.

3.2.1The Voltage Equation in dq Coordinate

The equation system for stator and rotor is expressed as:

$$V_{ds} = R_s i_{ds} + p\psi_{ds} - \omega_e \psi_{qs} \quad V_{qs} = R_s i_{qs} + p\psi_{qs} - \omega_e \psi_{ds} \qquad (3.1a)$$

$$V_{dr} = R_r i_{dr} + p\psi_{dr} - (\omega_{e-}\omega_r)\psi_{qr} \qquad (3.1b)$$

$$V_{qr} = R_r i_{qr} + p\psi_{qr} - (\omega_{e-}\omega_r)\psi_{dr} \qquad (3.1c)$$

Where p is the ${}^{d}/_{dt}$operator,R_s and $R_r$are the stator and rotor winding

resistances,ω_e is the motor synchronous speed and $\omega_r$is the rotor "electrical" Speed [27].

To squirrel-cage induction motor, the rotor is short circuit, so ($V_{dr} = V_{qr} = 0$).

3.2.2The Flux Equation in dq Coordinate

The relations of fluxes to currents can be given as:

$$\psi_{qs} = L_s i_{qs} + L_m i_{qr} \qquad (3.2a)$$

$$\psi_{ds} = L_s i_{ds} + L_m i_{dr} \qquad (3.2b)$$

$$\psi_{qr} = L_r i_{qr} + L_m i_{qs} \tag{3.2c}$$

$$\psi_{dr} = L_r i_{dr} + L_m i_{ds} \tag{3.2d}$$

Where $L_m$the mutual inductance between stator equivalent winding and the rotor in the dq coordinate,L_s is the self inductance in the equivalent 2-phase winding of the stator,$L_r$is the self inductance in the equivalent 2-phase winding of the rotor [27].

3.2.3The Torque Equation in dq Coordinate

The instantaneous torque produced in the electromechanical interaction is:

$$T_e = n_p L_m(i_{qs} i_{dr} - i_{ds} i_{qr}) \tag{3.3}$$

When the two-phase synchronous rolling coordinate is orientated by the rotor

flux $\psi_{dr} = \psi_r, \psi_{qr} = 0$

$$T_e = n_p \frac{L_m}{L_r} \left(i_{qs} \psi_{dr} \right) \tag{3.4}$$

Where n_p is the motor's pole-pair [27].

3.2.4 Motion Equation

Neglecting the viscous friction and turn-round resilience in the transmission mechanism of the electrical drive system, the motion equation

$$T_e = T_L + \frac{J}{n_p} \frac{d\omega_r}{dt} \tag{3.5}$$

Where isT_L the load torque, J is the moment of inertia [27].

I. Clarke transformation and inverse transformation Change the three-phase AC system to two-phase system is called Clarke transformation, also called 3/2 transformation, as shown below[27]

$$\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{3.6}$$

Conversely, change the 2-phase AC system to 3-phase AC system is called inverse Clarke transformation, also called 2/3 transformation, as shown [27]

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} \tag{3.7}$$

II.Park transformation and inverse transformation Change the two-phase AC system to rotating DC system is called Park transformation, as shown [27]

$$\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix} \tag{3.8}$$

Conversely, change the DC system to AC system is called inverse Park transformation, as shown [27]

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} \tag{3.9}$$

3.3 Vector Control of Induction Motor

The procedure of controlling the induction motor using field orientation involves important mathematical transformations which are summarily explained as follows in three main points [28]

I. The developed torque and the flux $T_e^* and \lambda_f^*$ should be set and then the corresponding i_{ds}^e and i_{qs}^e in synchronous reference frame are found [28].

II. Angular position ρ is then found to be used in the transformation from the synchronous reference frame to the stationary frame ($dqe \rightarrow dqs$) to achieve desired i_{qs}^s and i_{ds}^s [28].

III. The last step is to convert the gotten component of stator current in stationary reference frame to the desired three phase currents to be the base of control the inverter [28].

3.3.1Torque and flux equations for Vector Control

All vector control strategies agree that the machine torque and the flux linkage can be controlled using the stator current vector alone [29]. That is the flux linkage component on the q-axis of synchronous frame, and equal to zero because the rotor flux is aligned with the d-axis [29].

$$\lambda_{qr}^e = L_m i_{qs}^e + L_r' i_{qr}'^e = 0 \quad (3.10)$$

$$i_{qr}'^e = -\frac{L_m}{L_r'} i_{qs}^e \quad (3.11)$$

Substituting $\lambda_{qr}'^e$ =0 on the torque equation

$$T_e = \frac{3}{2}\frac{p}{2}\left(\lambda_{qr}'^e i_{dr}'^e - \lambda_{dr}'^e i_{qr}'^e\right) \quad (3.12)$$

The torque equation is simplified to that shown in equation

$$T_e = -\frac{3}{2}\frac{p}{2}\lambda_{dr}'^e i_{qr}'^e N.M \quad (3.13)$$

Substituting the expression of $i_{qr}'^e$ which is in equation 3.11 into the resultant torque equation 3.13, the torque will be [29]

$$T_e = \frac{3}{2}\frac{p}{2}\frac{L_m}{L_r'}\lambda_{dr}'^{e} i_{qs}'^{e} \tag{3.14}$$

=

Therefore, the torque can be controlled using the q-axis component of the stator current alone and the rotor flux linkage is not distressed. Applying the concept of zero q-axis rotor flux linkage, the rotor q-axis voltage can be expressed as: [29]

$$v_{qr}'^{e} = r_r' i_{qr}'^{e} + p\lambda_{qr}'^{e} - (\omega_e - \omega_r)\lambda_{dr}'^{e} \tag{3.15}$$

$$v_{qr}'^{e} = 0, p\lambda_{qr}'^{e} = 0, \lambda_{dr}'^{e} = 0 \tag{3.16}$$

If we arrange equation (3.15) as the difference of the two frequencies or the slip frequency [29]

$$(\omega_e - \omega_r) = \frac{r_r' i_{qr}'^{e}}{\lambda_{dr}'^{e}} \tag{3.17}$$

If we assume that $\lambda_{dr}'^{e}$ is constant, the derivative of it becomes zero and applying the previous conditions [29]

$$v_{dr}'^{e} = r_r' i_{dr}'^{e} + p\lambda_{dr}'^{e} - (\omega_e - \omega_r)\lambda_{qr}'^{e} \tag{3.18}$$

$$v_{dr}'^{e} = 0, p\lambda_{dr}'^{e} = 0, \lambda_{qr}'^{e} = 0 \tag{3.19}$$

This equation shows that $i_{dr}'^{e}$ has to be zero in order to satisfy the resultant equation of 3.18, Recalling the equation 3.15 when $i_{dr}'^{e}$ =0, so $\lambda_{dr}'^{e}= L_m i_{ds}'^{e}$ and if we substitute that into equation 3.17 the slip speed will simplify to [29]

$$(\omega_e - \omega_r) = \frac{r_r' i_{qr}'^{e}}{L_m i_{ds}'^{e}} \tag{3.20}$$

Because the rotor flux canbe changed by controllingi_{ds}^{e}, the orientation is accomplished by either keeping the slip speed or the q-axis stator current revolving in synchronous speed based on equation 3.20. Also, if we take the expression of$i_{dr}'^{e}$ from the equation $i_{dr}'^{e} = (\lambda_{dr}'^{e} - L_m i_{ds}^{e})/L_r'$ and substitute that into the equation 3.18 we get [29]

$$\lambda_{dr}'^{e} = \frac{r_r' L_m}{r_r' + L_r' p} i_{ds}^{e} \qquad (3.21)$$

3.4 Indirect Field Orientation Method (IFO):

This method does not depend on the measurement of air-gap magnetic flux but Torque can be controlled by either changingi_{qs}^{e} or the slip speed$(\omega_e - \omega_r)$,The rotor flux can also be controlled by varying i_{ds}^{e} , because if the desired rotor flux is measured or given, then the $i_{ds}'^{e*}$ is determined by using equation 3.21. Then, the desired torque can be found as equation [29]

$$T_e^* = \frac{3}{2}\frac{p}{2}\frac{L_m}{L_r'}\lambda_{dr}'^{e*} i_{qs}'^{e*} \qquad (3.22)$$

Equation $i_{dr}'^{e} = (\lambda_{dr}'^{e} - L_m i_{ds}^{e})/L_r'$ has shown that the optimum orientation of $i_{dr}'^{e}$ is to be zero, so the desired slip speed is as summarized equation [29]

$$\omega_2^* = (\omega_e - \omega_r) = \frac{r_r' i_{qr}'^{e*}}{L_m i_{ds}'^{e*}} \qquad (3.23)$$

Notice that that the angle of orientation (ρ) is the summation of the rotor angle (θr) and the slip speed angle $(\theta 2)$, if the values of the cosine and the sine of the rotor angle are known by the magnetic sensor [29].

The orientation angle can be determined using the equations

$$\cos\rho = \cos(\theta_r + \theta_2) = \cos\theta_r \cos\theta_2 - \sin\theta_r \sin\theta_2 \qquad (3.24)$$

$$\sin\rho = \sin(\theta_r + \theta_2) = \sin\theta_r \cos\theta_2 + \cos\theta_r \sin\theta_2 \qquad (3.25)$$

3.5Matlab model of fuzzy controller

The triangular membership function for the both input (error, error rate), output variables and control surface are shown in Figures (3.1- 3.4).

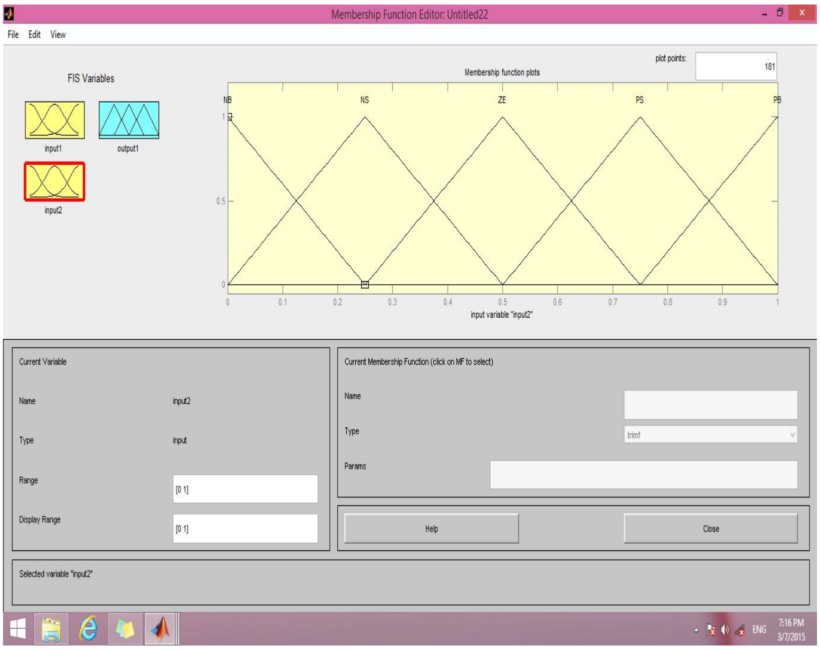

Fig. 3.1: Membership Functions for both the inputs

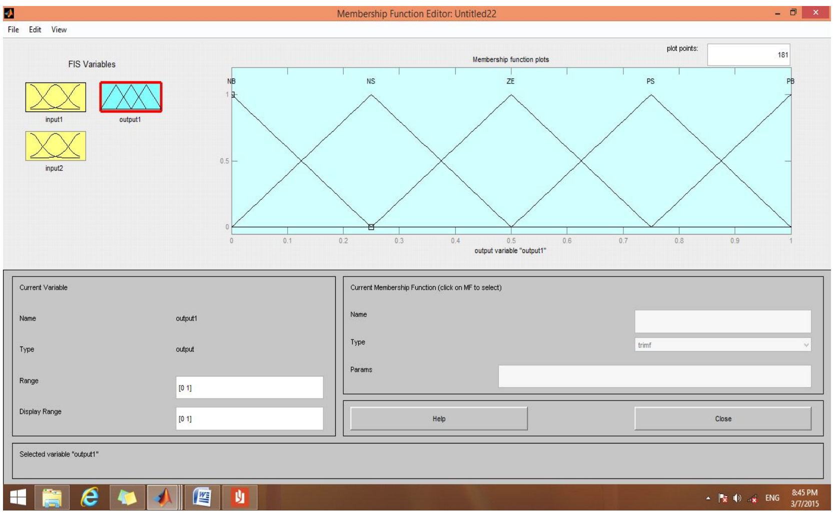

Fig.3.2: Membership Functions for the output

There are five MFs for inputs e and Δe signals, whereas there are five MFs for the output. All the MFs are symmetrical for positive and negative values of the variables [30].

Depending on these input variable values, the output variable value is to be decided from the experience encoded in the form of rules [30].

Table 1 shows the corresponding rule table for the speed controller.

The top row and left column of the matrix indicate the fuzzy sets of the variables e and Δe, respectively, and the MFs of the output variable (motor torque) operate according to the rule shown in the body of the matrix [30].

There are 5 x 5 = 25 possible rules in the matrix, where a typical rule reads as:

If e negative small (NS) and Δe is positive small (PS) then, T is zero (ZE) [30].

Δe \ e	NB	NS	ZE	PS	PB
NB	NB	NB	NS	NS	ZE
NS	NB	NS	NS	ZE	PS
ZE	NS	NS	ZE	PS	PS
PS	NS	ZE	PS	PB	PB
PB	ZE	PS	PS	PB	PB

Table1: the corresponding rule table for the speed controller.

In the FIS Editor: rules window, by clicking on view, rules and surface can also beseen.

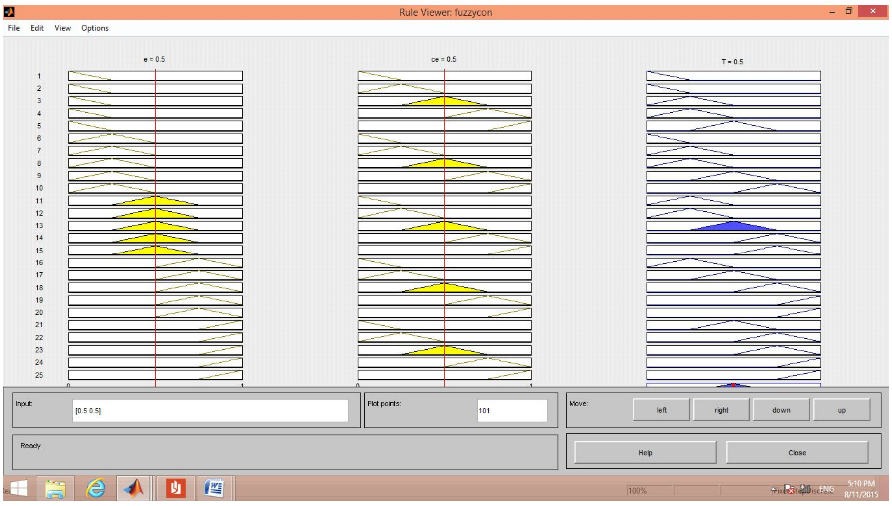

Fig.3.3Rule Viewer with e = 0.5 and Δe = 0.5

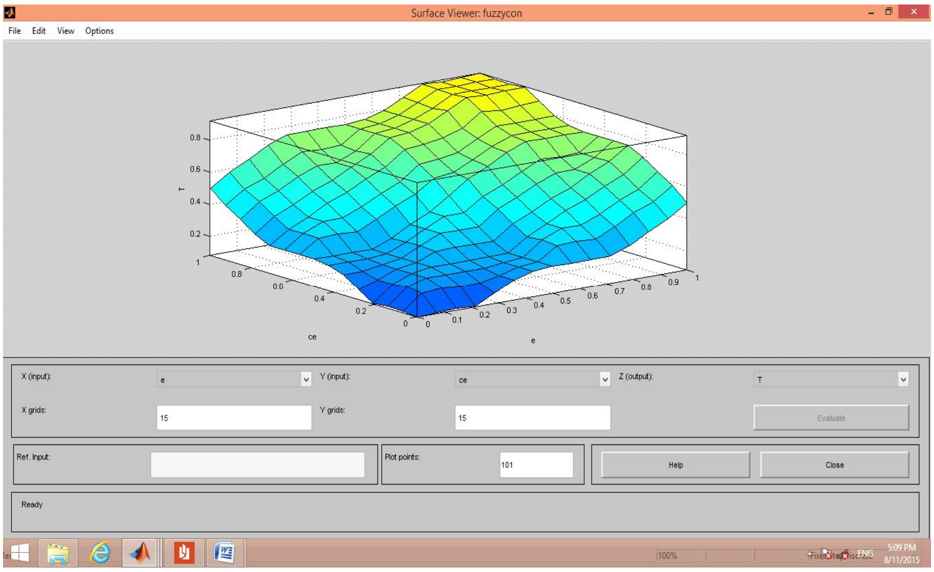

Fig.3.4 3-dimensional view of control surface

There are two types of fuzzy inference methods namely Mamdani's method and Sugeno or Takagi–Sugeno–Kang method of fuzzy inference process to calculate fuzzy output [30, 31].

The Mamdani implication is found suitable for DC machine and induction machine models. In order to convert fuzzy output in to a crisp value of the output variable, the de-fuzzification process is employed. The centre of area (COA) de-fuzzification method is generally used [30, 31].

Using this method, the centroid of each output membership function for each rule is first evaluated. The final output torque is then calculated as the average of the individual centroids, weighted by their heights (degree of membership) as Shawn below [30,31].

$$\Delta T = \frac{\sum_{i=1}^{n} \Delta T_i \mu_A(T_i)}{\sum_{i=1}^{n} \mu_A(\Delta T_i)} \quad (3.26)$$

Where:

n :Number of discrete elements

T_i: Value of discrete elements

$\mu_A(T_i)$: The corresponding MF Value at The point T_i

The fuzzy logic controller output torque is applied to the PWM using hysteresis controllers. The PWM controls the magnitude and frequency of the V/f scheme so that the desired speed of the motor can be obtained [32].