



بسم الله الرحمن الرحيم  
**Sudan University of Science and  
Technology**  
**College of Graduate Studies**



# **Servo Motor Position Control Using Fuzzy Controller**

محرك مؤازرة للتحكم في الوضع باستخدام  
المتحكم الغامض

**A Thesis Submitted In Partial Fulfillment For The Requirements Of  
The  
Degree Of M.Sc. In Electrical Engineering (Control and Microprocessors)**

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## Dedication

Dedication to my mother...

With warmth and faith...

Dedication to my father...

With love and respect ...

Dedication to my beloved...

Brothers and sisters...

Dedication to my friends...

Whom I cherish their friendship

Dedication to my husband

Who mean so much to me ...

Dedication to my children

With all my love ...

Dedication to all my teachers ...

In whom I believe so much ...

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## **Abstract**

The position control system is one of the interesting terms in control system engineering. Proportional-Integral-Derivative (PID) controller is a well known controller and widely used in feedback control in industrial processes. In position control system, PID controller sometimes cannot make this application accurate because of nonlinear properties. Therefore, in this thesis the fuzzy logic controller is proposed to overcome the problem of PID controller. Fuzzy logic controller has ability to control the nonlinear system because of the algorithm is implementing in language. Based on the simulation result the fuzzy logic controller designed is able to improve the performance of the position control system compared to the PID controller in term of rise time ( $T_r$ ) and settling time ( $T_s$ ).

## مستخلص

تعتبر أنظمة التحكم في الوضع من الفروع المهمة في هندسة نظم التحكم ويعتبر المتحكم التناسبي التفاضلي التكاملي من المتحكمات المعروفة والمستخدمه بكثرة في التحكم عن طريق التغذية العكسية في المجال الصناعي . عند استخدامه للتحكم في أنظمة التحكم في الوضع وجد انه لا يعطي الدقة المطلوبة نسبة لخواصها غير الخطية . ولذلك في هذا البحث تم تصميم المتحكم الغامض لتفادي هذه المشكلة . للمتحكم الغامض المقدرة علي التحكم في الأنظمة غير الخطية لان الخوارزمية المستخدمة تنفذ بتعليمات لغوية . واعتمادا علي نتائج المحاكاة نجد أن المتحكم الغامض له المقدرة علي تحسين أداء النظام والمتمثل في تقليل زمن الصعود وزمن السكون مقارنة بالمتحكم التناسبي التكاملي التفاضلي .

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## List of Symbols

|          |  |
|----------|--|
| $R_a$    | Armature resistance, $\Omega$ .                        |
| $L_a$    | Armature inductance, H.                                |
| $E_a(t)$ | Supply voltage, V.                                     |
| $I_a(t)$ | Motor current, A.                                      |
| $V_b(t)$ | Velocity of conductor normal to the magnetic force, V. |
| $w(t)$   | Angular speed.   |
| $T(t)$   | Motor output torque, Nm.                               |
| $J_m$    | Motor inertia, $\text{Kg.m}^2$ .                       |
| $B$      | Viscous friction coefficient, Nm/rad/s.                |
| $K_T$    | Torque constant, Nm/A.                                 |
| $K_B$    | Voltage constant, V/rad/s.                             |
| $K_p$    | Proportional gain                                      |
| $K_i$    | Integral gain  |
| $K_d$    | Derivative gain  |
| $T_r$    | Rise time, s.  |
| $T_s$    | Settling time, s.                                      |

# **Chapter one**

## **Introduction**

### **1.1 General**

The direct current (DC) motor is one of the first machines devised to convert electrical energy to mechanical power. Its origin can be traced to machines conceived and tested by Michael Faraday, the experimenter who formulated the fundamental concepts of electromagnetism. These concepts basically state that if a conductor, or wire, carrying current is placed in a magnetic field, a force will act upon it. The magnitude of this force is a function of strength of the magnetic field, the amount of current passing through the conductor and the orientation of the magnet and conductor. The direction in which this force will act is dependent on the direction of current and direction of the magnetic field . DC motor speeds can easily be varied, therefore they are utilized in applications where speed control or positioning needs exist. The DC motor has a lot of application in today's field of engineering and technology. Starting from an electric shaver to parts of automobiles, in all small or medium sized [1].

The types of DC motor can be listed as follows:

- Permanent Magnet DC Motor
- Separately Excited DC Motor
- Self Excited DC Motor

#### **Speed Control of DC Motor**

Speed of dc motor is controlled by the factors stated below:

#### **Field Control of DC Motor**

1-Field rheostat control of DC Motor: In this method, speed variation is accomplished by means of a variable resistance inserted in series with field . An increase in controlling resistance reduces the field current with a reduction in

flux and an increase in speed. This method of speed control is independent of load on the motor. Power wasted in controlling resistance is very less as field current is a small value.

2. Field voltage control: This method requires a variable voltage supply for the field circuit which is separated from the main power supply to which the armature is connected.

## **Armature Control of DC Motor**

### **Speed control by this method involves two ways:**

1. Armature resistance control : In this method armature circuit is provided with a variable resistance. Field is directly connected across the supply so flux is not changed due to variation of series resistance.

2. Armature voltage control: This method of speed control needs a variable source of voltage separated from the source supplying the field current. This method avoids disadvantages of poor speed regulation and low efficiency of armature-resistance control methods [1]. In this work the design and application of a fuzzy logic controller to DC-servomotor is investigated. The proposed strategy is intended to improve the performance of the original control system by use of a fuzzy logic controller (FLC) as the motor load changes. Computer simulation demonstrates that FLC is effective in position control of a DC-servomotor compared with conventional one. When attempting to carry out the control of a system applying the classical control theory; we need the mathematical model of the process and information about the evolution of the system variables to close the control loops. Normally both conditions are difficult to resolve sometimes because of the complexity of the process or lack of knowledge we have about it, and also because of the insufficient technological level reached at the moment in the sensor field. New process control techniques now combine advances in computer hardware and sensors with new

programming techniques. In this way they attempt to solve difficult control problems [2]. Fuzzy set theory, first formulated by Zadeh nearly 30 years ago, constitutes the fundamentals of fuzzy logic which has emerged as an outgrowth of fuzzy set theory and a generalization of the infinite-value logic that can be considered as a mathematical theory.

## **1.2 Problem statement**

When PID controller is used in position control to control servo motor system some obstacles appear such as behaviors in terms of nonlinearity system, time response and lastly engineering goals such as cost and reliability. The factors that motivate many researchers to use conventional control theory and techniques which are systems are nonlinear and may contain unknown parameters. That unknown parameters may not be estimated accurately if reliable experimental data is absent and also the delays present in the process of system might complicate achieving high performance control. Fuzzy logic controller has been used in control system when the mathematical model of the interested process is vague or exhibits uncertainties. The advantages of using the fuzzy logic control that the developed controller can deal with is increasingly a complex system. It also can implement without precise knowledge of the model structure of dynamic system.

## **1.3 Objectives**

The main objectives of this thesis are:

- 1) To study and understand servo motor.
- 2) To analyze the fuzzy logic controller for position control to control the servo motor system.
- 3) To investigate the application of fuzzy logic controller in servo motor control.

4) To test the performances of fuzzy logic controller by simulation results using MALAB/SIMULINK software.

## **1.4 Methodology**

The scopes of this project are:

- 1) Understanding of the servo motor control background, analyzing the problem and investigating fuzzy logic controller theory which has been applied to the servo motor system.
- 2) Designing and implementing of the control algorithm that is carried by fuzzy logic controller using MALAB/SIMULINK software.
- 3) Evaluate performance of position control of the servo motor based on simulation results.

## **1.5 Thesis Layout**

In this thesis five chapters are included. In chapter one an introduction along with the problem statement, objectives and methodology are included. Chapter two includes a background of system and control systems types. While in chapter three servo motor modeling is introduced. Furthermore, the main concept of PID controller and FLC are discussed. Chapter four includes simulation and results of the system. Finally, in chapter five the conclusion and recommendations presented.



# **Chapter Two**

## **Literature Review**

### **2.1 Technology Developments**

Interest in fuzzy systems was sparked in 1985 so it provided simulations that demonstrated the feasibility of fuzzy control systems for the Sendai railway. The fuzzy systems were used to control accelerating, braking, and stopping when the line opened in 1987 [2]. In 1987, the use of fuzzy control through a set of simple dedicated fuzzy logic chips appeared in an "inverted pendulum" experiment. This is a classic control problem, in which a vehicle tries to keep a pole mounted on its top by a hinge upright by moving back ward and forth ward [3].

An auto focusing camera was developed it uses a Charge Coupled Device (CCD) to measure the clarity of the image in six regions of its field of view and use the information provided to determine if the image is in focus. It also tracks the rate of change of lens movement during focusing, and controls its speed to prevent overshoot. The camera's fuzzy control system uses 12 inputs: 6 to obtain the current clarity data provided by the CCD and 6 to measure the rate of change of lens movement. The output is the position of the lens. The fuzzy control system uses 13 rules and requires 1.1 kilobytes of memory . An industrial air conditioner was designed using 25 heating rules and 25 cooling rules. A temperature sensor provides input, with control outputs fed to an inverter, a compressor valve, and a fan motor. Compared with the previous design, the fuzzy controller heats and cools five times faster, reduces power consumption by 24%, increases temperature stability by a factor of two, and uses fewer sensors.

Other applications investigated or implemented include: character and handwriting recognition; optical fuzzy systems; robots, voice robot helicopters (hovering is a "balancing act" rather similar to the inverted pendulum problem),

elevator systems, and so on. A fuzzy logic controller has been investigated for energy-efficient motors, and it has been studied for automated space docking: simulations show that a fuzzy control system can greatly reduce fuel consumption [4].

In 1995 an "intelligent" dishwasher was introduced, based on a fuzzy controller and a "one-stop sensing module" that combines a thermistor, for temperature measurement; a conductivity sensor, to measure detergent level from the ions present in the wash; a turbidity sensor that measures scattered and transmitted light to measure the soiling of the wash; and a magnet astrictive sensor to read spin rate. The system determines the optimum wash cycle for any load to obtain the best results with the least amount of energy, detergent, and water. It even adjusts for dried-on foods by tracking the last time the door was opened, and estimates the number of dishes by the number of times the door was opened.

Research and development is also continuing on fuzzy applications in software, as opposed to firmware, design, including fuzzy expert systems and integration of fuzzy logic with neural-network and so-called adaptive "genetic" software systems, with the ultimate goal of building "self-learning" fuzzy-control systems [5].

## **2.2 Control System**

A control system is a system of devices or set of devices, that manages commands, directs or regulates the behavior of other device(s) or system(s) to achieve desire results. In other words the control system can be rewritten as a control system is a system, which controls other systems [6].

## **2.3 Types of Control Systems**

There are various types of control systems but all of them are created to control outputs. The systems used for controlling such as position, velocity, acceleration, temperature, pressure, voltage and current are examples of control.

### 2.3.1 Open loop control systems

A control system in which the control action is totally independent of output of the system is called open loop control system. Manual control system is also an open loop control system. Figure 2.1 shows the block diagram of open loop control system in which process output is totally independent of controller action.

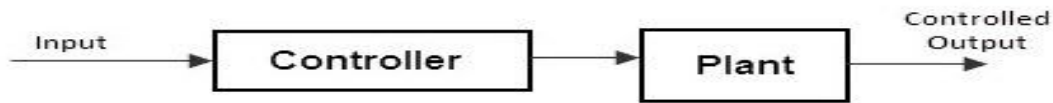


Figure 2.1: Open Loop Control System

### 2.3.2 Closed loop control system

Control system in which the output has an effect on the input quantity in such a manner that the input quantity will adjust itself based on the output generated is called closed loop control system. Open loop control system can be converted into closed loop control system by providing a feedback. This feedback automatically makes the suitable changes in the output due to external disturbance. In this way closed loop control system is called automatic control system. Figure 2.2 shows the block diagram of closed loop control system [6].

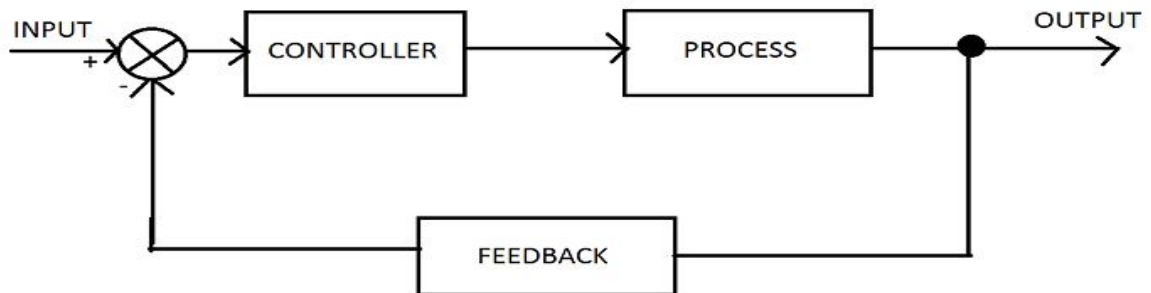


Figure 2.2: Closed loop control system

## 2.4 Linear Systems

It is a mathematical model of a system based on the use of a linear operator. Linear systems typically exhibit features and properties that are much simpler than the general, nonlinear case. As a mathematical abstraction or idealization, linear systems find important applications in automatic control theory, signal processing, and telecommunications. General deterministic system can be described by operator  $H$ , that maps an input  $x(t)$  as a function of  $(t)$  and output  $y(t)$  [7]. Systems satisfy the property of superposition. Given two valid inputs  $x_1(t)$  and  $x_2(t)$ . As well as their respective outputs as:

$$y_1(t) = H\{x_1(t)\} \quad (2.1)$$

$$y_2(t) = H\{x_2(t)\} \quad (2.2)$$

Then a linear system must satisfy:

$$\alpha y_1(t) + \beta y_2(t) = H\{\alpha x_1(t) + \beta x_2(t)\} \quad (2.3)$$

For any scale values  $\alpha$  and  $\beta$ . The system is then defined by the equation  $H(x(t)) = y(t)$ , where  $y(t)$  is some arbitrary function of time, and  $x(t)$  is the system state. Given  $y(t)$  and  $H$ ,  $x(t)$  can be solved. For example, a simple harmonic oscillator obeys the differential equation defined as:

$$m \frac{d^2(x)}{dt^2} + kx = 0 \quad (2.4)$$

Where  $m$  is a mass and  $k$  is spring constant. If  $[H(x(t)) = m \frac{d^2(x)}{dt^2} + kx]$  then  $H$  is a linear operator. Letting  $y(t) = 0$ , we can rewrite the differential equation as  $H(x(t)) = y(t)$ , which shows that a simple harmonic oscillator is a linear system. The behavior of the resulting system subjected to a complex input can be described as a sum of responses to simpler inputs. In nonlinear systems, there is no such relation.

## 2.5 Nonlinear Systems

It is a system which does not satisfy the superposition principle– meaning that the output of a nonlinear system is not directly proportional to the input. In mathematics, a nonlinear system of equations is a set of simultaneous equation in which the unknowns (or the unknown functions in the case of differential equation) appear as variables of a polynomial of degree higher than one or in the argument of a function which is not a polynomial of degree one. In other words, in a nonlinear system of equations, the equation(s) to be solved cannot be written as a linear combination of the unknown variables or function that appears in them known. It does not matter if nonlinear functions appear in the equations. In particular differential equation is *linear* if it is linear in terms of the unknown function and its derivatives, even if nonlinear in terms of the other variables appearing in it [6].

## 2.6 Superposition Principle

Also known as superposition property, states that, for all linear system, the net response at a given place and time caused by two or more inputs is the sum of the responses which would have been caused by each input individually. So that if input A produces response X and input B produces response Y then input

(A + B) produces response (X + Y). The homogeneity and additively properties together are called the superposition principle. A linear function is one that satisfies the properties of superposition. This is defined as:

**Additively:**  $F(x_1 + x_2) = f(x_1) + f(x_2)$  (2.5)

**Homogeneity:**  $F(\alpha x) = \alpha F(x)$  (2.6)

For scalar  $\alpha$ .

## 2.7 Time-variant system

Is a system that is not Time Invariant. Roughly speaking, its output characteristics depend explicitly upon time. Generally, the input–output characteristics vary with time. Let:

$x(t)$  be an excitation signal.

$T(x(t), t)$  describe the input–output map of a system in relaxed state.

$y(t)$  be the system's output response.

$$y(t) = T(x(t), t). \quad (2.7)$$

If the excitation signal is delayed by time  $k$  (i.e.,  $x(t-k)$ ) and the output response  $T(x(t-k), t)$  is *not* equivalent to a delayed version of the original output. The

$y(t-k)$  defined as:

$$y(t-k) = T(x(t-k), t-k) \quad (2.8)$$

then the system is time variant [7].

## 2.8 Time-invariant system

Is a system whose output does not depend explicitly on time, Such systems are regarded as a class of systems in the field of system analysis. Lack of time dependence is captured in the following mathematical property of such a system:

If the input signal  $x(t)$  produces an output  $y(t)$  then any time shifted input.  $x(t+\delta)$  results in a time-shifted output  $y(t+\delta)$ . This property can be satisfied if the transfer function of the system is not a function of time except expressed by the input and output.

## 2.9 Fuzzy Logic

Fuzzy logic was first proposed by Lotfi A. Zadeh of the University of California at Berkeley in a 1965 paper. He elaborated on his ideas in a 1973 paper that

introduced the concept of linguistic variables, which in this article equates to a variable defined as a fuzzy set. The term fuzzy refers to the fact that the logic involved can deal with concepts that cannot be expressed as the true or false but rather as partially true. Although alternative approaches such as genetic algorithms and neural networks can perform just as well as fuzzy logic in many cases. Fuzzy logic has the advantage that the solution to the problem can be cast in terms that human operators can understand, so that their experience can be used in the design of the controller. This makes it easier to mechanize tasks that are already successfully performed by humans [8].

## 2.10 The Basics of Fuzzy Logic

A fuzzy control is a controller that is intended to manage some vaguely known or vaguely described process. The controller can be used with the process in two modes:

- i. Feedback mode when the fuzzy controller acts as a control device.
- ii. Feed forward mode where the controller can be used as a prediction device.

Figure 2.3 illustrates the basic components of fuzzy logic controller.

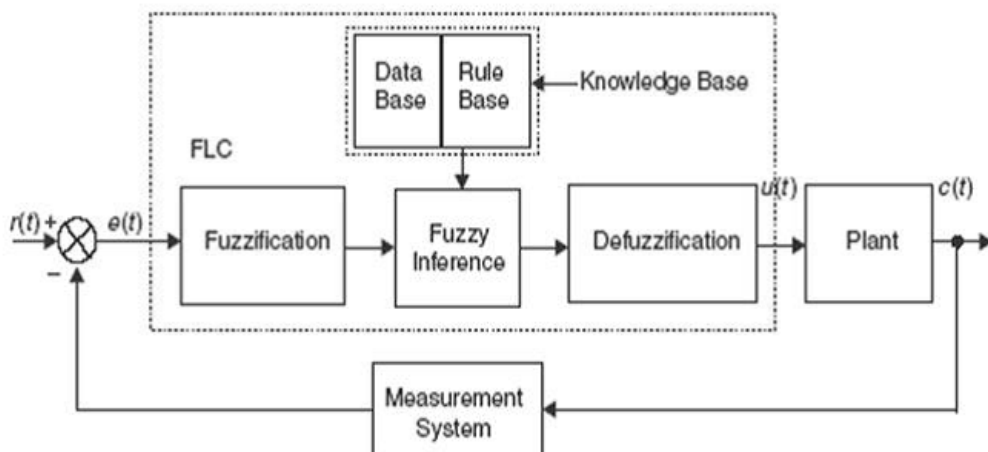


Figure 2.3: Fuzzy logic control system

The plant output are denoted by  $c(t)$ , the input are denoted by  $u(t)$ , and the reference input to the fuzzy controller is denoted by  $r(t)$  [9]. The fuzzy controller has four main components:

### **2.10.1 Fuzzification**

The first block of the fuzzy controller is fuzzification, which converts each piece of input data to degrees of membership by a lookup in one or several membership functions. The fuzzification block thus matches the input data with the conditions of the rules to determine how well the condition of each rule matches that particular input instance. There is a degree of membership for each linguistic term that applies to that input variable.

### **2.10.2 Inference mechanism**

Inference mechanism or engine is the processing program in a fuzzy control System. It derives a conclusion from the facts and rules contained in the knowledge base using various human expert techniques.

### **2.10.3 Rule-base**

A group of rules may use several variables both in the condition and the conclusion of the rules. They are based on a set of rules that a human expert would follow in diagnosing a problem. Rule-base also where the knowledge is stored.

### **2.10.4 Defuzzification**

Defuzzification is a process that maps a fuzzy set to a crisp set and has attracted far less attention than other processes involved in fuzzy systems and technologies. Four most common defuzzification methods are:

- Maximum membership method.
- Center of gravity method.
- Weight average method.
- Mean-maximum membership method.



## 2.11 Fuzzy Set Operations

There are three types of fuzzy set operation[10].

### 2.11.1 Union

The membership function of the Union of two fuzzy sets A and B with membership functions  $\mu_A$  and  $\mu_B$  respectively is defined as the maximum of the two individual membership functions. This is called the maximum criterion. Figure 2.4 show the union operation.

$$\mu_{A \cup B} = \max(\mu_A, \mu_B) \quad (2.9)$$

The union operation in Fuzzy set theory is the equivalent of the **OR** operation in Boolean algebra.

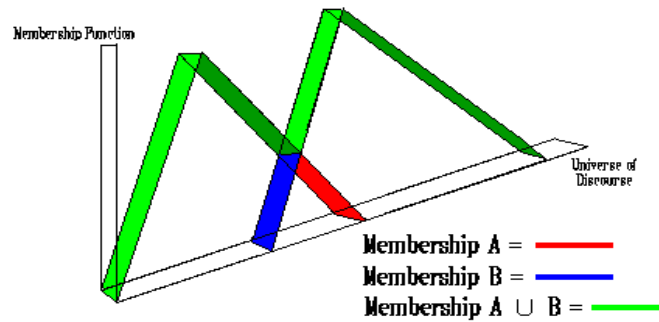


Figure 2.4: the union operation

### 2.11.2 Intersection

The membership function of the intersection of two fuzzy sets A and B with membership functions  $\mu_A$  and  $\mu_B$  respectively is defined as the minimum of the two individual membership functions. This is called the minimum criterion. Figure 2.5 shows intersection operation.

$$\mu_{A \cap B} = \min(\mu_A, \mu_B) \quad (2.10)$$

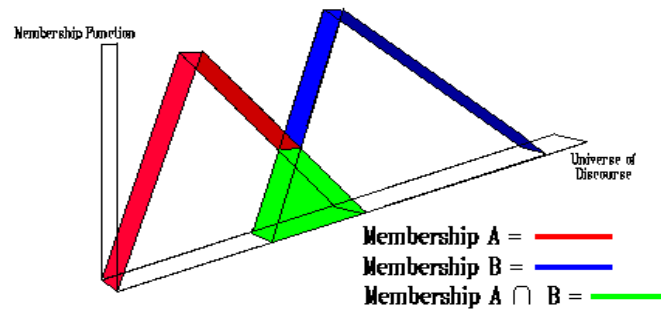


Figure 2.5: Intersection operation

The intersection operation in fuzzy set theory is the equivalent of the AND operation in Boolean algebra.

### 2.11.3 Complement

The membership function of the complement of a fuzzy set A with membership function  $\mu_A$  is defined as the negation of the specified membership function. This is called the negation criterion. Figure 2.6 shows complement operation.

$$\mu_{\bar{A}} = 1 - \mu_A \quad (2.11)$$

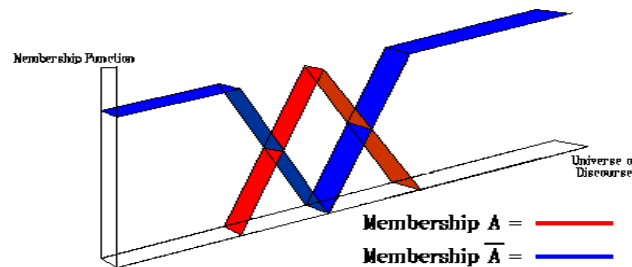


Figure 2.6: Complement operation

The complement operation in fuzzy set theory is the equivalent of the NOT operation in Boolean algebra.

# Chapter Three

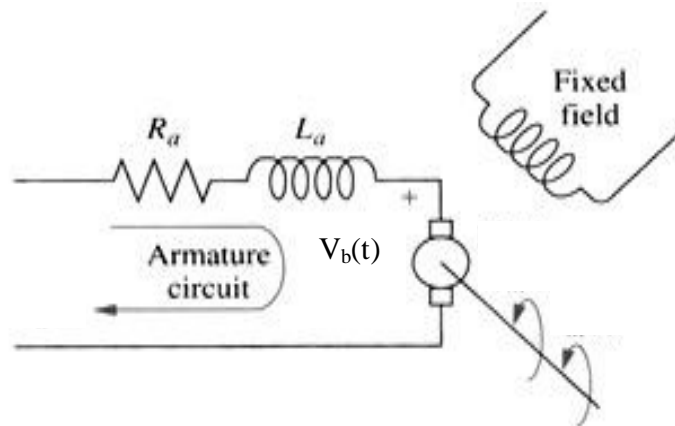
## Control of Servomotor

### 3.1 Introduction

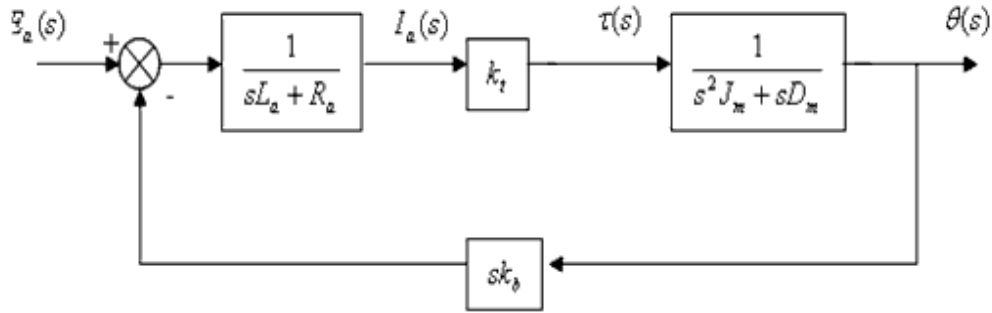
A servomotor is a packaged combination of several components: a motor, a gear train to reduce the many rotations of the motor to a higher torque rotation, a position encoder that identifies the position of the output shaft and an inbuilt control system. The input control signal to the servo indicates the desired output position. Any difference between the position commanded and the position of the encoder gives rise to an error signal that causes the motor and gear train to rotate until the encoder reflects a position matching that commanded. A simple low-cost servo of this type is widely used for radio-controlled model.

### 3.2 Servomotor Modeling

Servomotor is used for position or speed control in closed loop control systems. The equivalent circuit diagram of servomotor is presented in Figure 3.1. The armature is modeled as a circuit with resistance  $R_a$  connected in series with an inductance  $L_a$  and a voltage source  $V_b(t)$  representing the back emf in the armature when the rotor rotates.



(a) Schematic diagram



(b) Block diagram

Figure 3.1: Servo motor system

Kirchhoff's voltage law is used to map the armature circuitry dynamic of the motor. Thus, assume the inductance  $L_a$  can be ignored, which in the case for servomotor, the supply voltage  $E_a(t)$  will be:

$$E_a(t) = I_a(t)R_a + V_b(t) \quad (3.1)$$

Since the current carrying armature is rotating in a magnetic field, its back electromotive force is proportional to speed.  $V_b(t)$  is the velocity of the conductor normal to the magnetic field.

$$V_b(t) = K_B \omega(t) \quad (3.2)$$

The typical equivalent mechanical loading on a motor that connected to the motor shaft including total moment of inertia  $J_m$  and total viscous friction.

Assume that  $T(t)$  is the torque developed by the motor.

$$T(t) = J_m \alpha(t) + B \alpha(t) \quad (3.3)$$

The developed motor output torque for the servo motor can given by:

$$T(t) = K_T I_a(t) \quad (3.4)$$

By using Laplace transforms on the equation (3.1), (3.2), (3.3) and (3.4) and neglecting initial condition we have:

$$E_a(s) = R_a I_a(s) + V_b(s) \quad (3.5)$$

$$V_b(s) = K_B s \theta_m(s) \quad (3.6)$$

$$T(s) = J_m s^2 \theta_m(s) + B s \theta_m(s) \quad (3.7)$$

$$T(s) = K_T I_a(s) \quad (3.8)$$

Substitute Equation (3.8) into Equation (3.7), we have:

$$K_T I_a(s) = J_m s^2 \theta_m(s) + B s \theta_m(s) \quad (3.9)$$

Equation (3.5) is rearranged to obtain:

$$I_a(s) = \frac{E_a(s) - V_b(s)}{R_a} \quad (3.10)$$

Substitute equation (3.10) into Equation (3.9), we get:

$$K_T \left[ \frac{E_a(s) - V_b(s)}{R_a} \right] = J_m s^2 \theta_m(s) + B s \theta_m(s) \quad (3.11)$$

From equation (3.11), the transfer function between the input voltage  $E_a(s)$  and the output  $\theta_m(s)$  can be obtained as:

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_T}{J_m R_a s^2 + (B R_a + K_T K_B) s} \quad (3.12)$$

The physical parameters of the servo motor used for simulation testing are given in table 3.1.

Table 3.1: The parameters of the servo motor

|       |               |
|-------|---------------|
| $K_T$ | 0.01 N.m/A    |
| $K_B$ | 0.01 V/rad/s  |
| $R_a$ | 1 $\Omega$    |
| $B$   | 0.1 N.m/rad/s |

Substitute there parameters in Equation (3.12), the transfer function becomes as follow:

$$\frac{\theta_m(s)}{E_a(s)} = \frac{3.839}{0.004 s^2 + 0.34 s + 1} \quad (3.13)$$

### 3.3 PID Controller Design

A PID controller is one of the most commonly used controllers because it is simple and robust. This controller is extremely popular because it can usually provide good closed loop response characteristics that can be tuned using relatively simple rules and easy to construct using either analogue or digital components. Figure 3.2 illustrates the block diagram of PID controller.

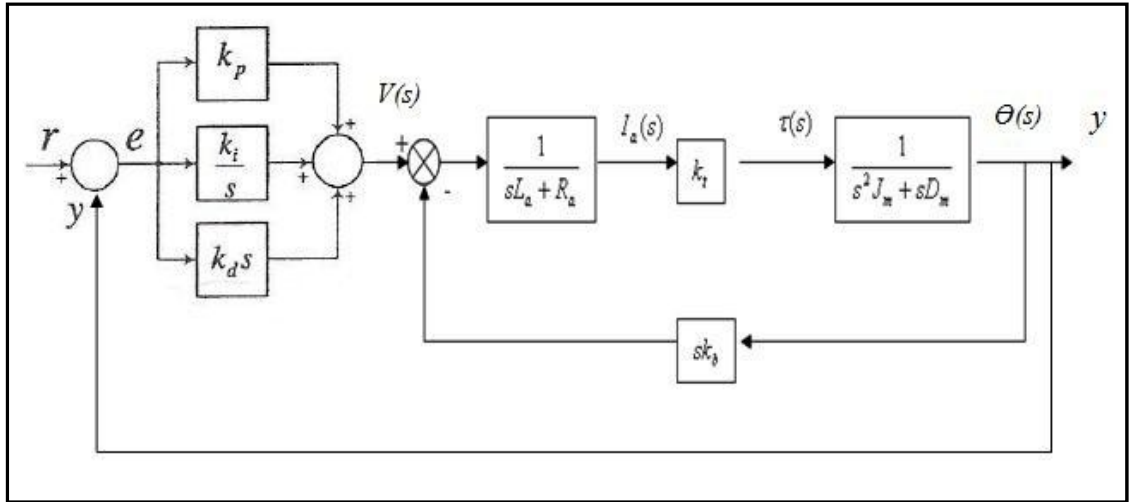


Figure 3.2: Block diagram of the closed loop servomotor with PID controller

The PID controller can be defined as equation (3.10) by the following relationship between controller input  $e(t)$  and the controller output  $V(t)$  that is applied to the motor armature [11].

$$V(t) = K_p e(t) + K_i \int_0^t e(t) dt + \frac{K_d}{dt} de(t) \quad (3.10)$$

Where  $K_p$ ,  $K_i$ ,  $K_d$  are the proportional, integral and derivative gain values of the PID controller ,respectively. By using the Laplace transform, the transfer functions of PID controller is obtained as:

$$\frac{V(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s \quad (3.11)$$

### 3.4 Fuzzy Logic Controller Design

A basic structure of fuzzy logic controller system block diagram for position control is clearly shown in Figure 3.3.

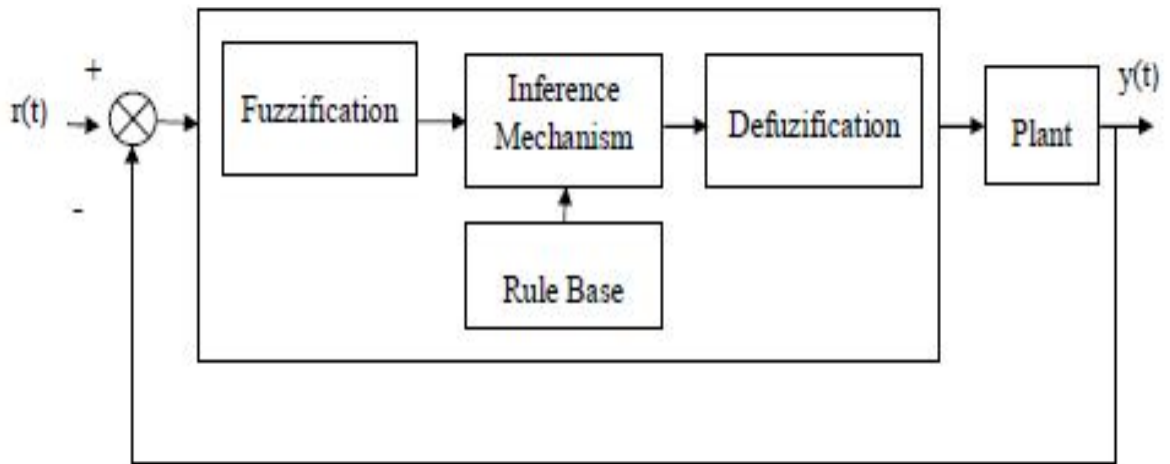


Figure 3.3: Structure of fuzzy logic controller system

A fuzzy logic controller input variables involves receiving the error signal and change of error .These variables evaluate the fuzzy control rules using the compositional rules of inference and the appropriately computed control action is determined by using the defuzzification [12]. The essential steps to design the fuzzy logic controller are illustrate in Figure 3.4.

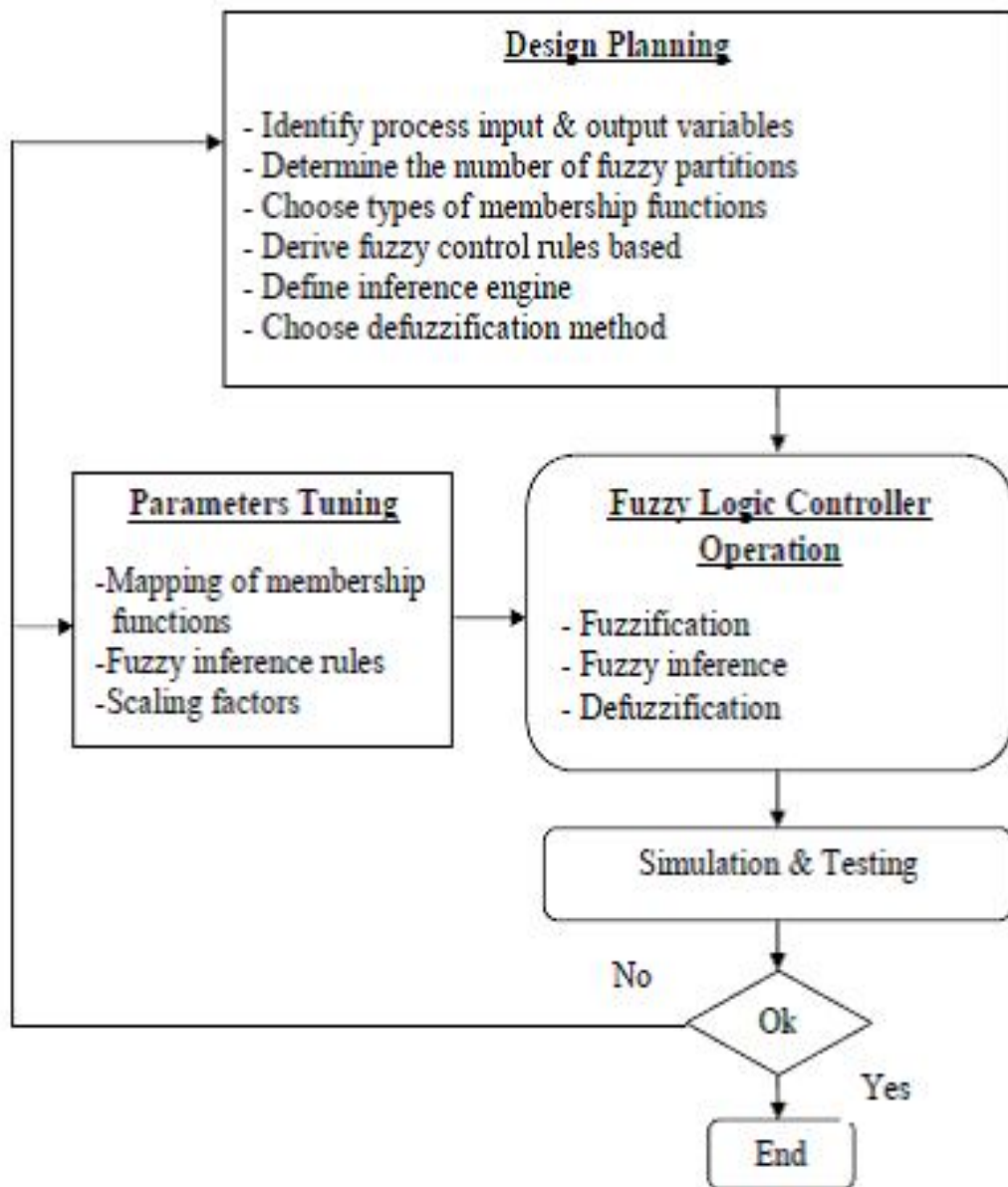


Figure 3.4: Design procedure of the fuzzy logic controller

### 3.5 Fuzzy Sets

In mathematics a set, by definition, is a collection of things that belong to some definition. Any item either belongs to that set or does not belong to that set. Let us look at another example; the set of tall men. We shall say that people taller than or equal to 6 feet are tall. This set can be represented graphically as in figure 3.5 bellow.



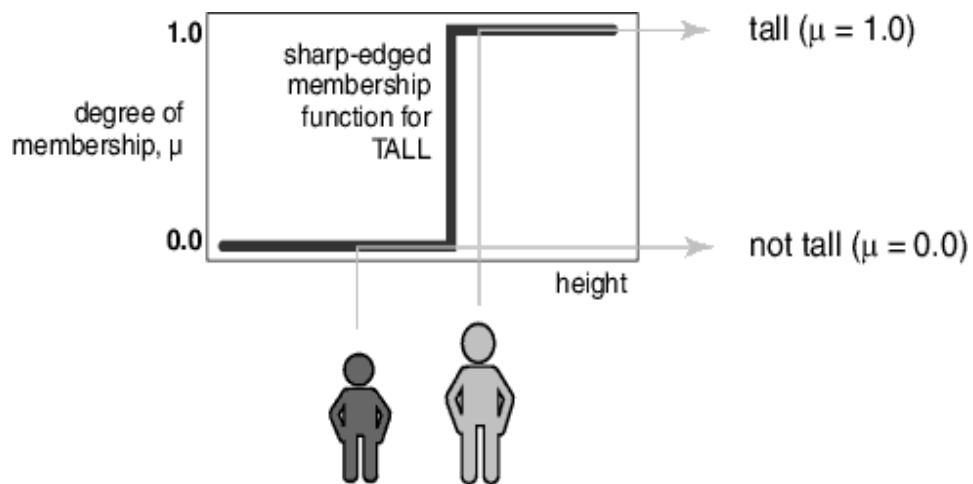


Figure 3.5: The membership of the 'tall' set

The input variables in a fuzzy control system are in general mapped by sets of membership functions known as "fuzzy sets". A membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The input space is sometimes referred to as the universe of discourse, a fancy name for a simple concept or it is a graphical representation of fuzzy sets,  $\mu_F(x)$ .

### 3.6 Membership Functions

The simplest membership functions are formed using straight lines. Of these, the simplest is the *triangular* membership function, and it has the function name *trimf*. It is nothing more than a collection of three points forming a triangle. The *trapezoidal* membership function, *trapmf*, has a flat top and really is just a truncated triangle curve as shown in figure 3.6 these straight line membership functions have the advantage of simplicity.

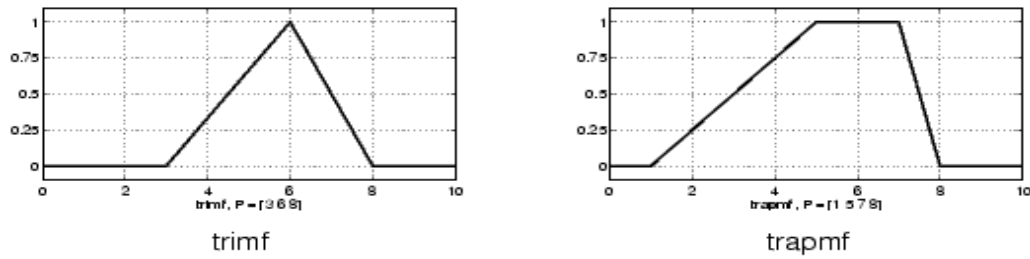


Figure 3.6: The simplest membership function

## 3.7 Linguistic Variable

A linguistic variable is a linguistic expression (one or more words) labeling an information granular. For example, a membership function is labeled by the expressions like hot temperature or rich customer or it is a variable whose values are words or sentences in a natural or artificial language.

## 3.8 Inference Mechanisms

Inference mechanism allows mapping given input to an output using fuzzy logic. It uses all pieces described in previous sections: membership functions, logical operations and IF-THEN rules. The most common types of inference systems are Mamdani and Sugeno. They vary in ways of determining outputs.

### 3.8.1 Fuzzy inference systems (Mamdani)

An example of a Mamdani inference system is shown in Figure 3.7. To compute the output of this FIS given the inputs, one must go through six steps:

1. Determining a set of fuzzy rules.
2. Fuzzifying the inputs using the input membership functions.
3. Combining the fuzzified inputs according to the fuzzy rules to establish rule strength.
4. Finding the consequence of the rule by combining the rule strength and the output membership function.
5. Combining the consequences to get an output distribution.

6. Defuzzifying the output distribution (this step is only if a crisp output (class) is needed).

The following is a more detailed description of this process.

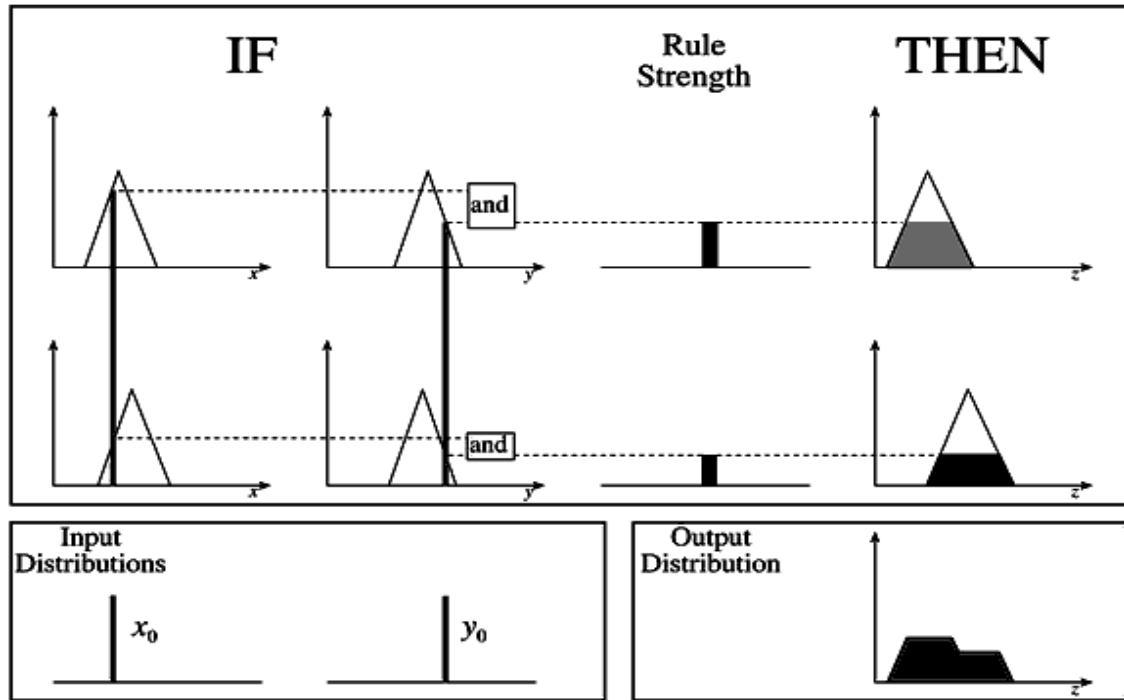


Figure 3.7: A two input and two rule Mamdani FIS with crisp inputs

### 3.9 Creating Fuzzy Rules

Fuzzy rules are a collection of linguistic statements that describe how the FIS should make a decision regarding classifying an input or controlling an output.

Fuzzy rules are always written in the following form:

IF (input 1 is membership function 1) and/or (input 2 is membership function 2)

THEN (output<sub>n</sub> is output membership function<sub>n</sub>).

For example, one could make up a rule that says:

***IF** temperature is high **and** humidity is high **THEN** room is hot.*

### 3.10 Fuzzification

Fuzzification is a process of producing a fuzzy input on the base of a crisp one. It involves the conversion of the input and output signals into a number of fuzzy

represented values (fuzzy sets). Figure 3.8 describes the input and output variables that are used in this system.

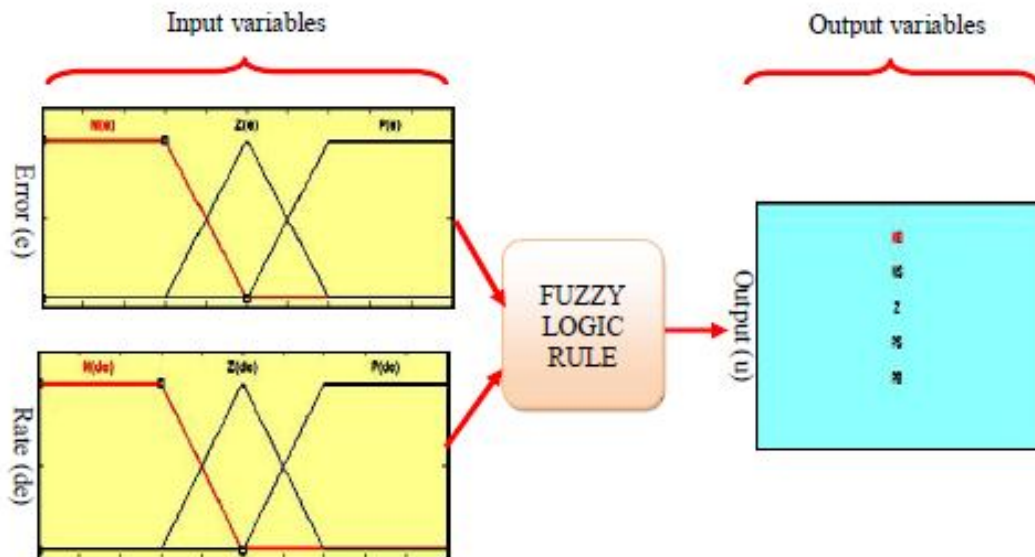


Figure 3.8: MF for input and output of fuzzy logic controller

### 3.10.1 Input variables:

There are two types of input:

- Error (e)

Quantized into 3 and 5 membership function: Negative N (e), Negative Small NS (e), Negative Big NB (e), Zero Z (e), Positive P (e), Positive Small PS (e) and Positive Big PB (e).

- Rate (de)

Quantized into 3 and 5 membership function: Negative N (de), Negative Small NS (de), Negative Big NB (de), Zero Z (de), Positive P (de), Positive Small PS (de) and Positive Big PB (de).

### 3.10.2 Output Variables:

Quantized into 5 and 7 membership function: Negative Small (NS), Negative

Medium (NM), Negative Big (NB), Zero (Z), Positive Small (PS), Positive Medium (PM) and Positive Big (PB).

### 3.11 Rule Base

The basic function of the rule based is to represent the expert knowledge in a form of IF-THEN rule structure. The fuzzy logic can be derived into combination of input ( $3 \times 3$  and  $5 \times 5$ ) respectively. The rules of fuzzy logic controller ( $3 \times 3$  and  $5 \times 5$ ) are listed in Table 3.2 and Table 3.3

Table 3.2: Rule of fuzzy logic controller ( $3 \times 3$ )

| Rate (de) \ Error (e) | P(de) | Z(de) | N(de) |
|-----------------------|-------|-------|-------|
| P(e)                  | PB    | PS    | Z     |
| Z(e)                  | PS    | Z     | NS    |
| N(e)                  | Z     | NS    | NB    |

The fuzzy logic control rules based on Table 3.2 are:

- 1) IF Error (e) is N (e) AND Rate (de) is P (de) THEN Output (u) is Z (u)
- 2) IF Error (e) is Z (e) AND Rate (de) is P (de) THEN Output (u) is PS (u)
- 3) IF Error (e) is P (e) AND Rate (de) is P (de) THEN Output (u) is PB (u)
- 4) IF Error (e) is N (e) AND Rate (de) is Z (de) THEN Output (u) is NS (u)
- 5) IF Error (e) is Z (e) AND Rate (de) is Z (de) THEN Output (u) is Z (u)
- 6) IF Error (e) is P (e) AND Rate (de) is Z (de) THEN Output (u) is PS (u)
- 7) IF Error (e) is N (e) AND Rate (de) is N (de) THEN Output (u) is NB (u)
- 8) IF Error (e) is Z (e) AND Rate (de) is N (de) THEN Output (u) is NS (u)
- 9) IF Error (e) is P (e) AND Rate (de) is N (de) THEN Output (u) is Z (u)

Table 3.3: Rule of fuzzy logic controller (5 × 5)

| Rate(de)<br>Error (e) | PB(de) | PS(de) | Z(de) | NS(de) | NB(de) |
|-----------------------|--------|--------|-------|--------|--------|
| PB(e)                 | Z      | NS     | NM    | NM     | NB     |
| PS(e)                 | PS     | Z      | NS    | NS     | NM     |
| Z(e)                  | PM     | PS     | Z     | NS     | NM     |
| NS(e)                 | PM     | PS     | PS    | Z      | NS     |
| NB(e)                 | PB     | PM     | PM    | PS     | Z      |

The fuzzy logic control rules based on Table 3.3 are:

- 1) IF Error (e) is NB (e) AND Rate (de)is PB(de)THEN Output(u)is PB(u)
- 2) IF Error (e) is NS(e) AND Rate(de)is PB(de)THEN Output(u)is PM(u)
- 3) IF Error (e) is Z (e) AND Rate (de)is PB (de)THEN Output (u) is PM (u)
- 4) IF Error (e) is PS (e) AND Rate (de)is PB (de)THEN Output (u) is PS (u)
- 5) IF Error (e) is PB (e) AND Rate (de) is PB (de)THEN Output (u) is Z(u)
- 6) IF Error (e) is NB (e) AND Rate (de)is PS (de) THEN Output (u) is PM (u)
- 7) IF Error (e) is NS (e) AND Rate (de) is PS(de) THEN Output(u) is PS(u)
- 8) IF Error (e) Is Z (e) AND Rate (de)is PS (de)THEN Output(u) is PS(u)
- 9) IF Error (e) is PS (e) AND Rate (de) is PS (de) THEN Output(u) is Z(u)
- 10) IF Error(e) is PB(e)AND Rate(de) is PS(de) THEN Output(u) Is NS(u)
- 11) IF Error(e) is NB(e) AND Rate(de) is Z(de) THEN Output(u) is PM(u)
- 12) IF Error (e) is NS (e) AND Rate (de) is Z (de) THEN Output (u) Is PS (u)
- 13) IF Error (e) is Z (e) AND Rate (de) is Z (de) THEN Output (u) is Z(u)
- 14) IF Error (e) is PS (e) AND Rate (de) is Z (de) THEN Output(u) is NS(u)
- 15) IF Error(e) is PB(e) AND Rate(de) is Z(de) THEN Output(u) is NM(u)

- 16) IF Error(e) is NB(e) AND Rate(de) is NS(de) THEN Output(u) is PS(u)
- 17) IF Error(e) is NS(e) AND Rate(de) is NS(de) THEN Output(u) is Z(u)
- 18) IF Error (e) is Z(e) AND Rate(de) is NS(de) THEN Output(u) is NS(u)
- 19) IF Error(e) is PS(e) AND Rate(de) is NS(de) THEN Output(u) is NS(u)
- 20) IF Error(e) is PB(e) AND Rate(de) is NS (de) THEN Output(u) is NM(u)
- 21) IF Error(e) is NB(e) AND Rate(de) is NB(de) THEN Output(u) is Z(u)
- 22) IF Error(e) is NS(e) AND Rate (de) is NB(de) THEN Output(u) is NS(u)
- 23) IF Error(e) is Z(e) AND Rate(de) is NB(de) THEN Output(u) is NM(u)
- 24) IF Error(e) is PS(e) AND Rate(de) is NB(de) THEN Output(u) is NM(u)
- 25) IF Error(e) is PB(e) AND Rate (de) is NB(de) THEN Output(u) is NB(u)

### 3.12 MF Editor of FLC (3 × 3)

Figures 3.9, 3.10 and 3.11 show memberships function editor of two inputs and one output, respectively.

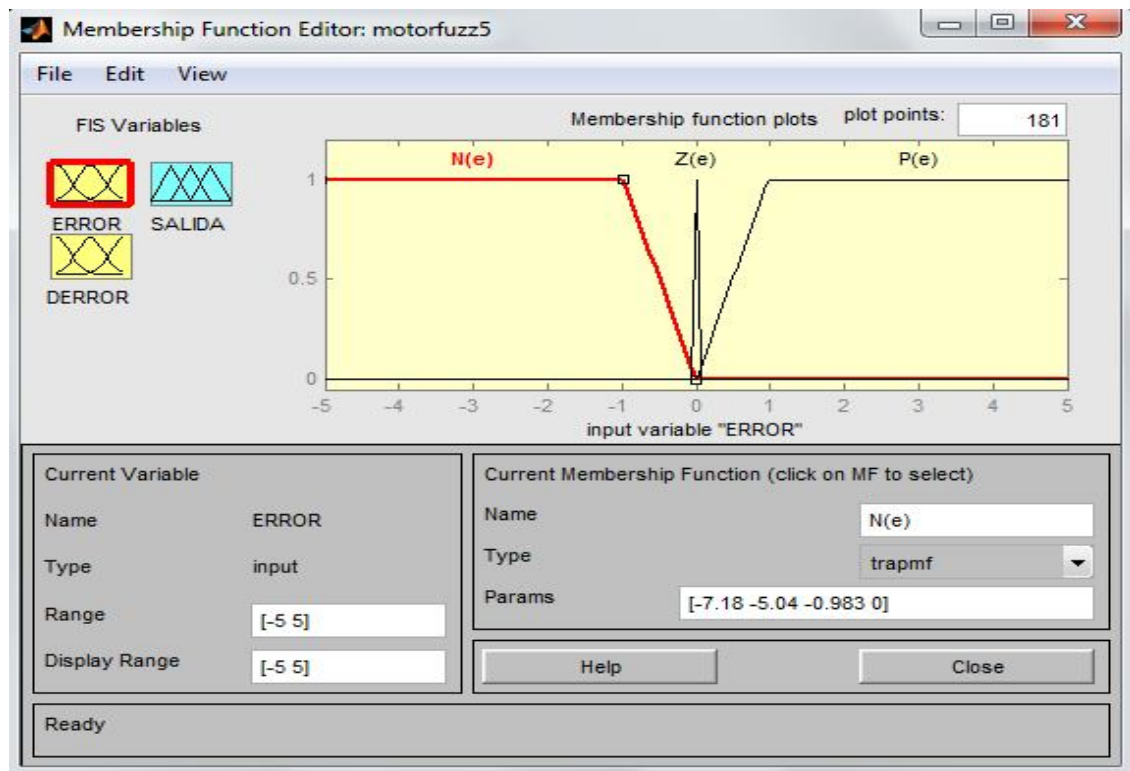


Figure 3.9: Input (ERROR)

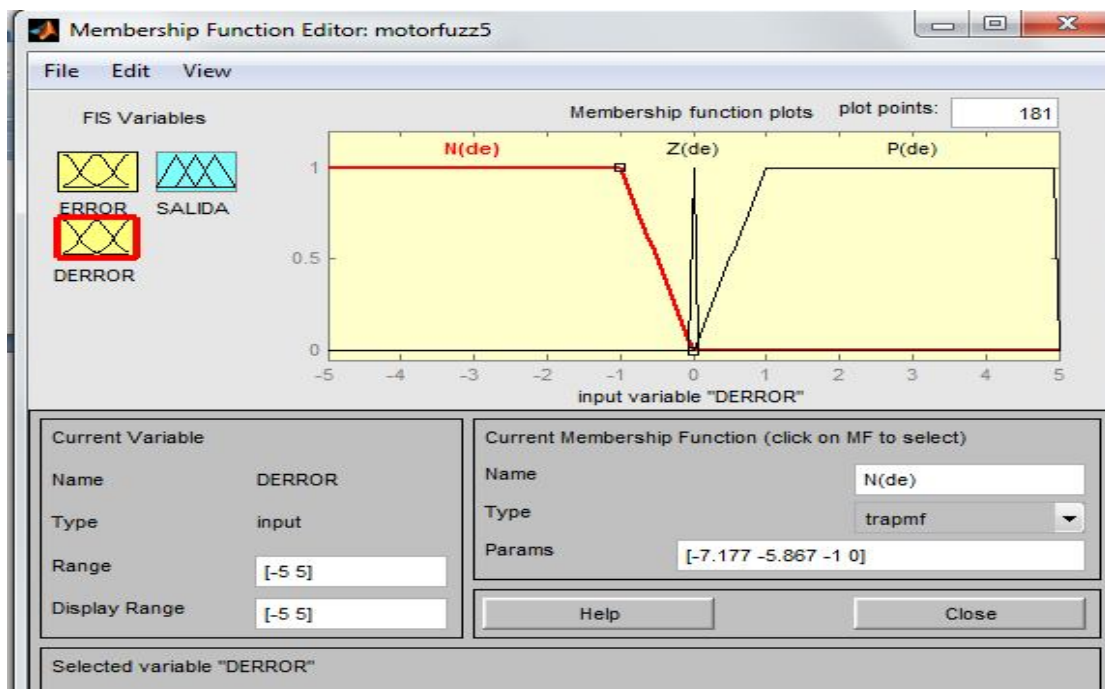


Figure 3.10: input (DERROR)

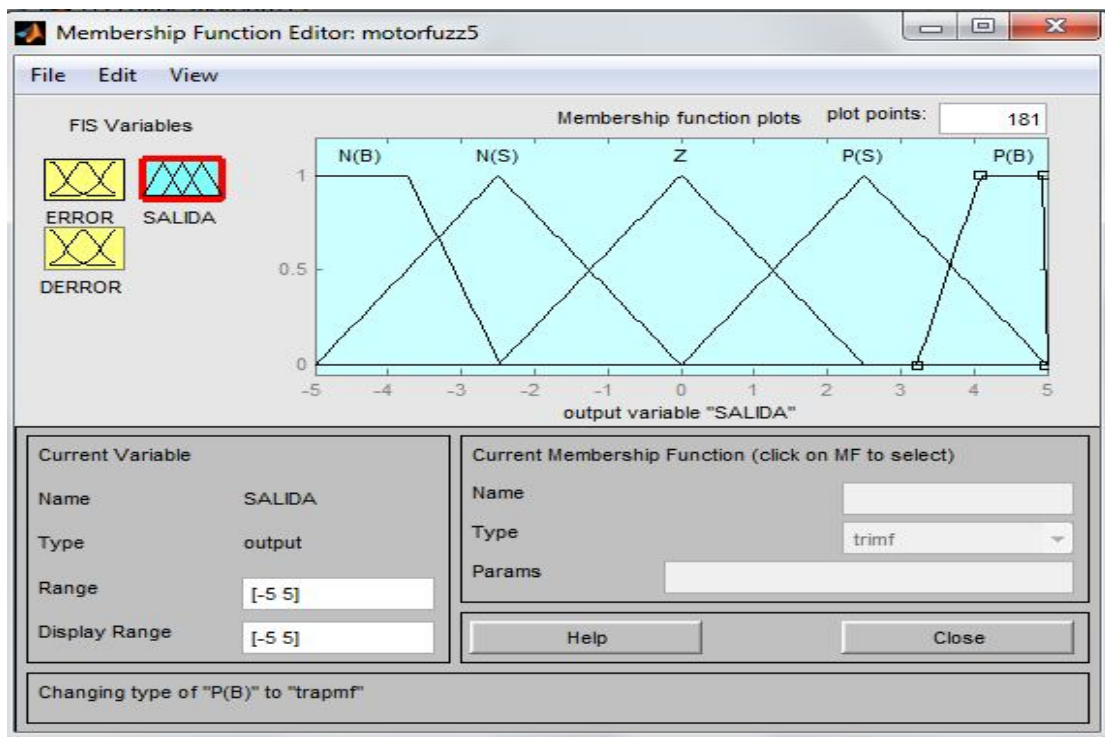


Figure3.11: Output (SALIDA)



### 3.12.1 Rule editor

Figure 3.12 shows the rule editor for FLC ( $3 \times 3$ ).

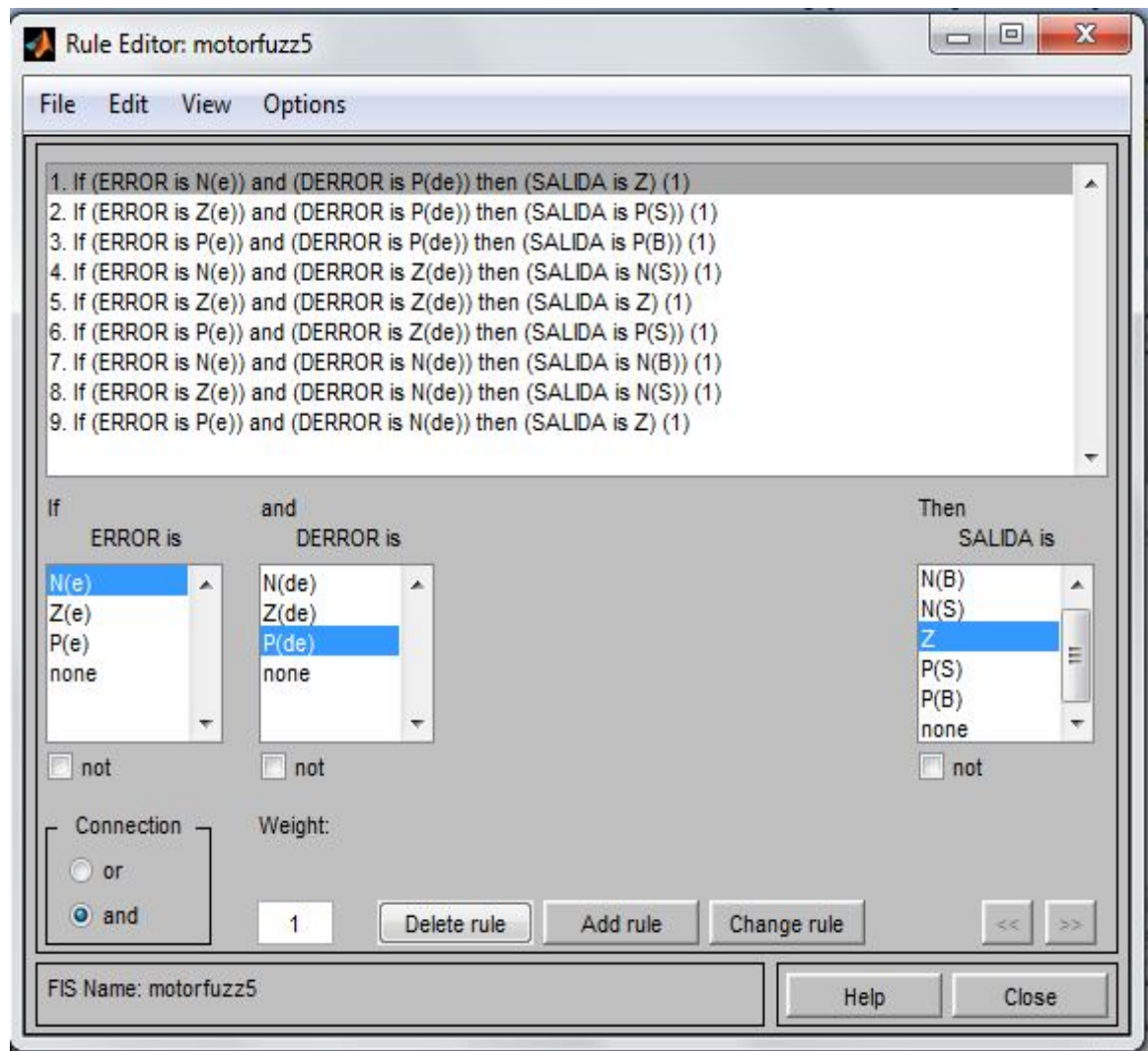


Figure 3.12: Rule editor

### 3.12.2 Rule viewer

Figure 3.13 shows rule viewer of FLC (3×3).

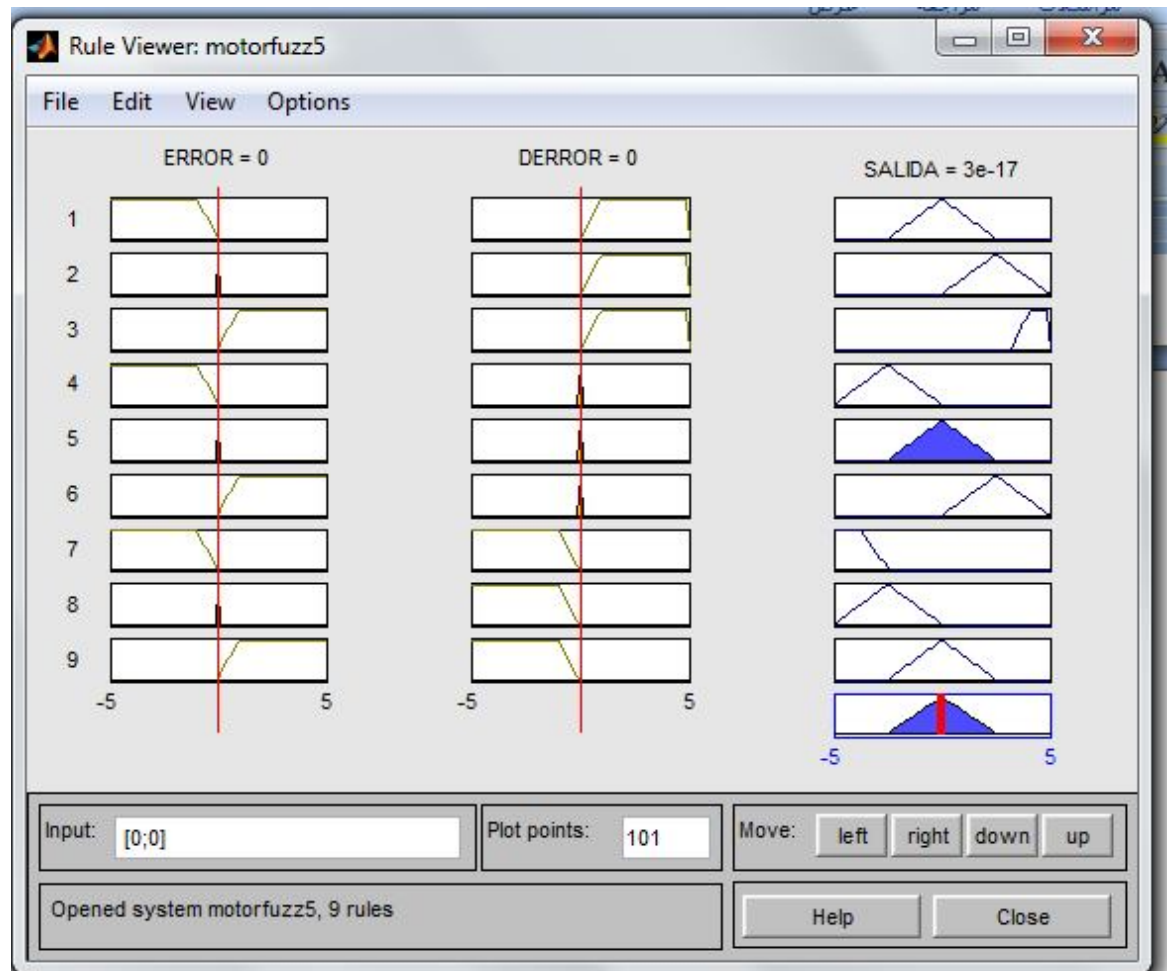


Figure 3.13: Rule viewer

### 3.12.3 Surface viewer

Figure 3.14 shows surface viewer of FLC (3×3).

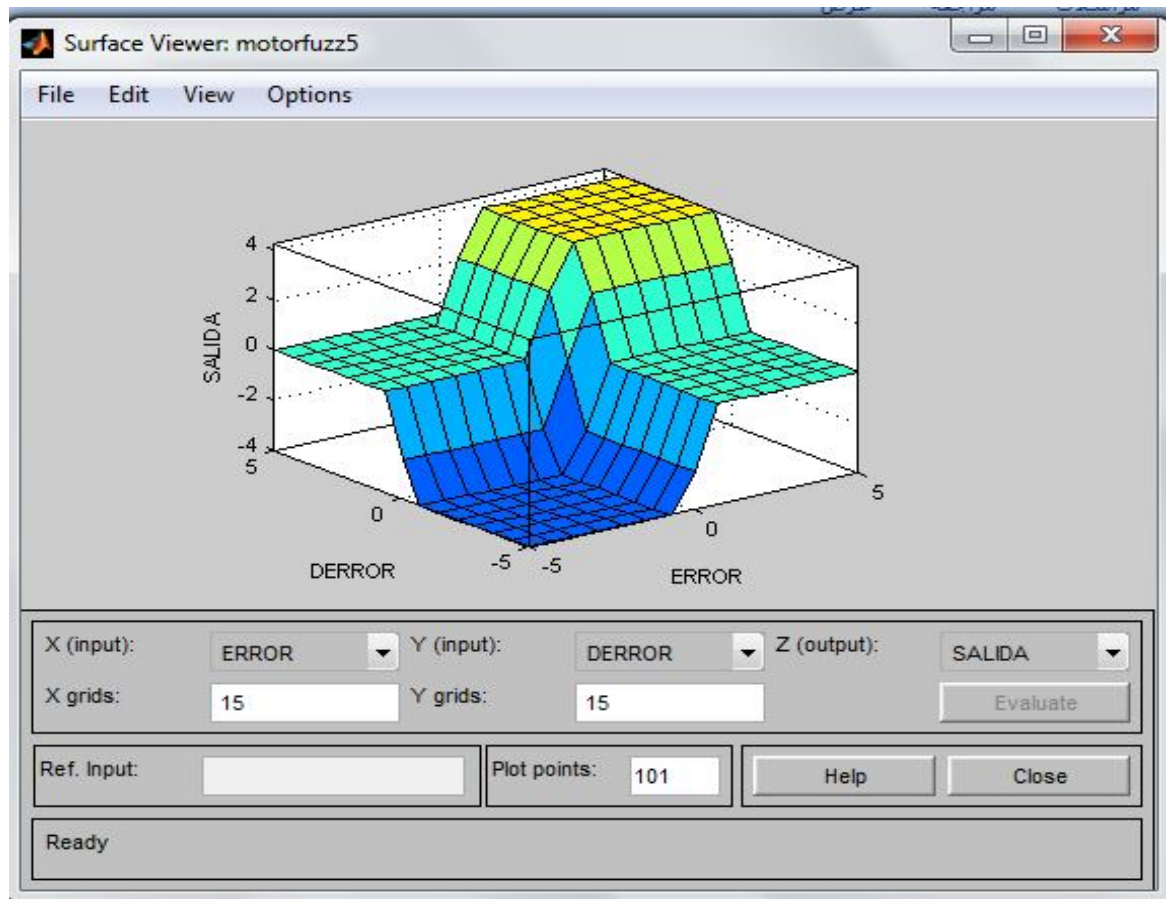


Figure 3.14: Surface viewer

### 3.13 MF Editor of FLC ( $5 \times 5$ )

Figures 3.15, 3.16 and 3.17 show the membership function editor for two inputs and one output respectively.

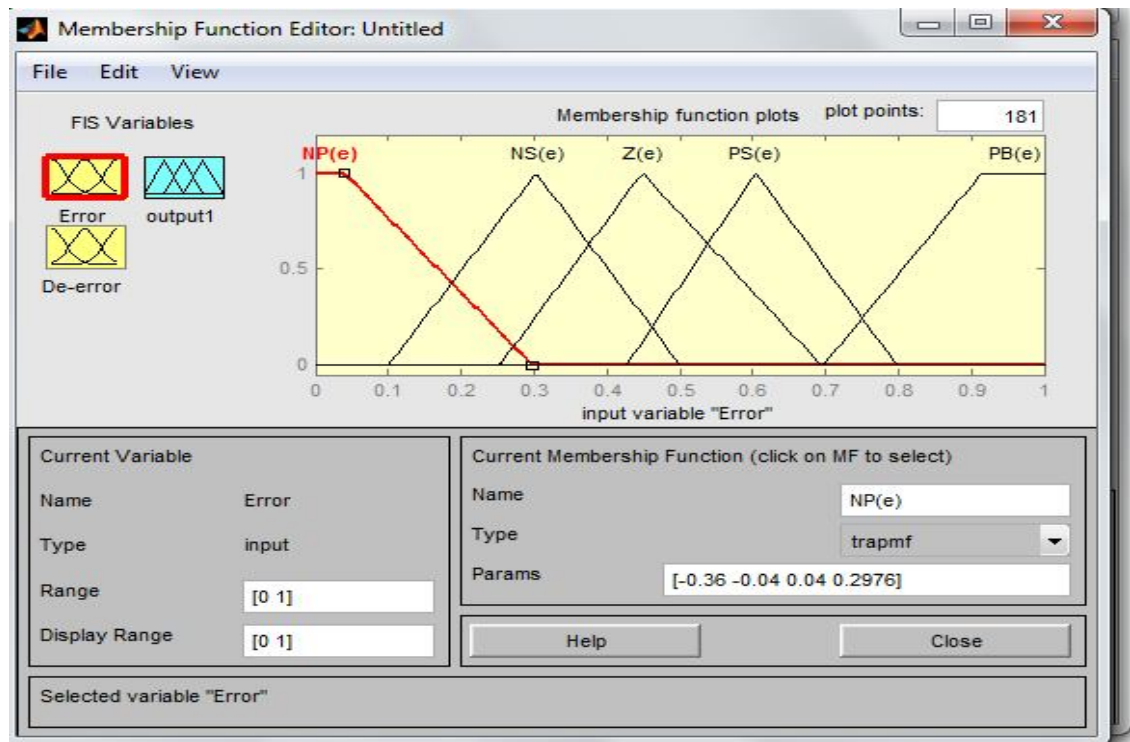


Figure3.15:Input (error)

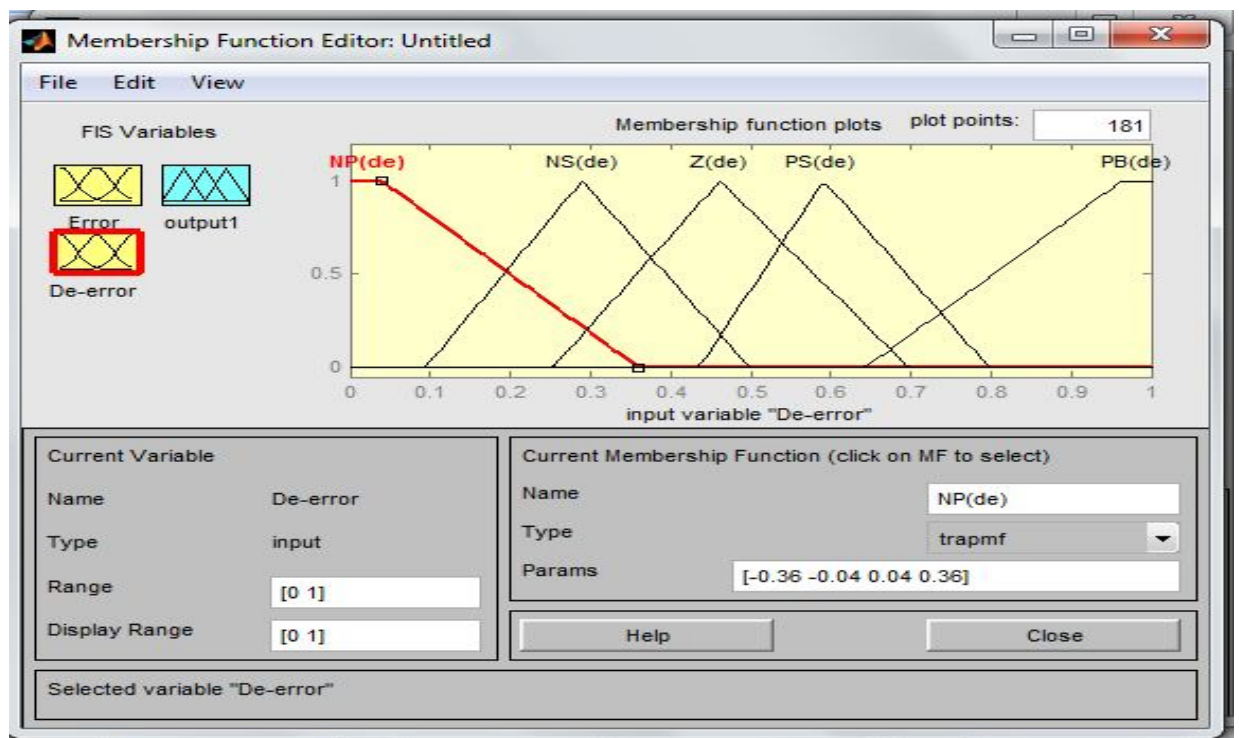


Figure 3.16: Input (DERROR)

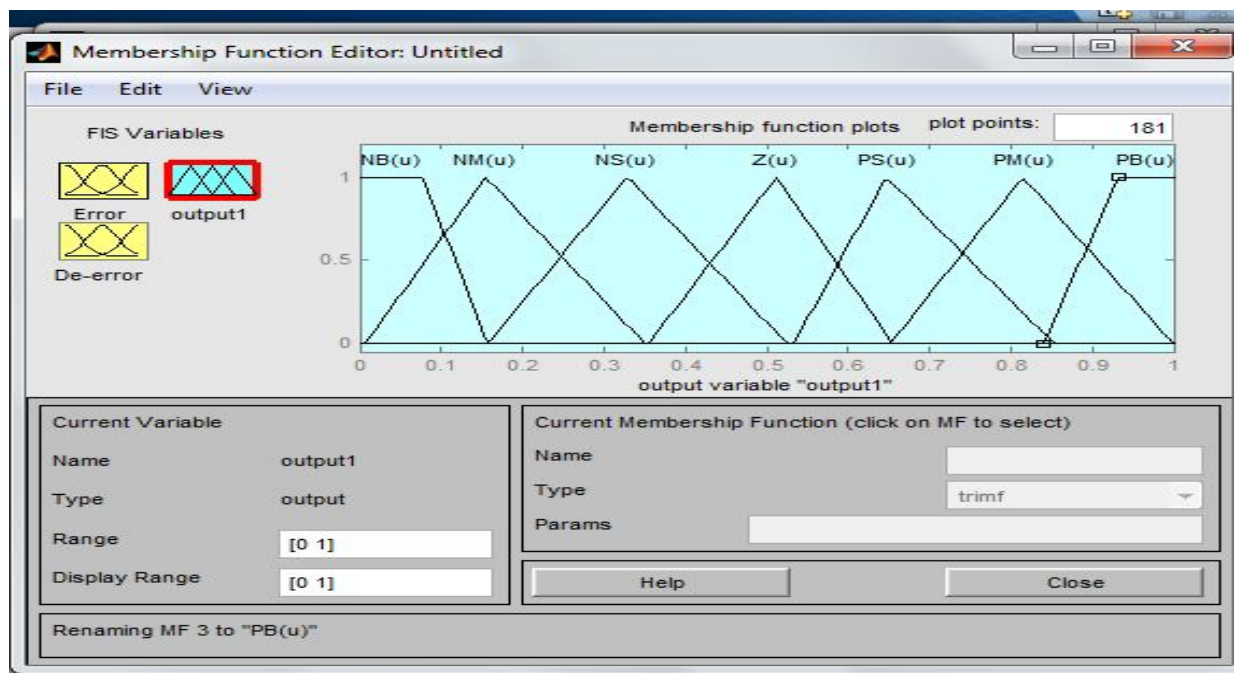


Figure 3.17: Output (SALIDA)

### 3.13.1 Rule editor

Figure 3.18 shows rule editor for FLC (5×5).

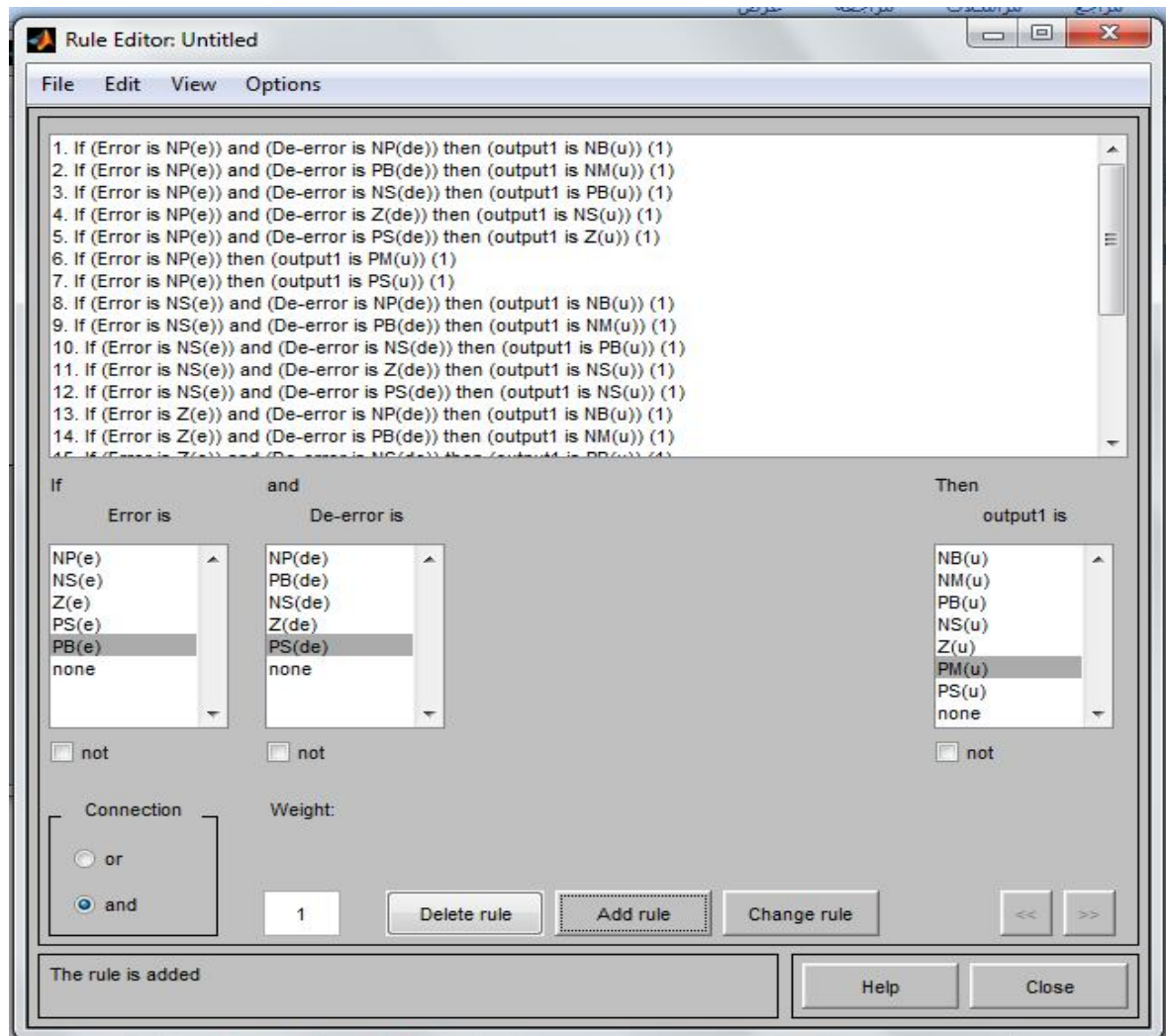


Figure 3.18: Rule editor



### 3.13.2 Rule viewer

Figure 3.19 shows rule viewer for FLC (5×5).

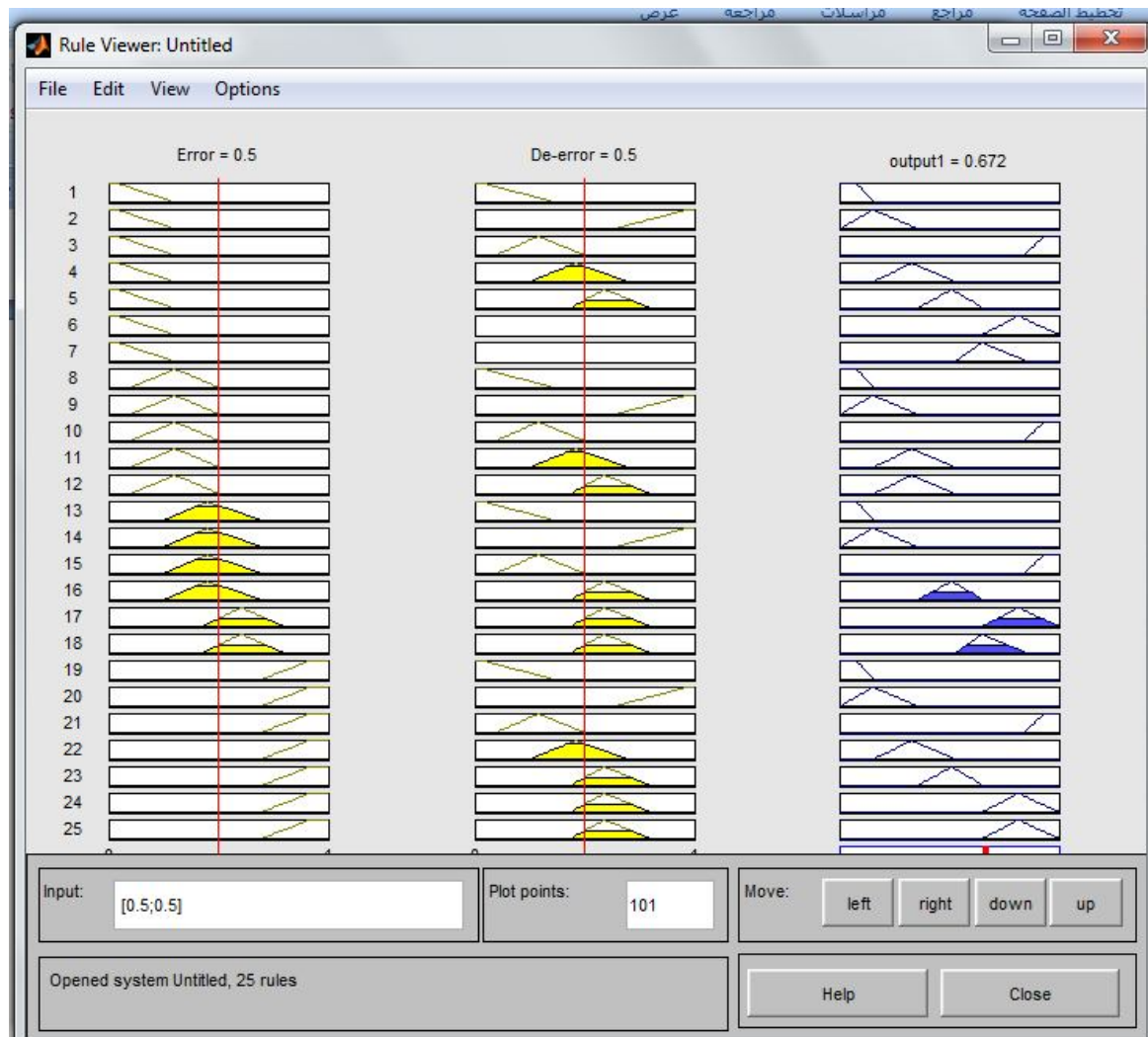


Figure3.19: Surface viewer

### 3.13.3 Surface Viewer

Figure 3.20 shows surface viewer for FLC (5×5).

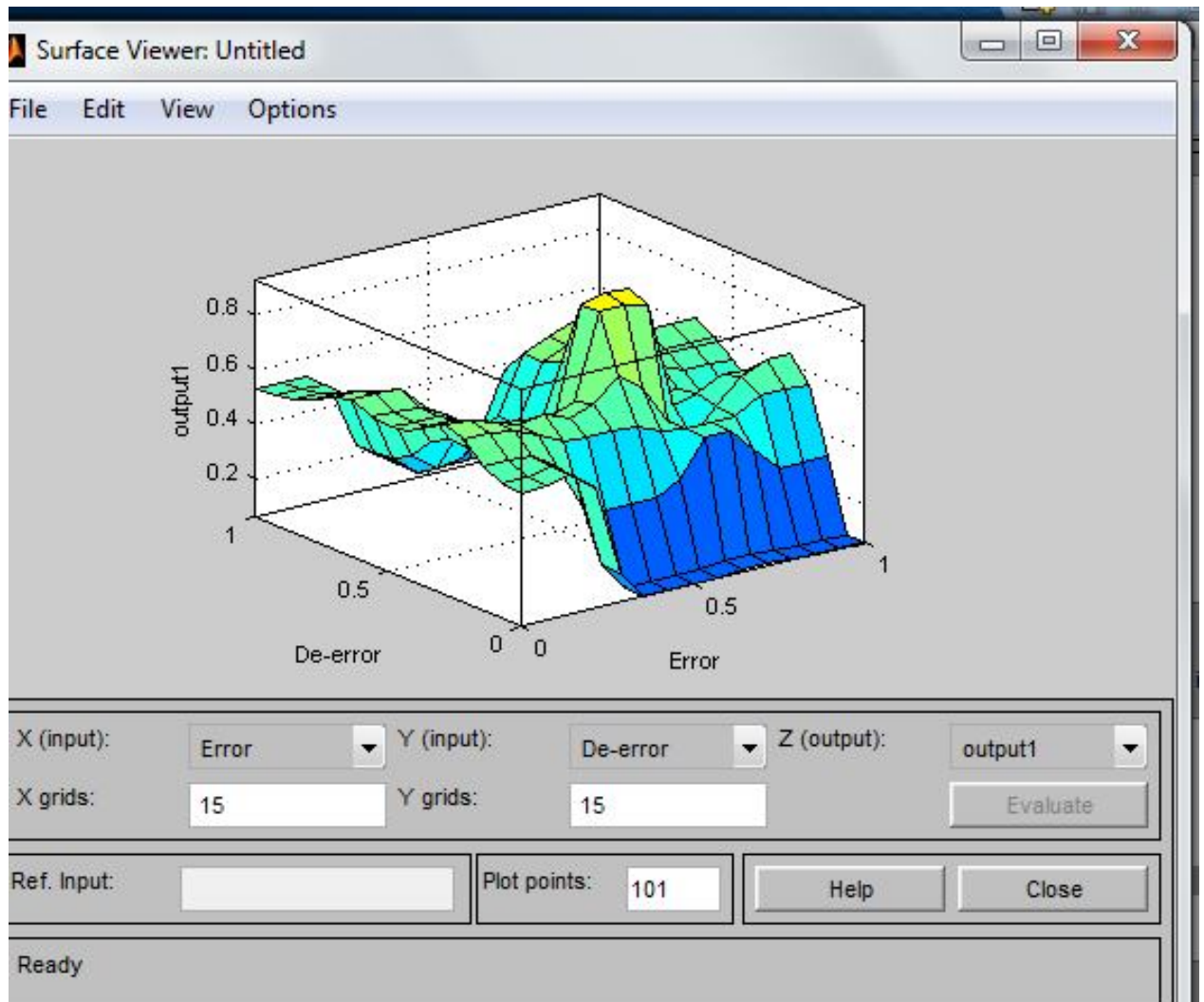


Figure 3.20: Surface viewer



# Chapter Four

## Simulation and Results

### 4.1 Introduction

The MATLAB/SIMULINK model of the servo motor under study with PID controller and FLC is shown in Figure 4.1.

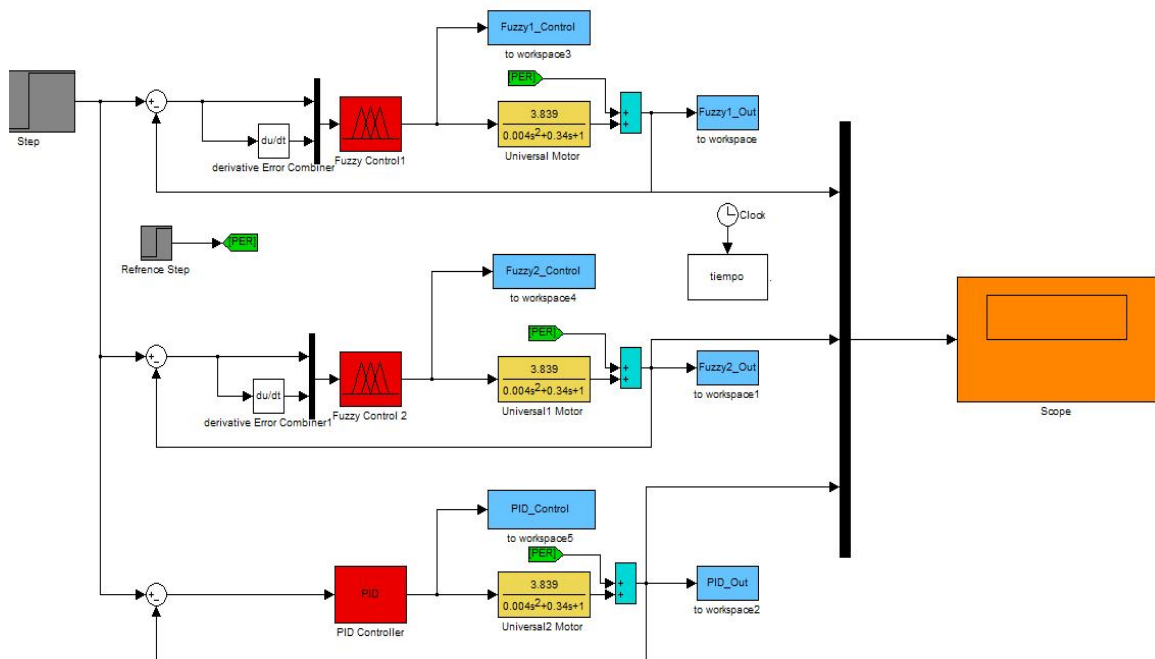


Figure 4.1: MATLAB/SIMULINK model of servo motor using FLC and PID controller.

### 4.2 Simulation of PID Controller

In this section, simulation result is presented to evaluate the effectiveness of the PID controller. Figure 4.2 shows the SIMULINK block diagram of position

control of servo motor using PID controller .The PID controller gain selected as  $K_p= 2$ ,  $K_i= 6.25$  and  $K_d= 0.01$ .Figure 4.3 shows the step response of PID controller.

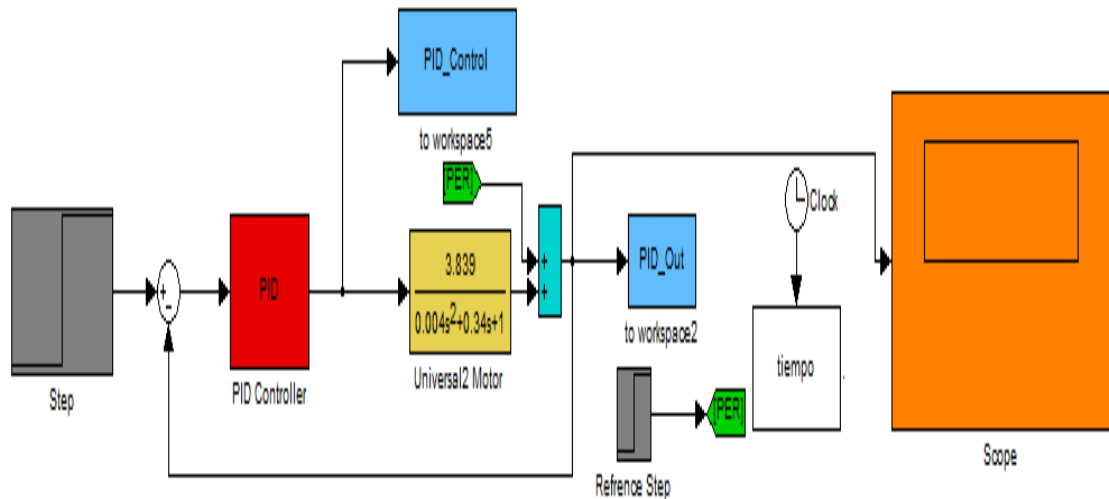


Figure 4.2: The simulink block diagram of PID controller

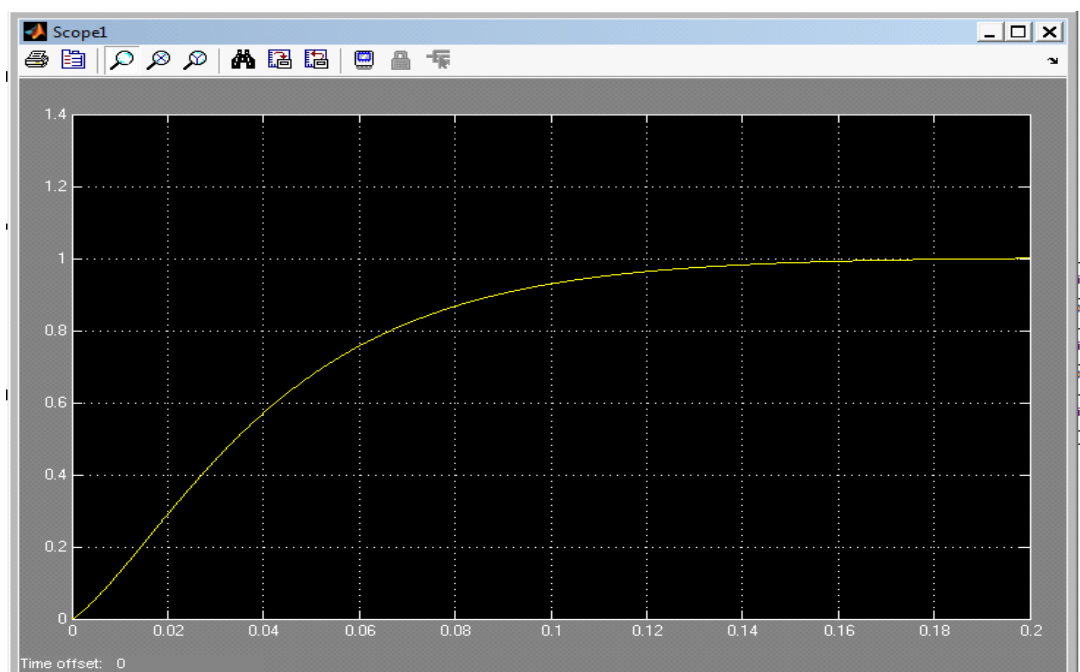


Figure 4.3: The step response of PID controller

### 4.3 Simulation of (3x3) Fuzzy Logic Controller

Figure 4.4 shows the simulink block diagram of (3x3) Fuzzy logic controller. Figure 4.5 shows the step response of (3x3) Fuzzy logic Controller.

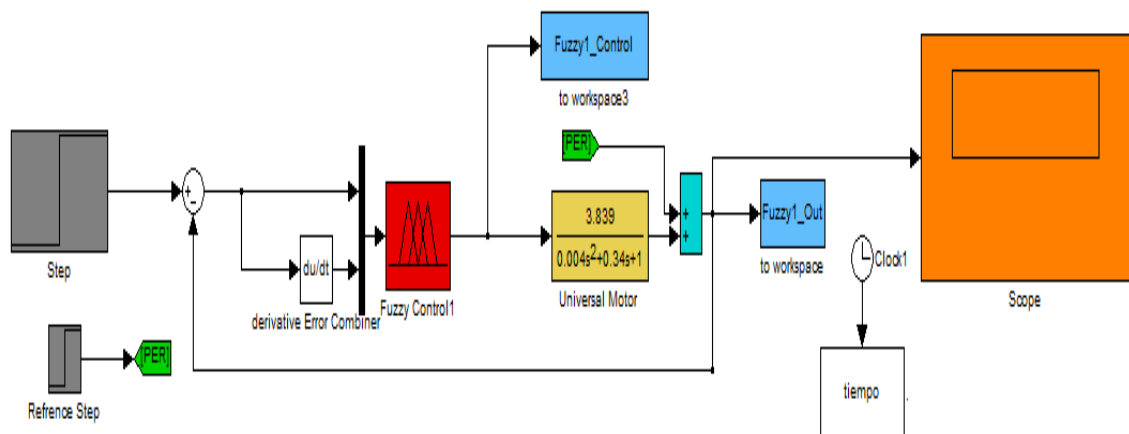


Figure 4.4: The simulink block diagram of (3x3) fuzzy logic controller

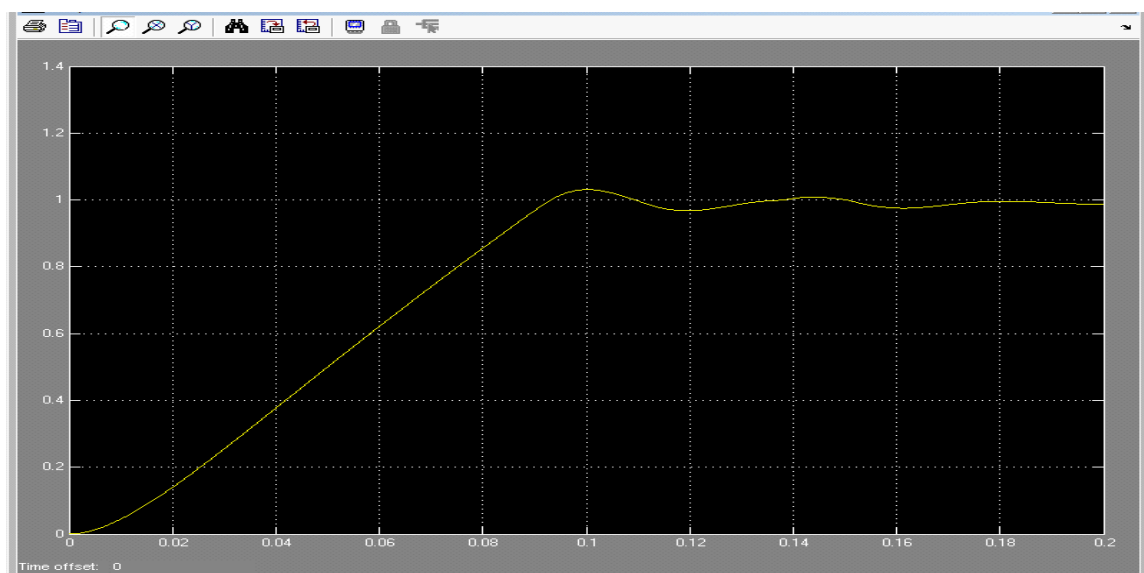


Figure 4.5: The step response of (3x3) fuzzy logic controller

## 4.4 Simulation of (5x5) Fuzzy Logic Controller

Figure 4.6 shows the SIMULINK block diagram of (5x5) fuzzy logic controller. Figure 4.7 shows the step response of (5x5) fuzzy logic controller .

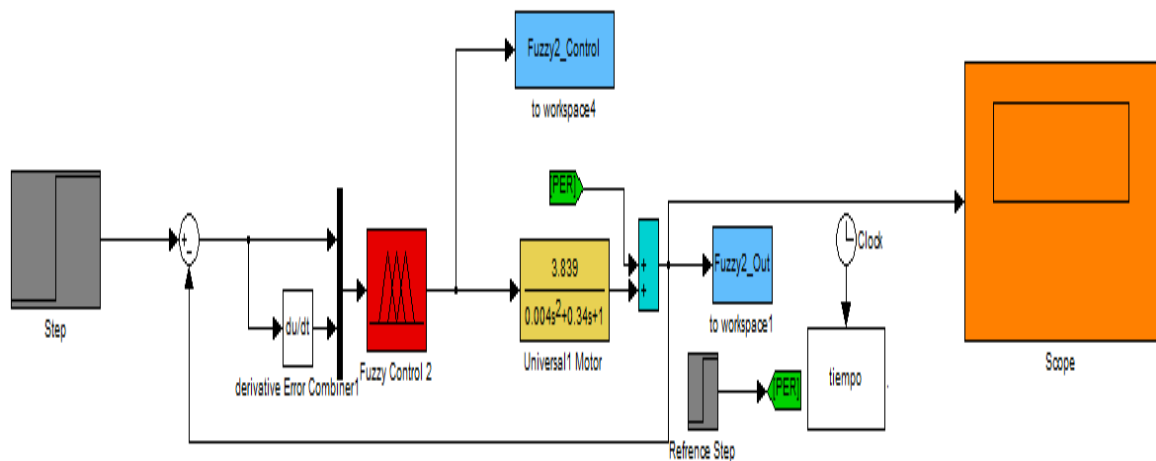


Figure 4.6: The block diagram of (5x5) fuzzy logic controller

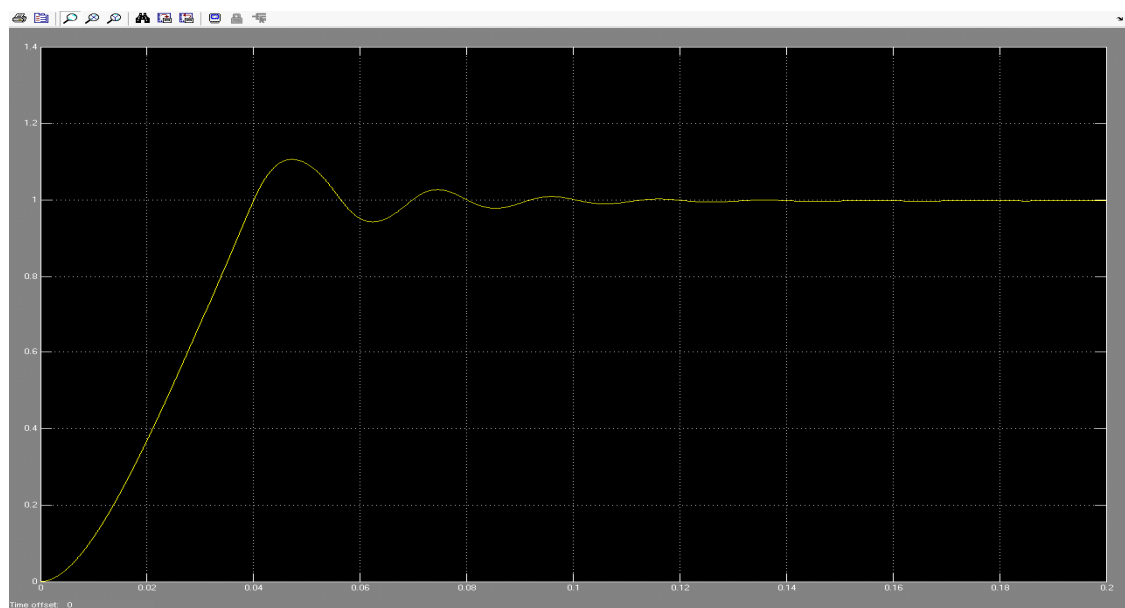


Figure 4.7: The step response of (5x5) fuzzy logic controller

## 4.5 Comparison between two Controllers

Figure 4.8 shows the step response of position control system using two controllers.

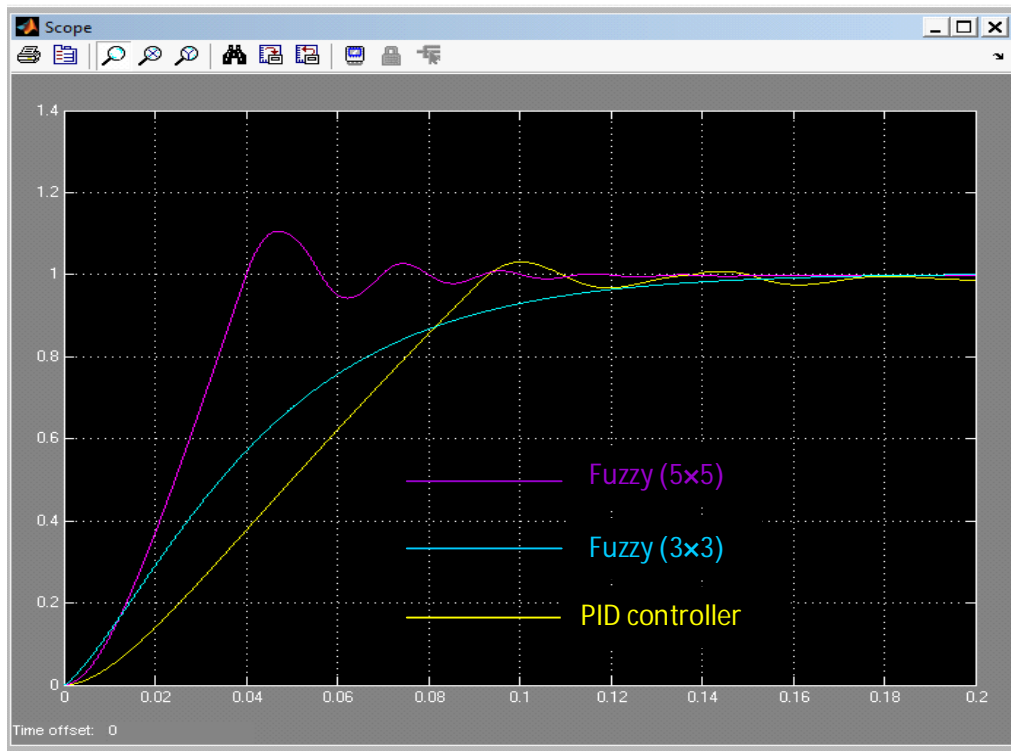


Figure 4.8: The step response comparison between two controllers

# **Chapter Five**

## **Conclusion and Recommendations**

### **5.1 Conclusion**

Fuzzy controllers have the advantage that can deal with nonlinear systems and use the human operator knowledge. Here we tested it with a linear system of second order with known parameters. In order to compare it with one classical controller we simulated the same system controlled by PID. PID controller has only three parameters to adjust. Controlled system shows good results in terms of response time and precision when these parameters are well adjusted.

Fuzzy controller has a lot of parameters. The most important is to make a good choice of rule base and parameters of membership functions. Once a fuzzy controller is given, the whole system can actually be considered as a deterministic system. When the parameters are well chosen, the response of the system has very good time domain characteristics. The fuzzy controlled system is very sensitive to the distribution of membership functions but not to the shape of membership functions. Fuzzy controlled system doesn't have much better characteristics in time domain than PID controlled system, but its advantage is that it can deal with nonlinear systems.

One of the most important problems with fuzzy controller is that the computing time is much more long than for PID, because of the complex operations as fuzzification and particularly defuzzification. Some optimization can be done if the defuzzification method is simplified. It means that it is recommended to avoid center of gravity method.

PID controller cannot be applied with the systems which have a fast change of parameters, because it would require the change of PID constants in the time. It is necessary to further study the possible combination of PID and fuzzy controller. It means that the system can be well controlled by PID which is supervised by a fuzzy system. As a conclusion the advantages of a fuzzy based controller over a PID controller are derived from results. Better control performance, robustness and overall stability can be expected from the fuzzy controller. Fuzzy controllers have better stability, small overshoot, and fast response. From the results the following parameters can be observed. Hence, fuzzy logic controller is introduced for controlling fluid flows.

1) Even though, the PID controller produces the response with lower delay time and rise time compared with fuzzy logic controller, but it offers very high settling time due to the oscillatory behavior in transient period. It has severe oscillations with a very high peak overshoot of 13% which causes the damage in the system performance.

2) The proposed Fuzzy logic controller can effectively eliminate these dangerous oscillations and provides smooth operation in transient period.

Hence, it is concluded that the conventional PID controller could not be used for the control of nonlinear processes like fluid flows. So, the proposed fuzzy logic based controller design can be a preferable choice for this.

## **5.2 Recommendations**

The researcher recommends that the following points should be applied

- Simulate the performance of the fuzzy logic control using closed/open loop control.
- Compare fuzzy logic with neural networks.
- Compare fuzzy with genetic algorithms

- Increase number of parameters to be compared to increase accuracy of testing the performance.



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