Dedication

To my father and mother ,brother and sister, to all my dear family…

I dedicate my effort.

Acknowledgments

Abstract

We show the approximation of ground state eigenvalues and eigenfunctions, with properties of convergence, of Dirichlet Laplacians and structures. The extension property and boundary regularity for the poisson equation of Refenberg-flat domains are considered. We study the non-selfadiont spectral problems for liner pencils of ordinary differential operators with λ- linear boundary conditions and the characterizations of the spectral decomposition of the non self-adjont block operater matrices. We apply the integration of positive constructible functions against Euler characteristic and dimension with the loci of integrability, zero loci, and stability under integration for the preparation of the constructible functions on Euclidean space with Lebesque measure and classes. The quantitative and spectral stability for the first Dirichlet eigenvalue and Neuman Laplacian in Refenberg-flat and rough domains in Euclidean space are shown. We discuss the class of non-selfadjoint quadratic matrix operator pencils arising in elasticity theory and the spectral sharp bounds and basis results for non selfadjoint pencils with an application to Hagen-poiseuille flow.

الخـلاصــة

أوضحنا تقريب حالة الثبات للقيم الذاتية والدوال الذاتية مع خواص التقارب للابلسيان وبناءات درشلت . اعتبرنا خاصية التمديد وانتظامية الحدودية لمعادلة بواسون لمجالات سطح ريفنبيرج . تمت دراسة مسائل الطيف غير المرافق الذاتي للأحزمة الخطية للمؤثرات التفاضلية العادية مع شروط الحدودية الخطية ـ λ والتشخيصات للتفكيك الطيفي لمصفوفات مؤثر المربعات غير المرافق الذاتي. طبقنا تكامل دوال البناء الموجبة المقابلة لتشخيص اويلر والبعد طبقا للبؤر التكاملية وبؤر الصفر والاستقرار تحت التكامل لتجهيز دوال البناء على الفضاء الاقليدى مع قياس لبيق وعائلاته. أوضحنا الاستقرار الكيفي والطيفي لأجل القيم الذاتية الاولى لدرشلت ولابلسيان نيومان في مجالات سطح ريفنبيرج والمجالات الخشنة في الفضاء الاقليدي. تمت دراسة العائلة لأحزمة مؤثر مصفوفة الدرجة الثانية غير المرافقة الذاتية الناتجة من نظرية المرونة والحديات القاطعة الطيفية والنتائج الاساسية لأجل أحزمة غير المرافق الذاتية مع التطبيق لانسياب هاجن- بواسيللي.

The Contents

Introduction

We study some approximations of eigenvalues and eigenfunctions. In univariate settings, we show a strong reinforcement of the energy image density criterion for local Dirichlet forms admitting square field operators. This criterion enables us to redemonstrate classical results of Dirichlet forms theory. Besides, when $X = (X_1, \ldots, X_p)$ belongs to the $\mathbb{D}$ domain of the Dirichlet form, and when its square field operator matrix $\Gamma[X; {}^t X]$ is almost surely definite, we show that $\mathcal{L}_X$ is Rajchman.

We provide a detailed proof of the fact that any domain which is sufficiently flat in the sense of Reifenberg is also Jones-flat, and hence it is an extension domain. We discuss various applications of this property, in particular we obtain L^∞ estimates for the eigen functions of the Laplace operator with Neumann boundary conditions. We devote to the investigation of the boundary regularity for the Poisson equation

$$\begin{cases} -\Delta u = f & \text{in} \quad \Omega \\ \quad u = 0 & \text{on} \ \partial\Omega \end{cases}$$

where f belongs to some $L^p(\Omega)$ and Ω is a Reifenberg-flat domain of $\mathbb{R}^n$.

We study nonselfadjoint spectral problems for ordinary differential equations $N(y) - \lambda P(y) = 0$ with λ-linear boundary conditions where the order p of the differential operator P is less than the order n of N. We address the question of the completeness of the eigen functions and associated functions in the Sobolev spaces $W_2^k(0,1)$ for $k = 0, 1, \ldots, n$. To this end we associate a pencil $\mathbb{K} - \lambda \mathbb{H}$ of operators acting from $L_2(0,1)$ to the larger space $L_2(0,1) \times \mathbb{C}^n$ with the given problem. We study spectral properties of a block operator matrix $\tilde{A}$, where A and $-D$ are m-accretive, and B, D are bounded operators.

We develop a direct image formalism for positive constructible functions in the globally subanalytic context. This formalism is generalized to arbitrary first-order logic models and is illustrated by several examples on the p-adics, on the Presburger structure and on o-minimal expansions of groups. We introduce and study loci of integrability. We show a correspondence between zero loci and loci of integrability for constructible functions on Euclidean space, where a function is called constructible if it is a sum of products of globally subanalytic functions and of logarithms of globally subanalytic functions. We call a function constructible if it has a globally subanalytic domain and can be expressed as a sum of products of globally subanalytic functions and logarithms of positively-valued globally subanalytic functions. For any $q > 0$ and constructible functions f and μ on $E \times \mathbb{R}^n$, We also show a closely related preparation theorem for f and μ. These results relate analysis (the study of L^p-spaces) with geometry (the study of zero loci).

We show that if Ω and Ω' are close enough for the complementary Hausdorff distance and their boundaries satisfy some geometrical and topological conditions then

$$|\lambda_1 - \lambda'_1| \leq C |\Omega \, \Delta \, \Omega'|^{\frac{\alpha}{N}}$$

where λ_1 ($resp. \lambda'_1$) is the first Dirichlet eigenvalue of the Laplacian in Ω ($resp. \Omega'$) and $|\Omega \, \Delta \, \Omega'|$ is the Lebesgue measure of the symmetric difference. We establish new quantitative stability estimates with respect to domain perturbations for all the eigenvalues of both the Neumann and the Dirichlet Laplacian. The main results follow from an abstract lemma stating that it is actually sufficient to provide an estimate on suitable projection operators. Whereas this lemma could be applied under different regularity assumptions on the domain, here we use it to estimate the spectrum in Lipschitz and in so-called Reifenberg flat domains.

We deal with a class of nonselfadjoint quadratic pencils of block operator matrices. Finally, an application to the problem of vibrations of a rotating beam is given. We obtain eigenvalue enclosures and basisness results for eigen- and associated functions of a non-selfadjoint unbounded linear operator pencil $\mathcal{A} - \lambda \mathcal{B}$ in which $\mathcal{B}$ is uniformly positive and the essential spectrum of the pencil is empty.