

CHAPTER ONE

INTRODUCTION

1.1 Background

Stability of power system is the core of power system security protection which is one of the most important problems researched by electrical engineers, and has been and continues to be of major concern in system operation. This arises from the fact that in steady state (under normal conditions) the average electrical speed of all the generators must remain the same anywhere in the system. This is termed as the synchronous operation of a system. Any disturbance small or large can affect the synchronous operation. For example, there can be a sudden increase in the load or loss of generation. Another type of disturbance is the switching out of a transmission line, which may occur due to overloading or a fault. The stability of a system determines whether the system can settle down to a new or original steady state after the transients disappear [1].

The first step in a stability study is the derivation of a mathematical model for the system during the transient. The elements included in the model are those affecting the acceleration (or deceleration) of the machine rotors. Generally, the components of the power system that influence the electrical and mechanical torques of the machines should be included in the model. Thus the basic ingredients for solution are the knowledge of the initial conditions of the power system prior to the start of the transient and the mathematical description of the main components of the system that affect the transient behavior of the synchronous machines. The number of power system components included in the study and the complexity of their mathematical description will depend upon many factors. In general, however, differential

Equations are used to describe the various components. Study of the dynamic behavior of the system depends upon the nature of these differential Equations. If the system Equations are linear (or have been linearized), the techniques of linear system analysis are used to study dynamic behavior. The most common method is to simulate each component by its transfer function. The various transfer function blocks are connected to represent the system under study. The system performance may then be analyzed by such methods as root-locus plots. Frequency domain analysis (Nyquist criteria), and Routh's criterion. The above methods have been frequently used in studies pertaining to small systems or a small number of machines. For larger systems the state-space model has been used more frequently in connection with system studies described by linear differential equations. Usually time solutions of the nonlinear differential equations are obtained by numerical methods with the aid of digital computers, and this is the method usually used in power system stability studies [2].

Low frequency oscillations in range of (0.1- 0.8) Hz are observed when large power systems are interconnected by weak tie lines. These oscillations may sustain and grow, causing system separation if no adequate damping is available. Moreover, low frequency oscillations present limitations on the power transfer capability [3].

The voltage regulator introduces a damping torque component but it observed w that under heavy loading conditions the voltage regulator give negative damping. To offset this effect and to improve the system damping in general, “power system stabilizer” (PSS) has been used [3]. Power system stabilizers (PSSs) are now routinely used in the industry to damp out oscillations. An appropriate selection of PSS parameters results in satisfactory performance during system disturbances [4].

1.2 Project Objectives

- Analyze the response of multi-machines system when subject to small and large disturbance.
- Design power system stabilizer (PSS) controller.
- Study the influence of PSS in power system stability.

1.3 Statement of Problem

The stability problem is concerned with the behavior of the synchronous machines after they have been perturbed. If the perturbation does not involve any net change in power, the machines should return to their original state. If an unbalance between the supply and demand is created by a change in load, in generation, or in network conditions, a new operating state is necessary. In any case all interconnected synchronous machines should remain in synchronism if the system is stable; i.e., they should all remain operating in parallel and at the same speed.

One of major problem in power system operation is related to small signal instability caused by insufficient damping in the system the AVR view of the system usually increases the value of synchronous torque but gives negative damping. PSS or the power system stabilizer damping out low frequency oscillations in the power system and contributes to the electro-mechanical torque components without effecting in the synchronizing torque in the system; so it is suitable for the small signal analysis modes.

1.4 Methodology

First, differential Equations describing the power system, state space model were derived devolved and linearized. Eignvalues has been computed from state matrix (A).

The frequency modes have been determined from this eigenvalues . The low frequency mode(inter area mode) with low damping ratio identified. Finally the PSS has been added in optimal location to increase the damping ratio which return the system in stability condition.

1.5 Thesis Lay-out

- **Chapter One:** This chapter represents a general introduction to the power system stability, project objectives and statement of problem.
- **Chapter Two:** Includes the definition, classification of stability and modeling of power system.
- **Chapter Three:** Linearization of multi-machines system and power system stabilizer (PSS) design.
- **Chapter Four:** Represent the results of the eigenvalue analysis and time domain simulation of small signal and transient stability. The case study (4-machines, 10-busbars).
- **Chapter Five:** Represent the project conclusion and recommendations.

CHAPTER TWO

DEFINITION OF STABILITY AND POWER SYSTEM MODELING

In this chapter stability of power system has been defined and classified into rotor angle stability, frequency stability and voltage stability. After that all of power system component have been modeled.

2.1 Definition of Stability

Stability is known as the tendency of the power system to develop restoring forces equal to or greater than the disturbing forces to maintain the state of equilibrium [5].

2.2 Classification of Power System Stability

The understanding of stability problems is greatly facilitated by the classification of stability into various categories. While classification of power system is an effective and convenient means to deal with the complexities of the problem, the overall stability of the system should always be kept in mind. Solution to stability problems of one category should not be at the expense of another [3].

Figure 2.1 gives an overall picture of the power system stability problem

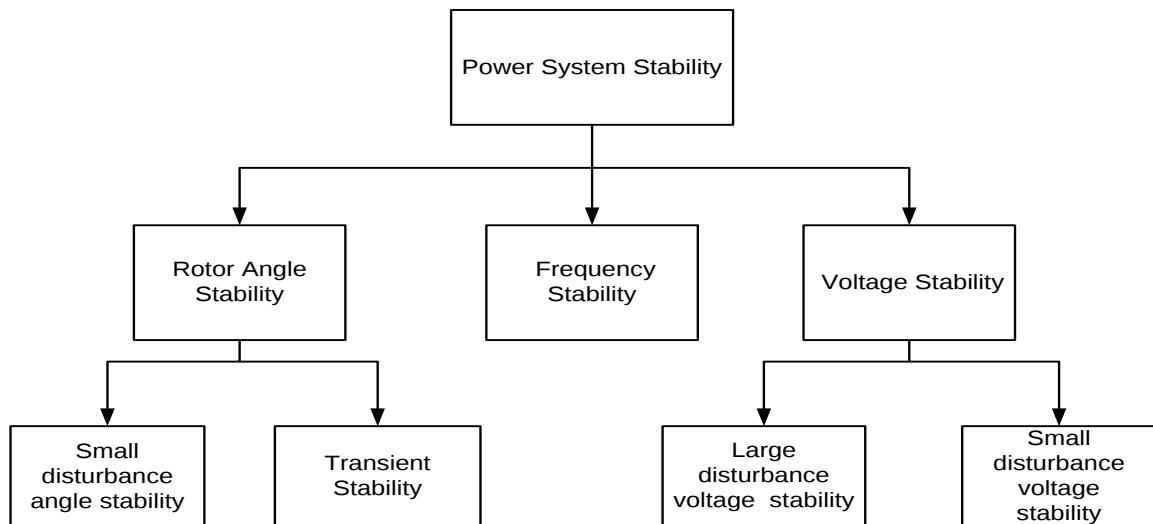


Figure 2.1 Classification of Power System Stability

2.3 Rotor Angle Stability

Rotor angle stability is the ability of interconnected synchronous machines of a power system to remain in synchronism. The stability problem involves the study of the electromechanical oscillation inherent in power systems.

The rotor angle stability phenomena in terms of the following two types:

2.3.1 Small Signal (or Small-Disturbance)

Small signal (or small-disturbance) stability is the ability of the power system to maintain synchronism under small disturbances; it is caused by small variations in loads and generation [5].

2.3.2 Transient Stability

Transient stability is the ability of the power system to maintain synchronism when subjected to a severe transient disturbance. Disturbances of widely varying degrees of severity and probability of occurrence can occur on the system. The system is, however, designed and operated so as to be stable for a selected set of contingencies [5].

2.4 Types of Oscillations

In today's practical power system, small signal stability is largely a problem of insufficient damping of oscillations. The stability of the following types of oscillations is of concern:

- Local modes are machine-system modes associated with the swinging of units at a generating station with respect to the rest of the power system. The term local is used because the oscillations are localized at one station or a small part of the power system, Frequency Range is: 0.7 to 2 Hz.
- Inter-area modes are associated with the swinging of many machines in one part of the system against machines in other parts. They are caused by two or more groups of closely coupled machines being interconnected by weak ties, Frequency Range is: 0.1 to 0.7 Hz.
- Control modes are associated with generating units and other controls. Poorly tuned exciter, speed governors, HVDC converters and static VAR-compensator are the usual cause of instability of these modes.
- Torsional modes are associated with the turbine-generator shaft system rotational components. Instability of torsional modes may be caused by interaction with excitation controls, speed governors, HVDC controls, and series-capacitor-compensated lines, with frequencies ranging from 4 Hz [1].

2.5 Power System Modeling

In this section all of the power system component which uses in this project were modeled. Figure 2.2 represent the power components.

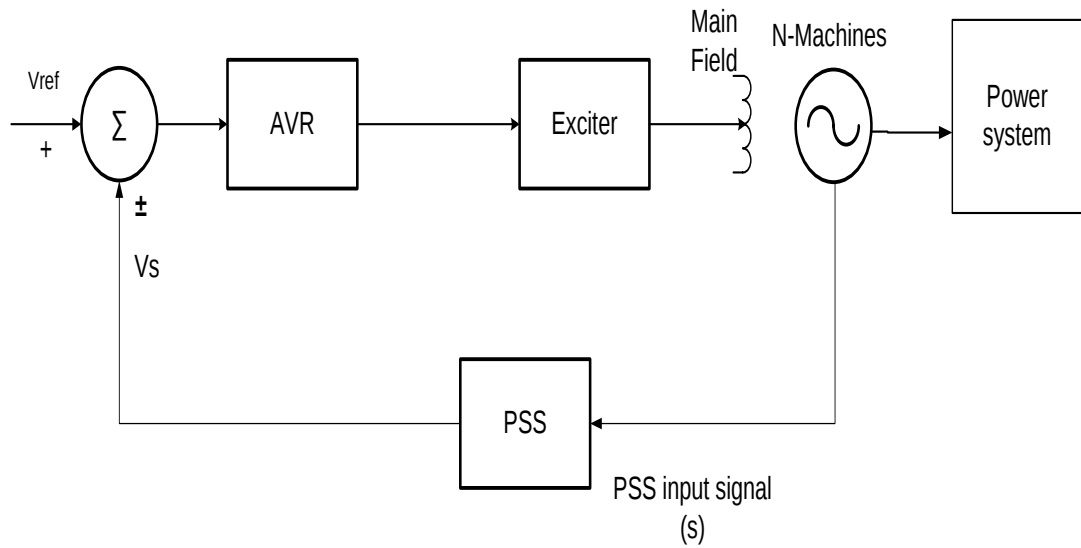


Figure 2.2 Power System Components

2.5.1 Synchronous Machine Modeling

Mathematical model of a synchronous machine was developed to use in stability computations, figure 2.3 illustrates a synchronous machine with salient pole [6].

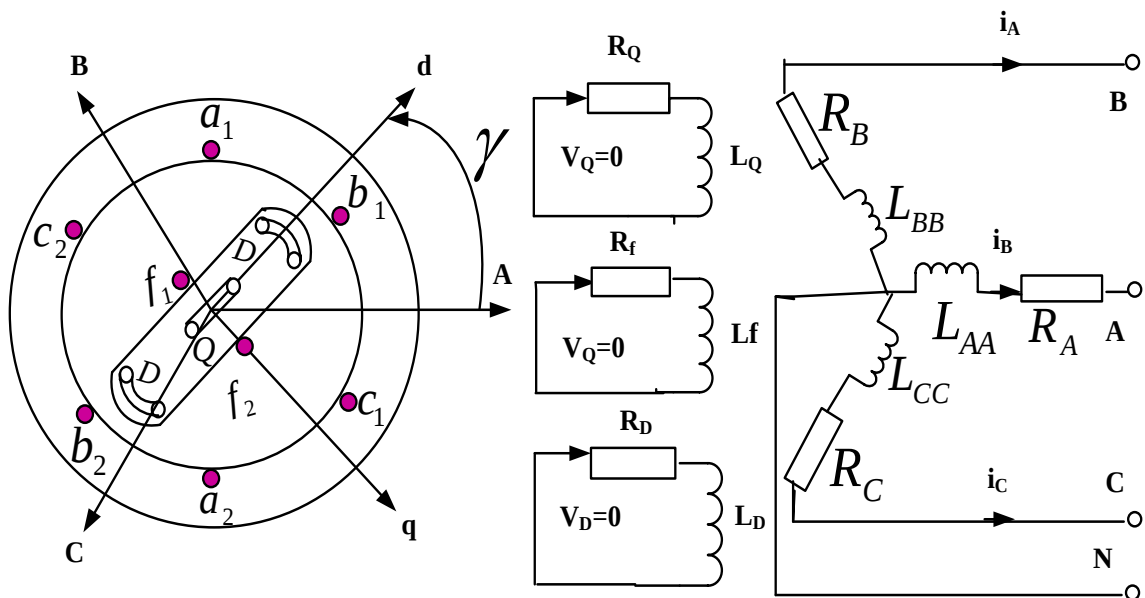


Figure 2.3: The windings in the synchronous generator and their axes.

Referring to figure (2.3) the following windings are depicted:

- The three stator windings denote a, b, and c.
- Field winding denoted F. This winding carries the field current, which gives rise to the field flux. This rotating flux induces the voltages in the stator windings.
- Short circuited damper winding in the d-axis denoted by D.
- Short circuited damper winding in the q-axis denoted by Q.

The basic voltage Equations which descript the machine in a-b-c reference system are:

Stator Voltage Equations

$$v_{ai} = i_{ai}r_{si} + \frac{d}{dt} \psi_{ai} \quad i= 1, \dots n \quad (2.1)$$

$$v_{bi} = i_{bi}r_{si} + \frac{d}{dt} \psi_{bi} \quad i= 1, \dots n \quad (2.2)$$

$$v_{ci} = i_{ci}r_{si} + \frac{d}{dt} \psi_{ci} \quad i= 1, \dots n \quad (2.3)$$

Rotor voltage equation

$$v_{fdi} = i_{fdi}r_{fdi} + \frac{d}{dt} \psi_{fdi} \quad i= 1, \dots n \quad (2.4)$$

Where ψ is flux linkage, v , i and r is winding voltage , current and resistance in the a-b-c reference system respectively.

Park's Transformation

It is convenient to transform all synchronous machine stator and network variable into a reference frame that converts balance three-phase sinusoidal variations into constant. Such a transformation is

$$T_{dqos} = \frac{2}{3} \begin{bmatrix} \cos \omega_s t & \cos(\omega_s t - \frac{2\pi}{3}) & \cos(\omega_s t + \frac{2\pi}{3}) \\ -\sin \omega_s t & -\sin(\omega_s t - \frac{2\pi}{3}) & -\sin(\omega_s t + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (2.5)$$

$$T_{dqos}^{-1} = \begin{bmatrix} \cos \omega_s t & -\sin \omega_s t & 1 \\ \cos(\omega_s t - \frac{2\pi}{3}) & -\sin(\omega_s t - \frac{2\pi}{3}) & 1 \\ \cos(\omega_s t + \frac{2\pi}{3}) & -\sin(\omega_s t + \frac{2\pi}{3}) & 1 \end{bmatrix} \quad (2.6)$$

The machine stator voltage, current, and flux linkage in the synchronously rotating reference frame are

$$\begin{aligned} \begin{bmatrix} V_{Di} \\ V_{Qi} \\ V_{oi} \end{bmatrix} &= T_{dqos} T_{dqos}^{-1} \begin{bmatrix} V_{di} \\ V_{qi} \\ V_{oi} \end{bmatrix} = T_{dqos} \frac{V_{BABCi}}{V_{BDQi}} \begin{bmatrix} V_{ai} \\ V_{bi} \\ V_{ci} \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} T_{dqos} \begin{bmatrix} V_{ai} \\ V_{bi} \\ V_{ci} \end{bmatrix} \end{aligned} \quad i=1, \dots, n \quad (2.7)$$

$$\begin{aligned} \begin{bmatrix} I_{Di} \\ I_{Qi} \\ I_{oi} \end{bmatrix} &= T_{dqos} T_{dqos}^{-1} \begin{bmatrix} I_{di} \\ I_{qi} \\ I_{oi} \end{bmatrix} = T_{dqos} \frac{I_{BABCi}}{I_{BDQi}} \begin{bmatrix} I_{ai} \\ I_{bi} \\ I_{ci} \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} T_{dqos} \begin{bmatrix} I_{ai} \\ I_{bi} \\ I_{ci} \end{bmatrix} \end{aligned} \quad i=1, \dots, n \quad (2.8)$$

$$\begin{aligned} \begin{bmatrix} \psi_{Di} \\ \psi_{Qi} \\ \psi_{oi} \end{bmatrix} &= T_{dqos} T_{dqos}^{-1} \begin{bmatrix} \psi_{di} \\ \psi_{qi} \\ \psi_{oi} \end{bmatrix} = T_{dqos} \frac{\psi_{BABCi}}{\psi_{BDQi}} \begin{bmatrix} \psi_{ai} \\ \psi_{bi} \\ \psi_{ci} \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} T_{dqos} \begin{bmatrix} \psi_{ai} \\ \psi_{bi} \\ \psi_{ci} \end{bmatrix} \end{aligned} \quad i=1, \dots, n \quad (2.9)$$

Where T_{dqoi} is the machine i transformation [7].

Stator Voltage Equations

$$\varepsilon \frac{d}{dt} \psi_{Di} = i_{Di} r_{si} + \psi_{Qi} + v_{Di} \quad i=1, \dots, n \quad (2.10)$$

$$\varepsilon \frac{d}{dt} \psi_{Qi} = i_{Qi} r_{si} - \psi_{Di} + v_{Qi} \quad i=1, \dots, n \quad (2.11)$$

$$\varepsilon \frac{d}{dt} \psi_{Oi} = i_{Oi} r_{si} + v_{Oi} \quad i=1, \dots, n \quad (2.12)$$

Where v_{Di} , v_{Qi} , v_{Oi} , i_{Di} , i_{Qi} , i_{Oi} , ψ_{Di} , ψ_{Qi} and ψ_{Oi} are the stator voltage, current, and flux linkage in the dq0 reference system. ω_i is the rotor speed and r_{si} is the stator resistance and $\varepsilon = 1/\omega_s$.

Rotor Voltage Equation

$$v_{fdi} = i_{fdi} r_{fdi} + \frac{d}{dt} \psi_{fdi} \quad i=1, \dots, n \quad (2.13)$$

Where v_{fdi} , i_{fdi} , r_{fdi} and ψ_{fdi} are the field voltage, current, resistance and flux components in the dq0 reference system respectively. ω_i is the rotor speed.

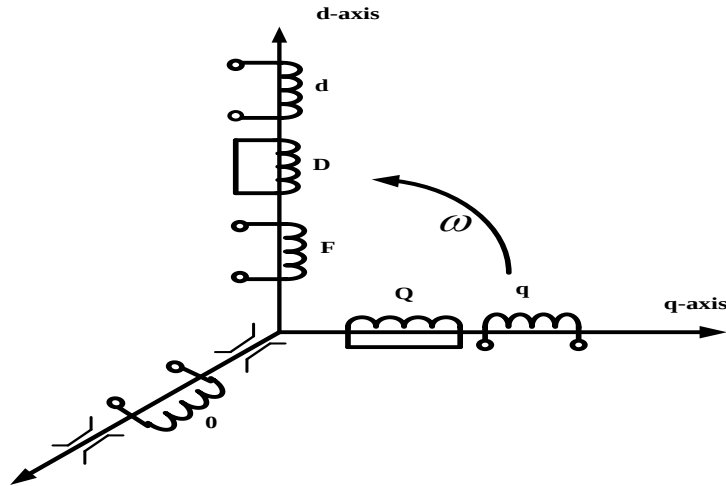


Figure 2.4: Three sets of fictitious perpendicular windings representing the synchronous generator.

Equation of motion

The mechanical dynamics in the rotor are represented by the following equations:

$$\dot{\delta}_i = \omega_i - \omega_s \quad i=1, \dots, n \quad (2.14)$$

$$\dot{\omega}_i = (\omega_s / 2H_i) * (T_{mi} - T_{ei} - T_{fwi}) \quad i=1, \dots, n \quad (2.15)$$

Where

H is inertia time constant in seconds.

T_m is the mechanical torque.

T_e is the electrical torque.

T_{fw} is the damping torque coefficient given in per unit.

δ is the angular position of the rotor.

ω_s is the synchronous speed.

ω is the rotor speed.

2.5.2 Transformer Modeling

The transformers are generally used as inter-connecting (IC) transformers and generator transformers. These transformers are usually with off-nominal-turns-ratio and are modeled as equivalent π circuit [8].

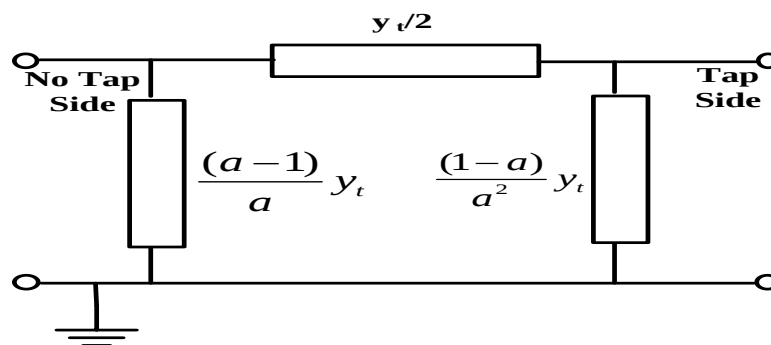


Figure 2.5: The transformer as equivalent π circuit

Where,

$$y_t = \frac{1}{z_t}$$

z_t : represents the series impedance at nominal – turns – ratio.

a : represents per unit off-nominal tap position.

The transmission network is represented by an algebraic equation given by

$$Y_{BUS}\bar{V} = \bar{I}$$

Where,

Y_{BUS} : Bus admittance matrix.

$\bar{V}$: Vector of bus voltages.

$\bar{I}$: Vector of injected bus currents.

The above equation is obtained by writing the network equations in the node-frame of reference taking ground as the reference.

2.5.3 Transmission Line Modeling

Transmission Lines are modeled as a nominal π circuit [8] as shown in Figure (2.6).

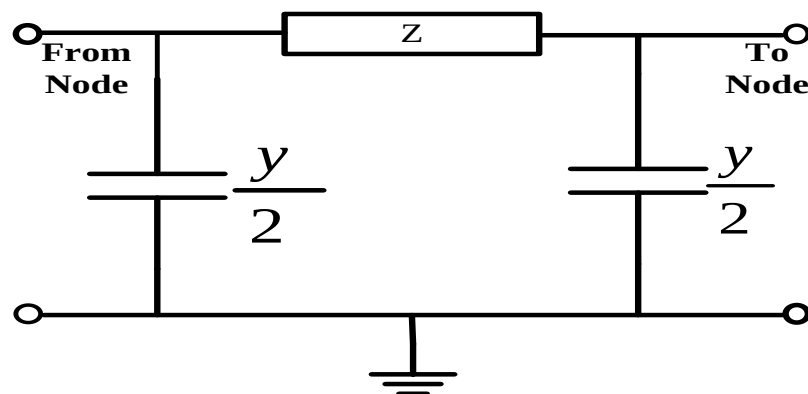


Figure 2.6: Transmission Lines as a nominal π circuit.

Z : represents the series impedance of the line.

$\frac{y}{2}$: represents half of the total line charging y , at each node.

2.5.4 Excitation System Modeling

The main objective of the excitation system is to control the field current of the synchronous machine. The field current is controlled so as to regulate the terminal voltage of the machine.

There are three distinct types of excitation systems based on the power source for exciter:

- 1) DC Excitation Systems:** (DC) which utilize a DC generator with commutator.
- 2) AC Excitation Systems:** (AC) which use alternators and either stationary or rotating rectifiers to produce the direct current needed.
- 3) Static Excitation Systems :**(ST) in which the power is supplied through transformers and rectifiers.

The first two types of exciters are also called rotating exciters which are mounted on the same shaft as the generator and driven by the prime mover[1].

The exciters can be of the following types

1. Field controlled dc generator – commutator.
2. Field controlled alternator.
 - a) Field controlled alternator with non-controlled rectifier (using diodes).
 - i) With slip rings and brushes (stationary rectifier).
 - ii) Brushless, without slip rings (rotating rectifier).
 - b) Alternator with controlled rectifier.
3. Static exciter with
 - a) Potential source controlled rectifier in which the excitation power is supplied through a potential transformer connected generator terminals.

b) Compound source (using both current and voltage transformers at the generator terminals) with

i) Non-controlled rectifier (control using magnetic elements such as saturable reactors).

ii) Controlled rectifier (for controlling the voltage).

In the static excitation systems all components are static or stationary. Static rectifiers, controlled or uncontrolled, supply the excitation current directly to the field of main synchronous generator through slip rings. The supply of power to the rectifiers is from the main generator (or the station auxiliary bus) through a transformer to step down the voltage to an appropriate level, or in some cases from auxiliary windings in the generator.

The advantages of the static excitation system are

- 1) Expanded range for excitation voltage and current with high amplification.
- 2) Very fast response.
- 3) Simplicity of design.

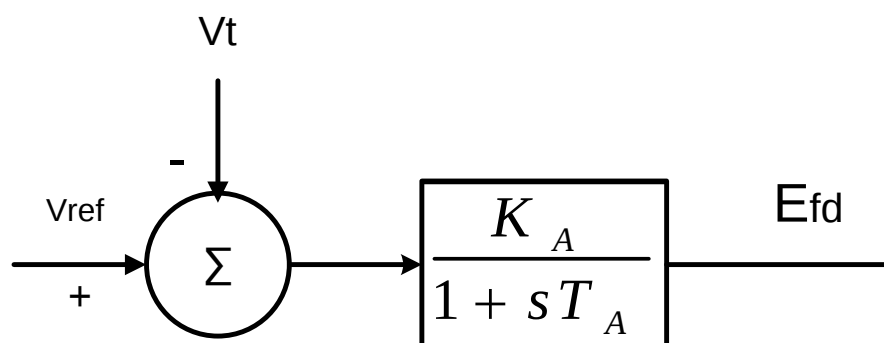


Figure 2.7: Static Excitation System

A signal coming from a PSS can be added to the voltage reference in the AVR input, this signal is used to modulate the excitation of the generator with the aim of achieving rotor oscillation damping.

2.5.5 Power System Stabilizer (PSS) Modeling

A PSS can help to damp generator rotor oscillations by providing an additional input signal that produces a torque component that is in phase with the rotor speed deviation [9].

It consists of a washout circuit, dynamic compensator, torsional filter and limiter as shown in figure (2.8).

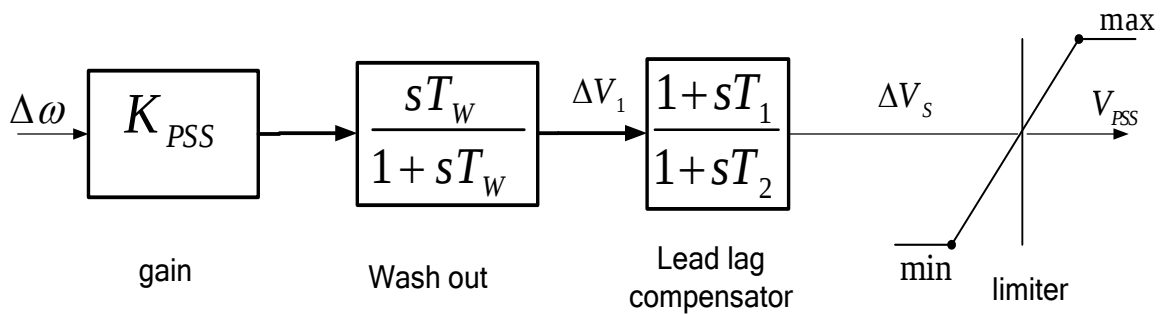


Figure 2.8: Power System Stabilizer Construction

➤ Washout Circuit

The washout circuit is provided to eliminate steady-state bias in the output of PSS which will modify the generator terminal voltage. The PSS is expected to respond only to transient variations in the input signal (say rotor speed). This is achieved by subtracting from it the low frequency components of the signal obtained by passing the signal through a low pass filter. The washout circuit acts essentially as a high pass filter and it must pass all frequencies that are of interest.

➤ Phase Compensator

The compensator is made up to single lead-lag stage where K_s is the gain of PSS and the time constants, T_1 was chosen to provide a phase lead for the input signal in the range of frequencies that are of interest (0.1 to 3.0 Hz). With static exciters, only one lead-lag stage may be adequate.

➤ **Gain**

The gain of PSS is to be chosen to provide adequate damping of all critical mode under various operating conditions. Although PSS may be tuned to give optimum damping under such condition, the performance will not be optimal under other conditions. The critical modes include not only local and inter area modes, but other modes.

➤ **PSS output limits**

The output of the PSS must be limited to prevent the up normal change in terminal voltage of PSS. The positive limit of PSS output is of importance during the change of load for higher value of the terminal voltage (due to the increase in speed or frequency). The negative limit of PSS output is of importance during the back swing of the rotor (after initial acceleration is over). we uses a -0.05 per unit as the lower limit and 0.1 to 0.2 as the higher limit.

➤ **Input signal**

The common signal which used as input to the PSS are: rotor speed deviation ($\Delta\omega$), bus frequency (Δf), electrical power (ΔP_e). The signal must be obtained from local measurements. Since the basic function of power system stabilizer (PSS) is to add damping to the rotor oscillations [1]. In this work a speed signal is used as input signal. The overall transfer function of the PSS is:

$$T(s) = \frac{K_{PSS} (1 + sT_1)(sT_w)}{(1 + sT_2)(1 + sT_w)} \quad (2.16)$$

CHAPTER THREE

LINEARIZED MODEL OF MULTI MACHINES SYSTEM AND PSS DESIGN

3.1 Introduction

When the system is subjected to a small load change, it tends to acquire a new operating state. During the transition between the initial state and the new state the system behavior is oscillatory. If the two states are such that all the state variables change only slightly (i.e., the variable x_i changes from x_{i0} to $x_{i0} + \Delta x_i$ where Δx_i is a small change in x_i), the system is operating near the initial state. The initial state may be considered as a quiescent operating condition for the system. To examine the behavior of the system when it is perturbed such that the new and old equilibrium states are nearly equal, the system equations are linearized about the quiescent operating condition. By this we mean that first-order approximations are made for the system Equations. The new linear Equations thus derived are assumed to be valid in a region near the quiescent condition.

As an example of product nonlinearities, consider the product $x_i x_j$. Let the state variables x_i and x_j have the initial values x_{i0} and x_{j0} . Let the changes in these variables be Δx_i and Δx_j . Initially their product is given by $x_{i0} x_{j0}$. The new value becomes

$$(x_{i0} + \Delta x_i)(x_{j0} + \Delta x_j) = x_{i0} x_{j0} + x_{i0} \Delta x_j + x_{j0} \Delta x_i + \Delta x_i \Delta x_j$$

The last term is a second-order term, which is assumed to be negligibly small. Thus for a first-order approximation, the change in the product $x_i x_j$ is given by

$$(x_{i0} + \Delta x_i)(x_{j0} + \Delta x_j) - x_{i0} x_{j0} = x_{j0} \Delta x_i + x_{i0} \Delta x_j$$

We note that x_{j0} and x_{i0} are known quantities and are treated here as coefficients, while Δx_i and Δx_j are “incremental” variables [2]. The trigonometric nonlinearities are treated in a similar manner as

$$\cos(\delta_0 + \Delta\delta) = \cos\delta_0 \cos\Delta\delta - \sin\delta_0 \sin\Delta\delta$$

With $\cos\Delta\delta \cong 1$ and $\Delta\sin\delta \cong \Delta\delta$.therefore

$$\cos(\delta_0 + \Delta\delta) - \cos\delta_0 \cong (-\sin\delta_0)\Delta\delta$$

The incremental change in $\cos\delta$ is then $(-\sin\delta_0)\Delta\delta$; the incremental variable is $\Delta\delta$ and its coefficient is $-\sin\delta_0$. Similarly, we can show that the incremental change in the term $\sin\delta$ is given by

$$\sin(\delta_0 + \Delta\delta) - \sin\delta_0 \cong (\cos\delta_0)\Delta\delta$$

The equivalent circuit of multi-machine system can be shown below

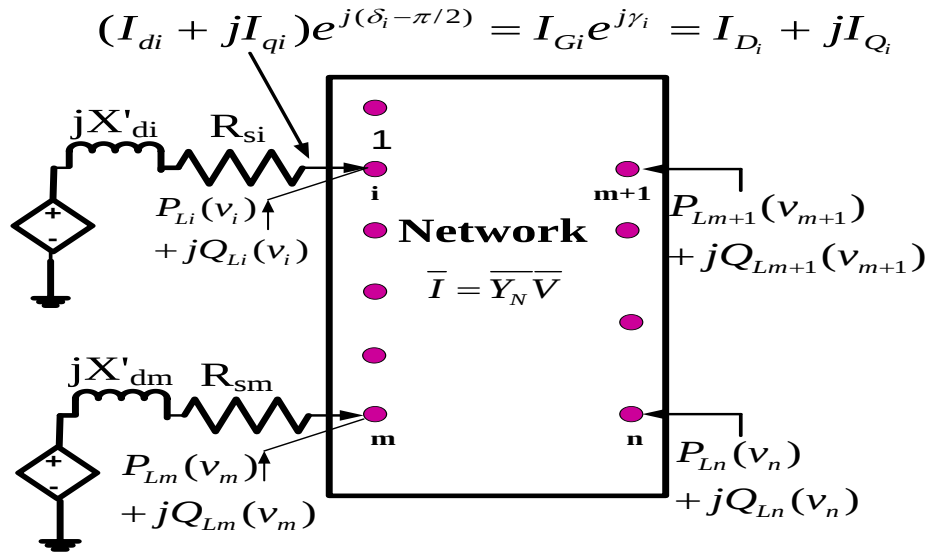


Figure 3.1: Interconnection of the synchronous machine dynamic circuit and the rest of the network.

Six Order Model Differential Equations

$$\dot{\delta}_i = \omega_i - \omega_s \quad (3.1)$$

$$\dot{\omega}_i = \frac{1}{M_i} \left[T_{mi} - E_{qi}'' I_{qi} - E_{di}'' I_{di} - (x_{di}'' - x_{qi}'') I_{di} I_{qi} - D_i (\omega_i - \omega_s) \right] \quad (3.2)$$

$$\dot{E}_{qi}' = \frac{1}{T_{doi}'} \left[-E_{qi}' + (x_{di} - x_{di}') I_{di} + E_{fdi} \right] \quad (3.3)$$

$$\dot{E}_{di}'' = \frac{1}{T_{qoi}''} \left[-E_{di}'' - (x_{qi}' - x_{qi}'') I_{qi} \right] \quad (3.4)$$

$$\dot{E}_{qi}'' = \frac{1}{T_{doi}''} [E_{qi}' - E_{qi}'' + I_{di} (x_{di}' - x_{di}'')] \quad (3.5)$$

$$\dot{E}_{di}'' = \frac{1}{T_{qoi}''} [E_{di}' - E_{di}'' - I_{qi} (x_{qi}' - x_{qi}'')] \quad (3.6)$$

A fast exciter has been added; whose state space Equation is

$$\dot{E}_{fdi} = -\frac{1}{T_{Ai}} E_{fdi} + \frac{K_{Ai}}{T_{Ai}} (V_{ref} - V_i) \quad (3.7)$$

For $i=1, \dots, n$

Algebraic Equations

(a) Stator Algebraic Equations

$$0 = v_i e^{j\theta_i} + jx_{di}'' (I_{di} + jI_{qi}) e^{j(\delta_i - \frac{\pi}{2})} - \left[E_{di}'' + (x_{qi}'' - x_{di}'') I_{qi} + jE_{qi}'' \right] e^{j(\delta_i - \frac{\pi}{2})} \quad (3.8)$$

In polar form

$$E_{qi}'' - V_i \cos(\delta_i - \theta_i) - x_{di}'' I_{di} = 0 \quad (3.9)$$

$$E_{di}'' + V_i \sin(\delta_i - \theta_i) - x_{qi}'' I_{qi} = 0 \quad (3.10)$$

For $i=1, \dots, n$

(b) Network Equations

The network Equation separate into generator buses Equations and load buses Equations

Generator Buses

$$\begin{aligned} & I_{di} V_i \sin(\delta_i - \theta_i) + I_{qi} V_i \cos(\delta_i - \theta_i) \\ & + P_{li} V_i - \sum_{k=1}^n V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) = 0 \end{aligned} \quad (3.11)$$

$$\begin{aligned} & I_{di} V_i \cos(\delta_i - \theta_i) - I_{qi} V_i \sin(\delta_i - \theta_i) \\ & + Q_{li} V_i - \sum_{k=1}^n V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = 0 \end{aligned} \quad (3.12)$$

For $i=1, \dots, n$

Load Buses

$$P_{li} V_i + Q_{li} V_i = \sum_{k=1}^n V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) \quad (3.13)$$

$$P_{li} V_i + Q_{li} V_i = \sum_{k=1}^n V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) \quad (3.14)$$

For $i=1, \dots, n$

3.2 Linearization

Let the state-space vector $\mathbf{x}$ have an initial state $\mathbf{x}_0$ at time $t=t_0$, at the occurrence of a small disturbance, i.e., after $t=t_0^+$, the states will change slightly from their previous positions or values. Thus

$$\mathbf{x} = \mathbf{x}_0 + \Delta \mathbf{x}$$

Note that $\mathbf{x}_0$ need not be constant, but we do require that it be known. The state-space model is in the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$$

Which

$$\dot{\mathbf{x}}_0 + \Delta \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}_0 + \Delta \mathbf{x}, t)$$

From which we obtain the linearized state-space Equation

$$\Delta \dot{\mathbf{x}} = \mathbf{A}(\mathbf{x}_0) \Delta \mathbf{x} + \mathbf{B}(\mathbf{x}_0) \mathbf{u}$$

Where $\mathbf{A}$ and $\mathbf{B}$ are state matrixes. Therefore the system variable be

$$\begin{aligned} V_i &= V_{i0} + \Delta V_i & \delta_i &= \delta_{i0} + \Delta \delta_i & \omega_i &= \omega_{i0} + \Delta \omega_i \\ \theta_i &= \theta_{i0} + \Delta \theta_i & E'_{qi} &= E'_{qi0} + \Delta E'_{qi} & E'_{di} &= E'_{di0} + \Delta E'_{di} \\ E_{fdi} &= E_{fdi0} + \Delta E_{fdi} & E''_{qi} &= E''_{qi0} + \Delta E''_{qi} & E''_{di} &= E''_{di0} + \Delta E''_{di} \end{aligned}$$

3.2.1 Linearization of The Differential Equations

The linearization of the fourth order differential equations and the static excitation system (3.1) – (3.7) yields

$$\frac{d \Delta \delta_i}{dt} = \Delta \omega_i \tag{3.15}$$

$$\frac{d \Delta \omega_i}{dt} = \frac{1}{M_i} [\Delta T_{Mi} - E_{qi0}'' \Delta I_{qi} - (x_{di}'' - x_{qi}'') I_{qi} \Delta I_{di} - E_{di0}'' \Delta I_{di} - \Delta E_{di0}'' I_{di} - (x_{di}'' - x_{qi}'') I_{di} \Delta I_{qi} - D_i \Delta \omega_i - \Delta E_{qi0}'' I_{qi}] \quad (3.16)$$

$$\frac{d \Delta E_{qi}'}{dt} = \frac{1}{T_{doi}'} [-\Delta E_{qi}' + (x_{di} - x_{di}') \Delta I_{di} + \Delta E_{fdi}] \quad (3.17)$$

$$\frac{d \Delta E_{di}'}{dt} = \frac{1}{T_{qoi}'} [-\Delta E_{di}' - (x_{qi} - x_{qi}') \Delta I_{qi}] \quad (3.18)$$

$$\frac{d \Delta E_{qi}''}{dt} = \frac{1}{T_{doi}''} [\Delta E_{qi}' - \Delta E_{qi}'' + (x_{di}' - x_{di}'') \Delta I_{di}] \quad (3.19)$$

$$\frac{d \Delta E_{di}''}{dt} = \frac{1}{T_{qoi}''} [\Delta E_{di}' - \Delta E_{di}'' - (x_{qi}' - x_{qi}'') \Delta I_{qi}] \quad (3.20)$$

$$\Delta \dot{E}_{fdi} = -\frac{1}{T_{Ai}} \Delta E_{fdi} + \frac{K_{Ai}}{T_{Ai}} (\Delta V_{ref} - \Delta V_i) \quad (3.21)$$

Writing (3.15) through (3.21) in matrix notation, we obtain

$$\begin{aligned}
\begin{bmatrix} \Delta \dot{\delta}_i \\ \Delta \dot{\omega}_i \\ \Delta \dot{E}'_{qi} \\ \Delta \dot{E}'_{di} \\ \Delta \dot{E}''_{qi} \\ \Delta \dot{E}''_{di} \\ \Delta \dot{E}_{fdi} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{D_i}{M_i} & 0 & 0 & -\frac{I_{qio}}{M_i} & -\frac{I_{dio}}{M_i} & 0 \\ 0 & 0 & -\frac{1}{T'_{qoi}} & 0 & 0 & 0 & \frac{1}{T'_{qoi}} \\ 0 & 0 & 0 & -\frac{1}{T'_{qoi}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{T''_{doi}} & 0 & -\frac{1}{T''_{doi}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T''_{qoi}} & 0 & -\frac{1}{T''_{qoi}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{Ai}} \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta \omega_i \\ \Delta E'_{qi} \\ \Delta E'_{di} \\ \Delta E''_{qi} \\ \Delta E''_{di} \\ \Delta E_{fdi} \end{bmatrix} + \\
&\begin{bmatrix} 0 & 0 \\ -\frac{(\mathbf{x}''_{di} - \mathbf{x}''_{qi}) \mathbf{I}_{qi} - E''_{dio}}{M_i} & -\frac{(\mathbf{x}''_{di} - \mathbf{x}''_{qi}) \mathbf{I}_{di} - E''_{qio}}{M_i} \\ \frac{(\mathbf{x}_{di} - \mathbf{x}'_{di})}{T'_{doi}} & 0 \\ 0 & -\frac{(\mathbf{x}_{qi} - \mathbf{x}'_{qi})}{T'_{qoi}} \\ \frac{(\mathbf{x}'_{di} - \mathbf{x}''_{di})}{T''_{doi}} & 0 \\ 0 & -\frac{(\mathbf{x}'_{qi} - \mathbf{x}''_{qi})}{T''_{qoi}} \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta \mathbf{I}_{di} \\ \Delta \mathbf{I}_{qi} \end{pmatrix} + \\
&\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{K_{Ai}}{T_{Ai}} \end{bmatrix} \begin{pmatrix} \Delta \theta_i \\ \Delta V_i \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_i} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{K_{Ai}}{T_{Ai}} \end{bmatrix} \begin{pmatrix} \Delta T_{Mi} \\ \Delta V_{ref} \end{pmatrix}
\end{aligned} \tag{3.22}$$

Put

$$\Delta \mathbf{I}_{gi} = \begin{pmatrix} \Delta \mathbf{I}_{di} \\ \Delta \mathbf{I}_{qi} \end{pmatrix}, \quad \Delta \mathbf{V}_{gi} = \begin{pmatrix} \Delta \theta_i \\ \Delta V_i \end{pmatrix}, \quad \Delta V_i = \begin{pmatrix} \Delta T_{Mi} \\ \Delta V_{ref} \end{pmatrix}$$

Then

$$\Delta \dot{x}_i = A_{1i} \Delta x_i + B_{1i} \Delta I_{gi} + B_{2i} \Delta V_{gi} + E_i \Delta V_i \quad i=1, \dots, n \quad (3.23)$$

3.2.2 Linearization of Algebraic Equations

The linearization of algebraic equations (3.9) and (3.10) in matrix form yields

$$\begin{bmatrix} -V_{i0} \cos(\delta_{i0} - \theta_{i0}) & 0 & 0 & 0 & 0 & 1 & 0 \\ V_{i0} \sin(\delta_{i0} - \theta_{i0}) & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta \omega_i \\ \Delta E'_{qi} \\ \Delta E'_{di} \\ \Delta E''_{qi} \\ \Delta E''_{di} \\ \Delta E_{fdi} \end{bmatrix} + \begin{bmatrix} V_{i0} \cos(\delta_{i0} - \theta_{i0}) & -\sin(\delta_{i0} - \theta_{i0}) \\ -V_{i0} \sin(\delta_{i0} - \theta_{i0}) & -\cos(\delta_{i0} - \theta_{i0}) \end{bmatrix} \begin{pmatrix} \Delta \theta_i \\ \Delta V_i \end{pmatrix} = 0 \quad (3.24)$$

Rewriting (3.24) we obtain

$$0 = C_{1i} \Delta x_i + D_{1i} \Delta I_{gi} + D_{2i} \Delta V_{gi} \quad (3.25)$$

In matrix notation, (3.25) can be written as

$$0 = C_1 \Delta x + D_1 \Delta I_g + D_2 \Delta V_g \quad (3.26)$$

Where C_1 , D_1 , and D_2 are block diagonal.

3.2.3 Linearization of the Network Equations

For generator buses (3.11) and (3.12)

$$\begin{aligned}
 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} &= \begin{bmatrix} [I_{di}V_{i0} \cos(\delta_{i0} - \theta_{i0}) - I_{qi}V_{i0} \sin(\delta_{i0} - \theta_{i0})] & 0 & 0 & 0 & 0 & 0 & 0 \\ [-I_{di}V_{i0} \sin(\delta_{i0} - \theta_{i0}) - I_{qi}V_{i0} \cos(\delta_{i0} - \theta_{i0})] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta\omega_i \\ \Delta E'_{qi} \\ \Delta E'_{di} \\ \Delta E''_{qi} \\ \Delta E''_{di} \\ \Delta E_{fdi} \end{bmatrix} \\
 &+ \begin{bmatrix} V_{i0} \sin(\delta_{i0} - \theta_{i0}) & V_{i0} \cos(\delta_{i0} - \theta_{i0}) \\ V_{i0} \cos(\delta_{i0} - \theta_{i0}) & -V_{i0} \sin(\delta_{i0} - \theta_{i0}) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta I_{di} \\ \Delta I_{qi} \end{bmatrix} + \\
 &\begin{bmatrix} -I_{di}V_{i0} \cos(\delta_{i0} - \theta_{i0}) + I_{qi}V_{i0} \sin(\delta_{i0} - \theta_{i0}) & I_{di}V_{i0} \sin(\delta_{i0} - \theta_{i0}) + I_{qi}V_{i0} \cos(\delta_{i0} - \theta_{i0}) \\ + V_{i0} \sum_{\substack{k=1 \\ k \neq i}}^n V_{ko} Y_{ik} \sin(\theta_{i0} - \theta_{ko} - \alpha_{ik}) & + V_{i0} \sum_{k=1}^n V_{ko} Y_{ik} \cos(\theta_{i0} - \theta_{ko} - \alpha_{ik}) + \frac{\partial P_{li}(V_i)}{\partial V_i} \\ I_{di}V_{i0} \sin(\delta_{i0} - \theta_{i0}) + I_{qi}V_{i0} \cos(\delta_{i0} - \theta_{i0}) & I_{di}V_{i0} \cos(\delta_{i0} - \theta_{i0}) - I_{qi}V_{i0} \sin(\delta_{i0} - \theta_{i0}) \\ - V_{i0} \sum_{\substack{k=1 \\ k \neq i}}^n V_{ko} Y_{ik} \sin(\theta_{i0} - \theta_{ko} - \alpha_{ik}) & - V_{i0} \sum_{k=1}^n V_{ko} Y_{ik} \sin(\theta_{i0} - \theta_{ko} - \alpha_{ik}) + \frac{\partial Q_{li}(V_i)}{\partial V_i} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta\theta_i \\ \Delta V_i \end{pmatrix} \\
 &+ \begin{bmatrix} -V_{i0} \sum_{\substack{k=1 \\ k \neq i}}^n V_{ko} Y_{ik} \sin(\theta_{i0} - \theta_{ko} - \alpha_{ik}) & -V_{i0} \sum_{k=1}^n V_{ko} Y_{ik} \cos(\theta_{i0} - \theta_{ko} - \alpha_{ik}) \\ -V_{i0} \sum_{\substack{k=1 \\ k \neq i}}^n V_{ko} Y_{ik} \cos(\theta_{i0} - \theta_{ko} - \alpha_{ik}) & -V_{i0} \sum_{k=1}^n V_{ko} Y_{ik} \sin(\theta_{i0} - \theta_{ko} - \alpha_{ik}) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta\theta_K \\ \Delta V_K \end{pmatrix} \\
 &\quad \quad \quad i=1, \dots, n \quad (3.27)
 \end{aligned}$$

Rewriting (3.27) we obtain

$$\begin{aligned}
0 = & \begin{bmatrix} C_{21} & & \\ & \ddots & \\ & & C_{2n} \end{bmatrix} \begin{pmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{pmatrix} + \begin{pmatrix} D_{31} & & \\ & \ddots & \\ & & D_{3n} \end{pmatrix} \begin{pmatrix} \Delta I_{g1} \\ \vdots \\ \Delta I_{gm} \end{pmatrix} + \\
& \begin{bmatrix} D_{41,1} & \cdots & D_{41,n} \\ \vdots & \vdots & \vdots \\ D_{4n,1} & \cdots & D_{4n,n} \end{bmatrix} \begin{pmatrix} \Delta V_{g1} \\ \vdots \\ \Delta V_{gn} \end{pmatrix} + \begin{bmatrix} D_{51,n+1} & \cdots & D_{51,m} \\ \vdots & \vdots & \vdots \\ D_{5n,n+1} & \cdots & D_{5n,m} \end{bmatrix} \begin{bmatrix} \Delta V_{\ell n+1} \\ \vdots \\ \Delta V_{\ell n} \end{bmatrix} \quad (3.28)
\end{aligned}$$

Where the various sub matrices of (3.28) can be easily identified. In matrix notation, (3.28) is

$$0 = C_2 \Delta x + D_3 \Delta I_g + D_4 \Delta V_g + D_5 \Delta V_\ell \quad (3.29)$$

Where

$$\Delta V_\ell = \begin{pmatrix} \Delta \theta_K \\ \Delta V_K \end{pmatrix}$$

For the non-generator buses $i=1, \dots, n$.

Note that C_2 , D_3 are block diagonal, where D_4 , D_5 are full matrices.

For load buses (3.13) and (3.14) which are written of form matrix as shown below:

$$\begin{aligned}
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} &= \begin{bmatrix} \sum_{\substack{k=1 \\ k \neq i}}^n V_{io} V_{ko} Y_{ik} \sin(\theta_{io} - \theta_{ko} - \alpha_{ik}) & -\sum_{k=1}^n V_{ko} Y_{ik} \cos(\theta_{io} - \theta_{ko} - \alpha_{ik}) + \frac{\partial P_{li}(V_i)}{\partial V_i} \\ -\sum_{\substack{k=1 \\ k \neq i}}^n V_{io} V_{ko} Y_{ik} \cos(\theta_{io} - \theta_{ko} - \alpha_{ik}) & -\sum_{k=1}^n V_{ko} Y_{ik} \sin(\theta_{io} - \theta_{ko} - \alpha_{ik}) + \frac{\partial Q_{li}(V_i)}{\partial V_i} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta \theta_i \\ \Delta V_i \end{pmatrix} \\
&+ \begin{bmatrix} -V_{io} \sum_{\substack{k=1 \\ k \neq i}}^n V_{ko} Y_{ik} \sin(\theta_{io} - \theta_{ko} - \alpha_{ik}) & -V_{io} \sum_{k=1}^n Y_{ik} \cos(\theta_{io} - \theta_{ko} - \alpha_{ik}) \\ -V_{io} \sum_{\substack{k=1 \\ k \neq i}}^n V_{ko} Y_{ik} \cos(\theta_{io} - \theta_{ko} - \alpha_{ik}) & -V_{io} \sum_{k=1}^n Y_{ik} \sin(\theta_{io} - \theta_{ko} - \alpha_{ik}) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta \theta_K \\ \Delta V_K \end{pmatrix}
\end{aligned} \tag{3.30}$$

Rewriting (3.30) we obtain

$$0 = \begin{bmatrix} D_{6m+1,1} & \cdots & D_{6m+1,m} \\ \vdots & \vdots & \vdots \\ D_{6n,1} & \cdots & D_{6n,m} \end{bmatrix} \begin{pmatrix} \Delta V_{g1} \\ \vdots \\ \Delta V_{gm} \end{pmatrix} + \begin{bmatrix} D_{7m+1,m+1} & \cdots & D_{7m+1,n} \\ \vdots & \vdots & \vdots \\ D_{7n,m+1} & \cdots & D_{7n,n} \end{bmatrix} \begin{pmatrix} \Delta V_{\ell m+1} \\ \vdots \\ \Delta V_{\ell n} \end{pmatrix} \tag{3.31}$$

In matrix notation, (3.31) can be written as

$$0 = D_6 \Delta V_g + D_7 \Delta V_\ell \quad (3.32)$$

Where D_6 and D_7 are full matrices.

Rewritten equations (3.23),(3.26),(3.29) and (3.32) together as follow :

$$\Delta \dot{x}_i = A_{1i} \Delta x_i + B_{1i} \Delta I_{gi} + B_{2i} \Delta V_{gi} + E_i \Delta V_i \quad (3.33)$$

$$0 = C_1 \Delta x + D_1 \Delta I_g + D_2 \Delta V_g \quad (3.34)$$

$$0 = C_2 \Delta x + D_3 \Delta I_g + D_4 \Delta V_g + D_5 \Delta V_\ell \quad (3.35)$$

$$0 = D_6 \Delta V_g + D_7 \Delta V_\ell \quad (3.36)$$

From (3.34) we obtain

$$\Delta I_g = -D_1^{-1} C_1 \Delta X - D_1^{-1} D_2 \Delta V_g \quad (3.37)$$

Substitute (3.37) in (3.35) as shown

$$C_2 \Delta X + D_3 (-D_1^{-1} C_1 \Delta X - D_1^{-1} D_2 \Delta V_g) + D_4 \Delta V_g + D_5 \Delta V_\ell = 0 \quad (3.38)$$

Let

$$\left[D_4 - D_3 D_1^{-1} D_2 \right] = K_1$$

And

$$\left[C_2 - D_3 D_1^{-1} C_1 \right] = K_2$$

Then

$$K_2 \Delta X + K_1 \Delta V_g + D_5 \Delta V_\ell = 0 \quad (3.39)$$

Substitute (3.37) in (3.33) we find

$$\Delta \dot{X} = (A_1 - B_1 D_1^{-1} C_1) \Delta X + (-B_1 D_1^{-1} D_2 + B_2) \Delta V_g + E_1 \Delta u \quad (3.40)$$

$$K_2 \Delta X + K_1 \Delta V_g + D_5 \Delta V_\ell = 0 \quad (3.41)$$

$$0 = D_6 \Delta V_g + D_7 \Delta V_\ell \quad (3.42)$$

Equations (3.40)-(3.42) can be put in a form of matrix shown below

$$\begin{pmatrix} \Delta \dot{X} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} A_1 - B_1 D_1^{-1} C_1 & -B_1 D_1^{-1} D_2 + B_2 & 0 \\ K_2 & K_1 & D_5 \\ 0 & D_6 & D_7 \end{pmatrix} \begin{pmatrix} \Delta X \\ \Delta V_g \\ \Delta V_\ell \end{pmatrix} + \begin{pmatrix} E_1 \\ 0 \\ 0 \end{pmatrix} \Delta u \quad (3.43)$$

Let

$$A' = [A_1 - B_1 D_1^{-1} C_1]$$

$$B' = -B_1 D_1^{-1} D_2 + B_2$$

So,

$$\Delta \dot{X} = A' \Delta X + B' \Delta V_g + E_1 \Delta u \quad (3.44)$$

$$K_2 \Delta X + K_1 \Delta V_g + D_5 \Delta V_\ell = 0 \quad (3.45)$$

$$0 = D_6 \Delta V_g + D_7 \Delta V_\ell \quad (3.46)$$

From (3.46)

$$\Delta V_\ell = -D_7^{-1} D_6 \Delta V_g \quad (3.47)$$

Substitute (3.47) in (3.45) we find

$$K_2 \Delta X + F \Delta V_g = 0 \quad (3.48)$$

Where

$$F = K_1 - D_5 D_7^{-1} D_6 \quad (3.49)$$

From (3.48)

$$\Delta V_g = -F^{-1}K_2\Delta X \quad (3.50)$$

Substitute (3.50) in (3.44) we find

$$\Delta \dot{X} = A_{\text{SYS}} \Delta X + E_1 \Delta u \quad (3.51)$$

Where

$$A_{\text{SYS}} = A' - B'F^{-1}K_2 \quad (3.52)$$

[illegible]

3.3 Design of Power System Stabilizer

The steps involved in designing a PSS are as follows:

1. Computation of GEPS(s).
2. Design of compensator using phase compensation technique.
3. Determination of compensator gain.

3.3.1 Computation of GEPS(s)

A PSS acts through generator, exciter system, and power system (GEPS). Therefore, a PSS must compensate the phase lag through the GEPS. To obtain the phase information of GEPS, the frequency response of the transfer function between the exciter reference input (i.e., PSS output) and the generator electrical torque should be observed. In computing this response, the generator speed and rotor angle should remain constant, otherwise, when the excitation of a generator is modulated, the resulting change in electrical torque causes variations in rotor speed and angle and that in turn affect the electrical torque. As we are interested only in the phase characteristics between exciter reference input and electrical torque, the feedback effect through rotor angle variation should be eliminated by holding the speed constant. This is achieved by removing the columns and rows corresponding to rotor speed and angle from the state matrix.

$$\text{GEPS}(s) = \frac{\Delta T_e(s)}{\Delta v_s(s)} \quad (3.53)$$

Where

$\Delta T_e(s)$ is the electrical torque and $\Delta v_s(s)$ is the output of the PSS.

A PSS must compensate the phase lag through the GEPS.

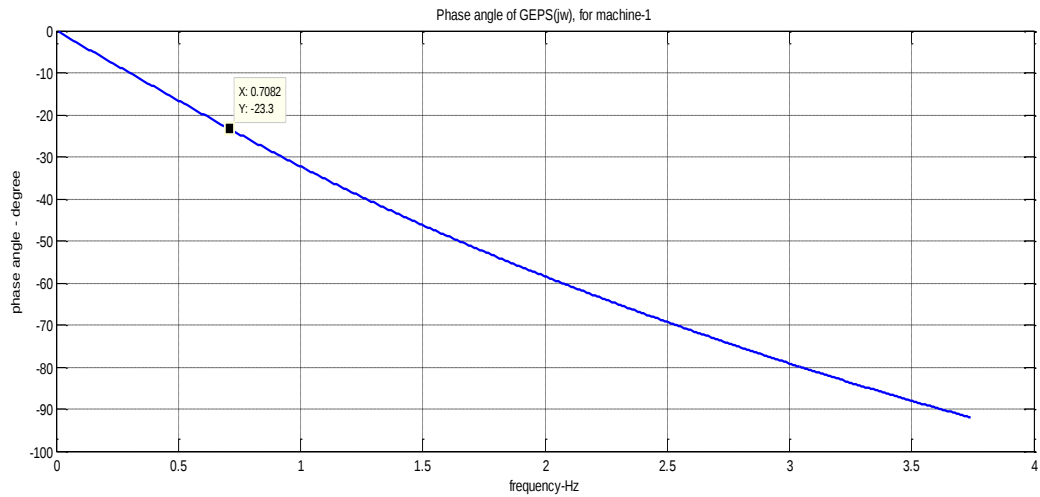


Figure 3.2 Phase Angle of GEPS

3.3.2 Design of Compensator $G_C(s)$

If a PSS is to provide pure damping torque at all frequencies, ideally, the phase characteristics of PSS must balance the phase characteristics of GEPS at all frequencies.

The following criteria are chosen to design the phase compensation for PSS:

- Compute the phase angle from the GEPs transfer function.
- Substitute the phase angle in the compensator transfer function

$$G_C(s) = \frac{(1+sT_1)K_s}{(1+sT_2)} \quad (3.54)$$

Where T_1 , T_2 are time constant of PSS and K_s is the gain of PSS.

To determine T_1 and T_2 the phase angle lead ϕ_m to be provided by the compensator is related to T_1 and T_2 as

$$\sin(\phi_m) = \frac{1-\alpha}{1+\alpha} \quad (3.55)$$

Where α is the ratio between T_2 and T_1

$$\alpha = \frac{T_2}{T_1} \quad (3.56)$$

The center frequency at which it offers a phase lead ϕ_m is given by

$$\omega_m = \frac{1}{\sqrt{\alpha}T_1} \quad (3.57)$$

3.3.3 Determination of compensator gain

To set the gain of the PSS, the following criterion are generally employed

1. Based on the gain for instability

$$K_{PSS} = \frac{K_{PSS}^*}{2}$$

Where,

K_{PSS}^* is the instability gain, which determine by trial and error.

2. Damping factor of the critical mode: Here, the gain is selected such that damping factor for the mode is above some typical value say 0.05.
3. High frequency gain: The high frequency gain of PSS is given by

$K_{PSS} \frac{T_1}{T_2}$ this should not be too high as it would lead to noise amplification decreasing the effectiveness of a PSS.

3.3.4 Stabilizer Limits

The positive output limits of the stabilizer is set at a relatively large value in the range of 0.1 to 0.2 per unit.

3.4 Interfacing PSS to the System Matrix

To see the performance of the system with PSS, the system matrix needs to be modified to account for PSS.

The state space equations for single stage PSS can be written as

$$\Delta \dot{V}_1 = K_{PSS} \left[\Delta \dot{\omega}_i - \frac{1}{T_w} \Delta V_1 \right] \quad (3.58)$$

$$\Delta \dot{V}_s = \frac{\Delta V_1}{T_1} - \frac{\Delta V_s}{T_2} + \frac{T_1}{T_2} \Delta V_1 \quad (3.59)$$

Substitute equation (3.16) in equation (3.58)

$$\Delta \dot{V}_1 = K_{PSS} \left[\begin{aligned} & \frac{1}{M_i} [\Delta T_{Mi} - E''_{qi0} \Delta I_{qi} - (x''_{di} - x''_{qi}) I_{qi} \Delta I_{di} - E''_{di0} \Delta I_{di} \\ & - \Delta E''_{di0} I_{di} - (x''_{di} - x''_{qi}) I_{di} \Delta I_{qi} - D_i \Delta \omega_i - \Delta E''_{qi0} I_{qi}] - \frac{1}{T_w} \Delta V_1 \end{aligned} \right] \quad (3.60)$$

Substitute equation (3.60) in equation (3.59), and then add it in the system matrix, i.e. equation (3.52) [8].

CHAPTER FOUR

SIMULATION AND RESULTS

In the previous chapter the power system stabilizer (PSS) was designed. This chapter is an important part of project. In this chapter the eigenvalue analysis and time-domain simulation have been represented for the multi-machines system. The results are obtained through simulations of the system response for different operating conditions: the system with static AVR and the system with PSS for different cases.

4.1 Methods of Analysis of Small Signal Stability

1. Eigenvalue analysis.
2. Synchronizing and damping torque analysis.
3. Frequency response- and residue-based analysis.
4. Time-domain solution analysis.

4.2 Eigenvalue Analysis for Small Signal Stability

Eigenvalue analysis is used to study oscillatory behavior of power systems and hence has been described in detail. The system is linearized about an operating point and typically involves computation of eigenvalues. After linearization of system equations the system can be described as

$$\Delta \dot{x} = A \Delta x(t) + B \Delta u(t) \quad (4.1)$$

$$\lambda = \sigma \pm j\omega \quad (4.2)$$

Where A is the state matrix and λ is the eigenvalues of the matrix A. The real component of the eigenvalues gives the damping ratio and the imaginary

component gives the frequency of oscillations. When at least one of the eigenvalues has positive real part (σ) the system is unstable.

The frequency of oscillation in Hz is given by

$$f = \frac{\omega}{2\pi} \quad (4.3)$$

The damping ratio is given by

$$\zeta = \frac{-\sigma}{\sqrt{(\sigma^2 + \omega^2)}} \quad (4.4)$$

The damping ratio ζ determines the rate of decay of the amplitude of the oscillation. The time constant of amplitude decay is $[1/|\sigma|]$.

4.3 Advantages of Eigenvalue or Modal Analysis

With eigenvalue techniques, oscillations can be characterized easily, quickly and accurately. Different modes, which are mixed with each other in curves of time-domain simulation, are identified separately. Root loci plotted with variations in system parameters or operating conditions provide valuable insight into the dynamic characteristics of the system.

Using eigenvectors coherent groups of generators which participate in a given swing mode can be identified. In addition, linear models can be used to design controllers that damp oscillations. Further, information regarding the most effective site of controller, tuning of existing one, installation of new controller can be decided.

Eigenvalue or modal analysis describes the small-signal behavior of the system about an operating point, and does not take into account the nonlinear behavior of components such as controller's limits at large system perturbations. Further, design and analysis carried out using various indices

such as participation factors, residues, etc. may lead to many alternate options. These options need to be verified for their effectiveness using system responses for small/large disturbances. In such cases, time-domain simulations are very essential. In this project, time-domain simulation, and modal analysis in the frequency domain should be used in a complement manner in analyzing small-signal stability of power systems [8].

4.4 Case Study

A 4 machines, 10-busbars power system has been used to demonstrate the modal analysis of a power system. The single line diagram of the system is shown in Figure (4.1).

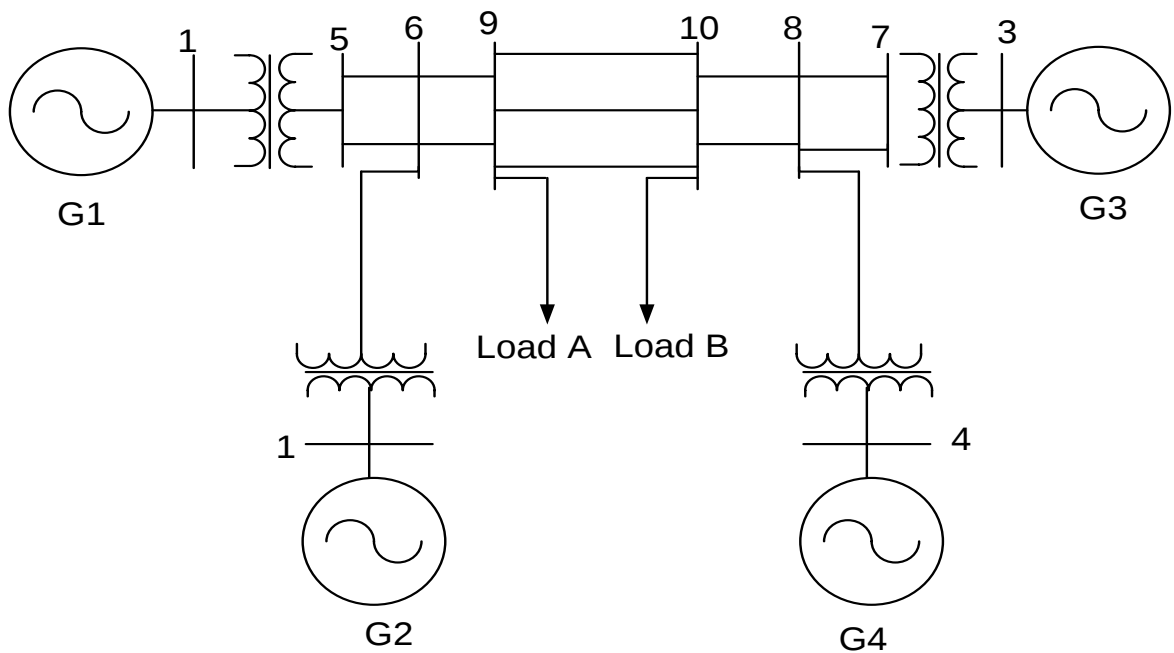


Figure 4.1: Four Machines Power System

4.5 Eignvalue Analysis

In this section eignvalue method has been used to analyze the stability problem of the system in two cases.

4.5.1 Base Case

In this case, generators are provided with a static exciter with no PSS. Further, constant impedance type load model has been employed for both real and reactive components of loads. The eigenvalues, damping ratio, frequency of oscillations, and the type of mode of oscillations are shown in Table 4.1.

All of the real parts are negative which mean the system is stable but it is observed that the system is poorly damped for inter area mode of oscillation and having (damping ratio less than 5%). There are a different oscillation frequencies but we interested here in a swing modes (local modes and inter area mode of oscillations (0.3-2 HZ)). The eigenvectors for swing modes have been determined by using slip-participation matrix; which represent the nature of oscillatory modes.

The figures (4.2)-(4.4) give a good picture for the types of swing modes.

Table 4.1: The eigenvalues of the system without PSS

SL_ Number	Eigenvalue	damping ratio	frequency(Hz)	Nature of mode
1	-42.919	1	0	Non swing
2	-42.713	1	0	Non swing
3	-16.025 + 17i	0.68594	2.7056	Non swing
4	-16.025 - 17i	0.68594	2.7056	Non swing
5	-38.695	1	0	Non swing
6	-37.619	1	0	Non swing
7	-33.931	1	0	Non swing
8	-33.606	1	0	Non swing
9	-16.4 + 12.228i	0.80167	1.9462	Non swing
10	-16.4 - 12.228i	0.80167	1.9462	Non swing
11	-27.453	1	0	Non swing
12	-25.333	1	0	Non swing
13	-1.1037 +7.4473i	0.1466	1.1853	Local
14	-1.1037 - 7.4473i	0.1466	1.1853	Local
15	-1.0488 + 6.7981i	0.15247	1.082	Local
16	-1.0488 - 6.7981i	0.15247	1.082	Local
17	-0.03722+4.4583i	0.0083487	0.7095	Inter area
18	-0.037222 - 4.4583i	0.0083487	0.70956	Inter area
19	-1e-006	1	0	Non swing
20	-17.541	1	0	Non swing
21	-15.619 + 0.90814i	0.99831	0.14454	Non swing
22	-15.619 - 0.90814i	0.99831	0.14454	Non swing
23	-13.619	1	0	Non swing
24	-4.5	1	0	Non swing

25	-5.0199	1	0	Non swing
26	-4.8931	1	0	Non swing
27	-4.686	1	0	Non swing
28	-1e-006	1	0	Non swing

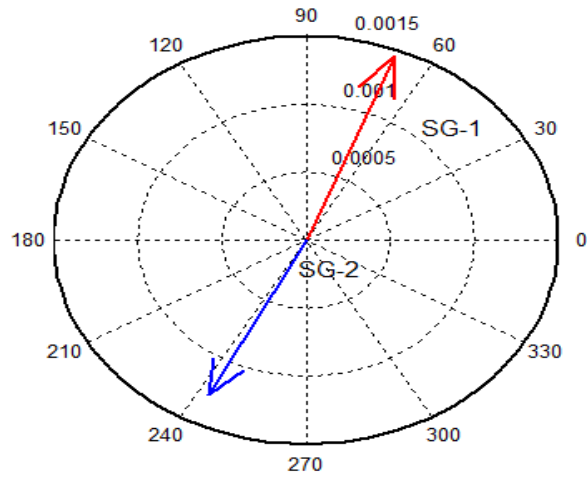


Figure 4.2: The swing for machines 1 and 2 for mode number 13

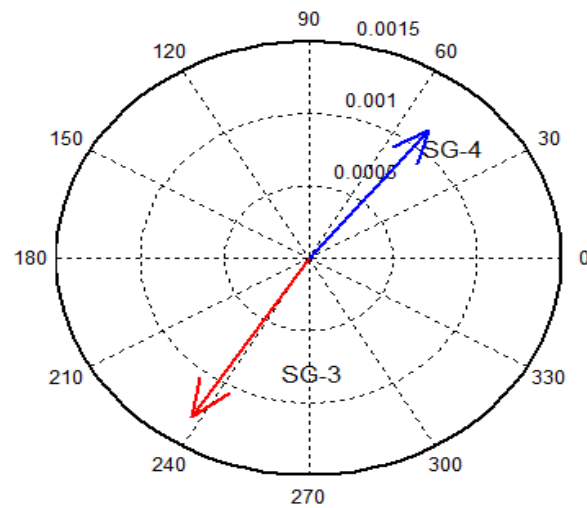


Figure 4.3: The swing for machines 3 and 4 for mode number 15

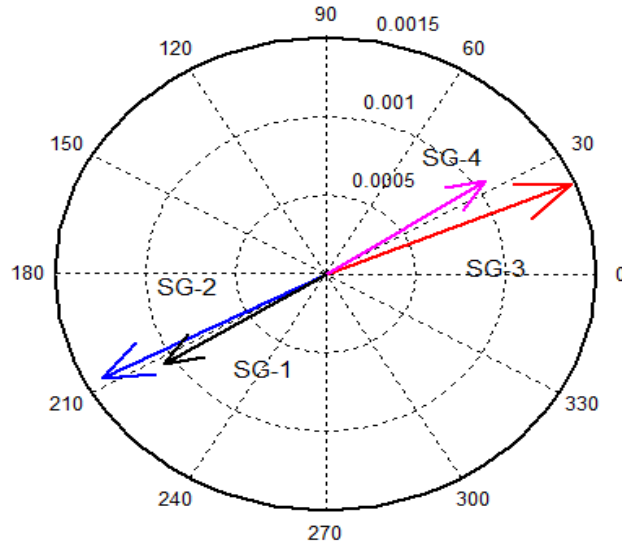


Figure 4.4: The swing for machines 1,2 and machines 3,4 for mode number17

4.5.2 Controlled Case

Here the PSS controller has been added to the system however, before adding it the optimal location of PSS has been determined.

❖ Participation Factor

Despite the fact that eigenvectors are a good indication of the relative activity of the states within a mode, it is not a good indicator of the importance of states to the mode from a control aspect. To overcome this shortcoming, a matrix containing the measures of the association between the state variables and the modes is formed by combining the right and left eigenvectors. This matrix is known as participation matrix P.

$$p = [p_1 \quad \dots \quad p_n] \rightarrow p_i = \begin{bmatrix} p_{1i} \\ \vdots \\ p_{ni} \end{bmatrix} = \begin{bmatrix} \phi_{1i} \\ \vdots \\ \phi_{ni} \end{bmatrix} \begin{bmatrix} \psi_{1i} \\ \vdots \\ \psi_{ni} \end{bmatrix} \quad (4.5)$$

Let a certain element p_{ki} , also known as the participation factor be defined as the measure of the relative participation of the k_{th} state variable in the i_{th} mode and given as follows

$$p_{Ki} = \frac{\partial \lambda_i}{\partial a_{KK}} \quad (4.6)$$

In power system, participation factors give good indications for power system stabilizers placement. As previously mentioned, PSS is a device use to provide supplementary signal to add damping at the generator shaft. Hence, the participation factor is use to identify the states that correspond have the highest participation in the mode (in general rotor speed and rotor angle). If the corresponding rotor angle and/or rotor speed participation factor of a generator in a mode is zero, then that particular generator state does not contribute to the damping of the mode. However if the participation factor is real positive, adding damping at the generator will increase the damping of the mode whereas if negative, it will have adverse effects [10].

The contribution greater than (0.5) has been assumed that it is high contribution in the mode not depend on the cost of the controllers. Table (4.2) shows the contribution of all generators in inter-area mode.

Table 4.2: The contribution of all generators in inter-area mode

State variable	Magnitude(Normal PF)	angle(Normal PF)degree
Slip-1	1.0000	-3.49
Slip-2	0.9561	2.79
Slip-3	0.6431	0.92
Slip-4	0.3798	-1.51

Form the method of design the PSS; which represented in the previous section the PSSs have been designed for machine 1, 2, and 3. Table 4.3 represent the parameters of PSSs.

Table 4.3: The parameters of PSSs

No_ gen	K_{PSS}	T_w	T_1	T_2	$V_{S \max}$	$V_{S \min}$
1	27.5	10	0.34147	0.14790	0.1	-0.1
2	27.5	10	0.33534	0.13216	0.1	-0.1
3	14.0	10	0.36956	0.17785	0.1	-0.1

All eignvalues are listed in Table (4.4), all of the real parts are negative which mean the system is stable; the PSSs raise the damping ratio from (.84%) to (29.03%) for inter area mode.

Figure (4.5) below give a comparison between base case and controlled case.

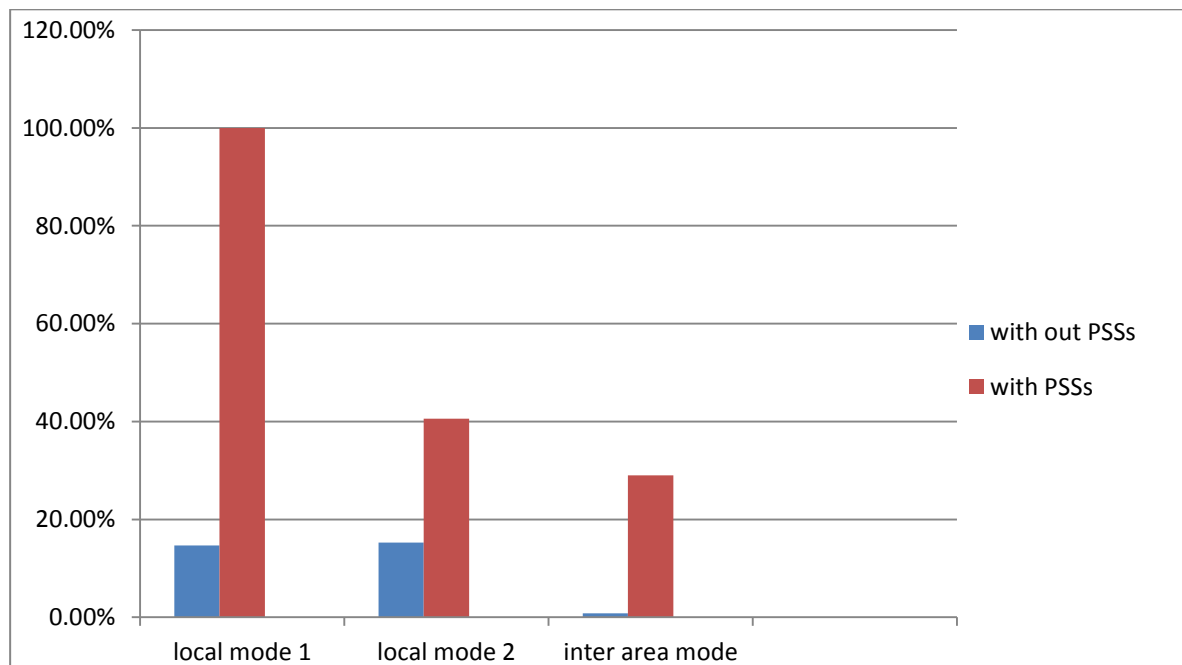


Figure 4.5: Comparison of damping ratio with and without PSSs

Table 4.4: The eigenvalues of the system with PSSs

SL_ number	Eigenvalue	Damping ratio	frequency(Hz)	Nature of modes
1	-48.055	1	0	Non swing
2	-43.774	1	0	Non swing
3	-41.253	1	0	Non swing
4	-38.598	1	0	Non swing
5	-2.5695 + 17.582i	0.1446	2.7983	Non swing
6	-2.5695 - 17.582i	0.1446	2.7983	Non swing
7	-33.98	1	0	Non swing
8	-33.643	1	0	Non swing
9	-9.2027 + 16.499i	0.48711	2.626	Non swing
10	-9.2027 - 16.499i	0.48711	2.626	Non swing
11	-13.867 + 14.013i	0.70338	2.2303	Non swing
12	-13.867 - 14.013i	0.70338	2.2303	Non swing
13	-27.522	1	0	Non swing
14	-25.378	1	0	Non swing
15	-22.887	1	0	Non swing
16	-19.997	1	0	Non swing
17	-17.438	1	0	Non swing
18	-3.7804 + 8.5207i	0.40555	1.3561	Local
19	-3.7804 - 8.5207i	0.40555	1.3561	Local
20	-1.0935 + 3.6044i	0.29031	0.57366	Inter area
21	-1.0935 - 3.6044i	0.29031	0.57366	Inter area
22	-7.4896	1	0	Non swing
23	-7.0925	1	0	Non swing
24	-4.343 + 2.5028i	0.86642	0.39834	Non swing
25	-4.343 - 2.5028i	0.86642	0.39834	Non swing
26	-5.3177	1	0	Non swing

27	-4.7351	1	0	Non swing
28	-2.3946 + 1.3382i	0.87293	0.21298	Non swing
29	-2.3946 - 1.3382i	0.87293	0.21298	Non swing
30	-1.3796	1	0	Non swing
31	-1e-006	1	0	Non swing
32	-0.10311	1	0	Non swing
33	-0.10119	1	0	Non swing
34	-1e-006	1	0	Non swing

The nature of oscillatory modes was shown in figures (4.6) and (4.7)

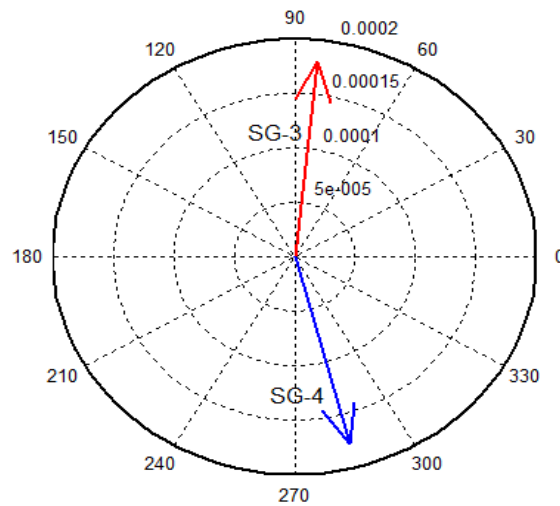


Figure 4.6: Plot of slip for machines 3 and machines 4

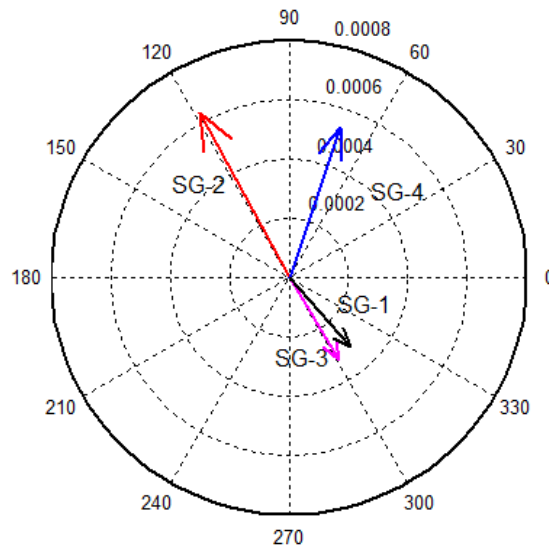


Figure 4.7: Plot of slip for machines 1,2 and machines 3,4

4.6 Time Domain Simulation

The time domain simulations were performed to validate the results of modal analysis. The effectiveness of the PSSs is assessed by their ability to damp low frequency oscillations under various operating conditions. Furthermore, the PSS must be able to stabilize the system under transient conditions. Therefore, two types of time domain simulations are performed; small signal and transient simulations.

4.6.1 Small Disturbance

The small disturbance has been assumed.

- **Perturbation of reference voltage for generator two**

The reference voltage perturbed due to change in excitation system voltage by magnitude (0.01) per unit.

➤ **The system without PSS**

The response of the rotor angle deviation, rotor speed deviation, generator field voltage and output active power were shown in figures (4.8)-(4.11).

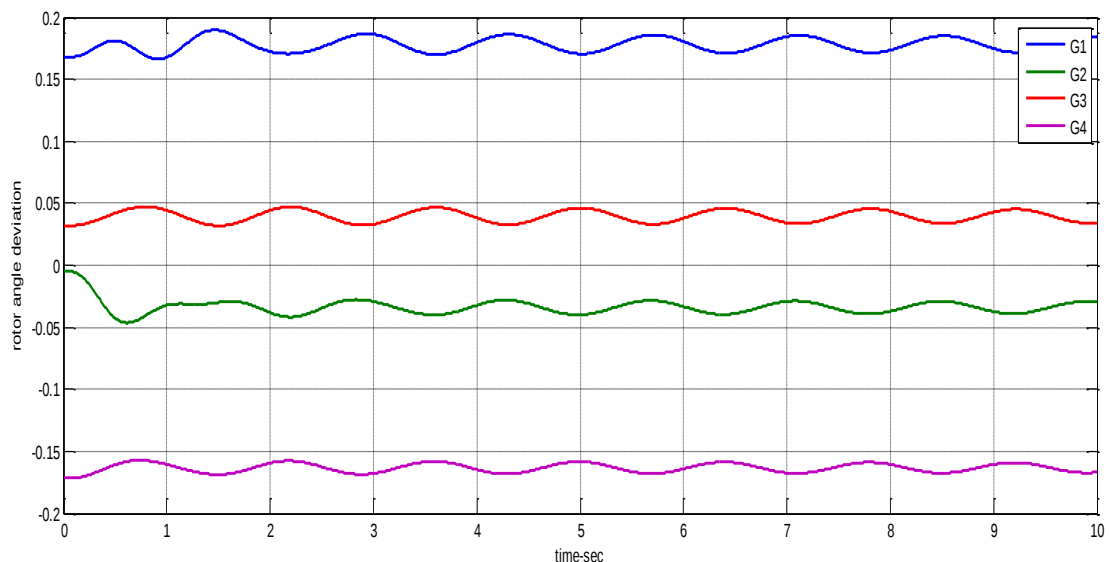


Figure 4.8: Response of rotor angle deviation

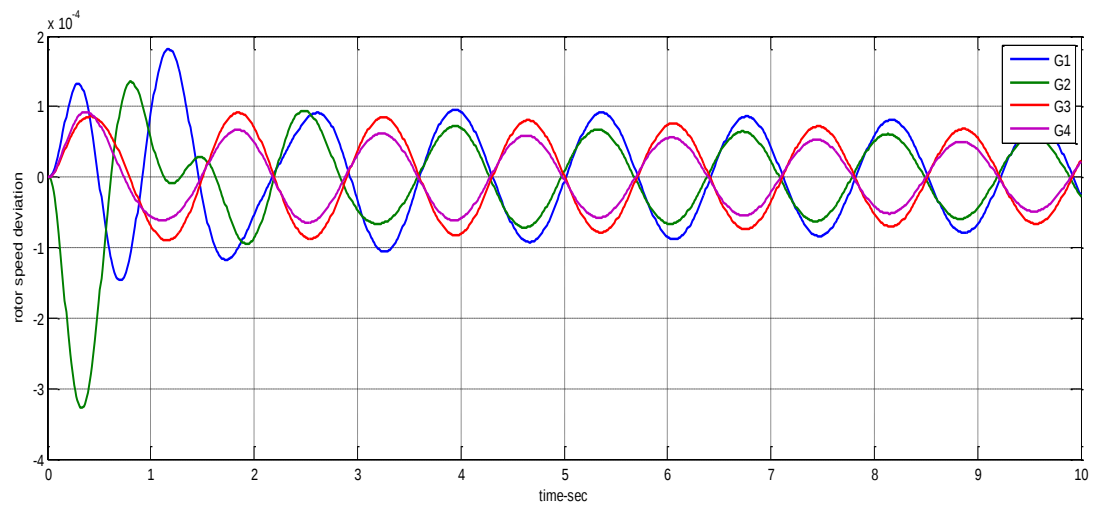


Figure 4.9: Response of rotor speed deviation

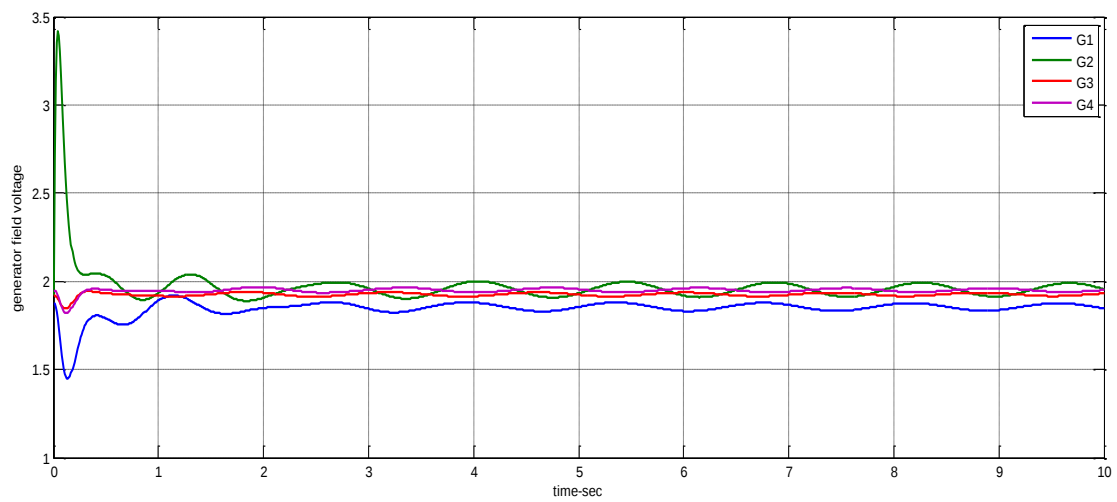


Figure 4.10: Response of generator field voltage

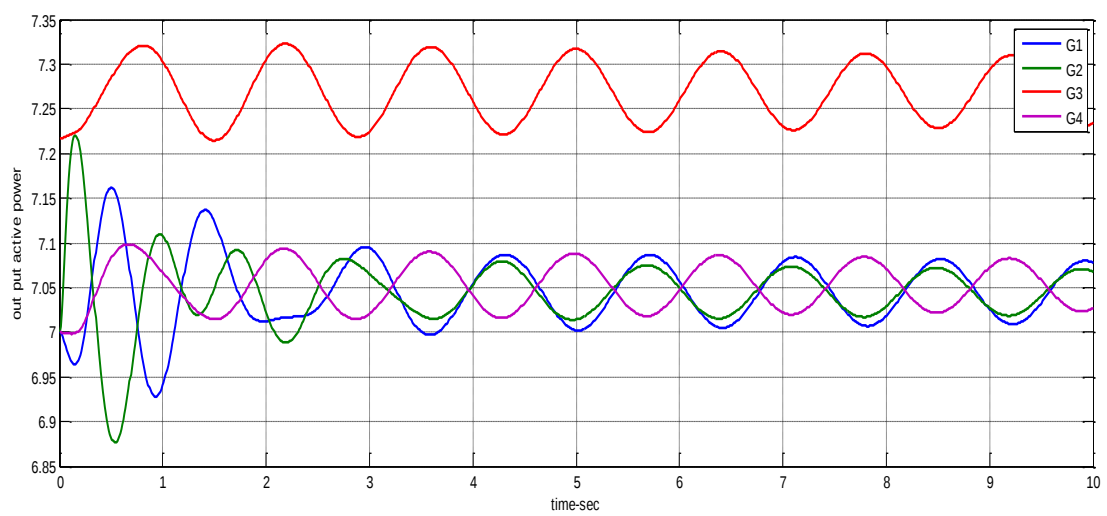


Figure 4.11: Response of output active power

➤ The system with PSS

The response of the rotor angle deviation, rotor speed deviation, generator field voltage and output active power were shown in figures (4.12)-(4.15).

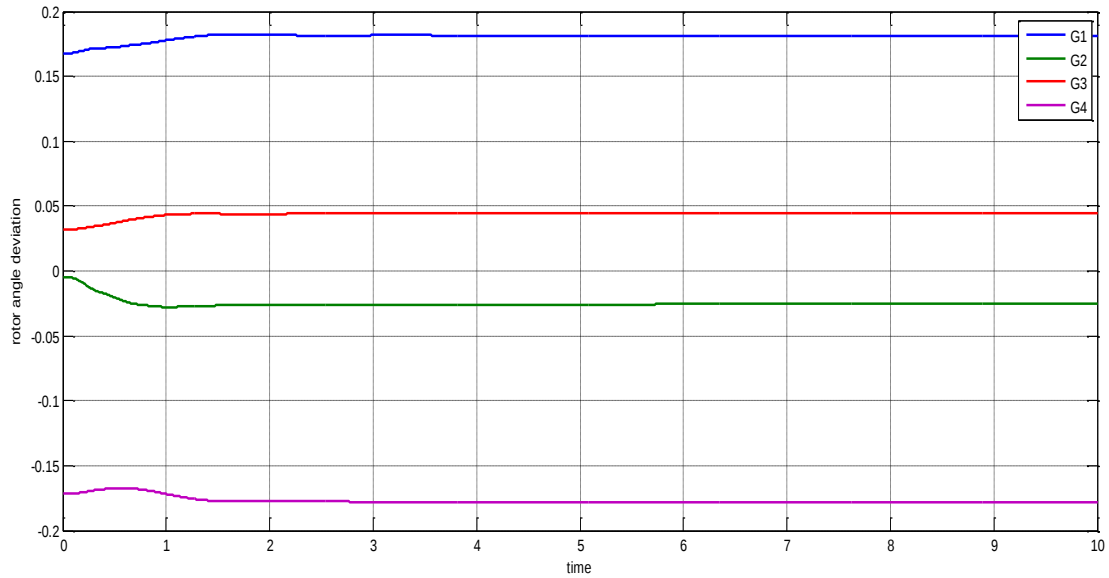


Figure 4.12: Response of rotor angle deviation

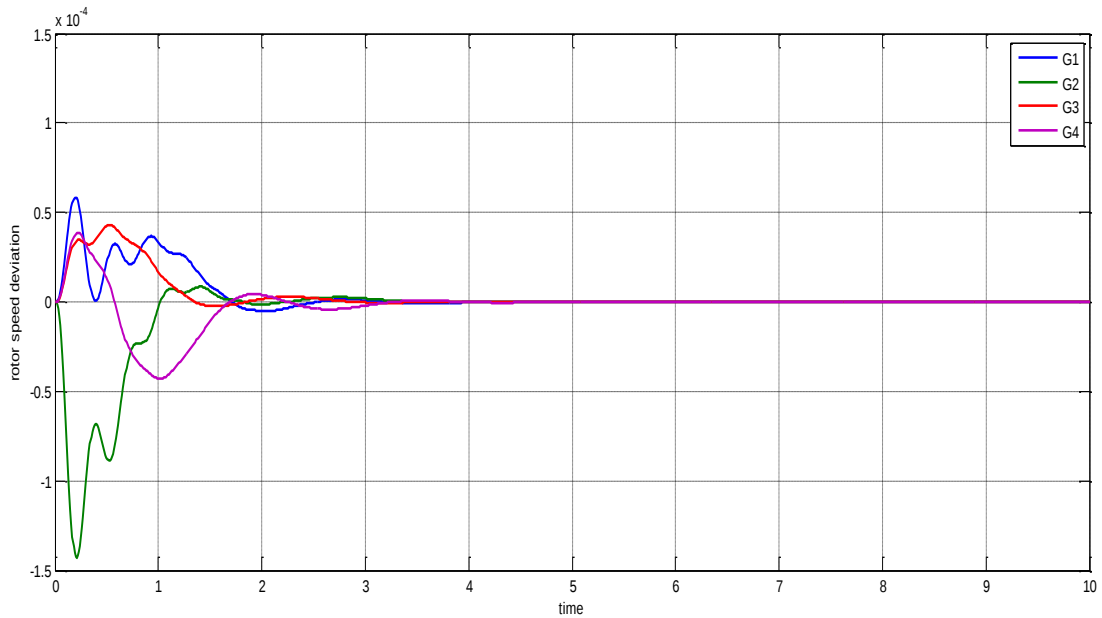


Figure 4.13: Response of rotor speed deviation

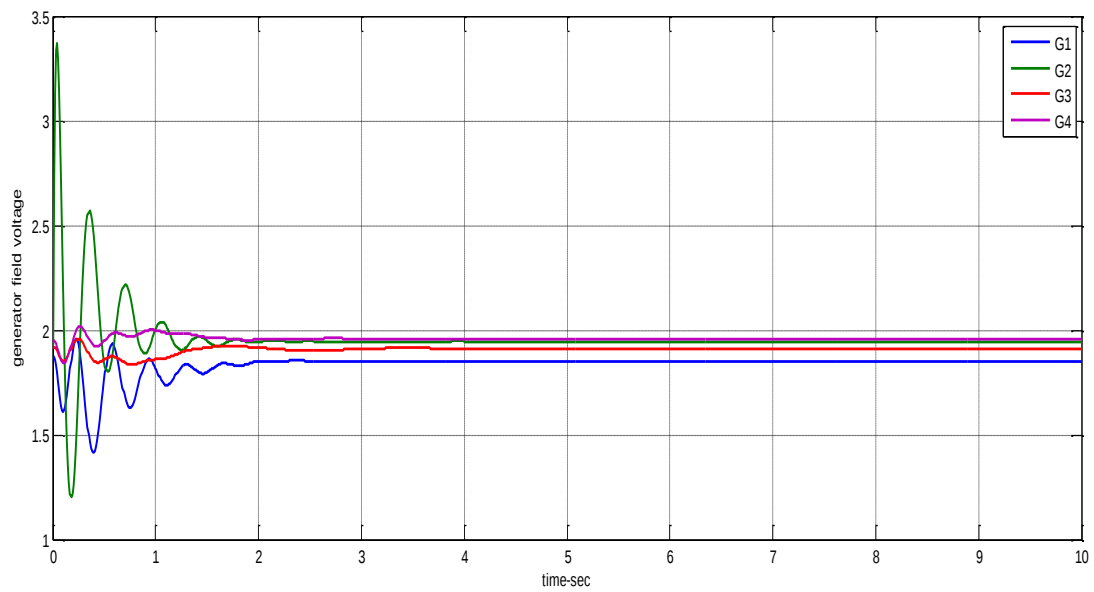


Figure 4.14:Response of generator field voltage

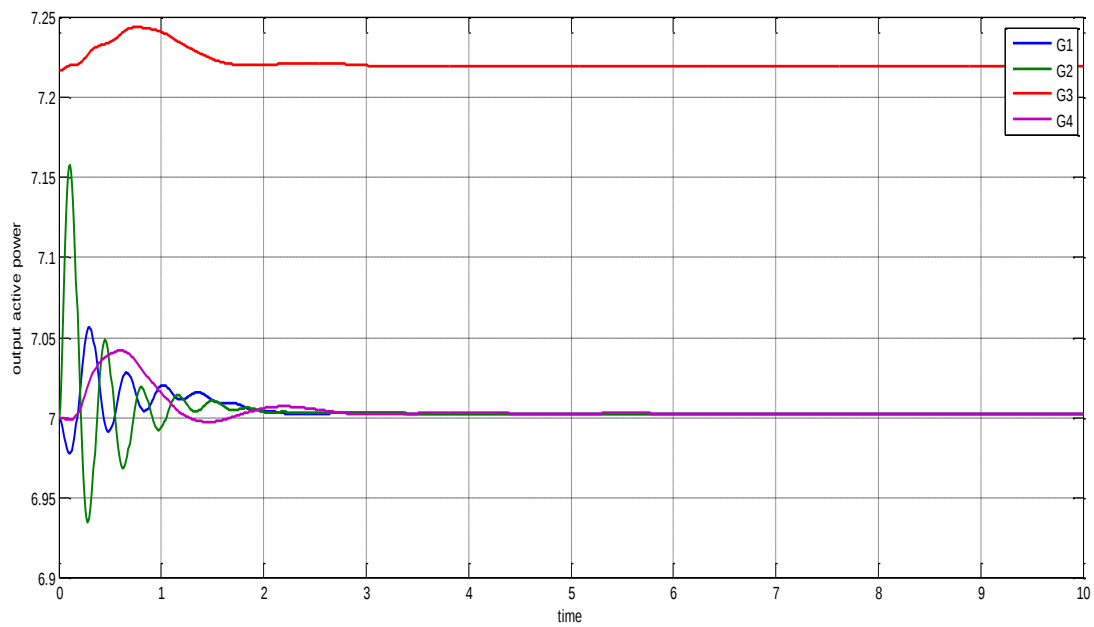


Figure 4.15: Response of output active power

4.6.2 Large Disturbances

Three cases have been assumed.

- **Three phase fault occurs at bus four**

➤ The system without PSS

The response of the rotor angle deviation, rotor speed deviation, generator field voltage and output active power were shown in figures (4.16)-(4.19).

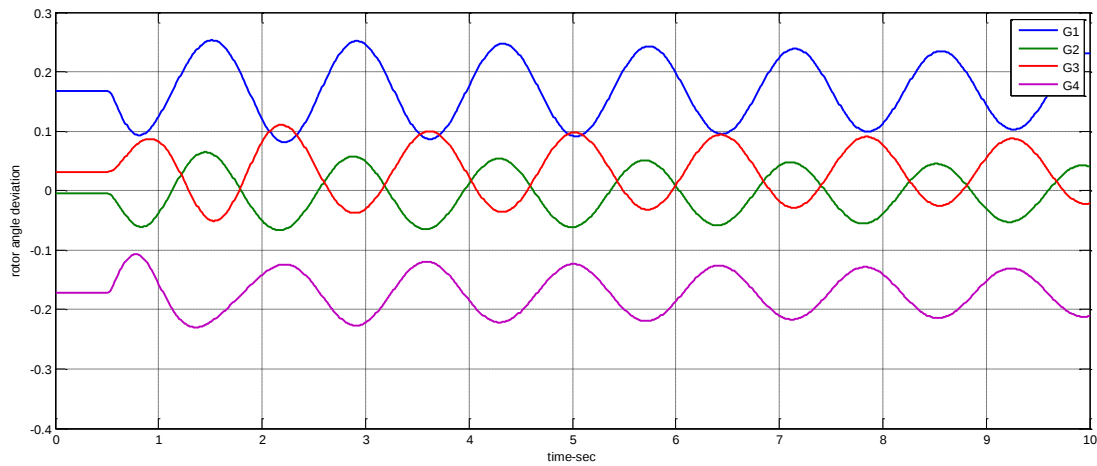


Figure 4.16: Response of rotor angle deviation

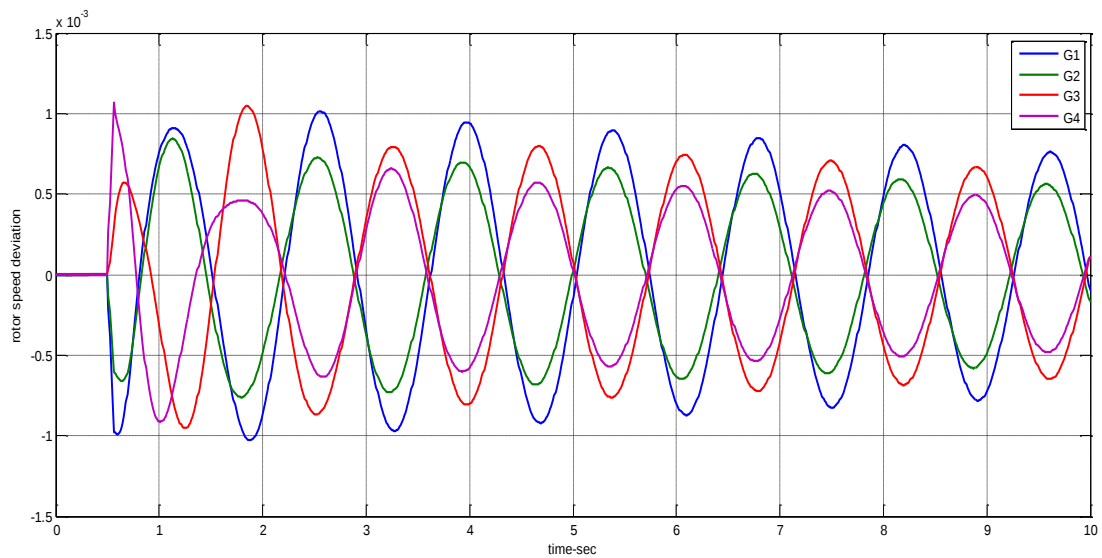


Figure 4.17: Response of rotor speed deviation

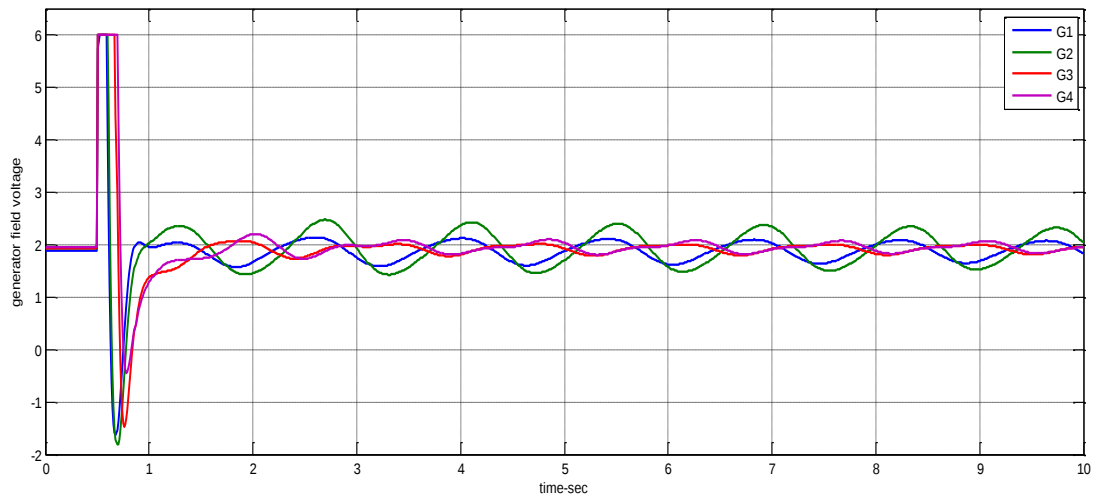


Figure 4.18: Response of generator field voltage

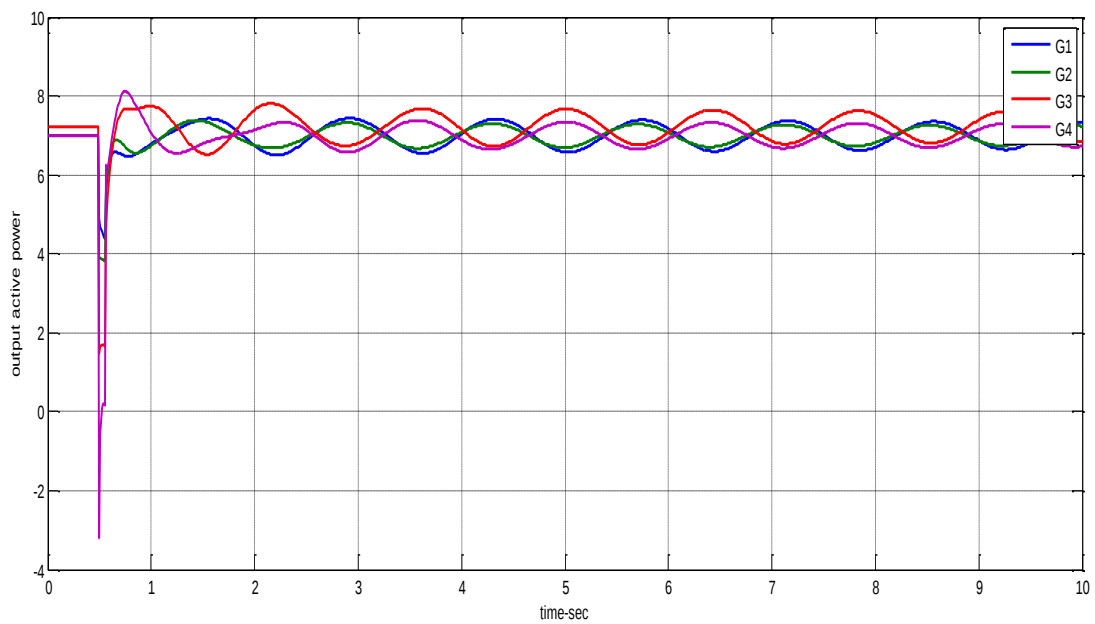


Figure 4.19: Response of output active power

➤ The system with PSS

The response of the rotor angle deviation, rotor speed deviation, generator field voltage and output active power were shown in figures (4.20)-(4.23).

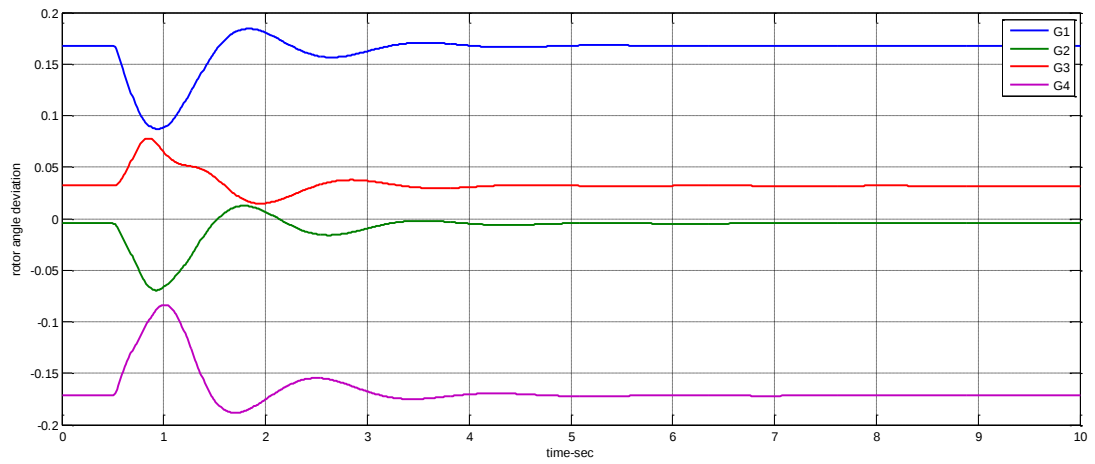


Figure 4.20: Response of rotor angle deviation

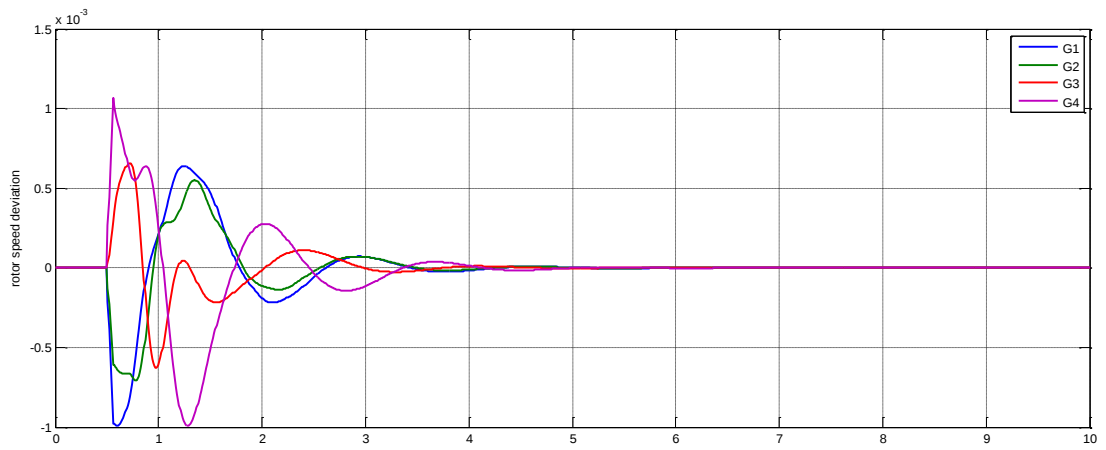


Figure 4.21: Response of rotor speed deviation

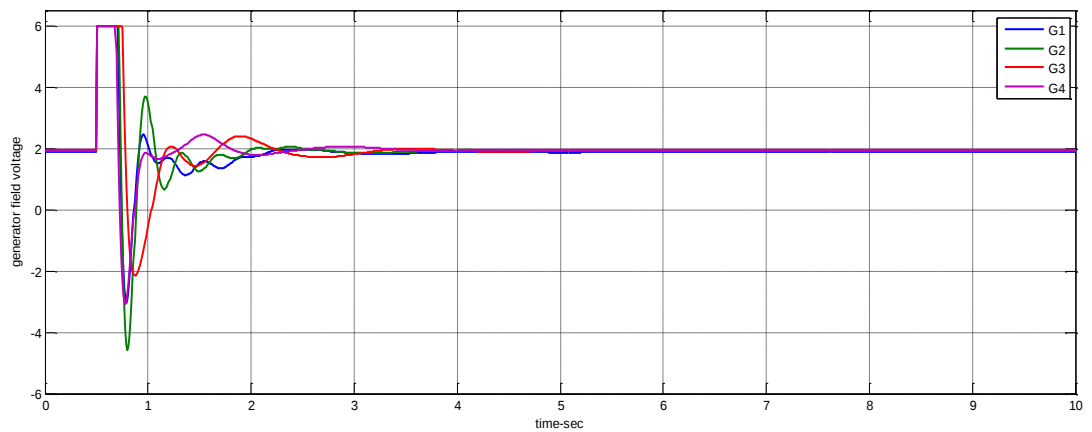


Figure 4.22: Response of generator field voltage

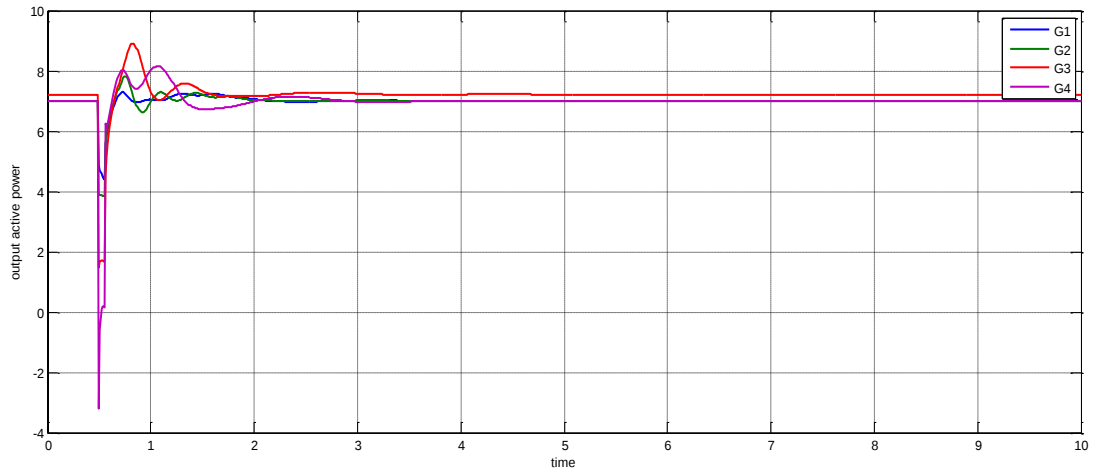


Figure 4.23: Response of output active power

- **Three phase fault occurs at bus nine and tripping in line (six – nine)**

➤ **The system without PSS**

The response of the rotor angle deviation, rotor speed deviation, generator field voltage and output active power were shown in figures (4.24)-(4.27).

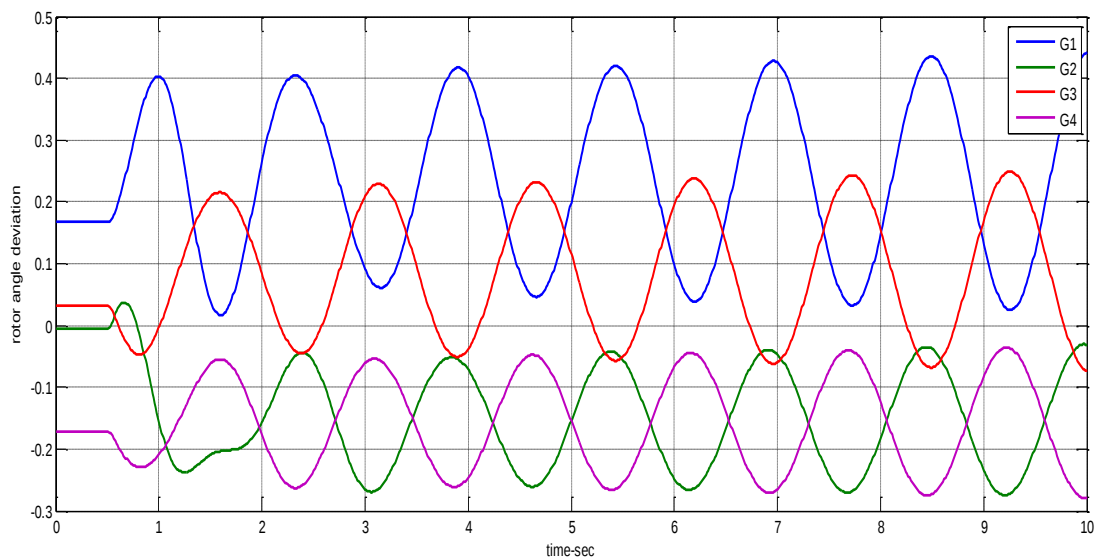


Figure 4.24: Response of rotor angle deviation

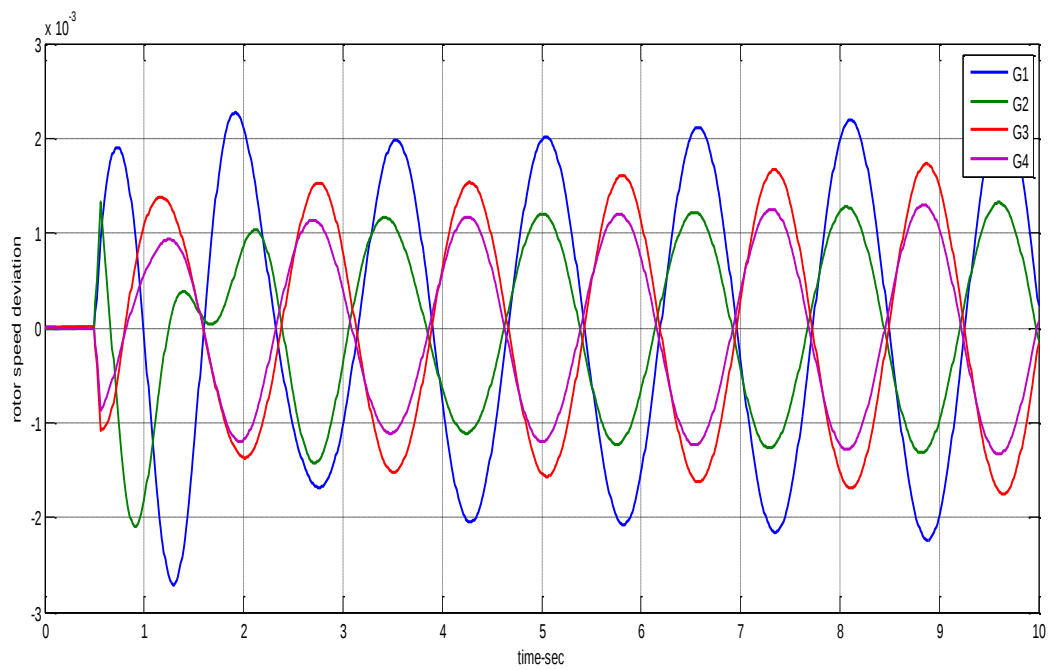


Figure 4.25: Response of rotor speed deviation

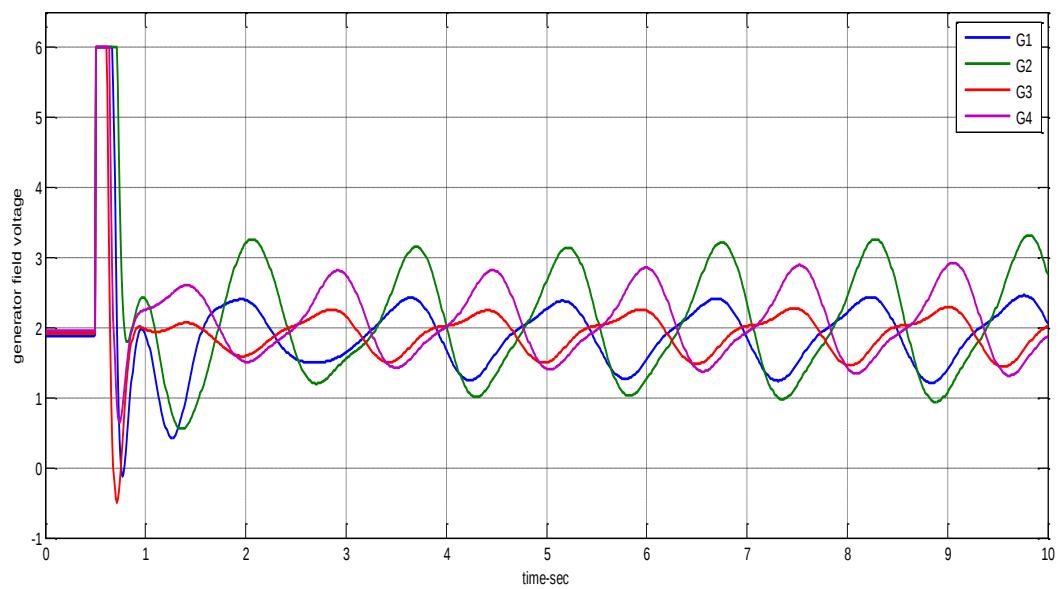


Figure 4.26: Response of generator field voltage

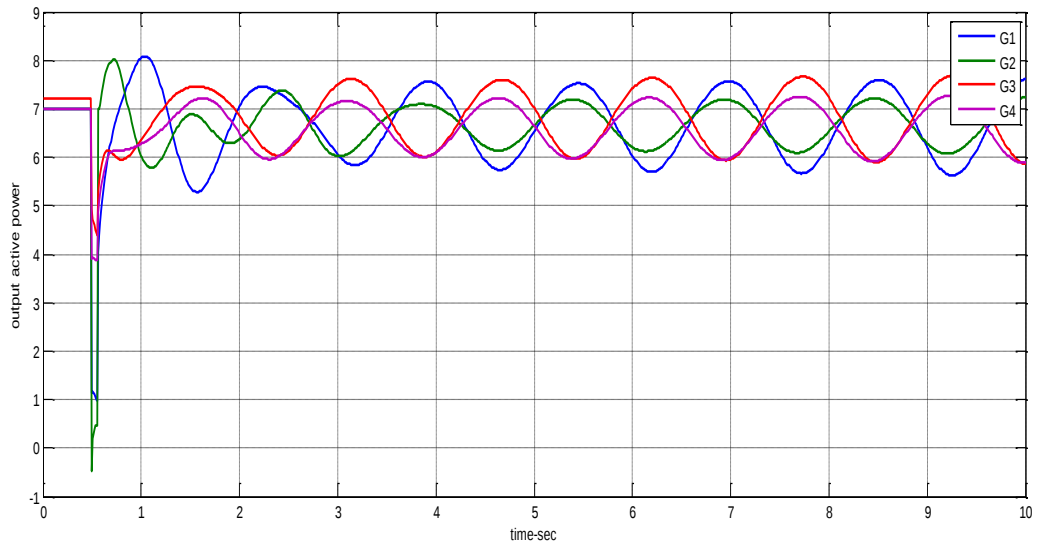


Figure 4.27: Response of output active power

➤ The system with PSS

The response of the rotor angle deviation, rotor speed deviation, generator field voltage and output active power were shown in figures (4.28)-(4.31).

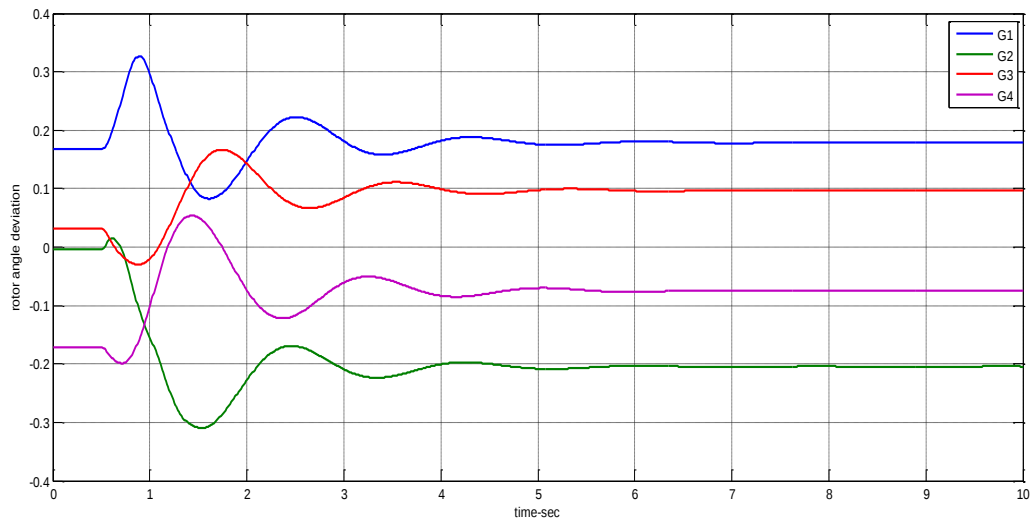


Figure 4.28: Response of rotor angle deviation

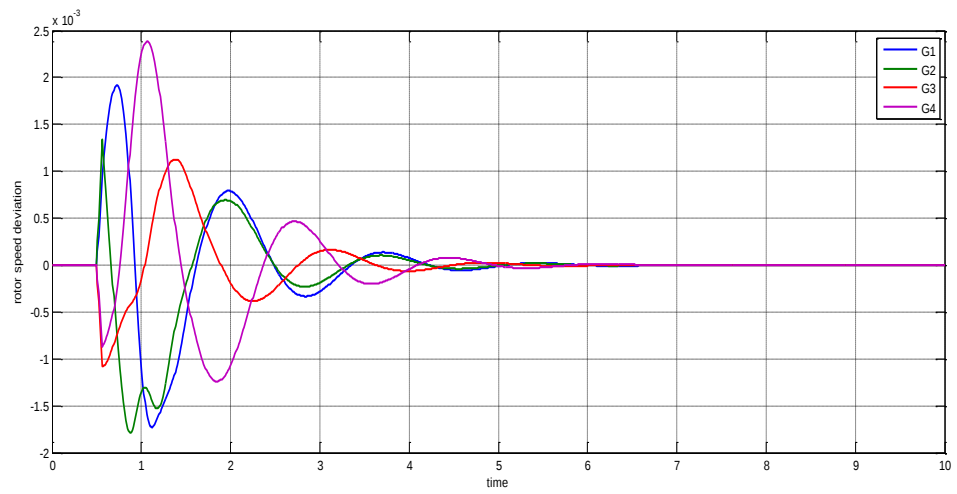


Figure 4.29: Response of rotor speed deviation

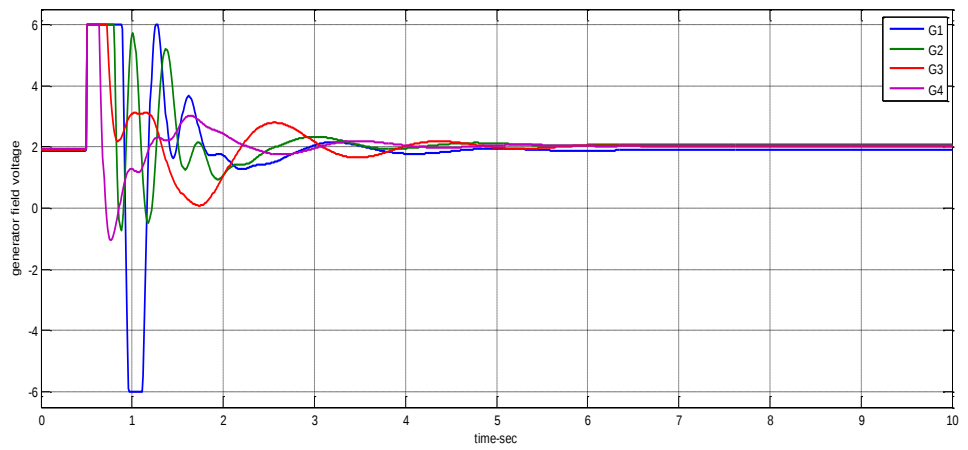


Figure 4.30: Response of generator field voltage

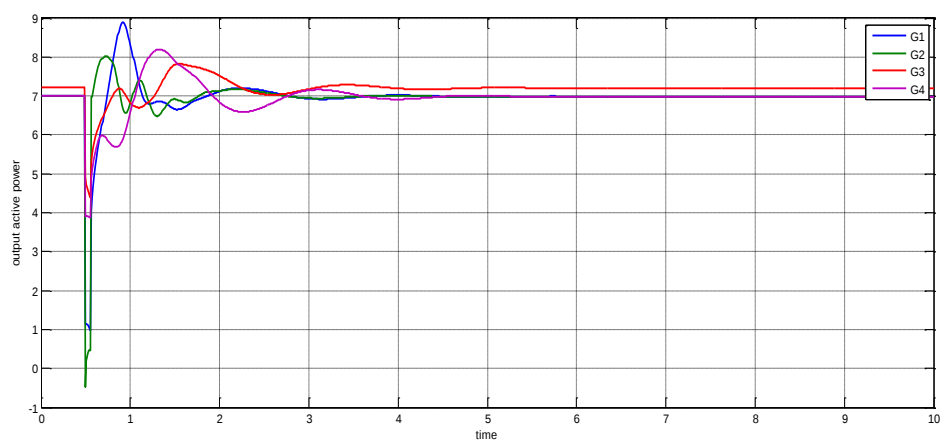


Figure 4.31: Response of output active power

- **Ramping of T_m for generator three**

The mechanical power decreased by ramping function to half value at 0.5 sec.

➤ **The system without PSS**

The response of the rotor angle deviation, rotor speed deviation, generator field voltage and output active power were shown in figures (4.32)-(4.35).

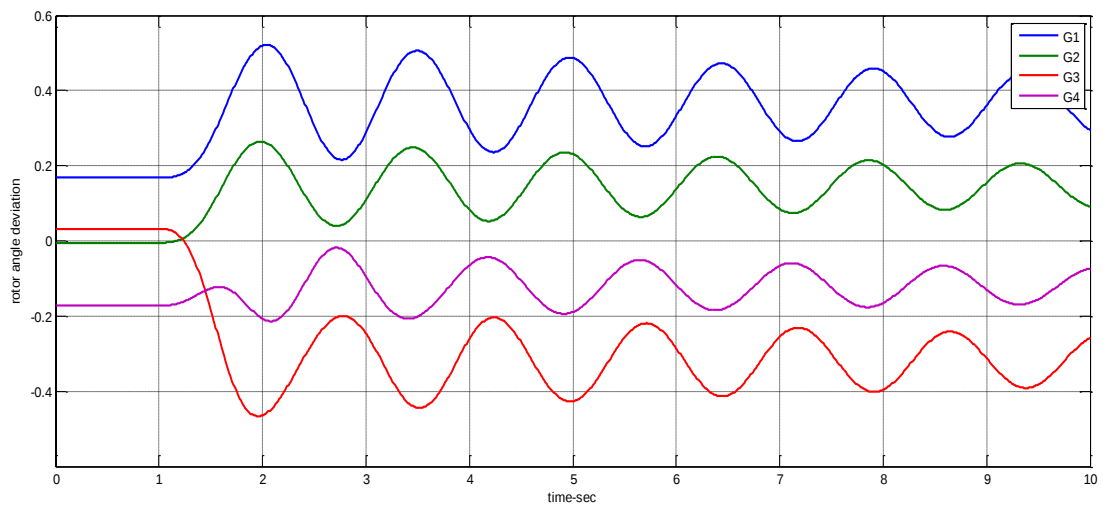


Figure 4.32: Response of rotor angle deviation

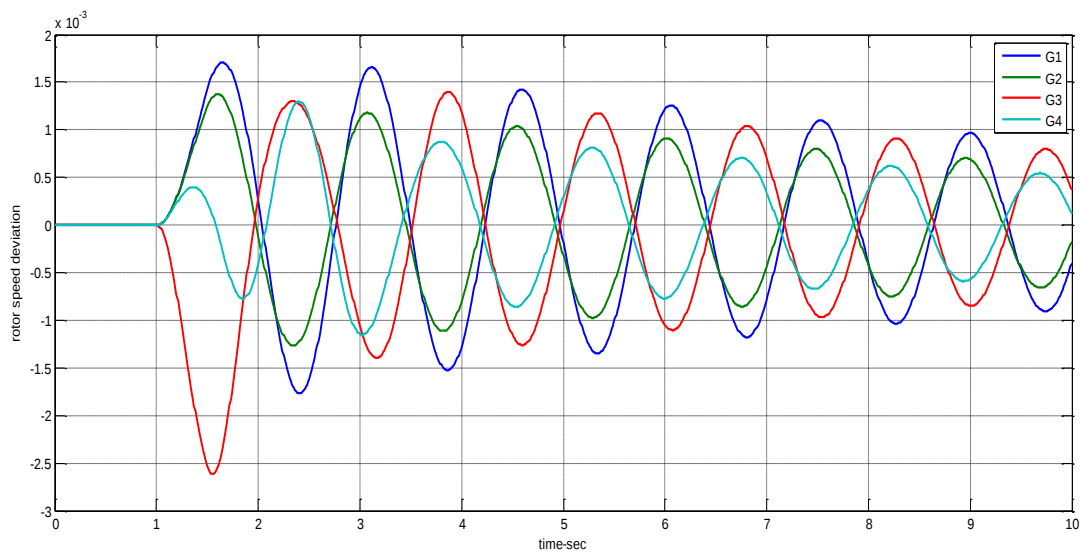


Figure 4.33: Response of rotor speed deviation

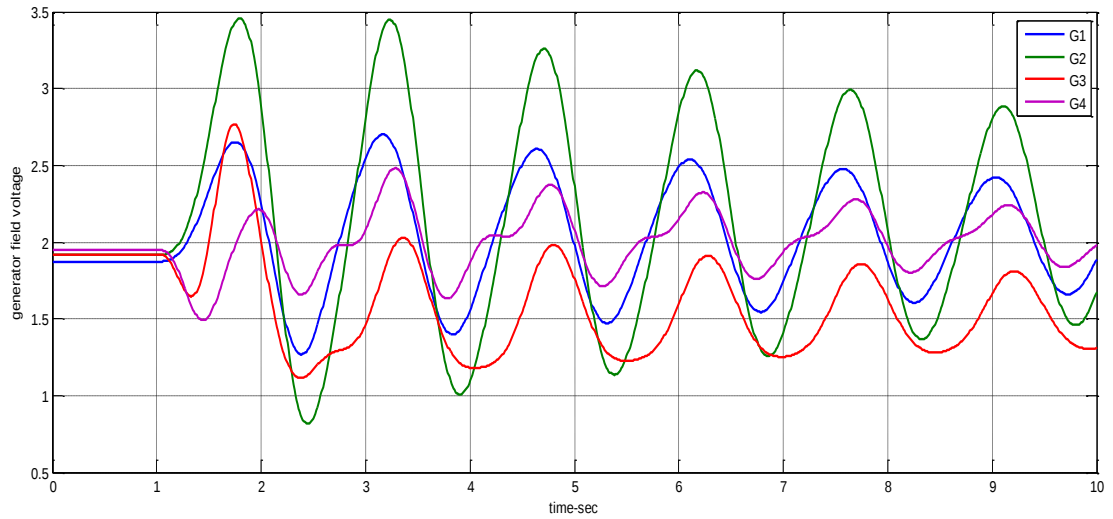


Figure 4.34: Response of generator field voltage

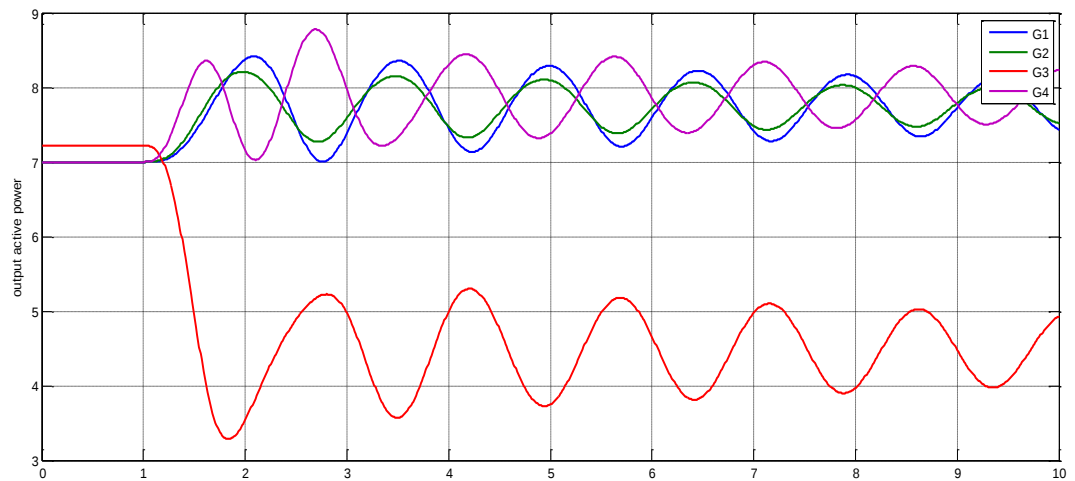


Figure 4.35: Response of output active power

➤ The system with PSS

The response of the rotor angle deviation, rotor speed deviation, generator field voltage and output active power were shown in figures (4.36)-(4.39).

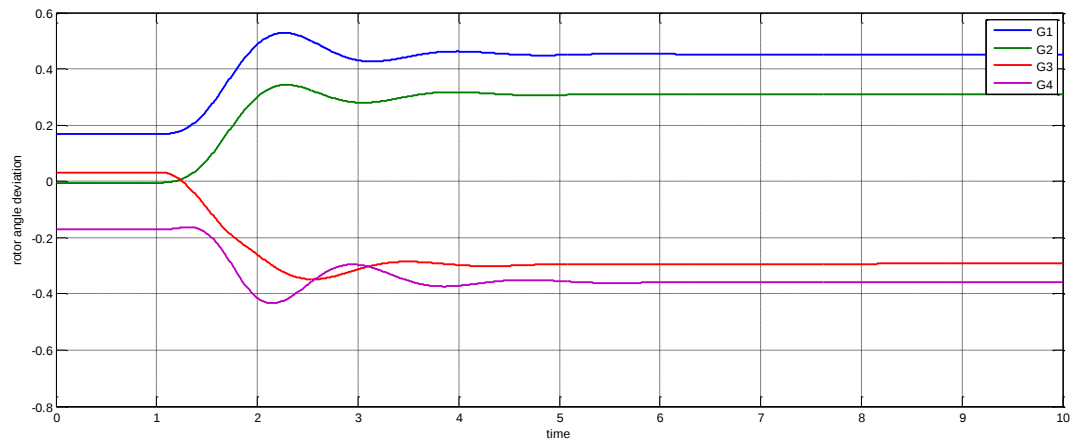


Figure 4.36: Response of rotor angle deviation

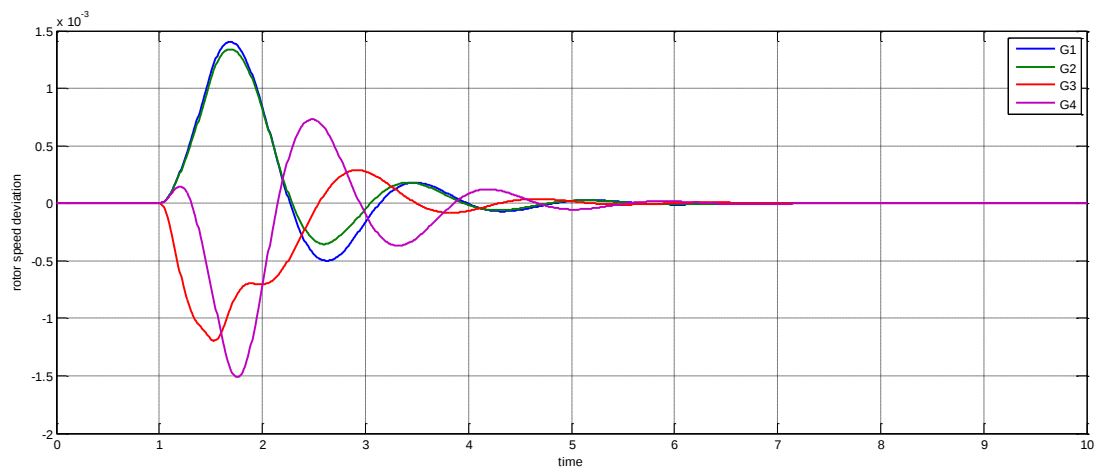


Figure 4.37: Response of rotor speed deviation

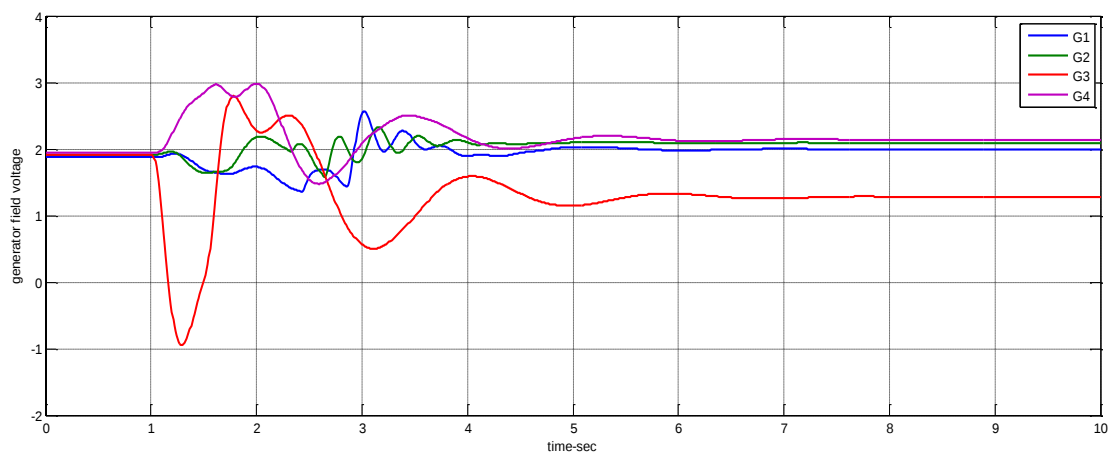


Figure 4.38: Response of generator field voltage

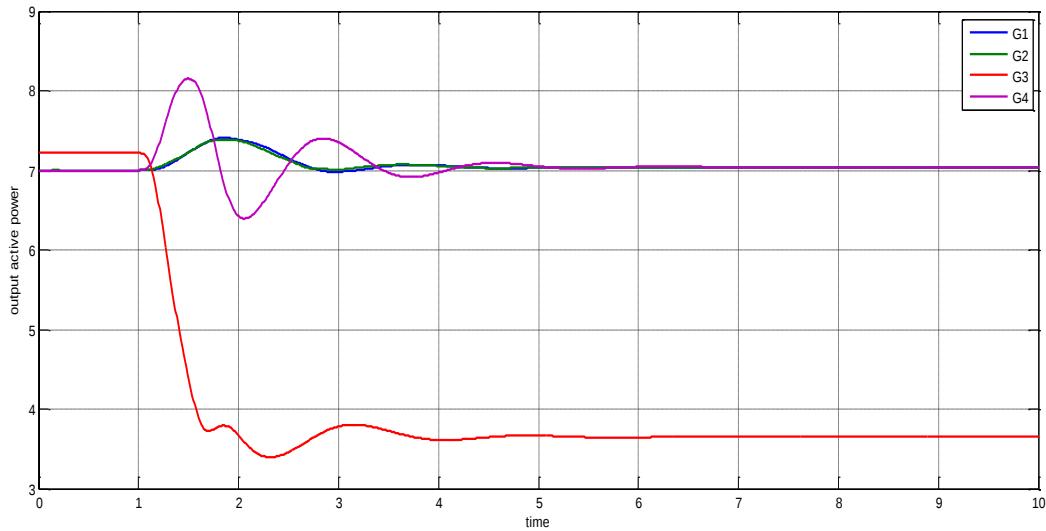


Figure 4.39: Response of output active power

4.8 Results Analysis

- ❖ Frequencies of swing modes in all cases have the same values approximately in eignvalue method, and time domain simulation method as shown in table (4.5).
- ❖ Implementation of PSSs reduces the settling time for all cases effectively specially at small disturbance. Table (4.6) represent the settling time for all conditions.
- ❖ PSSs decreases the maximum overshoot specially at small disturbance; but not effective at large disturbances except the reference generator, and the generator far away from the fault location; as example the maximum overshoot problem was discussed for rotor angle deviation; as shown in figures (4.40)-(4.43).

❖ Method of calculation the frequency from time domain simulation

1) Plot the response of speed deviation:

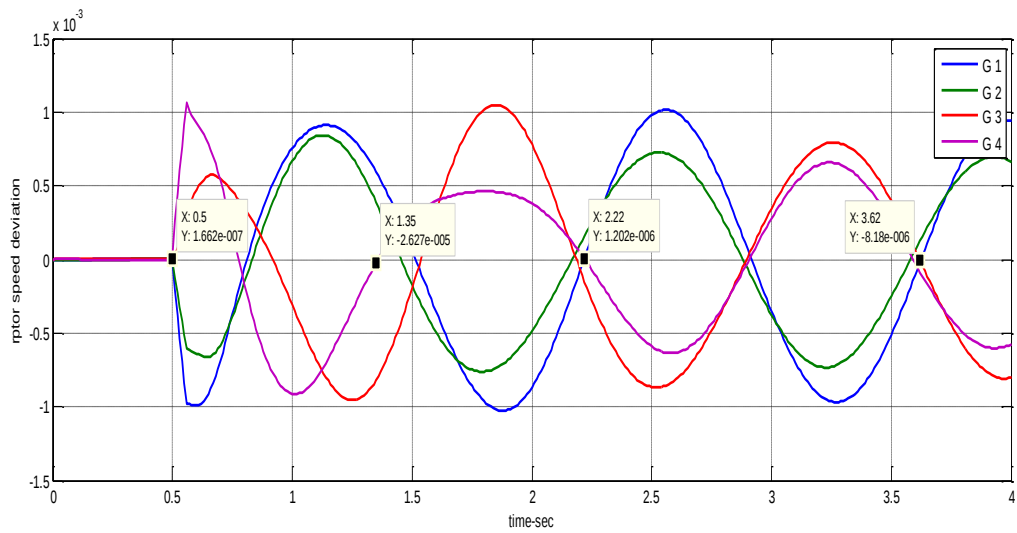


Figure 4.40: Method of calculation the frequency

$$T_1 = 1.35 - 0.5 = 0.85 \text{ sec/cycle.}$$

$$T_2 = 3.62 - 2.22 = 1.4 \text{ sec/cycle.}$$

2) the oscillation frequencies is the inverse of the cycle times

$$F_1 = \frac{1}{T} = \frac{1}{.85} = 1.1764 \text{ Hz.}$$

$$F_2 = \frac{1}{T} = \frac{1}{1.4} = 0.7142 \text{ Hz.}$$

3) repeat the above steps for all figure.

Table 4.5: Comparison in frequency of oscillation between eignvalue and time domain simulation methods

Cases	Oscillation frequencies			
	Without PSS		With PSS	
	Eigenvalue	Time domain	Eigenvalue	Time domain
Perturbation of V_{ref}	1.1853	1.400	1.3561	1.1000
	1.0820	1.0800		
	0.7096	0.7100	0.57366	0.55000
Fault at bus 4	1.1853	1.1628	1.3561	1.4000
	1.0820	1.0500		
	0.7096	0.6993	0.57366	0.5900
Fault at bus 9 And trip line (six-nine)	1.1853	1.2400	1.3561	1.1000
	1.082	0.9200		
	0.70956	0.6800	0.57366	0.5700
Ramping of T_m	1.1853	1.100	1.3561	0.9800
	1.082	0.9810		
	0.70956	0.69444	0.57366	0.59000

Table 4.6: The settling time for all cases

The cases	Variables	Settling time (sec)	
		Without PSS	With PSS
Perturbation of V_{ref}	$\Delta\delta$	Oscillatory	1.0
	$\Delta\omega$	Oscillatory	3.0
	E_{fd}	Oscillatory	2.0
	P	Oscillatory	2.0
Fault at bus 4	$\Delta\delta$	Oscillatory	3.5
	$\Delta\omega$	Oscillatory	4.0
	E_{fd}	Oscillatory	3.0
	P	Oscillatory	2.0
Fault at bus 9 and trip line (six-nine)	$\Delta\delta$	Oscillatory	4.0
	$\Delta\omega$	Oscillatory	4.5
	E_{fd}	Oscillatory	4.5
	P	Oscillatory	4.5
Ramping of T_m	$\Delta\delta$	Oscillatory	5.0
	$\Delta\omega$	Oscillatory	5.0
	E_{fd}	Oscillatory	5.0
	P	Oscillatory	4.5

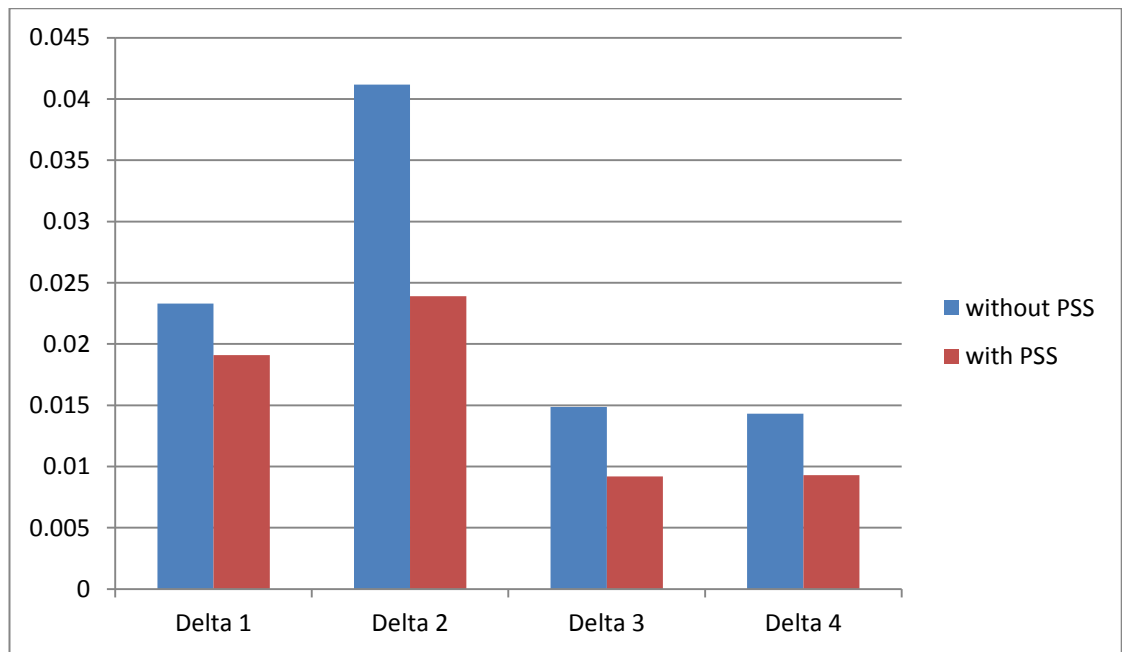


Figure 4.41: The maximum overshoot of rotor angle deviation at perturbation of V_{ref} for generator two

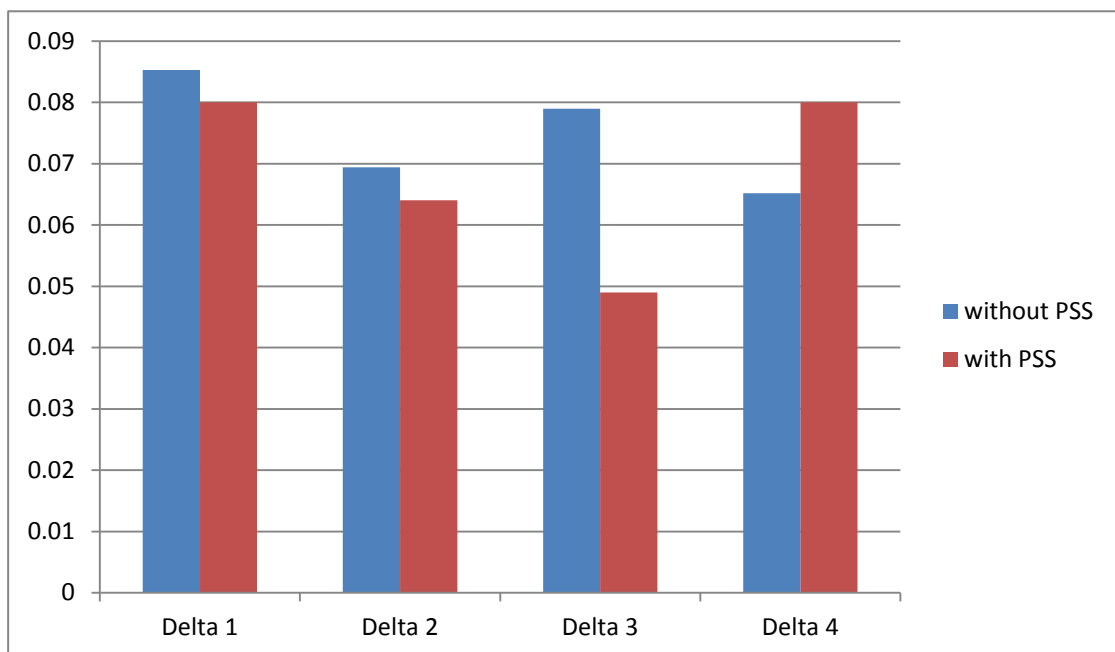


Figure 4.42: The maximum overshoot of rotor angle deviation at fault condition at bus 4

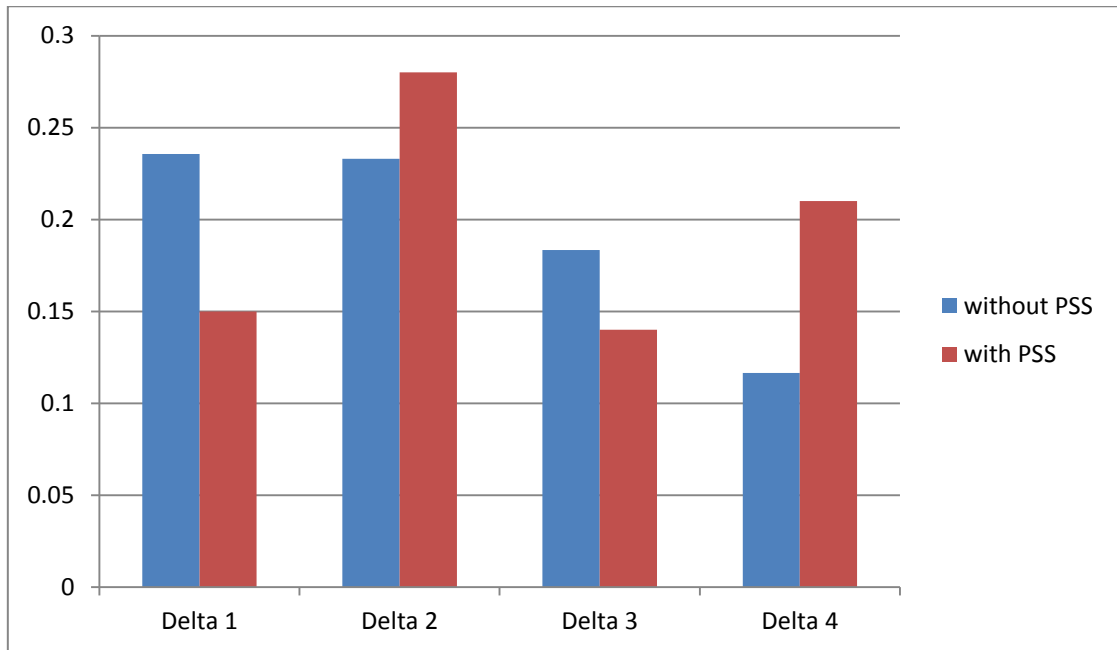


Figure 4.43: The maximum overshoot of rotor angle deviation at fault condition at bus 9 and trip line (6-9)

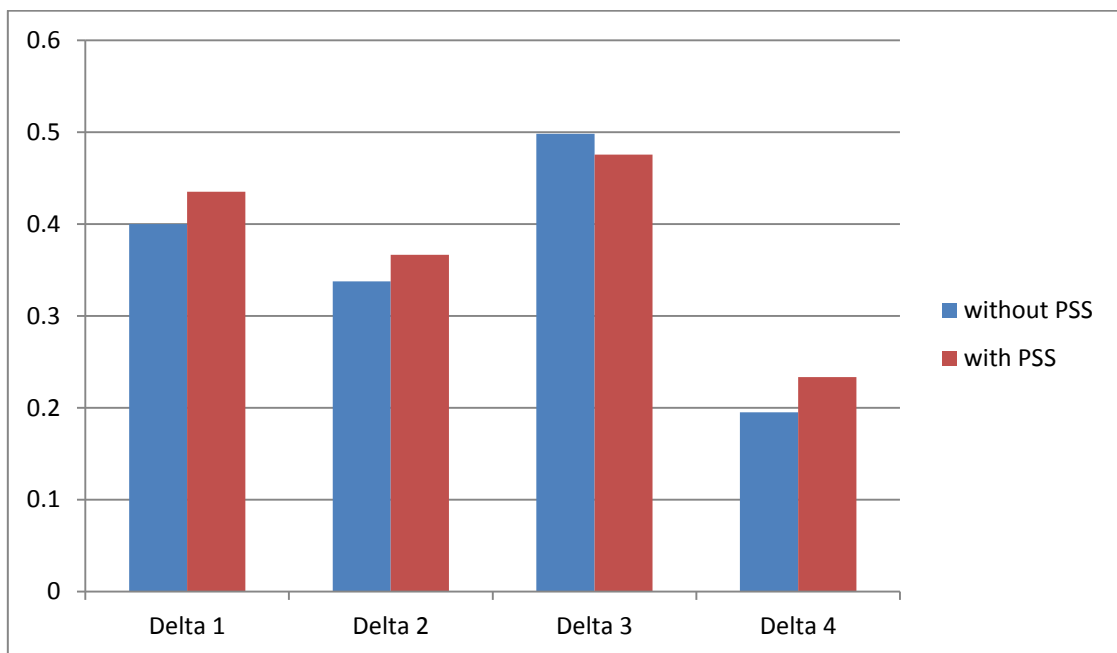


Figure 4.44: The maximum overshoot of rotor angle deviation at ramping of T_m in generator three

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

The problem of the power system stability (rotor angle stability) of multi-machines system has been addressed in this thesis by using eignvalue method, and time domain simulation. The system in base case (the system without PSS) is a stable at normal condition (no disturbance occurs) but have poor damping. To provide adequate damping ratio, the power system stabilizer has been designed by using phase compensation technique.

Before design of PSS the optimal location of PSS has been specified according to participation factor, which represent the contribution of machines in modes. After that the effect of PSS has been discussed for small and large disturbances.

Finally the power system stabilizer (PSS) is a cost effective way of improving the damping of electromechanical oscillations of rotor and return the stability to the system. Also it improves the power transfer capability of transmission lines.

5.2 RECOMMENDATIONS

In this project design and optimal location of PSS to improve dynamic stability were determined, speed deviation was taken as the input signal; other investigations:

- Change the input signal to PSS such as terminal bus frequency or electrical power output.
- Use dual input power system stabilizer signals such as speed and electrical power output are used.
- Use other methods to appropriate tuning of PSS parameters such as Particle Swarm Optimization (PSO) technique or Multi-Objective Honey Bee Mating Optimization (MOHBMO) [11], [12].
- Use other methods to specify the optimal location of PSS such as Genetic Algorithm (GA) method [13].
- Improve the stability by using another controller addition to PSS such as one of the flexible AC transmission system (FACTS) family.