

Dedication

To my Parents, wife, sons, daughters and the rest of family

Acknowledgements

I would like to express my deepest gratitude to my respected supervisor Prof: Shawgy Hussein AbdAlla for the assistance provided by him, and I am most appreciative of the time and effort he spent with me. His comments and suggestions were of great value to me in the preparation of this work.

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Abstract

We show the extensions of the abelian categories and the method of the strong no loops conjecture. We also show the strong no loop conjecture for special biserial algebras for algebras with two simples and radical cube zero and is true for mild algebras. The minimal projective resolutions with the decomposition of group-valued measures on orthoalgebras are considered. We study the uniform lattices with the topological MV-algebras and topological ℓ -groups. We give the categories of fractions and excision in K -Theory with the Thom isomorphism in Bivariant K -Theory and Weyl algebra. We determine the first Alexandroff decomposition theorem for topological lattice group valued measures with the universal cycles and homological invariants of locally convex algebras.

الخلاصة

أوضحنا التمديدات للطبقات الأبيلية وطريقة تخمين الا حلقات القوية. أيضا أوضحنا تخمين اللاحقة القوية للجبريات ثنائية التسلسل الخاصة وللجبريات طبقا لبسيطين والصفير المكعب نصف القطرى وصحيح للجبريات المعتدلة. تم اعتبار إعادة حلول الاسقاط الاصغرى مع تفكيك قيمة الزمرة على جبريات الاورثو. درسنا الشبكات المنتظمة مع جبريات MV -التبولوجية وزمر- ℓ التبولوجية. تم اعطاء الطبقات للكسور والاكشن في نظرية K مع ايزومورفيزم ثوم ثنائي الاختلاف في نظرية- K وجبر ويل. حددنا مبرهنة تفكيك ألكساندروف الأولى للقياسات قيمة زمر الشبكة التبولوجية مع الدورات الشاملة وغير الثوابت الهومولوجية للجبريات المحدبة الموضوعية.

Introduction

Given a ring A , we form its truncated $A \rtimes_l M$. We study the extension groups $\text{Ext}_{A \rtimes_l M}^n(X, Y)$ in relation to $\text{Ext}_A^n(X, Y), n \in \mathbb{N} \cup \{0\}$. If ${}_A M$ is flat, then using the theory of spectral sequences, we obtain a precise formula for $\text{Ext}_{A \rtimes_l M}^n(X, Y)$ for simple $(A \rtimes_l M)$ -modules. We present an algorithmic method for computing a projective resolution of a module over an algebra over a field. If the algebra is finite dimensional, and the module is finitely generated, we have a computational way of obtaining a minimal projective resolution, maps included.

There are strong analogies between the theory of topological Riesz spaces and the theory of topological Boolean rings. We study uniform lattices with the property (σ) or (F) . We present a general decomposition theorem for a positive inner regular finitely additive measure on an orthoalgebra L with values in an ordered topological group, not necessarily commutative. In the case where L is a Boolean algebra, we establish the uniqueness of such a decomposition.

We introduce a new version kk^{alg} of bivariant K -theory that is defined on the category of all locally convex algebras. A motivating example is the Weyl algebra, i.e. the algebra generated by two elements satisfying the Heisenberg commutation relation, with the fine locally convex topology.

We establish the strong no loop conjecture for some special cases, in particular, for special biserial algebras. Let Λ an artinian ring and let r denote its Jacobson radical. We show that a simple module of finite projective dimension has no self-extensions when Λ is graded by its radical. For A a finite dimensional associative algebra over an algebraically closed field with a simple module S of finite projective dimension. The strong no loop conjecture says that this implies $\text{Ext}_\Lambda^1(S, S) = 0$, i.e. that the quiver of Λ has no loops in the point corresponding to S .

We show an Alexandroff decomposition type theorem, which extends a decomposition theorem been in. We show a representation theorem for complete MV-algebras endowed

with a Hausdorff order continuous locally convex topology which admits a 0 neighbourhood base consisting of sublattices.

We give a simple proof of the smooth Thom isomorphism for complex bundles for the bivariant K -theories on locally convex algebras considered by Cuntz. Using an appropriate notion of locally convex Kasparov modules, we show how to induce isomorphisms under a large class of functors on the category of locally convex algebras; examples are obtained from spectral triples. Our considerations are based on the action of algebraic K -theory on these functors, and involve compatibility properties of the induction process with this action, and with Kasparov-type products. This is based on an appropriate interpretation of the Connes-Skandalis connection formalism.

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