CHAPTER ONE

Introduction

(1.1)History of superconductivity:

Superconductivity is the ability of certain materials to conduct electrical current with practically zero resistance. To his produces interesting and potentially useful effects. For a material to behave as a superconductor, low temperatures are required.

(1.1.1) Discovery of superconductivities:

Superconductivity was first observed in 1911 by H. K. Ones, a Dutch physicist. His experiment was conducted with elemental mercury at 4 degrees Kelvin (approximately -452 degrees Fahrenheit), the temperature of liquid helium. Since then, some substances have been made to act as superconductors at higher temperatures, although the ideal a material that can superconductor at room temperature remains elusive.

(1.2) Applications of superconductivity:

High-temperature superconductors may lead to many important technological Advances, Such as highly efficient, lightweight superconducting motors. However, any significant materials-science problems must be overcome before such applications Become reality. Perhaps the most difficult technical challenge is to mold the brittle Ceramic materials into useful shapes, such as wires and ribbons for large-scale applications and thin films for small devices (e.g., SQUIDs). Another major problem is the relatively low current density that bulk ceramic compounds

can carry assuming that such problems will be overcome; it is interesting to speculate on some of the Future applications of these newly discovered materials.
An obvious application using the property of zero resistance to direct currents is Low-loss electrical power transmission. A significant fraction of electrical power is lost as heat when current is passed through normal conductors. If power transmission lines could be made superconducting, these dc losses could be eliminated and substantial savings in energy costs would result.
The new superconductors could have a major impact in the field of electronics. Because of its switching properties, the Josephson junction can be used as a computer element. In addition, if one could use superconducting films to inter connect computer chips, chip size could be reduced and consequently speeds would be enhanced.
Thus information could be transmitted more rapidly and more chips could be contained on a circuit board with far less heat generation.
An important application of superconducting magnets is a diagnostic tool called magnetic resonance imaging (MRI). This technique has played a prominent role in diagnostic medicine because it uses relatively safe radio-frequency radiation, rather than x-rays, to produce images of body sections. Because the technique relies on intense magnetic fields generated by superconducting magnets, the initial and operating costs of MRI systems are high. A liquid-nitrogen-cooled magnet could reduce such costs significantly.

(1.3)Research problem

The quantization of magnetic field in superconductivity is complicate

(1.4) Aim of the work

The aim of this work is to deduce the quantization of magnetic field using simplified equation.

(1.5) Presentation of the thesis

The research included three chapters. In the first chapter, we introduced the detailed information about discovery, definitions and applications of super conductivity and also the research problem and aim of the work has been conducted. In chapter two types of superconductive, magnetic field flux quantization and the different ideas that related to superconductive such BCS theory, Meissners' effect and Copper pair are studied. In chapter three magnetic flux and quantization of magnetic flux.

CHAPTER TWO

Theory of Superconductivity

(2.1)Introduction

Superconductivity is characterized by a vanishing static electrical resistivity and an expulsion of the magnetic field from the interior of a sample where fore mainly this chapter is dealing with the theory of super conductivity. We want to understand superconductivity using methods of theoretical physics.

(2.2)The Cooper Pair

In 1956 Cooper [14] showed that the attraction, however weak, may bind a pair of electrons. He started with Cooper's equation:

$$\omega_q \alpha(k,q) = [\varepsilon\left(\left|k+\frac{q}{2}\right|\right) + \varepsilon(\left|-k+\frac{q}{2}\right|)\alpha(k,q)$$

$$\frac{1}{(2\pi h)^2} v_0 \int d^2 k\alpha(\acute{k},q) \qquad (2.1.1)$$

Where is the energy of a pairon, $\alpha(k,q)$. the wavefunction and the interaction strength. The solution of Eq. (2-1-1) for small moment yields:

$$\omega_q = \omega_0 + cq < 0 , \omega_0 = \frac{-2h\omega_D}{\exp\left[\frac{2}{\upsilon_0 N(0)}\right]-1} \qquad (2.1.2)$$

Where for $\frac{c}{v_F} = 1/2(\frac{2}{\pi})$ (2.1.3)

For 3D. The constant strength can be justified for the acoustic phonon exchange:

$$\nu_0 = |v_q|^2 \frac{1}{2\omega_q} = A_q^2 \frac{h_q^2}{2\omega_q} = A_q^2 \frac{1}{2c_s^2} \quad (2.1.4)$$

Where $\omega_q = c_s q$ and Cs= the sound speed.

(2.3) Flux quantization:

We now two concentric superconducting cylinders that are thick compared to the London penetration depth λ_l a magnetic flux

$$\phi = \int_o d^2\tau B \quad (2.2.1)$$

Penetrates the inner hole and a thin surface layer on the order of λ_l of the inner cylinder the only purpose of the inner cylinder is to prevent the magnetic field from touching the outer cylinder which we are really interested in the outer cylinder is completely field- free we want to find the possible values of the flux ϕ.

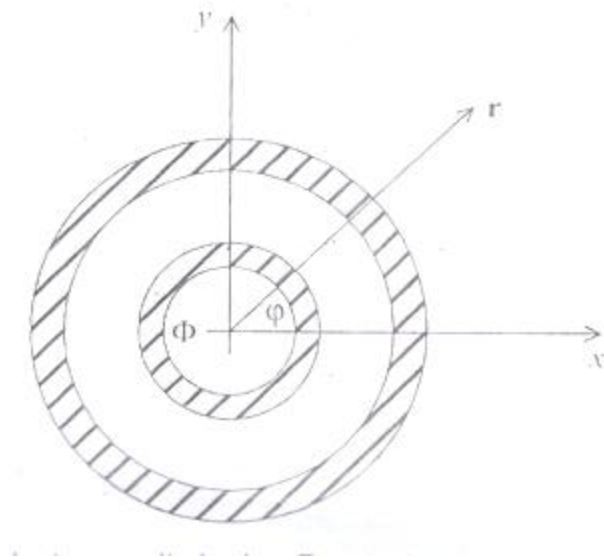


Figure (2.1).the surface layer

Although the region outside of the inner cylinder has B=0 the vector potential does not vanish the relation $\nabla x A = B$ implies.

$$\oint dsA - \iint d^2 rB = \phi \qquad (2.2.2)$$

By symmetry the tangential part of A is

$$A_\varphi = \frac{\phi}{2\pi\tau} \qquad (2.2.3)$$

The London gauge requires this to be the only non-zero component. Thus outside of the inner cylinder we have in cylinder.

$$A = \frac{\phi}{2\pi\tau}\varphi = \nabla\frac{\phi\varphi}{2\pi} \qquad (2.2.4)$$

Since these is a pure gradient we can get from A=0to A= $(\phi/2\pi\tau)\varphi$ by a gauge transformation

$$A \to A + \nabla x \qquad (2.2.5)$$

With

$$x = \frac{\phi}{2\pi\tau} \qquad (2.2.6)$$

X is continuous but multivolume outside of the inner cylinder we recall that gauge transformation of A must be accompanied by a transformation of the wave function.

$$\psi_s^{\Phi} = \exp(\frac{ie}{he}\sum_j x(r_j)\psi_s \qquad (2.2.7)$$

This is most easily seen by noting that this guarantees the current to remain invariant under gauge transformation thus the wave function at $\Phi = 0(A = 0)$ and at non-zero flux Φ are related by:

$$\psi_s^{\Phi} = \exp(\frac{ie}{he}\sum_j \frac{\Phi\varphi_j}{2\pi})\psi_s^{0} = \exp(-i\frac{e}{hc}\Phi\sum_j \varphi_j)\psi_s^{0} \qquad (2.2.8)$$

Where φ_j is the polar angle of electron j for ψ_s^{Φ} as well as ψ_s^{0} to be single valued and continuous the exponential factor must not change for $\varphi_j \to \varphi_j + 2\pi$ for any j this case if

$$\frac{e}{hc}\Phi \in z \Leftrightarrow \Phi = n\frac{hc}{e} with.n \in z \qquad (2.2.9)$$

We find that the magnetic flux is quantized in unites of he /e note that the inner cylinder can be dispensed with assume we are heating it enough to become normal conducting then the flux will fill the whole interior of the outer cylinder plus a thin (on the order of λ_j)layer on inside but if the outer cylinder is much thicker than λ_j this should not affect ψ_s^{0} appreciably away from this thin layer .

The quantum he /e is actually not correct based in the idea that two electrons could form a boson that could Bose –Einstein condense on sager suggested that the relevant charge is 2einsted of e leading to the superconducting flux quantum

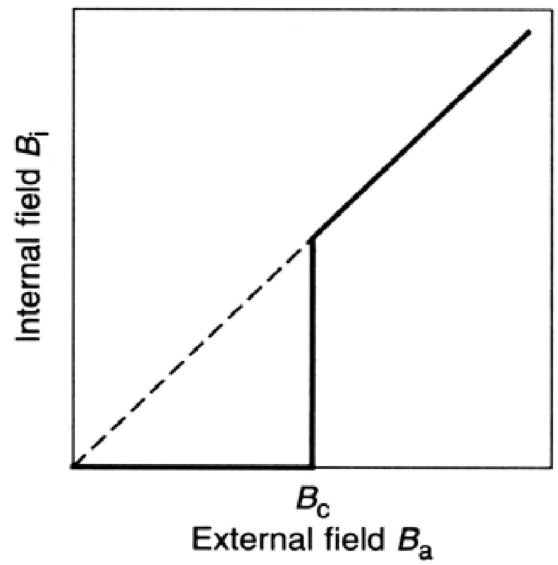
$$\Phi_0 = \frac{he}{2e} \qquad (2.2.10)$$

This is indeed found in experiments

(2.3)Types of superconductivity

(2.3.1) Type I and II superconductors:

High magnetic fields destroy superconductivity and restore the normal conducting state. Depending on The character of this transition, we may distinguish between type I and II superconductors. The graph shown in Figure 4 illustrates the internal magnetic field strength, Bi, with increasing applied magnetic field. It is found that the internal field is zero (as expected from the Meissner effect) until a critical magnetic field, Bc, is reached where a sudden transition to the normal state occurs. `This results in the penetration of the applied field into the interior. Superconductors that undergo this abrupt transition to the normal state above a critical magnetic field are known as type I superconductors. Most of the pure elements in Figure 2 tend to be type I superconductors. Type II superconductors; on the other hand, respond differently to an applied magnetic field. An increasing field from zero results in two critical fields, Bc1 and Bc2. At Bc1 the applied field begins to partially penetrate the interior of the superconductor. However, the superconductivity is maintained at this point. The superconductivity vanishes above the second, much higher, critical field, Bc2. For applied fields between Bc1 and Bc2, the applied field is able to partially penetrate the superconductor, so the Meissner effect is incomplete, allowing the superconductor to tolerate very high magnetic fields.



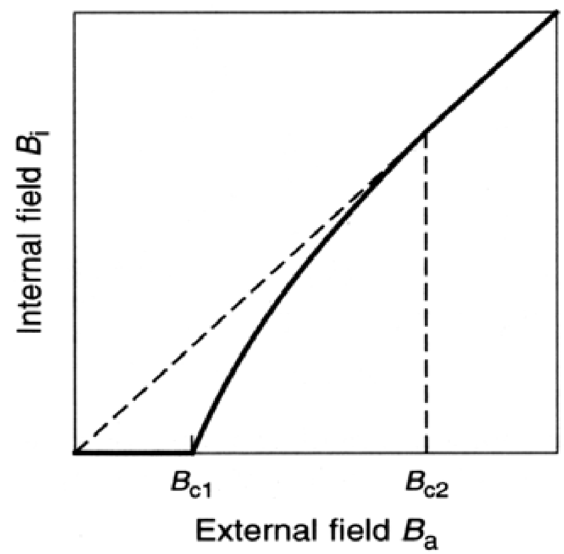


Figure (2.2), type-I superconductor behavior. **Figure (2.3), type-II superconductor behavior.**

Type II superconductors are most technologically useful because the second critical field can be quite high, enabling high field electromagnets to be made out of superconducting wire. Most compounds shown in Figure (2.2) are type-II superconductors. Wires made from say niobium-tin (Nb3Sn) have a Bc2 as high as 24.5 Tesla – in practice it is lower. This makes them useful for applications requiring high magnetic fields, such as Magnetic Resonance Imaging (MRI) machines. The advantage of using superconducting electromagnets is that the current only has to be applied once to the wires, which are then formed into a closed loop and allow the current (and field) to persist indefinitely – as long as the superconductor stays below the critical temperature. That is, the external power supply can be switched off. As a comparison, the strongest permanent magnets today may be able to produce a field close to 1 Tesla. However, it is possible to obtain up to 24.5 Tesla from a niobium–tin superconductor.

There is a misconception amongst some non-specialists that the term "Type II" refers to the copper oxide based high temperature superconductors discovered in the late 1980s. While these are type II superconductors, so are many superconductors discovered before that time

(2.5)The Meissner effect:

In 1933, Walter Meisrsne and Robert Ochsenfeld discovered a magnetic phenomenon that showed that superconductors are not just perfect conductors. Figure 3 illustrates a thought experiment that highlights this difference. Imagine that both the ideal conductor and superconductor are above their critical temperature, Tc. That is, they both are in a normal conducting state and have electrical resistance magnetic field, Ba, is then applied. This results in the field penetrating both materials. Both samples are then cooled so that the ideal conductor now has zero resistance. It is found that the superconductor expels the magnetic field from inside it, while the ideal conductor maintains its interior field. Note that energy is needed by the superconductor to expel the magnetic field. This energy comes from the exothermic Superconducting transition. Switching off the field induces currents in the ideal conductor that Prevent changes in the magnetic field inside it – by Lenz's law. However, the initial superconductor returns to its State, i.e. no magnetic field inside or outside it.

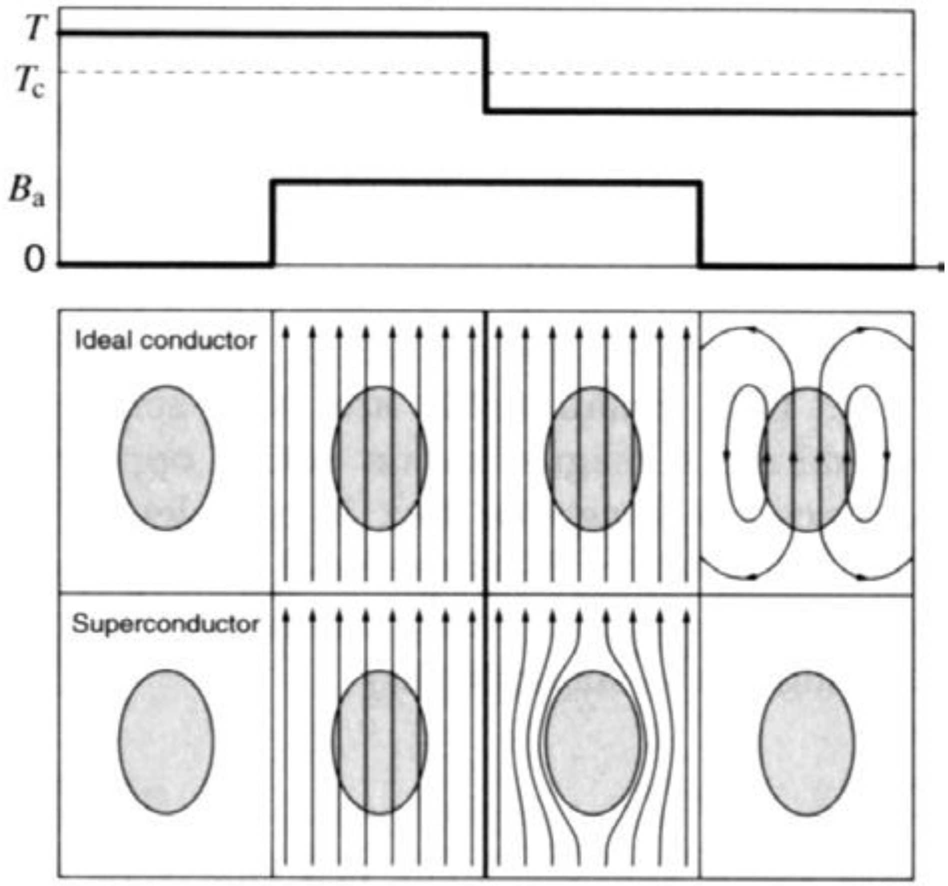


Figure (2.4), the meissner effect

(2.6)The BCS theory:

According to classical physics, part of the resistivity of a metal is due to collision between free electrons and thermally displaced ions of the metal lattice, and part is due to scattering of electrons from impurities or defects in the metal Soon after the discovery of superconductivity, scientists recognized that this classical model could never explain the superconducting state, because the electrons in a material always suffer some collisions, and therefore resistivity can never be zero. Nor could superconductivity bounder stood through a simple microscopic quantum mechanical model, where one views an individual electron as an independent wave traveling through the material. Although many phenomenological theories based on the known properties of superconductors were proposed, none could explain why electrons enter the superconducting state and why electrons in this state are not scattered by impurities and lattice vibrations. Several important developments in the 1950s led to better understanding of superconductivity. In particular, many research groups reported that the critical temperatures of isotopes of a metal decreased with increasing atomic mass.

This observation, called the isotope effect, was early evidence that lattice motion played an important role in superconductivity. For example, in the case of mercury, *Tc* _ 4.161 K for the isotope 199Hg, 4.153 K for 200Hg, and 4.126 K for 204Hg. The characteristic frequencies of the lattice vibrations are expected to change with the mass *M* of the vibrating atoms. In fact, the lattice vibration frequencies are expected to be proportional to *M*_1/2 [analogous

To the angular frequency _ of a mass-spring system, where _ _ (*k*/*M*)1/2]. On this basis, it became apparent that any theory of superconductivity for metals must include electron-lattice interactions, which is somewhat surprising because electron-lattice interactions increase the resistance of normal metals.

The full microscopic theory of superconductivity presented in 1957 by Bardeen, Cooper, and Schrieffer has had good success in explaining the features of superconductors. The details of this theory, now known as the BCS theory, are beyond the scope of this text, but we can describe some of its main features and predictions.

The central feature of the BCS theory is that two electrons in the superconductor are able to form a bound pair called a Cooper pair if they somehow experience an attractive interaction. This notion at first seems counterintuitive since electrons normally repel one another because of their like charges. However, anet attraction can be achieved if the electrons interact with each other via the motion of the crystal lattice as the lattice structure is momentarily deformed

By a passing electron.13 to illustrate this point, Figure 12.20 shows two electrons moving through the lattice. The passage of electron 1 causes nearby ions to move inward toward the electron, resulting in a slight increase

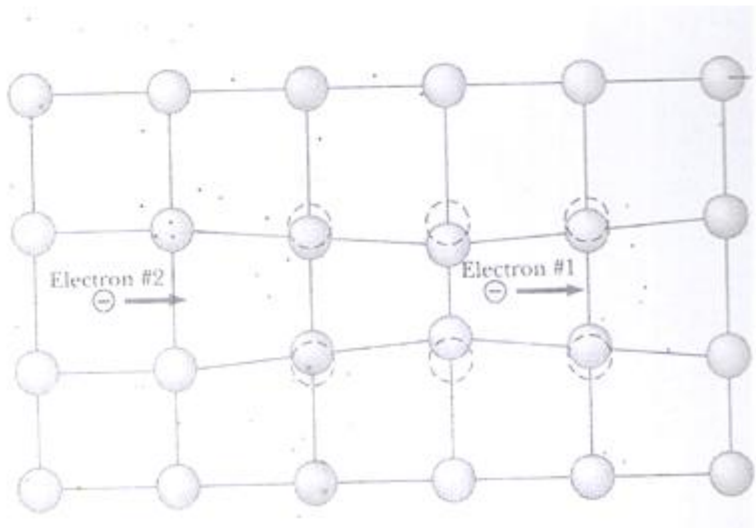
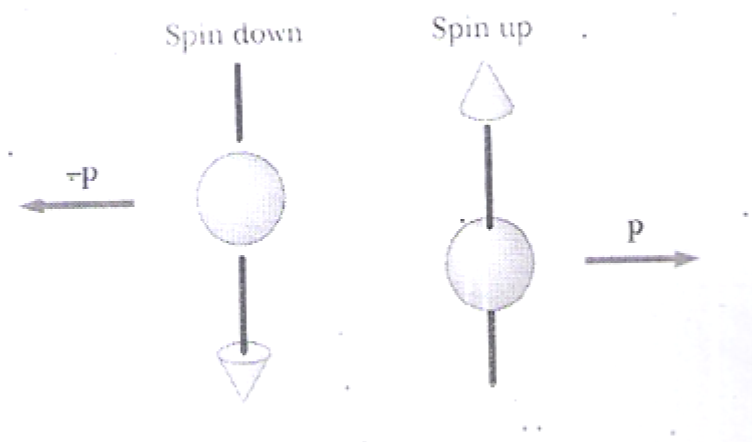


Figure (2.5). the basis for the attractive interaction between two electrons via the lattice deformation. Electron 1 attracts the positive ions, which move inward from their equilibrium positions (d). This distorted region of the lattice has a net positive charge, and hence electron 2 is attracted to it

The concentration of positive charge in this region. Electron 2 (the second electron of the Cooper pair), approaching before the ions have had a chance to return to their equilibrium positions, is attracted to the distorted (positively charged) region. The net effect is a weak delayed attractive force between the two electrons, resulting from the motion of the positive ions. as one researcher has beautifully put it, “the following electron surfs on the virtual lattice wake of the leading electron.” 14 In more technical terms, one can say that the attractive force between two Cooper electrons is an *electron-lattice-electron interaction*, where the crystal lattice serves as the mediator of the attractive force. Some scientists refer to this as a *phonon-mediated mechanism*, because quantized lattice vibrations are called *phonons*.



Figure(2 .6), a schematic diagram Spin down of a Cooper pair. The electron moving to the right has a momentum p and its spin is up, while the electron moving to the left has a momentum _p and its spin is down. Hence the total momentum of the system is zero and the total spin is zero.

A Cooper pair in a superconductor consists of two electrons having opposite momentum and spin, as described schematically in Figure 6. In the superconducting state, the linear momentum can be equal and opposite, corresponding to no net current, or slightly different and opposite, corresponding to a net superconducting current. Because Cooper pairs have zero spin, they can all be in the same state. This is in contrast with electrons, which are fermions (spin) that must obey the Pauli exclusion principle. In the BCS theory, a 12 ground state is constructed in which *all electrons form bound pairs*. In effect, all Cooper pairs are "locked" into the *same quantum state*. One can view this state of affairs as a condensation of all electrons into the same state. Also note that, because the Cooper pairs have zero spin (and hence zero angular momentum), their wave functions are spherically symmetric (like the *s*-states of the hydrogen atom.) In a "semi classical" sense, the electrons are always undergoing head-on collisions and as such are always moving in each other's wakes. Because the two electrons are in a bound state, their trajectories always change directions in order to keep their separation within the coherence length.

The BCS theory has been very successful in explaining the characteristic superconducting properties of zero resistance and flux expulsion. From a qualitative point of view, one can say that in order to reduce the momentum of any single Cooper pair by scattering, it is necessary to simultaneously reduce the momentum of all the other pairs—in other words, it is an all-or-nothing situation. One cannot change the velocity of one Cooper pair without changing those of all of them.15 Lattice imperfections and lattice vibrations, which effectively scatter electrons in normal metals, have no effect on Cooper pairs! In the absence of scattering, the resistivity is zeros and the current persists forever. It is rather strange, and perhaps amazing, that the mechanism of lattice vibrations that is responsible (in part) for the resistivity of normal metals also provides the interaction that gives rise to their superconductivity. Thus, copper, silver, and gold, which exhibit small lattice scattering at room temperature, are not superconductors, whereas lead, tin, mercury, and other modest conductors have strong lattice scattering at room temperature and become superconductors at low temperatures.

As we mentioned earlier, the superconducting state is one in which the Cooper pairs act collectively rather than independently. The condensation of all pairs into the same quantum state makes the system behave as a giant quantum mechanical system or macromolecule that is quantized on the macroscopic level. The Condensed state of the Cooper pairs is represented by a single coherent wave function that extends over the entire volume of the superconductor.

The stability of the superconducting state is critically dependent on strong correlation between Cooper pairs. In fact, the theory explains super conducting behavior in terms of the energy levels of a kind of "macro molecule" and the existence of an energy gap E_g between the ground and excited states of the system, as in Figure 12.22a. Note that in Figure there is no energy gap for a normal conductor. In a normal conductor, the Fermi energy *E*F represents the largest kinetic energy the free electrons can have at 0 K.

The energy gap in a superconductor is very small, of the order of *k T* B*c* (_10_3 eV) at 0 K, as compared with the energy gap in semiconductors (_1 eV) or the Fermi energy of a metal (_5 eV). The energy gap represents the energy needed to break apart a Cooper pair.

CHAPTER THREE

Flux Quantization for electron moving in a circular orbit

(3.1) Introduction:

Magnetic field plays an important role in many applications; therefore it is important to study the properties of magnetic field in different materials.

(3.2) Magnetic Flux produced by:

Consider an electron reviving around a nucleus in a circular orbit of radius(r) . The magnetic flux density produced is given by:

$$v = wr \qquad (3.2.1)$$

$$B = \frac{\mu_0 i}{2r} \qquad (3.2.2)$$

If the electron number of revolution per seconds is (f), then the current produced is giving by

$$i = ef \quad , \quad i = \frac{ew}{2\pi} \qquad (3.2.3)$$

Where (e) is electron charge .but since $w = 2\pi$

It follows that

$$i = \frac{ew}{2\pi} \qquad (3.2.3)$$

But the angular velocity

Can used to rewrite (i) in terms of it, where

$$i = \frac{ew}{2\pi} = \frac{ev}{2\pi r} \qquad (3.2.4)$$

Where

$$v = w\,r \qquad (3.2.5)$$

The flux enclosed by the orbit of electron is given by ϕ = flux $\phi = B\ A$

$$\phi = B\ A = B(2\pi r^2) \qquad (3.2.6)$$

Substitute (3.2.4) in(3.2.1) to get $B = \frac{\mu_0 e\ v}{4\pi r^2}$

$$\phi = B(2\pi r^2) = \frac{\mu_0 e\ v}{4\pi r^2}(2\pi r^2) = \frac{\mu_0 e\ v}{2} \qquad (3.2.7)$$

$$\phi = \frac{\mu_0 e\ v}{4\pi r^2}(2\pi r^2) = \frac{\mu_0 e\ v}{2} \qquad (3.2.8)$$

(3.3) Quantization of Magnetic Flux:

The momentum P is given bys

$$p = mv \qquad (3.3.1)$$

From the laws of quantum mechanics the Eigen equation of the momentum (p) is given by

$$\hat{p}\psi = p \qquad (3.3.2)$$

$$\frac{\hbar}{i}\frac{\partial\psi}{\partial x} = \frac{\hbar}{i}\left[\frac{\partial}{\partial x} e^{ikx}\right] = \hbar k \qquad (3.3.3)$$

Comparing (3.3.2) and (3.3.3) the momentum is given by

$$p = \hbar\, k_x = mv \qquad (3.3.4)$$

But from Bragg's law the condition of reflection is given by

$$2d \sin\theta = n\lambda \qquad (3.3.5)$$

$$2d \sin\theta = n\left(\frac{\lambda}{2\pi}\right)2\pi = \frac{2\pi n}{k} \qquad (3.3.6)$$

Rearranging equations to get:, $2d\sin\theta = \frac{h}{2}\left(\frac{1}{k}(2\pi)\right) = \frac{n\pi}{k}$ from which

$$k_x = k\sin\theta \qquad (3.3.7)$$

Where n=1, 2,3,…..

Thus the wave number is quantized .(n=integer) from(3.3.4)

$$v = \frac{\hbar k_x}{m} = \frac{n\hbar}{2md} \quad (3.3.8)$$

Sub. In (3.2.8) are gets

$$\phi = \frac{\mu_0 e}{2}\left(\frac{n\hbar}{2md}\right) \quad (3.3.9)$$

Also considering particle in a box. The solution of Schrodinger equation.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial \mathrm{x}^2} = E\Psi \quad (3.3.10)$$

Is

$$\Psi = A\sin\alpha\, x$$

$$\frac{\partial^2\Psi}{\partial \mathrm{x}^2} = -\alpha^2 A\sin\alpha\, x = -\alpha^2\, \Psi \quad (3.3.11)$$

Sub (3.3.9)in(3.3.10) yields

$$-\alpha^2\frac{\hbar^2}{2m} = E\Psi \quad (3.3.12)$$

$$\alpha = \frac{\sqrt{2mE}}{\hbar^2} \quad (3.3.13)$$

For free particle

$$E = \frac{\hbar^2 k^2}{2m} \quad (3.3.14)$$

Thus

$$\alpha = \mathrm{k} \qquad (3.3.15)$$

Sub in (3.3.11)

$$\Psi = A \sin \mathrm{k}\, x \qquad (3.3.16)$$

For particle in a box

$$|\Psi(\mathrm{x} = \mathrm{d})|^2 = 0$$

$$\Psi(\mathrm{x} = \mathrm{d}) = 0$$

$$\sin ka = 0$$

$$kd = 2n\pi$$

$$k = \frac{2n\pi}{d}$$

But from equation (3.3.4)

$$mv = p = \hbar k = \frac{2n\pi\hbar}{d}$$

Thus

$$v = \frac{2n\pi\hbar}{md} \quad (3.3.17)$$

Sub equation (3.3.4) in (3.2.9)

$$\phi \;=\; \frac{\mu_0 e\, v}{2} = \frac{\mu_0 e\, nh}{2md} \quad (3.3.18)$$

n=1, 2,3,… (n: integer number)

Conclusion

Magnetic field plays an important role in many applications. Based on this research the following conclusion we can result it. The quantization of magnetic field in super conductivity is complicate .wherefore the Equations of quantization of magnetic field are simplified by using the relations of flux density and current intensity and momentum.

Recommendations

Scientists are currently working on developing superconductors that are closer to room temperature, an important which would make superconductors important to everybody .as result of many applications of superconductors we can recommend to education more equation related to this technology ,and more researches about the superconductors.

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