CHAPTER FOUR

SYSTEM SIMULATION AND RESULTS

4.1 Simulation Modeling

For the purpose of controller design and evaluation the inverted pendulum systems were modeled in MATLAB /SIMULINK. The inverted pendulum was modeled using parameterized SIMULINK model and an 'm-File' as a run script for defining the physical parameters and control gains. From the MATLAB model it was possible to build and test controllers for the pendulum systems and to optimize their performance before implementation on the actual pendulum equipment.

4.2 System Identification

To implement the controllers as designed using the mathematical models, the real parameters of the system needed to be determined accurately. The majority of the parameters describing the system were determined accurately from the mechanical drawings, material properties and data sheets for the respective components. These parameters are as shown in Table 4.1.

Table 4.1: Parameters of inverted pendulum system

System Parameters	**Value**
Mass of the cart (M)	0.5 kg
Mass of the pendulum (*m*)	0.5 kg
Length to pendulum center of mass(L)	0.3 m
Inertia of the pendulum (I)	0.006 kg.m^2
Friction of the cart(b)	0.1N/m/sec
Gravity (g)	9.81 m/s^2

Figure 3.2 is repeated in Figure 4.1.

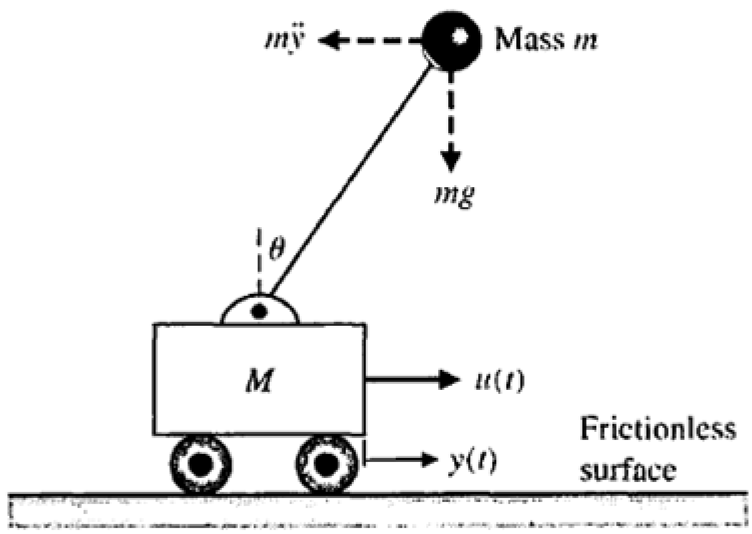

Figure 4.1: A cart and an inverted pendulum.

4.3 System Analysis

The simulation of the dynamic behavior for the system under step force, analyzing the stability, and controllability condition of the system were considered. The design requirements for the inverted pendulum project are:

- Settling time ($\boldsymbol{T_s}$) for x and θ of less than 5 seconds.
- Rise time ($\boldsymbol{T_r}$) for x of less than 0.5 seconds.
- Overshoot **(% OS)** of θ less than 20 degrees (0.35 radians).

4.3.1 Open-loop step response of the system

SIMULINK is used as an interactive tool for modeling, simulation, and analyzing the system. Figure 4.1 shows the SIMULINK model of the system.

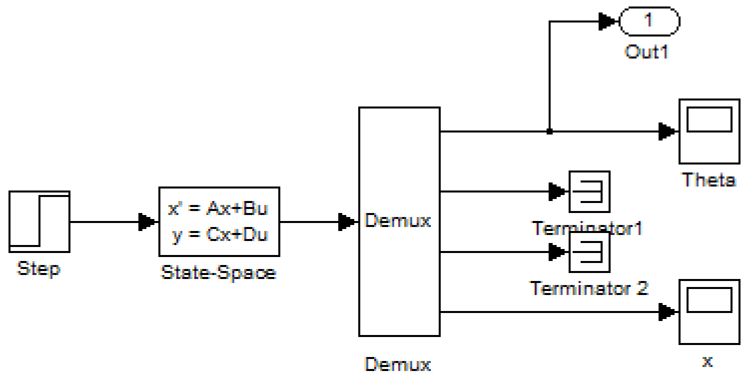

Figure 4.2: SIMULINK model of inverted pendulum system

Examining on how the system respond to step force applied to the cart, the following results are obtained as shown in Figure 4.2.

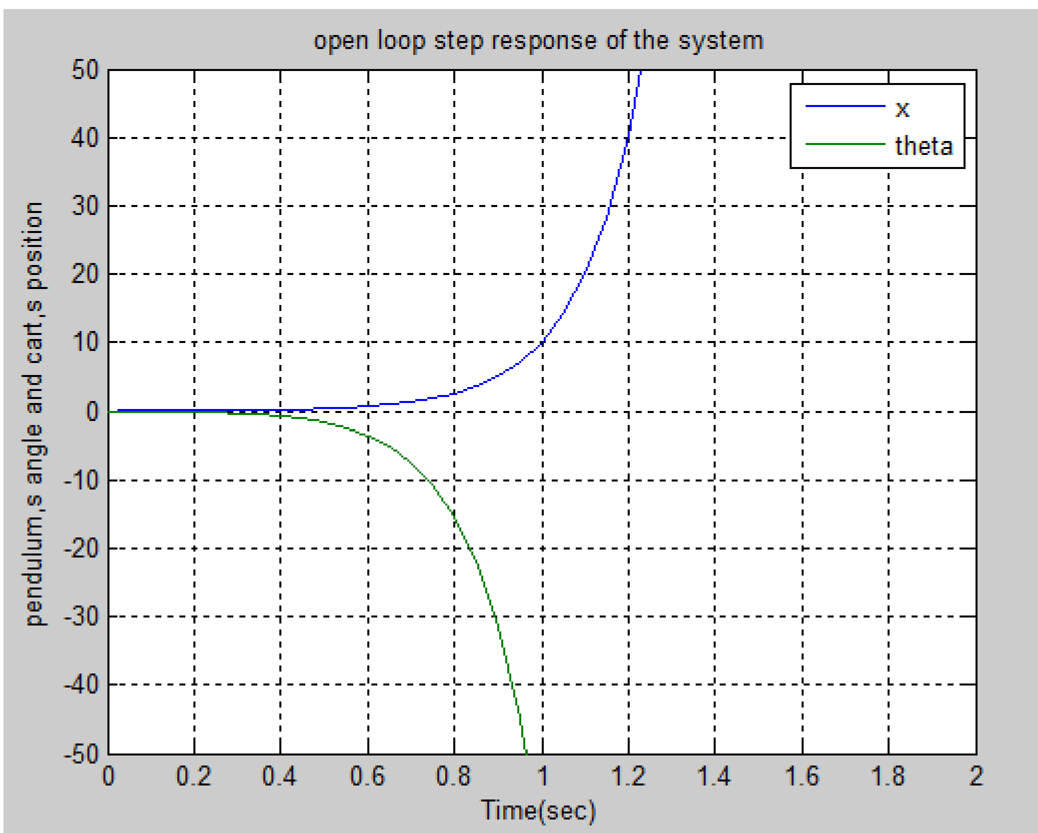

Figure 4.3: Open-loop response of an inverted pendulum

From the Figure 4.2 the response is unsatisfactory. Both outputs never settle , the angle of the pendulum goes to several hundred radians in a clockwise direction though it should be less than 0.35 rad; the cart goes to the right infinitely. So, this system is unstable in an open loop condition when there is step force applied to the cart. The outputs are found by using SIMULINK, and *lsim* command of MATLAB which can be employed to simulate the response to arbitrary inputs.

4.3.2 Stability of the system

To check stability means to analyze whether the open-loop system (without any control) is stable. That has partly done by the above simulations under the step forces. But as per the definition, the eigenvalues of the system state matrix (A) can determine the stability. That is equivalent to finding the poles of the transfer function of the system. The eigenvalues of the matrix (A) are the values of s where *det (sI-A) =0.* A system is stable if all its poles have lied in the left half of the s- plane. The poles (0, 7.1467, -7.2257, and -0.1) are found by using ***eig*** command in MATLAB. Or it can be shown by pole-zero mapping of the system as shown in Figure 4.3 using *Pzmap* in MATLAB command.

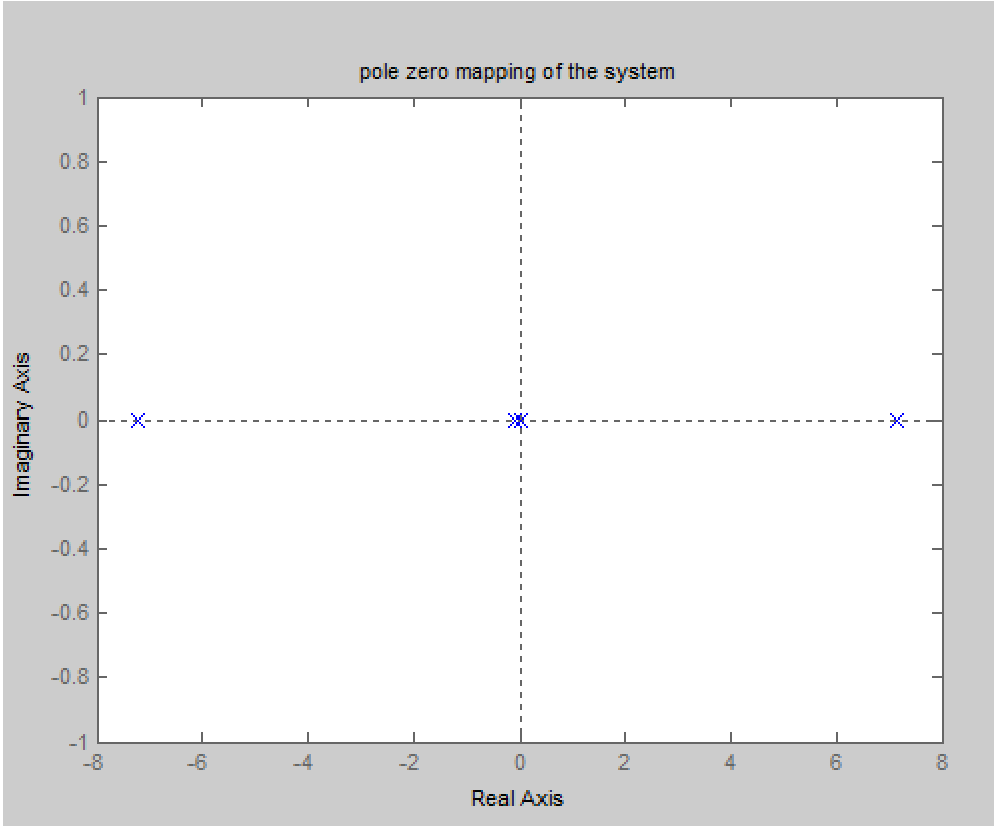

Figure 4.4: Pole-zero mapping of the system

As can be seen from the output, there is one pole on the right-half plane at 7.1430. This confirms that the intuition that the system is unstable in open loop system.

4.3.3 Controllability of the system

A system is controllable if there exist a control input *u (t)* that transfers any state of the system to zero in finite time. It can be shown that if and only if the controllability matrix has full rank.i.e. If *rank* $(M_c) = n$, where n the number of states. Using *rank (ctrb (A, B)* in MATLAB command, the controllability matrix of the system has full rank. So, the system is controllable.

4.4 Designing Full-state Feedback Controller

The main purpose is to design a state feedback controller to stabilize the system by improving the system using pole placement. So that when a step reference is given to the system, the pendulum should be displaced, but eventually return to zero (i.e. vertical) and the cart should move to its new commanded position. After checking the pair (A, B) is controllable and constructing equations that will govern the controller dynamics, then placing the eigenvalues of the controller matrix in a desired position by finding an arbitrary vector state feedback control gain vector *(K)* assuming that all of the state variables are measurable. This can be accomplished using pole placement method. The state of the system is to be feedback as an input, the controller dynamics will be:

$$u=r-K\,x \tag{4.1}$$

$$x = Ax + B(r-K\,x) = (A-BK)\,x + Br \tag{4.2}$$

4.4.1 Pole placement method

This method depends on the performance criteria, such as rise time, settling time, and overshoot used in the design. The design procedures are:

1. Using time domain specifications to locate dominant poles-roots of $s^2+2\zeta\omega_n s+\omega_n^2 = 0$. This is done by using the following formulas and finding the dominant poles at $-\sigma \pm j\omega_d$.

- $\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\ln^2+\pi^2+(\%OS/100)}}$ (4.3)
- $T_{settling} = \frac{4}{\zeta\omega_n}$, valid up to $\zeta \sim 0.7$ (4.4)
- $\omega_d = \omega_n\sqrt{1-\zeta^2}$ (4.5)

- $\sigma = \zeta\, \omega_n$ (4.6)
- $\beta = \cos^{-1} \zeta$ (4.7)

2. Then placing rest of pole so they are much faster than the dominant second order behavior.

- Typically, keeping the same damped frequency ω_d and then moving the real part to make them faster than the real part of the dominant poles so that the transient response of the real poles of the system will decay exponentially to insignificance at the settling time generated by the second order pair.
- While taking care of moving the poles too far to the left because it takes a lot of control effect (needs large actuating signal). The parameters as shown in Table 4.2.

Table 4.2: The parameters of transient response

Parameter	Value
Over shoot, %OS	20 Degree (0.35 radian)
Settling time , T_s	5 sec
Damping ratio , ζ	0.456
Un damping natural frequency, ω_n	1.78 rad /sec
Damping frequency, ω_d	1.584 rad /sec

Then the dominant poles are:

$-\sigma \pm j\omega_d$ = $-\zeta\, \omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ = -0.811+j 1.584 and -0.811-j 1.584 (which are complex conjugates). Using MATLAB software, the schematic diagram and SIMULINK block diagram of inverted pendulum with state feedback control via pole placement shown in Figure 4.4 and Figure 4.5 respectively. MATLAB calculates the state space matrices according to the system parameters shown in

Table 4.1, and using a state feedback controller to place the closed loop poles which are calculated above with Table 4.2 at: p= [-0.811± j1584] and using assumed remaining poles. Then evaluates the gain matrix *K* to obtain the response of the pendulum's angle and cart position (according to step input).

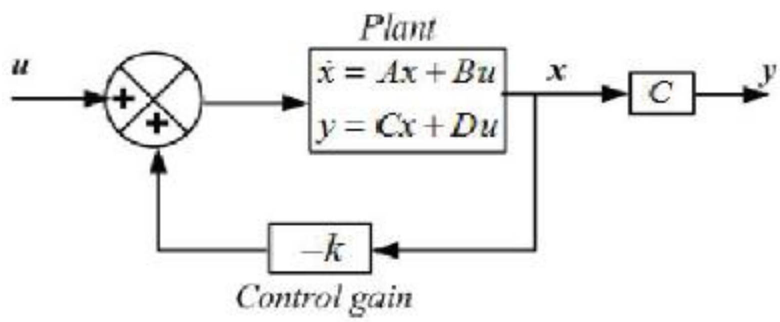

Figure 4.5: Schematic diagram for inverted pendulum in state-space form with state feedback

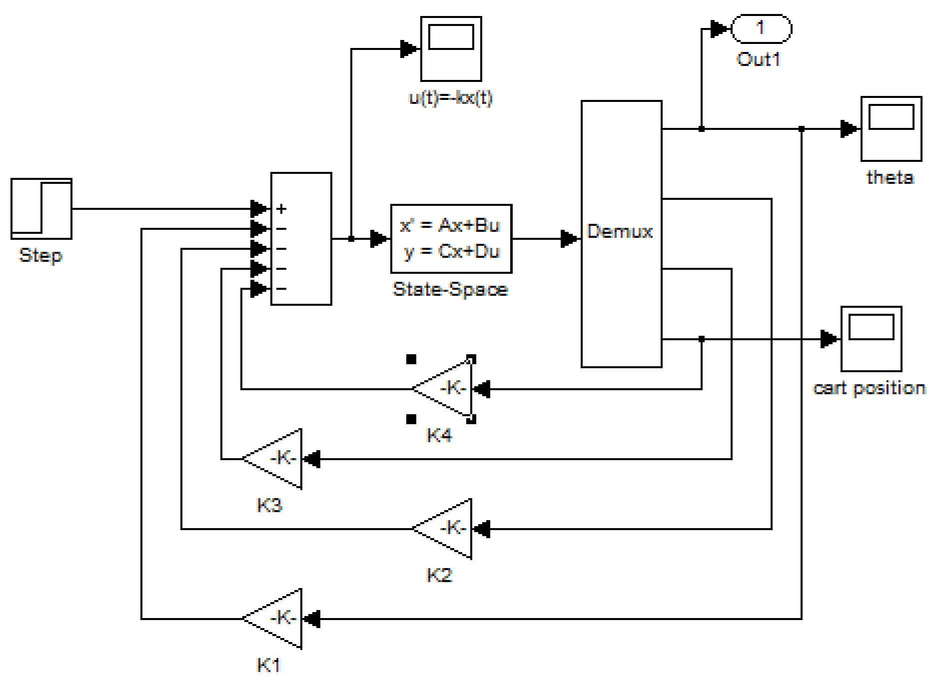

Figure 4.6: SIMULINK diagram for inverted pendulum in state-space form with state Feedback

4.4.2 Simulation results

The controller was applied to inverted pendulum; the following are the gain vectors for different sets of desired poles (P), and using MATLAB command: place *(A, B, P)* to find the value of gain vector *K*.

Test 1:

Table 4.3 shows the desired poles and gain vector which the remaining poles two and three times faster than the real part of the dominant poles:

Table 4.3: Desired poles and gain vectors for test 1

P_1	[-2.433 -1.622 -0.811± j1.584]
K_1	[-12.4935 -1.2053 -0.2420 -0.4727]

Figure 4.6 shows the step response using pole placement for cart' position and pendulum' angle:

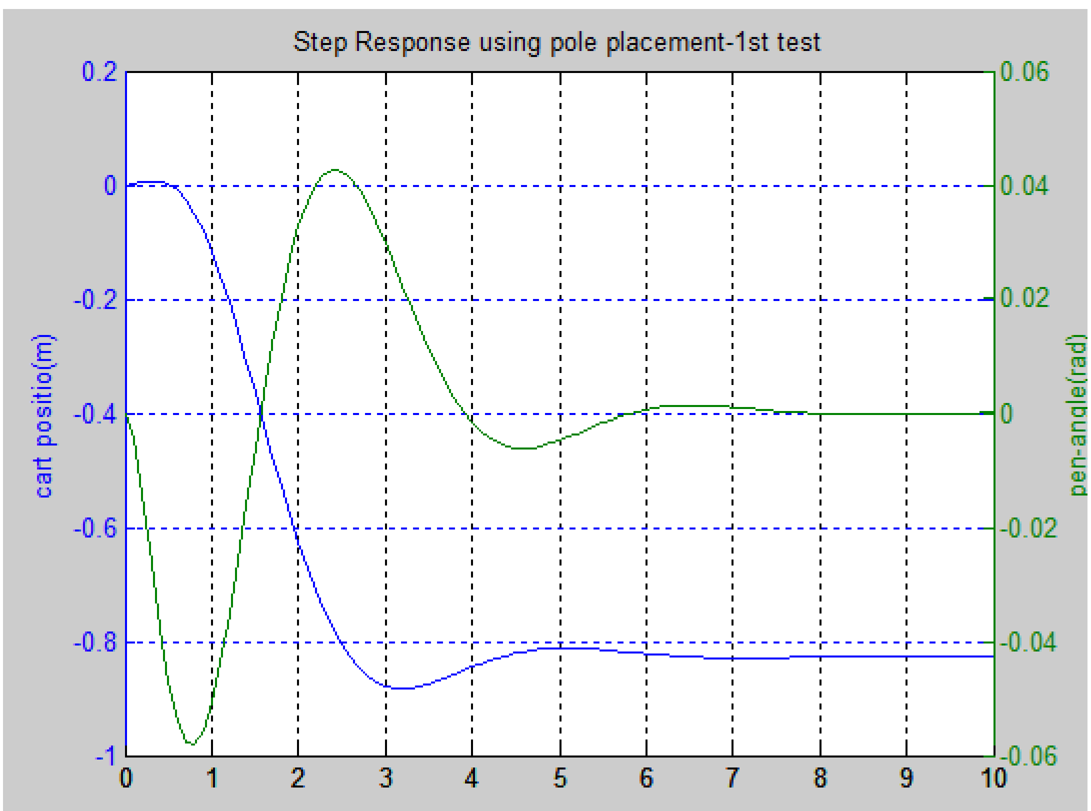

Figure 4.7: Step response using pole placement-1st test

Test 2:

Table 4.4 shows the desired poles and gain vector which the remaining poles ten and twelve times faster than the real part of the dominant poles:

Table 4.4: Desired poles and gain vectors for test 2

P_2	[-11.354 -9.732 -0.811± j1.584]
K_2	[-40.2087 -5.9345 -6.7772 -4.8645]

Figure 4.7 shows the step response using pole placement for cart' position and pendulum' angle.

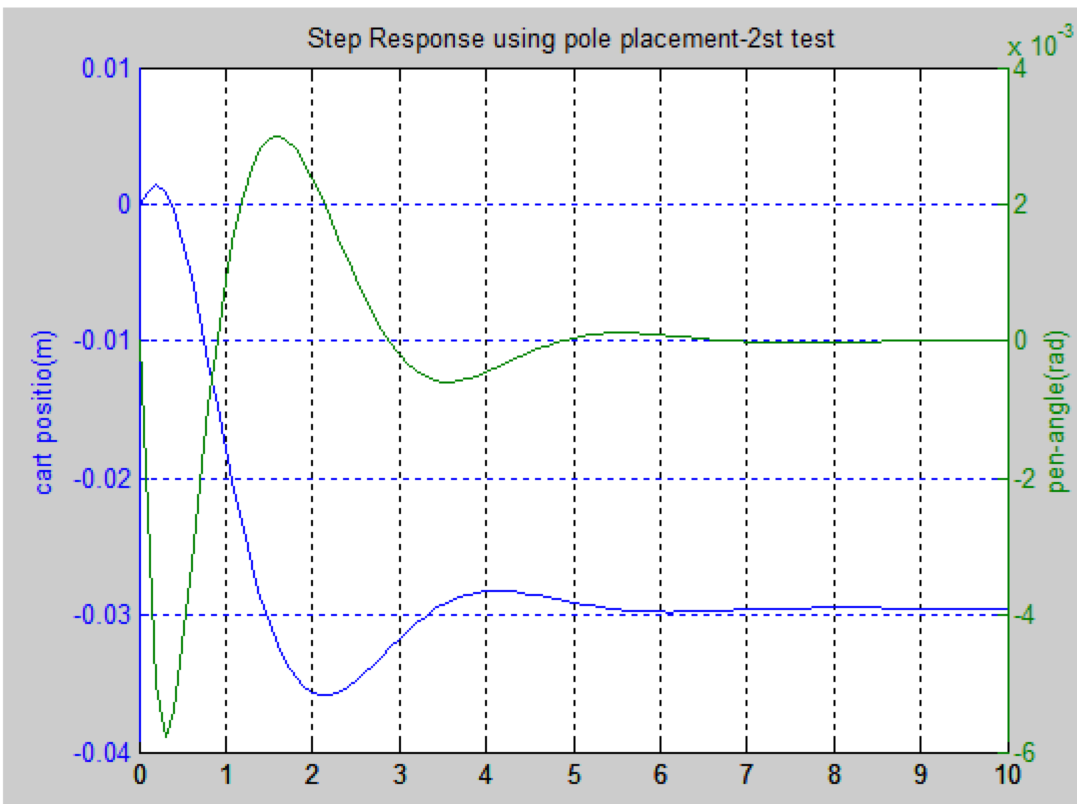

Figure 4.7: Step response using pole placement-2^{st} test

4.4.3 Discussion

As can be seen from the respective plots of the system step response for each calculated gain vectors as per the desired pole locations, system design requirements are satisfied in the two tests. The system response tends to be faster when the real poles go farther to the left from the real part of the dominant pole. The inverted pendulum has been stabilized and the final angle θ go to zero. From the position cart x plots, it obvious that the cart move to the right away from $x=0$ position, while in the same time, the pendulum has swung back and forth about $\theta = 0$ before it finally settles down at stable position. This means the using pole placement is effective control law which stabilized the system.