CHAPTER TWO

CONTROL SYSTEM

2.1 Previous Work

Several research works recently have discussed the inverted pendulum control throughout these research works, some mathematical theories have been developed and utilized for addressing this issues.

This paper [2] is about the Inverted pendulum which is a classic control problem, involves the mechanism of balance the inverted pendulum system. For display purposes, it is like trying to balance broomstick on the finger. To study this problem, we have to take into account the experimental system which consists of a pendulum, which is free to rotate 360 degrees. This study involves the complete system including all mechanical hardware, software and design at minimal cost. Three major subsystems that make this design: a mechanical system, feedback network which includes sensors and methods for reading and a driver and its interface with mechanical system.

The objective is to design a mechanical system for the problem of the inverted pendulum, and then implement a feasible controller. The controller should minimize both the displacement of the carriage and the angle of the pendulum. The system should be standalone. This study provides a chance of designing a controller for a system that has a good dynamic behavior and hence the consideration for the transient response is emphasized. The model for a pendulum system which used feedback control to keep the pendulum in the upright position and design a controller for this system and show that if the delay time of sampling is not too large, then still the local controller stabilizes the system. When the time delay is large enough, stability is lost.

This paper [3] is about control of an inverted pendulum which is one of the most classical problems for control engineering. The objective of this study is to design a controller which is capable of driving the pendulum from its (hanging-down) position to (upright) position and then holding it there. In order to get some sense about how well the linear model represents the original nonlinear system by simulate the dynamics of the system using both the linear and non-linear models and then compare their simulation outputs. The simulation will not only verify the linear model, but also establish a threshold for us to know the threshold of the linear model.

The controller of the whole system consists of three parts: destabilizing controller, stabilizing controller, and mode controller. The destabilizing controller, as the name implies, oscillates the arm until it has built up enough energy to break the initial stable (hanging-down) state and get the pendulum into an almost upright but unstable state. Then the stabilizing controller is turned on to stabilize the pendulum in its upright sate. The mode controller determines when to switch between the destabilizing controller and stabilizing controller. Destabilizing controller will essentially drive the position of the arm in order to get away from the stable "hanging-down" position of the pendulum. It simply makes sense that, by moving the arm back and forth strongly enough, it can eventually swing up the pendulum. Hence, the first thing we need to do is to design a position controller which can swing the arm to achieve the destabilizing goal and design a positive feedback controller to destabilize the pendulum and eventually swing up it.

The purpose of the mode controller is to track the pendulum angle (θ) and facilitate switching between the destabilizing controllers and stabilizing controller. This controller is to be enabled when (θ) is in the neighborhood of zero, within the threshold of (θ) (currently set to 10 degrees). The rotary inverted pendulum system has two degrees of freedom. In general, to maintain the pendulum in the upright position by feedback (θ) only not enough. Using the SISO design tool in the control system toolbox, and design a stabilizing PID controller for the system described by the transfer function, by simulate the stabilizing controller by feeding back (θ) only. From the simulation results, the pendulum can reject a pulse disturbance, but the rotating arm will rotate crazily by feedback (θ) only.

This paper [4] is about the design a controller that stabilizes an inverted pendulum. The first step in designing the controller was to identify the system. Lead and lag compensators were created to help stabilize the system. A transfer function in the continuous time domain was derived first for the hanging pendulum system which inverted for uses in the inverted pendulum system. The inverted pendulum transfer function is use in the microcontroller. Calculations for the lead controller are done using the MATLAB function pole location and the lag compensator was designed to be a balance between rise time effects and stability issues. The controller is found that by increasing the gain and moving the lead pole farther left, increased a smaller overshoot could be attained. The theoretical model of the pendulum did a very good job of predicting rise time and settling time. However the SIMULINK model way under predicted the amount of overshoot experienced by the system. This difference in overshoot can be explained in the way that the microcontroller inputs torque as well as assumptions made in the creation of the transfer function. To improve the accuracy of the controller, a few things could be done. A different microcontroller could be used that accepts decimal inputs. Additionally, the efficiency of the evaluation of the system could be increased by purchasing a microcontroller that uploads significantly faster. A majority of the analysis time for the pendulum was spent waiting for the program to upload.

2.2 Introduction to Control System

Control system is an important mechanism which alters the state of the system based on the input to the system. An efficient control system enhances the productivity of components by providing fine control over the desired range. In the nature everything is controlled or otherwise leads to catastrophic manner which creates a huge damage. In the same way in and around you everything is controlled by some component to ensure the stability. For example a light in a room is controlled by an electric switch. When a switch is on electricity flows and the light will be on, same for the reverse operation. The basic of control system consists of three components:

1. Input.
2. Logic operation.
3. Output or decision device.

Input is the cause parameter on which the control system acts, the logic operation is the intended or desired operation to perform on the input for generating a new output state, and the output is drive parameter which actuates the end component to perform the desired task. These can be open loop or closed loop control system depends on output feedback. Block diagram reduction helps in analysis and simplification of control system.

2.2.1 Open loop and closed loop system

There are two common classes of control systems, open loop control systems and closed loop control systems. In open loop control systems output is generated based on inputs, and utilizes an actuating device to control the process directly without using feedback as shown in Figure2.1.

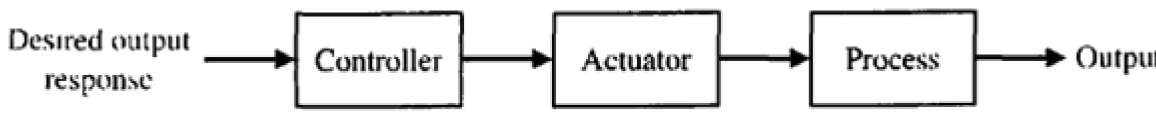

Figure 2.1: Open loop control system

In closed loop control systems current output is taken into consideration and corrections are made based on feedback. A closed loop system is also called a feedback control system. The human body is a classic example of feedback systems**.** A closed-loop control system uses a measurement of the output as feedback of this signal to compare it with the reference or command signal as illustrated in Figure 2.2.

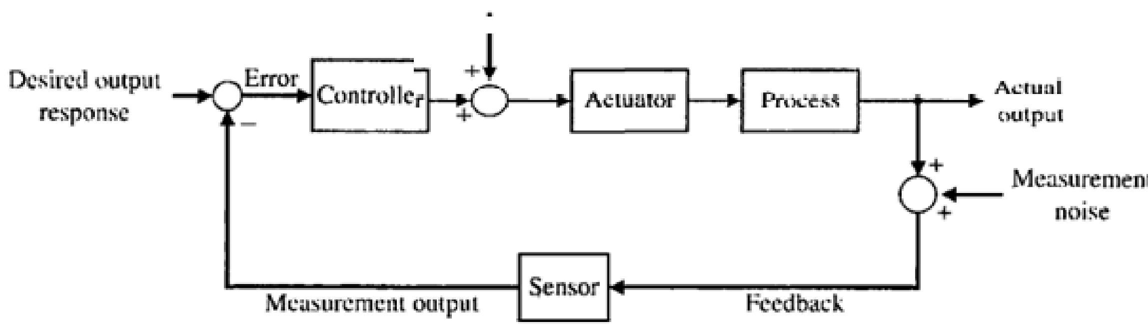

Figure 2.2: Closed-loop feedback system with external disturbances and measurement noise

In case of feedback systems, a control loop, including sensors, control algorithms and actuators, is arranged in such a fashion as to try to regulate a variable at a_set point or reference value. An example of this may increase the fuel supply to a furnace when a measured temperature drops. Control systems that include some sensing of the results they are trying to achieve are making use of feedback and so can, to some extent, adapt to varying circumstances. Open-loop control systems do not make use of feedback, and run only in pre-arranged ways.

2.2.2 Classical and modern control system

There are essentially two methods to approach the problem of designing a new control system: the classical and modern approaches.

Classical and modern control methodologies are named in a misleading way, because the groups of techniques called “classical” were actually developed later than the techniques labeled "modern". However, in terms of developing control systems, modern methods have been used to great effect more recently, while the classical methods have been gradually falling out of favor. Most recently, it has been shown that classical and modern methods can be combined to highlight their respective strengths and weaknesses.

Classical methods are methods involving the Laplace transform domain. Physical systems are modeled in the so-called "time domain", where the response of a given system is a function of the various inputs, the previous system values, and time. As time progresses, the state of the system and its response change. However, time-domain models for systems are frequently modeled using high-order differential equations which can become impossibly difficult for humans to solve and some of which can even become impossible for modern computer systems to solve efficiently. To counteract this problem, integral transforms, such as the Laplace transform and the Fourier transform, can be employed to change an Ordinary Differential Equation (ODE) in the time domain into a regular algebraic polynomial in the transform domain. Once a given system has been converted into the transform domain it can be manipulated with greater ease and analyzed quickly by humans and computers alike. Modern control methods, instead of changing domains to avoid the complexities of time-domain ODE mathematics, converts the differential equations into a system of lower-order time domain equations called state equations, which can then be manipulated using techniques from linear algebra.

2.3 Analysis and Design of Control System

Control systems are designed to perform specific tasks. The requirements imposed on the control system are usually spelled out as performance specifications. The specifications may be given in terms of transient response requirements (such as the maximum overshoot and settling time in step response) and of steady-state requirements (such as steady-state error in following ramp input) or may be given in frequency-response terms. The specifications of a control system must be given before the design process begins.

In the process of designing a control system, mathematical model of the control system and adjust the parameters of a compensator were set up. The most time-consuming part of the work is the checking of the system performance by analysis with each adjustment of the parameters. The designer should use MATLAB or other available computer package to avoid much of the numerical drudgery necessary for this checking. The basic aspects of design compensation systems are the modification of the system dynamics to satisfy the given specifications. The approaches to control system design and analysis are:

2.3.1 Root-locus method

The basic characteristic of the transient response of a closed-loop system is closely related to the location of the closed-loop poles. The root-locus method proves to be quite useful, since it indicates the manner in which the open-loop poles and zeros should be modified so that the response meets the system performance specifications. This method is particularly suited to obtaining approximate results very quickly. By using *rlocus (num, den)* MATLAB command for plotting root locus response, both vectors *num* and *den* must be written in descending powers of *s*.

The design by the root-locus method is based on reshaping the root locus of the system by adding poles and zeros to the system's open-loop transfer function and forcing the root loci to pass through desired closed-loop poles in the s plane. The characteristic of the root-locus design is its being based on the assumption that the closed-loop system has a pair of dominant closed-loop poles. This means that the effects of zeros and additional poles do not affect the response characteristics very much. In designing a control system, if other than a gain adjustment (or other parameter adjustment) is required, the original root loci should be modify by inserting a suitable compensator.

Once the effects on the root locus of the addition of poles and/or zeros are fully understood, we can readily determine the locations of the pole(s) and zero(s) of the compensator that will reshape the root locus as desired. In essence, in the design by the root locus method, the root loci of the system are reshaped through the use of a compensator so that a pair of dominant closed-loop poles can be placed at the desired location. It should be clear that it is possible to sketch a reasonably accurate root-locus diagram for a given system by simple rules. At preliminary design stages, not need the precise locations of the closed-loop poles. Often their approximate locations are all that is needed to make an estimate of system performance. Thus, it is important that the designer have the capability of quickly sketching the root loci for a given system [5].

2.3.2 Frequency-response method

In frequency-response methods, the frequency of the input is varied signal over a certain range and study the resulting response. In fact, the frequency response and root-locus approaches complement each other. One advantage of the frequency-response approach is that we can use the data obtained from measurements on the physical system without deriving its mathematical model. In many practical designs of control systems both approaches are employed. Control engineers must be familiar with both. Frequency-response methods were developed by Nyquist, Bode, and many others. The frequency-response methods are most powerful in conventional control theory. The types of the frequency-response methods are:

(1) Nyquist plot: The Nyquist stability criterion determines the stability of a closed-loop system from its open-loop frequency response and open-loop poles. The Nyquist stability criterion provides a convenient method for finding the number of zeros of $1 + GH(s)$ in the right-half s-plane directly from the Nyquist plot of $GH(s)$. The Nyquist stability criterion is defined in terms of point (-1, 0) on the Nyquist plot or the zero-dB, 180° point on the Bode plot. The Nyquist criterion is based upon a theorem of complex variable mathematics due to Cauchy. The Nyquist diagram is obtained by mapping the Nyquist path into the complex plane via the mapping function $GH(s)$. The Nyquist path is chosen so that it encircles the entire right-half s-plane. When the s-plane locus is the Nyquist path, the Nyquist stability criterion is given by:

$$Z = N + P \tag{2.1}$$

Where

P = Number of pair of poles of $GH(s)$ in the right-half s-plane.

N = Number of clockwise encirclements of (-1, 0) point by the Nyquist diagram.

Z = Number of zeros of $1 + GH(s)$ in the right-half s-plane.

For the closed-loop system to be stable, Z must be zero, that is:

$$N = -P \tag{2.2}$$

If the open-loop transfer function $GH(s)$ does not have poles in the right-half s-plane ($P = 0$), it is not necessary to plot the complete Nyquist diagram; the polar plot for ω increasing from 0+ to ∞ is sufficient. Such an open-loop transfer function is called minimum- phase transfer function. For minimum-phase open-loop transfer functions the closed-loop system is stable if and only if the polar plot lies to the right of (-1, 0) point. For a minimum-phase open-loop transfer function the criterion is defined in terms of the polar plot crossing with respect to (-1, 0) point as follows:

If Nyquist locus not enclosed the point (-1, 0) the system is stable, if it is passed through the point (-1, 0) the system is critically stable and finally if it enclosed the point (-1, 0) the system is unstable.

If P is not zero, the closed-loop system is stable if and only if the number of counterclockwise encirclements of the Nyquist diagram about (-1, 0) point is equal to P. The MATLAB control system toolbox function *nyquist(num, den,ω)* can obtain the Nyquist diagram by mapping the Nyquist path. However, the argument ω is specified as a real number. We must specify $\omega = -js$, in order to map a complex number s = a+ jb, since the above function automatically multiplies ω by the operator j. To avoid this, the developed function *cnyquist(num, den, s*) can be used, where the argument s must be specified as a complex number. In defining the Nyquist path care must be taken for the path not to pass through any poles or zeros of $GH(s)$ [6].

(2) Bode plot: Bode diagram consists of two graphs: One is a plot of the logarithm of the magnitude of a sinusoidal transfer function; the other is a plot of the phase angle; both are plotted against the frequency on a logarithmic scale. The standard representation of the logarithmic magnitude of $G(j\omega)$ is 20 log $|G(j\omega)|$ where the base of the logarithm is 10.The unit used in this representation of the magnitude is the decibel, usually abbreviated dB. In the logarithmic representation, the curves are drawn on semi log paper, using the log scale for frequency and the linear scale for either magnitude (but in decibels) or phase angle (in degrees). The frequency range of interest determines the number of logarithmic cycles required on the abscissa. The main advantage of using the Bode diagram is that:

1) It is based on the asymptotic approximation, which provides a simple method for sketching an approximate log-magnitude curve is available.
2) The multiplication of various magnitude appears in the transfer function can be treated as an addition, while division can be treated as subtraction as we are using a logarithmic scale.
3) With the help of this plot only, the comment on the stability of the system can be done without any calculations.
4) Bode plots provides relative stability in terms of gain margin and phase margin**.**
5) It also covers from low frequency to high frequency range.

The main advantage in using the logarithmic plot is the relative ease of plotting frequency-response curves. The basic factors that very frequently occur in an arbitrary transfer function $G(j\omega)\ H(j\omega)$ are :

1. Gain K**:** This factor has a slope of zero dB per decade. There is no corner frequency at which the two asymptotes cuts or meet each other is known as break frequency or corner frequency corresponding to this constant term. The phase angle associated with this constant term is also zero.

2. Integral and derivative factor $1/(j\ \omega)^{n}$**:** This factor has a slope of -20 × n (where n is any integer) dB per decade. There is no corner frequency corresponding to this integral factor. The phase angle associated with this integral factor is -90 × n here n is also an integer.

3. First order factor $1/(1+j\omega T)$: This factor has a slope of -20 dB per decade. The corner frequency corresponding to this factor is $1/T$ radian per second. The phase angle associated with this first factor is $-\tan^{-1}(\omega T)$.

4. Second order or quadratic factor $[\{1/(1+(2\zeta/\omega)\} \times (j\omega) + \{(1/\omega^2)\} \times (j\omega)^2)]$: This factor has a slope of -40 dB per decade. The corner frequency corresponding to this factor is ω^n radian per second. The phase angle associated with this first factor is $-\tan^{-1}\{(2\zeta\omega/\omega_n)/(1-(\omega/\omega_n)^2)\}$.

Once the logarithmic plots of these basic factors had became familiar, it is possible to utilize them in constructing a composite logarithmic plot for any general form of $G(j\omega)H(j\omega)$ by sketching the curves for each factor and adding individual curves graphically, because adding the logarithms of the gains corresponds to multiplying them together.

- **Gain margin and phase margin**

Gain margin(G_M) is the change in open-loop gain, required at 180° of phase shift to make the closed-loop system unstable. It is usually expressed in dB.

Phase margin($\emptyset_M$) is the change in open-loop phase shift required at unity gain to make the closed loop system unstable. It is usually expressed in phase. Phase crossover point in the Bode plot is the point where phase plot intersect with the 180° horizontal line. The frequency at which the point of intersection occurs is called phase crossover frequency. Gain crossover point in the Bode plot is the point where the magnitude plot intersects with 0 dB line. The frequency at which the intersection occurs is called gain crossover frequency. In the Bode plot, gain margin (dB) is the total difference between the magnitude plots at 0dB line till the phase crossover frequency. The phase margin is the total difference between the phase plots at 180 degree line till the gain cross over frequency.

- **Stability conditions of Bode plots**

Stability conditions are given below:

1. Stable system**:** Phase crossover frequency should be greater than gain crossover frequency.

2. Marginal stable system**:** Both phase crossover frequency and gain crossover frequency should be zero.

3. Unstable system**:** If any of them is negative, gain crossover frequency should be greater than phase crossover frequency.

An advantage of the frequency-response approach is that: frequency-response tests are, in general, simple and can be made accurately by use of readily available sinusoidal signal generators and precise measurement equipment. Often the transfer functions of complicated components can be determined experimentally by frequency-response tests. In addition, the frequency-response approach has the advantages that a system may be designed so that the effects of undesirable noise are negligible and that such analysis and design can be extended to certain nonlinear control systems.

2.4 System Compensation

Setting the gain is the first step in adjusting the system for satisfactory performance. In many practical cases, however, the adjustment of the gain alone may not provide sufficient alteration of the system behavior to meet the given specifications. As is frequently the case, increasing the gain value will improve the steady-state behavior but will result in poor stability or even instability. It is then necessary to redesign the system (by modifying the structure or by incorporating additional devices or components) to alter the overall behavior so that the system will behave as desired. Such a redesign or addition of a suitable device is called compensation which inserted into the system for the purpose of satisfying the specifications is called a compensator. The compensator compensates for deficient performance of the original system [7]. The types of compensation are:

- **Lead Compensation**

Essentially yields an appreciable improvement in transient response and a small change in steady-state accuracy. It may accentuate high-frequency noise effects. The lead compensation transfer function can be obtained with the electrical RC network as shown in Figure 2.3.

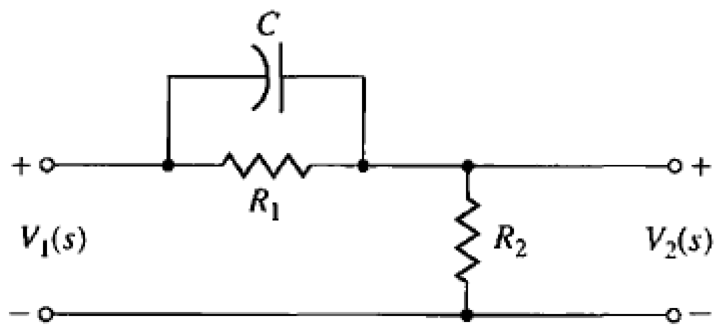

Figure 2.3: Lead compensation circuit

The transfer function:

$$\mathbf{G_c(s)} = \frac{V_{2(s)}}{V_{1(s)}} = \frac{R_2}{R_{2+}\frac{R_1/(Cs)}{R_{1+1/(Cs)}}} = \frac{R_2}{R_1+R_2}\,\frac{R_1Cs+1}{[R_1R_2/(R_1+R_2)]cs+1} \qquad (2.3)$$

Let $\boldsymbol{\tau} = \frac{R_1R_2}{R_1+R_2}$, $\boldsymbol{\alpha} = \frac{R_1+R_2}{R_2}$

$$G_c(s) = \frac{1+\alpha\tau s}{\alpha(1+\tau s)} \qquad (2.4)$$

Which is equal to Equation (2.5) when an additional cascade gain *K* is inserted,

p = 1/τ and z = 1/ (ατ).

$$G_c(s) = \frac{k(s+z)}{s+p} \tag{2.5}$$

Figure 2.4 show that the pole-zero diagram of lead compensation.

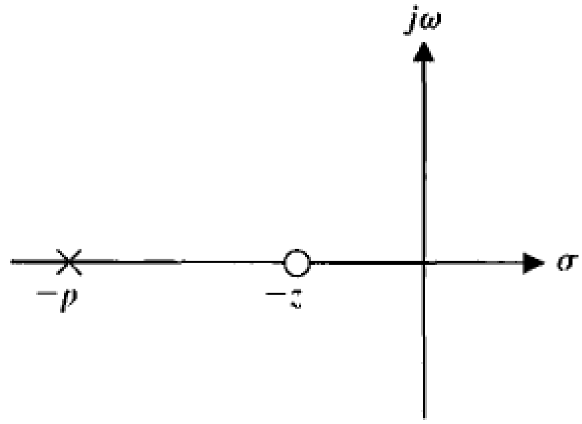

Figure 2.4: Pole-zero diagram of lead compensation

- **Lag compensation**

On the other hand, yields an appreciable improvement in steady-state accuracy at the expense of increasing the transient-response time. Lag compensation will suppress the effects of high-frequency noise signals. The lag compensation transfer function can be obtained with the network shown in Figure 2.5.

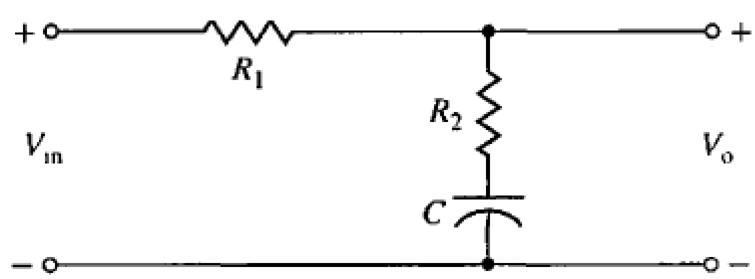

Figure 2.5: Lag compensation circuit

The transfer function:

$$G_c(s) = \frac{R_1 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{R_1 Cs + 1}{(R_1 + R_2)Cs + 1} \tag{2.6}$$

When $\tau = R_2 C$ and $\alpha = (R_1 + R_2)/R_2$, we have the lag compensation transfer function:

$$G_c(s) = \frac{1 + \tau s}{1 + \alpha \tau s} = \frac{1}{\alpha} \frac{s + z}{s + p} \tag{2.7}$$

Where $z = 1/\tau$ and $p = 1/(\alpha\tau)$. Figure 2.6 shows the pole-zero diagram of lag compensation.

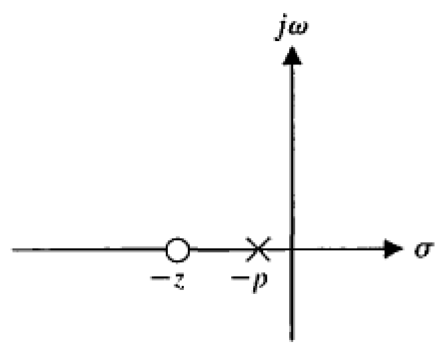

Figure 2.6: Pole-zero diagram of lag compensation

- **Lead–lag compensation**

Lead–lag compensation combines the characteristics of both lead and lag compensations. The use of a lead or lag compensator raises the order of the system by 1 (unless cancellation occurs between the zero of the compensator and a pole of the uncompensated open-loop transfer function). The use of a lag–lead compensator raises the order of the system by 2 [unless cancellation occurs between zero(s) of the lag–lead compensator and pole(s) of the uncompensated open-loop transfer function], which means that the system becomes more complex and it is more difficult to control the transient-response behavior. The particular situation determines the type of compensation to be used [7]. The circuit network of lead-lag compensator as shown in Figure 2.7.

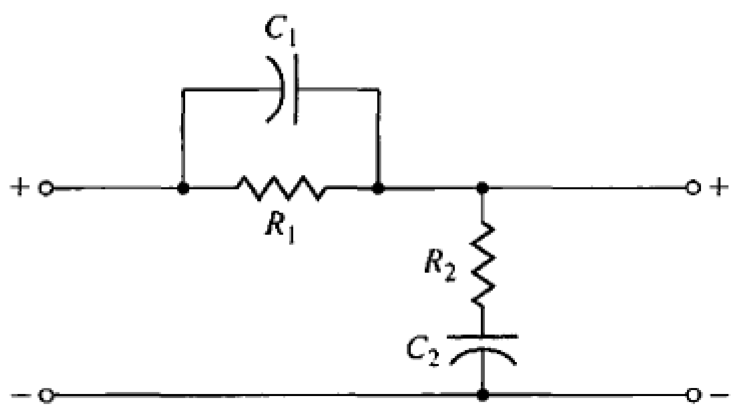

Figure 2.7: Lead–lag compensation circuit

The transfer function of lead-lag compensator is:

$$G_c(s) = \frac{V_{2(s)}}{V_{1(s)}} = \frac{(R_1C_1s+1)(R_2C_2s+1)}{R_1C_1R_2C_2s^2+(R_1C_1+R_1C_{2+R_2C_2})s+1} \tag{2.8}$$

Let $\tau_1 = R_1C_1$, $\beta\tau_2 = R_2C_2$,$\tau_1 + \tau_2 = R_1C_1 + R_1C_2 + R_2C_2$, and $\tau_1\tau_2 = R_1R_2C_1C_2$.Then $\alpha\beta = 1$, and Equation (2.8) becomes:

$$\frac{V_{2(s)}}{V_{1(s)}} = \frac{(1+\alpha\tau_1 s)(1+\beta\tau_2 s)}{(1+\tau_1 s)(1+\tau_2 s)} \tag{2.9}$$

2.5 State Space Analysis and Design

The state-space method is the modern approach for control system design and analysis. The controllability and observability are important structural properties of a control system. The controllability and observability analysis can be used to check whether the system is controllable or observable. The controllability and observability staircase are useful when the system has uncontrollable or unobservable states. The function grammian computes the controllability and observability grammian of a state-space model.

Pole placement design is one of the attractive features of the state-space design method. It allows us to place closed-loop poles to locations that will correspond to desired dynamic response. Given the state equation and a desired closed-loop pole vector, pole placement computes a gain matrix K according to (A, B) and p such that the full-state feedback places the closed-loop poles at the desired locations.

2.5.1 Controllability and observability

If the system is controllable and observable, then we can accomplish the design objective of placing the poles precisely at the desired locations to meet the performance specifications .Full-state feedback design commonly relies on pole-placement techniques. It is important to note that a system must be completely controllable and completely observable to allow the flexibility to place all the closed-loop system poles arbitrarily. The concepts of controllability and observability were introduced by Kalman in the 1960s. Rudolph Kalman was a central figure in the development of mathematical systems theory upon which much of the subject of state variable methods rests. Kalman is well known for his role in the development of the so-called Kalman filter. The state variable compensator employing full-state feedback in series with full- state observer is shown in Figure 2.8.

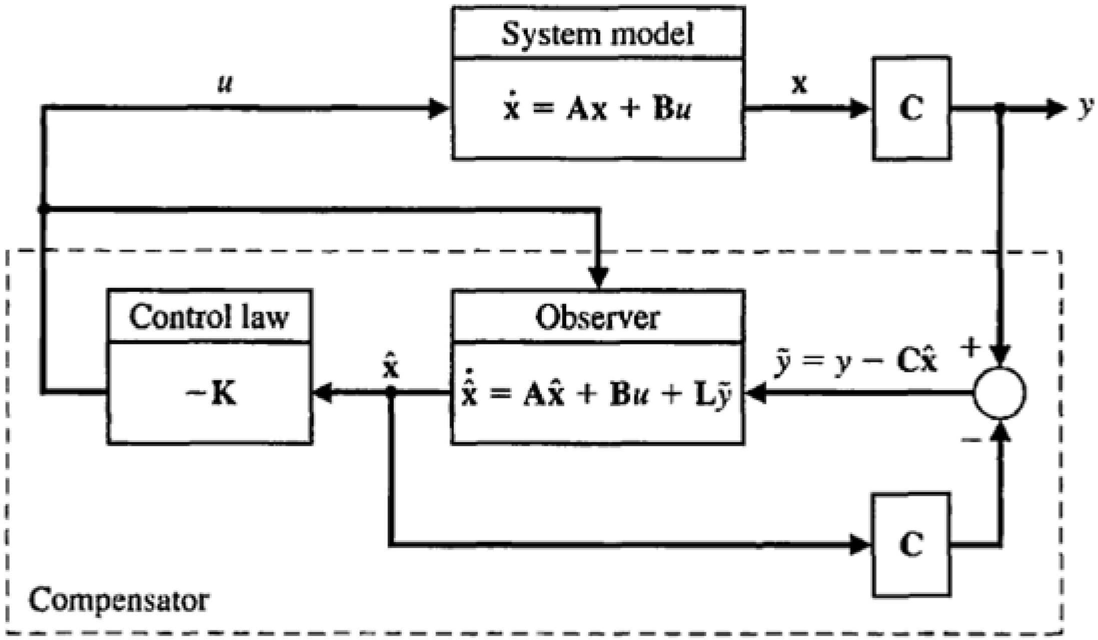

Figure 2.8: State variable compensator employing full-state feedback in series with a full- state observer

A system is completely controllable if there exists an unconstrained control *u(t)* that can transfer any initial state $x(t_0)$to any other desired location $x(t)$ in a finite time, $t_0 \leq t \leq T$.

For the system

$$\dot{x}=Ax + Bu \tag{2.10}$$

The system is controllable by examining the algebraic condition

$$\text{Rank}[B\ AB\ \ A^2B\ ...A^{n-1}B]= n. \tag{2.11}$$

The matrix A is an $n\times n$ constant matrix and B is an $n\times 1$ matrix.

For multi-input systems, B can be $n \times m$, where m is the number of input for a single-input, single-output system, the controllability matrix (M_c) is described in terms of A and B as:

$$M_c = [B\ AB\ \ A^2B\ \ldots A^{n-1}B] \tag{2.12}$$

Which is an $n \times n$ matrix Therefore, if the determinant of (M_c) is nonzero, the system is controllable. Advanced state variable design techniques can handle situations where in the system is not completely controllable, but where the states (or linear combinations thereof) that cannot be controlled are inherently stable. These systems are classified as stabilizable**.** If a system is completely controllable, it is also stabilizable. The Kalman state-space decomposition provides a mechanism for partitioning the state-space so that it becomes apparent which states (or state combinations) is controllable .The controllable subspace is thus exposed, and if the system is stablizable, the control system design can be completed. A system is completely observable if and only if there exists a finite time T such that the initial state $x(0)$can be determined from the observation history *y(t)* given the control *u(t)* ($0 \leq t \leq T$).

Consider the single-input, single-output system:

$$\dot{x} = Ax + Bu \quad \text{and} \quad y = Cx, \tag{2.13}$$

Where C is a *1* × n row vector and x is an n ×*1* column vector. This system is completely observable when the determinant of the observability matrix M_o is nonzero.

$$M_O = \begin{bmatrix} C \\ CA \\ \vdots \\ C^{n-1}A \end{bmatrix} \tag{2.14}$$

This is an $n \times n$ matrix.

The approach to state-variable design involves first verifying that the system under consideration is completely controllable and completely observable. If so, the pole placement design technique considered here can provide acceptable closed-loop system performance [7].

2.5.2 Full-state feedback control design

The first step in full-state variable feedback design process requires assuming that all the states are available for feedback, means the complete state $x(t)$ for all t. The control feedback given by is given by:

$$u = -K\,x \tag{2.15}$$

Determining the gain matrix K is the objective of the full-state feedback design procedure. The beauty of the state variable design process is that the problem naturally separates into a full-state feedback component and an observer design component.

These two design procedures can occur independently, and in fact, the separation Principle provides the proof that this approach is optimal. The stability of the closed-loop system is guaranteed if the full-state feedback control law stabilizes the system (under the assumption of access to the complete state) and the observer is stable (the tracking error is asymptotically stable). The full-state feedback block diagram is illustrated in Figure 2.9.

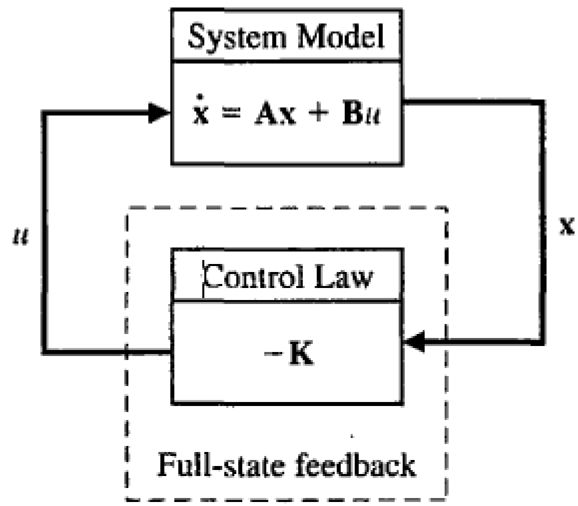

Figure 2.9: Full-state feedback block diagram (with no reference input)

The closed-loop system:

$$\dot{x} = Ax + Bu = Ax - BKx = (A - BK)\,x \tag{2.16}$$

The characteristic equation associated with Equation (2.16) is:

$$\det(\lambda I - (A - BK)) = 0 \tag{2.17}$$

If all the roots of the characteristic equation lie in the left half-plane, then the closed-loop system is stable. In other words, for any initial condition $x(t_0)$, it follows that:

$$x(t) = e^{(A-BK)t}x(t_0) \rightarrow 0 \text{ as } t \rightarrow \infty \tag{2.18}$$

Given the pair (A, B),to determine *K* to place all the system closed loop poles in the left half-plane after checking the system is completely controllable.

If and only if the controllability matrix (M_c) is full rank (for a single-input, single-output system, full rank implies that (M_c) is invertible). The addition of a reference input can be written as:

$$u(t) = -K\,x(t)+Nr(t) \qquad (2.19)$$

Where *r(t)* is the reference input. When *r(t)* = 0 for all $t > t_0$, the control design problem is known as the regulator problem. That is, we want to compute *K* so that all initial conditions are driven to zero in a specified fashion (as determined by the design specifications).

When using this state variable feedback, the roots of the characteristic equation are placed where the transient performance meets the desired response.