Chapter one

1.1 Introduction:-

To understand of semiconductor materials one most apply modern physics to solid materials more specifically. We are interested in semiconductor crystals. Crystals are solid materials consisting of atoms. Which are placed in a highly ordered structure called a lattice. Such a structure yields a periodic potential throughout the material. We need to understand the mobility of charge carriers in semiconductor and effect temperature and doping on these carriers.

In solid-state physics, the electron mobility characterizes how quickly an electron can move through a semiconductor, when pulled by an electric field. In semiconductor, there is an analogous quantity for holes, called hole mobility. The term carrier mobility refers in general to both electron and hole in semiconductors. Electron and hole mobility are special cases of electrical mobility of charged particles in a fluid under an applied electric field. When an electric field is applied a cross a piece of material, the electrons respond by moving with an average velocity called the drift velocity. Conductivity is product of mobility and proportional to the concentration. The same conductivity could come from small number of electrons with high mobility for each, or a large number of electrons with a small mobility for each. Therefore mobility is a very important parameter for semiconductor materials. Semiconductor mobility depends on the impurity concentration, temperature, defect concentration, and electron and hole concentration. It also depends on the electric field[1].

1.2 Importance of the study:-

The most important materials its used in industry is semiconductors, and mobility is a very important parameter for semiconductor materials. Another important of this study the random thermal motion of the electrons and holes. This type of motion is important in understanding *p-n* junctions and solar cells.

1.3 The aim of the study:-

The aim of this study is to understand carriers motion and reasons flow this carriers in semiconductors.

1.4 Thesis out line:-

This study includes four chapter, chapter one includes introduction, importance of the study, aim of the study and thesis out line. Chapter two includes general study in semiconductors and pn-junction. Chapter three includes introduction, electrons and holes in semiconductors, effective mass, conductivity of this carriers, carrier concentration, carrier drift, drift and diffusion current. Chapter four includes introduction, electron and hole mobilities, saturated drift velocity, mobility variation with temperature, carrier concentration at extremely high and low temperature, mechanism of carrier scattering, conclusion and reference.

Chapter two

2.1 Introduction:

The exist many different semiconductor materials. The most important parameter distinguishing these materials is the width of the energy band gap. The energy band gap of the most common semiconductors is 1.2ev (silicon). 0.67ev atoms in a pure silicon wafer contains four (germanium). electrons in outer orbit (called valence electrons). Germanium is another semiconductor material with four valence electrons. In the crystalline lattice structure of Si, the valence electrons of neighboring Si atoms every Si atom are locked up in covalent bonds with the valence electrons of Shows figure 2.1, difference four atoms. between this materials[2].

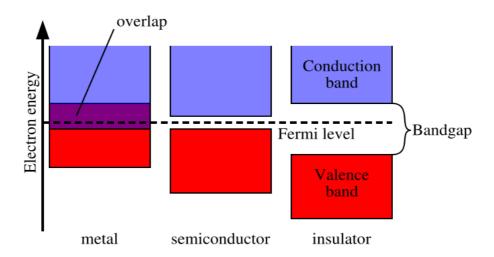


Figure 2.1 The energy gap between insulator, metal and semiconductor

2.2 Free electron in Semiconductor:-

Silicon (Si) is the most important semiconductor material. There are 14 Protons in the nucleus, and 14 electrons orbiting. An electron can exist in any of these orbits, but not outside their confines. The farthest 4 are known as Valence electrons. No free electrons: the electrons in the isolated Si atom cannot leave the atom. As shown figure 2.2, silicon atom [3].

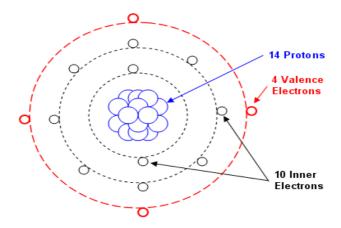


Figure 2.2 the silicon atom.

2.3 n-type semiconductors:-

The dopant atoms added to the semiconductor crystal in this case are donor atoms. For silicon, we can use phosphorus (P), arsenic (As) or antimony (Sb) as donors. These are column V elements, with five electrons in their outermost shell. When these atoms are included in the silicon crystal, one of the electrons in this shell can easily jump to the conduction band, leaving a positively charged atom behind. This process is sometimes called "activation" or "ionization" of the donor atoms. Figure 2.3a shows the required "activation" energies for these atoms in silicon; you can see that these are very small compared to the silicon band gap. The positively charged donor atom that is left behind after ionization is immobile and does not contribute to conduction. The electron leaving the atom by ionization does, and is counted in the electron concentration n. Because the activation energy is low, at room temperature almost all of the donor atoms included in the crystal will give an electron to the conduction band. So if $N_{\scriptscriptstyle D}$ is the donor concentration, for an n-type material at equilibrium:

$$n_0 \approx N_D \left(\frac{1}{c \, m^3}\right) (2.1)$$

where n_0 is thermal equilibrium concentration, N_D is donor concentration[4].

2.4 p-type semiconductor:-

The dopant atoms in this case are acceptor atoms. For silicon, we can use boron (B), Aluminum (Al) and Gallium (Ga) as acceptors. These are column III elements, with three

electrons in their outermost shell. When these atoms are included in the silicon crystal, one of the electrons in the silicon valence band can easily jump to the valence shell of one of the acceptor atoms, leaving a hole behind and making the acceptor atom negatively charged. Figure 2.3b shows where the valence shell energy level is for these atoms relative to the silicon valence band. The negatively charged acceptor atom after an electron joins its valence shell is immobile and does not contribute to conduction. The hole left behind by that electron does, and is counted in the hole concentration p. Because the activation energy is low, at room temperature almost all of the acceptor atoms included in the crystal will accept an electron from the valence band. So if N_A is the acceptor concentration, for a p-type material at equilibrium:

$$p_0 \approx N_A \left(\frac{1}{c m^3}\right) (2.2)$$

where p_0 is thermal equilibrium concentration, p_A is accepter concentration.

Figure 2.3, below shown donor and accepter ionization.

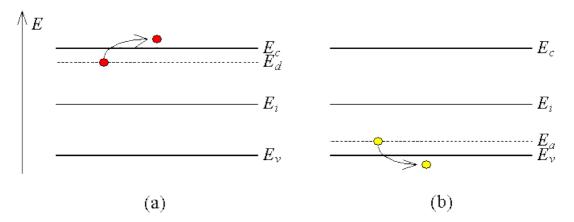


Figure 2.3 ionization of (a) a shallow donor and (b) a shallow accepter[4].

2.5 Energy band in Semiconductors:-

Electrons in an isolated atom can only have discrete energy levels, but when atoms are brought together as in crystalline solids, these degenerate energy levels will split into many separated levels due to the atomic interaction. Because the levels are so closely separated, they may be treated as a continuous band of allowed energy states. The two highest energy bands are the valence band and the conduction band. These bands are separated by a region which designates energies that the electrons in the solid cannot possess. This region is called the forbidden gap, or band gap E_{Q} . This is the energy difference between the valence band the minimum maximum energy and conduction For insulators the valence electrons form strong bonds between neighboring atoms. These bonds are difficult to break, and consequently there are no free electrons to current conduction. participate in Bonds neighboring atoms in a semiconductor are only moderately strong. Therefore thermal vibrations may break bonds. When a bond is broken, an electron is injected from the valence band into the conduction band. This is now a mobile negative charge carrier, and the atom from which the electron emerges is left with a negative charge deficiency, i.e. a positive net charge, also called a hole. The band gap for Si at 300 K is 1.12 eV. A characteristic property of semiconductors is that the band gaps have a negative temperature coefficient. The band gap in Si and GaAs as a function of temperature can be described by[5]:-

$$E(T)_g = 1.17 - \frac{4.73 \times 10^{-4} T^2}{T + 636}$$
 for Si (2.3)

$$E(T)_g = 1.52 - \frac{5.4 \times 10^{-4} T^2}{T + 204}$$
 for Ga As (2.4)

2.6 Doping of Semiconductors:-

Pure semiconductors, such as a Si crystal without any impurities, are called intrinsic semiconductors. when a semiconductor is doped with impurities, it becomes extrinsic and impurity energy levels are introduced. The doping occurs when some atoms in the lattice are replaced with foreign atoms, altering the lattice structure. Figure 2.4 shows a Si lattice where one of the atoms have been replaced by a type-V atom, e.g. phosphor. Four of the phosphor valence electrons form covalent bonds to the nearest

neighbouring Si-atoms, while the fifth becomes a conduction electron that is 'donated' to the conduction band. Thus, the material is called an n-type donator because of the additional negative charge carrier, or simply an n-type material. A complementary situation is seen in figure 2.5, where a Si atom is replaced by a type-III atom, e.g. boron. Here the local lattice deficiency of one electron can receive an electron from one of the neighbouring atoms, or a free

conduction band electron. Thus, the material is called a p-type acceptor because of the positive charge released by accepting an electron. The convention of viewing this occurrence as a movement of a positive charge, rather than a negative electron, will become clear. A silicon lattice with a single impurity atom (Phosphorus, P) added. As compared to Si, the Phosphorus has one extra valence electron which, after all bonds are made, has very weak bonding[5].

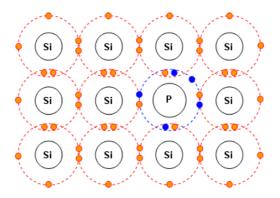


Figure 2.4 donor impurity

A silicon lattice with a single impurity atom (Boron, B) added. Boron has only three valence electrons, one electron less than the Si atom. Having only three valence electrons - not enough to fill all four bonds - it creates an excess hole that can be used in conduction[5].

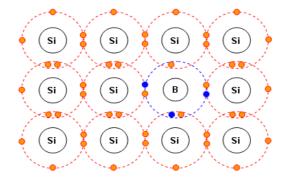


Figure 2.5 accepter impurities

2.7 Intrinsic semiconductors:-

The intrinsic semiconductors such as pure Ge or Si are undoped semiconductors. The electrical conductivity of this type of semiconductors is solely determined by thermally understand carriers. To the mechanism generated conduction, we consider the bonding between atoms in this semiconductors. Each silicon atom has four valence electrons and can form four covalent bonds with four neighbouring silicon atoms which are directed along the corners of a regular tetrahedron. Therefore, exhibits a three-dimensional regular network type structure which, for simplicity, is represented by a two-dimensional network as shown in fig. 2.6. apparently, all the valence electrons in a silicon crystal participate in the formation of covalent bonds and no electron is free to case conduction particularly at 0k. As the temperature increase above 0k, some of the valence electrons may acquire sufficient thermal energy to break their covalent bonds and become free from the influence of cores of the atoms[6].

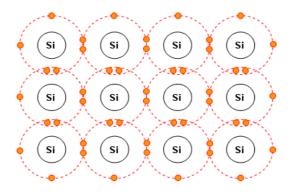


Figure 2.6 2-D representation of a silicon crystal

These electrons move randomly in the crystal. Each escaped electron leaves behind an empty space called a hole. Thus when a valence electron breaks away from a covalent bond, an

electron-hole pair is generated and two carriers of electricity are produced as shown in fig. 2.7.

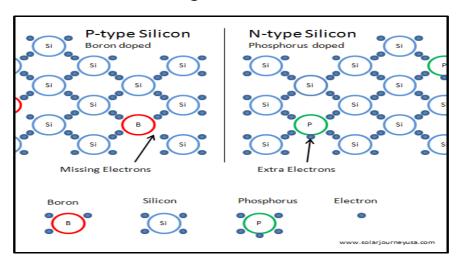


Figure 2.7 electron hole pair

When a valence electron located adjacent to a hole acquires sufficient thermal energy, it may jump into the hole position to reconstruct the broken covalent bond and a hole is created at the initial position of the electron. Thus the motion of an electron may also be regarded as the motion of a hole in the opposite direction. These electrons an holes move in opposite directions under the effect of an electric field and constitute the current. The electrons in the conduction band and holes in the valence band behave as free carriers and increase the conductivity of the material. The conditions for the movement of electrons are, however, different from the conditions for movement of holes, the electrons move when the conduction band nearly empty and the holes move when the valence band is nearly full. Thus the electrons move mainly under the influence of the applied field while the holes move under the combined effect of the applied electric field and the ionic field of the lattice[6].

2.8 Extrinsic semiconductors:-

Doping is the most efficient and convenient method of increasing and altering the conductivity of an intrinsic semiconductor. Depending on the type of doping, excess electrons or holes are generated in the material which are free to conduct electricity. The impurity atoms frequently employed to dope pure silicon or germanium are the elements of group III and group V of the periodic table. These impurity atoms are referred to as accepter or p-type and donor or n-type impurities as they contribute excess holes and electrons respectively to the semiconducting material. The dopants are added in the ratio of about 1 in

 $10^6 \, i \, 10^8$ atoms of the semiconducting material. Such a small quantity of dopants does not bring about any structural changes in the semiconductor as the impurity atoms replace the regular atoms in the crystal[6].

2.9 The pn junction:-

Semiconductor pn junction is a two-terminal device. It is the most fundamental device element that forms the basis of many electronic devices such as pn diodes, optoelectronic devices like light-emitting diode and photodetector, field effect transistors and bipolar transistor. PN junction conducts high current in one direction and conduct very small amount of current in the reversed direction. Thus, pn junction has the property of rectification. PN junction is formed in a single crystal of semiconductor by making one end of the crystal p-type by doping it with acceptor atom and making the other

end n-type by doping with donor atoms. The region where p-type and n-type meet is the junction[7].

2.9.1 The pn junction at equilibrium:-

The equilibrium state is the state where the pn junction is left without any external stimulant such as electrical potential connected to it. This is also the state of zero bias voltage condition. When two semiconductor material type, p-type and n-type are brought to contact, majority carrier of each type would diffuse across the junction. This shall mean that the majority carrier - hole from p-type diffuses to n-type material and the majority carrier of n-type diffuses to p-type material. The diffusion would stop after an electric field is built up sufficiently high to oppose diffusion. As the majority carrier such as hole diffuses across the junction, it combines with electron in the n-type side, which creates a net positive charge. Likewise, the majority carrier electron from n-type material diffuses across the junction recombines with hole in p-type side creates net negative charge. The net charge at each side creates an electric field in the direction, which would oppose further diffusion. The electric field created would drift the minority carrier in the opposite direction across the junction. Thus when equilibrium attained, the drift carriers and diffused carriers should be balanced in termed of magnitude and in opposite direction. Mathematically, current density I due to various carrier types can be written as shown in equation (2.5) and (2.6).

$$J_{p.drift} + J_{p.diffusion} = 0$$

 $J_{n,drift} + J_{n,diffusion} = 0$

(2.6)

where J_p is current density of type p, and J_n is current density of type n.

As the result of this process, a depletion region meaning a region that lack of carrier of certain thickness is created at both side of the junction. At time the depletion region is also termed as space charge region. Figure 2.8 to Fig. 2.10 illustrate the flow process of carriers in p-type and n-type materials and how equilibrium is attained when they come in contact[7].

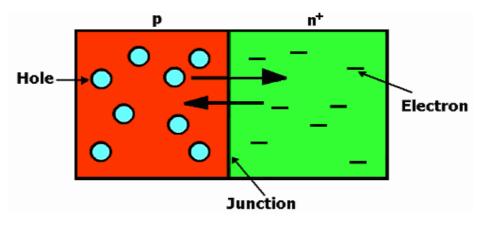


Figure 2.8 majority carrier diffusion in pn junction

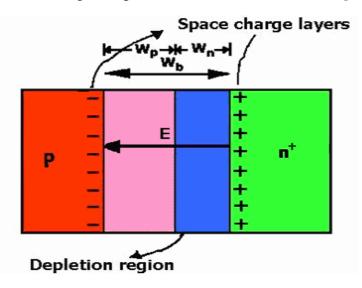


Figure 2.9 A pn junction showing depletion region, space charge layer and electric field

2.10 Conduction of pn junction:-

Conduction in pn junction physical involves in conduction band and valence band. Electron flows in conduction band, whereas the hole flows in valence band. At zero bias voltage equilibrium condition, the minority hole and electron can drift easily under the influence of built-in electric field E. The diffusion majority carriers have to overcome the potential barrier V of the junction created as the result of depletion region. This shall mean that majority carrier should at least acquire energy of q V_B electron volt before it can overcome the barrier and diffuse into either p or n region. Figure 2.11 illustrates the energy band diagram showing the location of the carrier at zero bias voltage equilibrium condition.

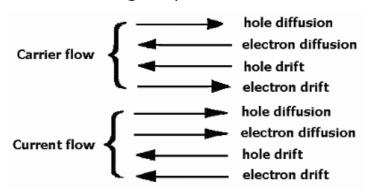


Figure 2.10 The carrier current at equilibrium cross a pn junction

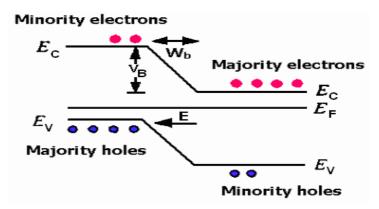


Figure 2.11 The energy band diagram of a pn junction showing the location of carriers under zero voltage bias

2.11 Forward bias pn junction:-

A pn junction can be forward biased to lower the voltage across the junction. If a positive forward voltage V is applied to the p-side of the pn junction relative to the n-side, the effective voltage across the junction is (V_B – V) not V_B . Thus, the energy required by the majority carrier to overcome the potential barrier is less than earlier zero voltage bias case. As the result more majority carrier will be able to diffuse across the junction. Figure 2.12 shows the condition of the carrier under forward bias voltage.

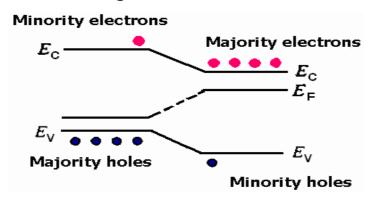


Figure 2.12 The energy band diagram of a pn junction showing the location of carriers under forward bias voltage

2.12 The reverse bias pn junction:-

Figure 2.13 shows that the barrier potential increases under reverse bias voltage –V. The bias voltage is effectively subtracted from the potential barrier. Consequently, the voltage across the junction is (V_B + V). As the result, lesser chance for majority carrier to diffuse across the junction and more easily for the minority carrier to drift across the junction would be happened. If the pn junction is forward biased, the diffusion would increase and drift current is negligible. If the pn junction is reverse biased, the diffusion current would be negligible and the drift current would increase drastically[7].

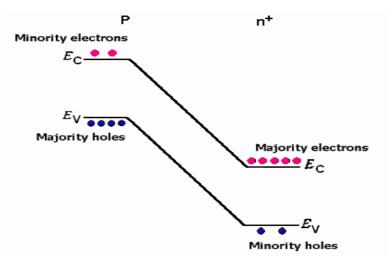


Figure 2.13 The energy band diagram of a pn junction showing the location of carriers under reverse bias voltage

Chapter three

3.1 Introduction:-

Current is the rate at which charge flows in a semiconductor, two types of charge carrier, the electron and the hole ,can contribute to a current. Since the current in a semiconductor is determined largely by the number of electrons in the conduction band and the number of holes in the valence band, an important characteristic of the semiconductor is the density of these charge carriers. The density of electrons and holes is related to the density of states function and the Fermi distribution function, both of which we have considered. A qualitative discussion of these relationships will be followed by a more rigorous mathematical derivation of the thermal-equilibrium concentration of electrons and holes[8].

3.2 Electrons and Holes:-

Concept of hole: an unoccupied electronic state in the valence band is called a hole. Although the term electrons can used in conjunction with the valence band as in "nearly all the energy states in the valence band are filled with electrons," we should that assume the term usually conduction-band electrons. Holes are the electron voids in the valence band. Electrons and holes carry negative and positive charge respectively. As shown in Fig. 3.1, a higher position in the energy band diagram represents minimum conduction a higher electron energy. The electron energy is E_{C} . Any energy above E_{C} is the electron kinetic energy. Electrons may gain energy by getting accelerated in an electric field and may lose energy through collisions with imperfections in the crystal. A lower location in the energy diagram represents a higher hole energy as shown in Fig. 3.1. It requires energy to move a hole "downward" because that is equivalent to moving an electron upward. E_V is the minimum hole energy. We may think of holes as bubbles in liquid, floating up in the energy band. Similarly, one may think of electrons as water drops that tend to fall to the lowest energy states in the energy band[9].

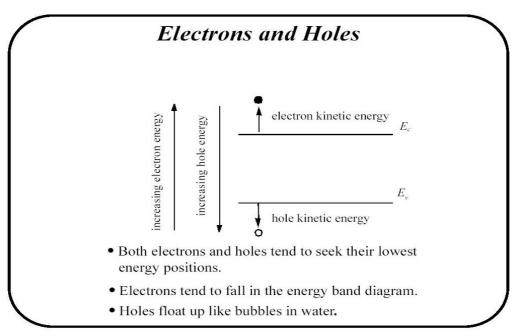


Figure 3.1 requires energy to move electron, hole upward and downward.

3.2.1 Effective Mass:-

When an electric field, is applied, an electron or a hole will accelerate according to:

Acceleration=
$$\frac{-q}{m_n}$$
 for electrons

(3.1)

Acceleration =
$$\frac{q}{m_p}$$
 for hols

(3.2)

Where m_n and m_p is mass of electron and hole respectively. In order to describe the motion of electrons and holes with the laws of motion of the classical particles, we must assign effective masses $(m_n$ and $m_p)$ to them. The electron and hole effective masses of a few semiconductors are listed in Table 3.1[9].

Table 3.1 Electron and hole effective masses, m_n and m_p , normalized to the free electron mass.

Effective	Si	Ge	GaAs	InAs	AlAs
masses					
m_n/m_0	0.26	0.12	0.068	0.023	2.0
m_p/m_0	0.39	0.30	0.50	0.30	0.3

3.3 conductivity of charge carriers:-

the number of electrons and holes in semiconductor is not very important but the density of charge carriers is significant. If n is number of electrons and p is the number of holes, then for an intrinsic semiconductor:

 $n_i = n = p$

(3.3)

where n_i is the carrier concentration in pure semiconductor. If σ_n is the conductivity due to free electrons and σ_p due to holes, then specific conductance(σ_i) is given by the expression:

 $\sigma_i = \sigma_n + \sigma_p$

(3.4)

Their corresponding mobilities are μ_n and μ_p , respectively. Therefore, Eq.(3.4) becomes:

 $\sigma_i = ne \mu_n + pe \mu_p$

(3.5)

Where μ_n is the mobility of electron and μ_p is the mobility of hole.

Since $^{n=p=n_i}$, Eq.(3.5) can be written as:

$$\sigma_i = n_i e(\mu_n + \mu_p)$$

(3.6) Since it has already been stated that the mobility of charge carriers depends on temperature, the power law($\mu\alpha 1/T^{\frac{3}{2}}i$ in case of the mobility of electrons and holes plays a role, i.e.,

$$\mu_n = \alpha T^{-3/2}$$
 and $\mu_p = \beta T^{-3/2}$

So,
$$(\mu_n + \mu_p) = (\alpha + \beta \dot{c} T^{-3/2})$$

Or
$$(\mu_n + \mu_p) = \gamma T^{-3/2}$$

(3.7)

Where $^{\alpha}$ and $^{\beta}$ are constants of proportionality and $^{\gamma}$ is equal to ($^{\alpha+\beta i}$. Substituting Eq.(3.7) in Eq.(3.6), we get:

$$\sigma_i = \gamma(n_i e) T^{-3/2}$$

(3.8)

On using certain equations for deriving the intrinsic carrier concentration, we get:

$$n_i = 4.83 \times 10^{21} T^{3/2} \exp\left(\frac{-E_g}{2 k_B T}\right)$$

(3.9)

Where E_g is energy band gap, k_B is Boltzmann constant.

Using this expression in Eq.3.8 we find:

$$\sigma_i = \gamma e \times 4.83 \times 10^{21} \exp\left(\frac{-E_g}{2k_BT}\right)$$
 therefore,

$$\sigma_i = A \exp\left(\frac{-E_g}{2k_BT}\right)$$

(3.10)

Where A is equal to $^{4.83\times10^{21}\gamma e}$. This becomes an expression for conductivity of an intrinsic semiconductor[10].

3.4 The intrinsic carrier concentration:-

For an intrinsic semiconductor, the concentration of in the conduction band is equal to concentration of holes in the valence band. We may n_i and p_i as the electron and denote concentration, in the intrinsic semiconductor. These parameters are usually referred to as the intrinsic electron concentration and intrinsic hole concentration. However, $n_i = p_i$, which refers to either the intrinsic electron or hole concentration. The Fermi energy level for[8] the intrinsic semiconductor is called the intrinsic Fermi energy, or $E_F = E_{Fi}$. The equation intrinsic can be written as:

$$n_0 = n_i = N_C \exp\left[\frac{-\left(E_C - E_{Fi}\right)}{kT}\right]$$

(3.11)

And

$$p_0 = p_i = N_v \exp\left[\frac{-\left(E_{Fi} - E_v\right)}{kT}\right]$$

(3.12)

Where

- n_0 electron density in thermal equilibrium.
- p_0 hole density in thermal equilibrium.
- n_i intrinsic carrier density of electron.
- $^{N_{_{\boldsymbol{v}}}}$ effective density of states in the valence band.
- N_c effective density of states in the conduction band.
 - p_i intrinsic carrier density of hole.
 - E_c conduction band energy.
 - E_{ν} valence band energy.
 - E_{Fi} intrinsic Fermi energy.

If we take product of equations (3.11)and(3.12), we obtain[8]:

$$n_i^2 = N_c N_v \exp \left[\frac{-(E_c - E_{Fi})}{kT} \right] \cdot \exp \left[\frac{-(E_{Fi} - E_v)}{kT} \right]$$

(3.13)

Or
$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$
 (3.14)

3.4.1 Electron concentration in the conduction band:-

The number of free electrons per unit volume in an energy range E and E+ dE can be written as:

$$dn = D(E)f(E)dE$$

(3.15)

Where D(E) is the density of states defined as the total number of allowed electronic states per unit volume in a semiconductors and f(E) is the Fermi distribution function representing the probability of occupation of a state with energy E. the expression for f(E) is given by[6]:

$$f(E) = 1/ \exp\left[\frac{E - E_F}{KT}\right] + 1$$

(3.16)

where

$$D(E) = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2}$$

(3.17)

Using Eq.(3.15) and (3.17), we obtain:

$$dn = \frac{4\pi}{h^3} (2m)^{\frac{3}{2}} E^{\frac{1}{2}} f i \quad E \quad i dE$$

(3.18)

Therefore in Eq.(3.17), E must be replaced by ($^{E-E_c}$). Thus Eq. (3.18) becomes

$$dn = \frac{4\pi}{h^3} \left(2 m_n^{i}\right)^{3/2} \left(E - E_C\right)^{1/2} \frac{1}{\exp\left[\frac{E - E_F}{KT}\right] + 1} dE$$

(3.19)

Where m_n^{i} is the effective mass of the electron in the conduction band. The concentration of electron, n, in the conduction band is obtained by integrating Eq.(3.19) from E=

$$E_C$$
 to E= ∞

$$n = \frac{4\pi}{h^3} (2m_n^i)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_C)^{1/2}}{\exp\left[\frac{E - E_F}{KT}\right] + 1} dE$$

(3.20)

Now near room temperature, KT=0.026 eV. Therefore, for energies greater than $^{E_{\it C}}$, we have

$$1 + \exp\left[\frac{E - E_F}{KT}\right] \cong \exp\left[\frac{E - E_F}{KT}\right] ,$$

$$n = \frac{4\pi}{h^3} (2m_n^i)^{3/2} \int_{E_c}^{\infty} (E - E_C)^{1/2} \exp\left[-(\frac{E - E_F}{KT})\right] dE$$

$$= \frac{4\pi}{h^3} (2m_n^i)^{3/2} \exp\left(\frac{E_F - E_C}{KT}\right) \int_{E_c}^{\infty} (E - E_C)^{1/2} \exp\left[-(\frac{E - E_C}{KT})\right] dE$$

$$\frac{E - E_C}{KT} = x : dE = KTdx, for E = E_C, X = 0$$

$$n = \frac{4\pi}{h^3} (2m_n^i)^{\frac{3}{2}} \exp\left(\frac{E_F - E_C}{KT}\right) \int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx.$$

Now
$$\int_{0}^{\infty} x^{\frac{1}{2}} e^{-x} dx = i \left(\frac{\pi}{4}\right)^{1/2}$$

$$\therefore \qquad \qquad \mathsf{n} = 2 \left[\frac{2\pi m_n^{\iota}}{h^2} \right]^{3/2} \exp \left[-\left(\frac{E_C - E_F}{KT} \right) \right]$$

(3.21)

From Eq.(3.16) , the probability of occupancy of level $^{E_{\mathcal{C}}}$ is given by:

$$f(\begin{array}{c} E_C \dot{\iota} = \frac{1}{1 + \exp\left(\frac{E_C - E_F}{KT}\right)} \\ = \begin{array}{c} \left(\frac{E_C - E_F}{KT}\right) \\ -\dot{\iota} \\ ex\dot{\iota} \end{array}$$
 Therefore Eq.(3.21)

becomes[6]

$$n = 2 \frac{\frac{2\pi \, m_n^i KT}{h^2}}{\frac{i}{i}}$$
 . Denoting it by N_c , we have

$$\begin{array}{ccc}
 & \left(\frac{E_C - E_F}{KT}\right) \\
 & -\dot{\iota} \\
 & N_C \exp \dot{\iota}
\end{array}$$

(3.22)

where $N_{c}=\dot{\iota} \qquad 2 \frac{\frac{2\pi \, m_{n}^{\dot{\iota}} KT}{h^{2}}}{\dot{\iota}_{\dot{\iota}}^{\dot{\iota}}}$

(3.23)

3.4.2 Hole Concentration in the Valence Band:-

An expression for the number of hole per unit volume in the energy range E and E+ dE can be written as:

$$1-f(E)dE$$
$$dp=D(E)\ddot{c}$$

(3.24)

Where we have replaced f(E) by [1-f(E)] which represent the probability of an energy state E not to be occupied by an electron, the probability of finding a hole in the energy state E.

Now
$$1-f(E) = 1 - \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)} = \frac{\exp\left(\frac{E - E_F}{KT}\right)}{1 + \exp\left(\frac{E - E_F}{KT}\right)}$$

In the valence band ,since E ${}^{\iota E_F}$ the exponential term in the denominator may be neglected in comparison to unity. Thus, we get:

$$1 - f(E) \cong \exp\left(\frac{E - E_F}{kT}\right)$$

(3.25)

it follows that the probability of finding holes decreases exponentially with increase in depth into the valence band. Also, the kinetic energy of a hole in the energy state E in the valence band is ($^{E_{V}}$ - E) . Therefore the density of state per unit volume can be written as:

$$D(E) = \frac{4\pi}{h^3} (2m_p^6)^{3/2} (E_V - E)^{1/2}$$

(3.26)

 where $^{m_p^c}$ is the effective mass of a hole in the valence band . Using Eq.(3.25) and (3.26) in Eq.(3.24) and integrating from E=- $^{\infty}$ to E= E_V , we obtain the hole concentration in the valence band as:

$$p = \frac{4\pi}{h^3} \left(2 m_p^{i} \right)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} \exp \left(\frac{E - E_F}{KT} \right) dE$$

$$= \frac{E_{V} - E}{\frac{\dot{\zeta}}{\dot{\zeta}}} = \frac{4\pi}{h^{3}} (2m_{p}^{i})^{3/2} \exp\left(\frac{E_{V} - E_{F}}{KT}\right) \int_{-\infty}^{E_{V}} \dot{\zeta}}$$
[6]

Let
$$\frac{E_V - E}{KT}$$
 , $dE = -KTdx$ for $E = E_V$, $x = 0$

$$\therefore p = \lambda \frac{4\pi}{h^3} (2m_p^{\lambda})^{\frac{3}{2}} \exp\left(\frac{E_V - E_F}{KT}\right) \int_{\infty}^{0} x^{\frac{1}{2}} KT^{\frac{1}{2}} e^{-X} (-KT) dx = \lambda$$

$$\frac{4\pi}{h^3} (2m_p^i)^{\frac{3}{2}} \exp\left(\frac{E_V - E_F}{KT}\right) (KT)^{3/2} \int_0^\infty x^{1/2} e^{-x} dx =$$

$$\frac{4\pi}{h^{3}} \left(2 m_{p}^{i} KT\right)^{\frac{3}{2}} \exp\left(\frac{E_{V} - E_{F}}{KT}\right) \left(\frac{\pi}{4}\right)^{1/2} =$$

$$\therefore \qquad p = \lambda \qquad 2 \qquad \frac{2\pi m_p^{\ell} KT}{h^2} \\ \therefore \qquad \qquad \lambda \qquad \qquad \lambda \qquad \qquad \lambda \qquad \qquad \lambda \qquad \qquad (3.27)$$

Or

$$p = N_V \exp \left[-\left(\frac{E_F - E_V}{KT}\right) \right]$$

(3.28)

Where

$$N_V = 2 \left(\frac{2 \pi m_P^i KT}{h^2} \right)^{3/2}$$

(3.29)

The electron and hole concentrations given by Eqs.

(3.23)and(3.29) respectively are valid for Both intrinsic and extrinsic materials. For intrinsic materials, these equations can also be written as[6]:

$$n_i = N_C \exp \left[-\left(\frac{E_C - E_i}{KT}\right) \right], p_i = N_V \exp \left[-\left(\frac{E_i - E_V}{KT}\right) \right]$$

(3.30)

3.4.3 Fermi Level:-

For an intrinsic semiconductor, $n=p=n_i$. Therefore, from Eqs.(3.22)and(3.28)

$$N_{C} \exp \left[-\left(\frac{E_{C} - E_{F}}{KT} \right) \right] = N_{V} \exp \left[-\left(\frac{E_{F} - E_{V}}{KT} \right) \right]$$

$$\exp\!\left(\frac{2\,E_F\!-\!E_C\!-\!E_V}{KT}\right)\!=\!\frac{N_V}{N_C}\quad\text{or}\quad \frac{2\,E_F\!-\!E_C\!-\!E_V}{KT}\!=\!\ln(\frac{N_V}{N_C}) \ ,$$

$$E_F\!=\!E_i\!=\!\frac{E_C\!+\!E_V}{2}\!+\!\frac{KT}{2}\ln(\frac{N_V}{N_C})$$

(3.32)

Using Eqs.(3.23) and(3.29) in Eq.(3.32) we obtain:

$$E_F = \frac{E_C + E_V}{2} + \frac{3}{4} KT \ln\left(\frac{m_p^c}{m_n^c}\right)$$

(3.33)

At 0k the Fermi level lies in the middle of the conduction band and valence band as shown in figure 3.2. This is also true at all other temperatures provided $m_p^i = m_n^i$ [12].

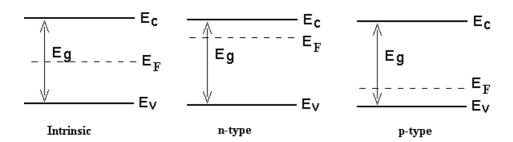


Figure 3.2 position of Fermi level in intrinsic, n and p type[6].

3.5 Carrier drift:-

Drift: the movement of charge due to the electric field.

Diffusion: the flow of charge due to density gradient.

An electric field applied to a semiconductor will produce a force on electrons and holes so that they will experience a net acceleration and net movement , provided there are available energy states in the conduction and valence band . This net

movement of charge due to an electric field is called drift. The net drift of charge gives rise to a drift current[8].

3.5.1 Drift Current :-

The current that flows in a semiconductor as a result of carrier drift. the current density, *J*, is the charge per second crossing a unit area plane normal to the direction of current flow. In the P-type semiconductor bar of unit area. The hole current density is[11]:

$$j_{p,drift} = qpv$$

(3.34)

but: $v=\mu_p E$, then:

where E is electric field and v is drift velocity and $^{\mu_{p}}$ is hole mobility. Eq.(3.34) becomes

$$j_{p.drift} = qp \mu_p E$$

(3.35)

Similarly, the electron current density can be expressed as:

$$j_{n,drift} = qnv = qn\mu_n E$$

(3.36)

The total drift current density is the sum of the electron and the hole components:

$$j_{drift} = j_{p,drift} + j_{n,drift} = (q p^{\mu_p} + q n^{\mu_n})E$$
 (3.37)

Where

- J current.
- j_p hole current density.
- j_n electron current density[11].

3.5.2 Diffusion Current:-

In addition to the drift current, there is a second component of current called the diffusion current. Diffusion current is generally not an important consideration in metals because of their high conductivities. The low conductivity and the ease of creating nonuniform carrier densities make diffusion an important process in semiconductors.

Diffusion is the result of particles undergoing thermal motion as depicted in Fig. 3.3. It is the familiar process by which particles move from a point of higher particle density toward a point of lower density. It is known that the rate of particle movement by diffusion is proportional to the concentration gradient. If the electron concentration is not uniform, there will be an electron diffusion current, which is proportional to the gradient of the electron concentration.

$$j_{n.diffusion} \propto \frac{dn}{dx}$$
 $j_{n.diffusion} = q D_n \frac{dn}{dx}$

(3.40)

Where D_n is electron diffusion, for holes[11]

$$j_{p.diffusion} = -q D_p \frac{dp}{dx}$$

(3.41)

Equation (3.41) has a negative sign, while Eq. (3.40) has a positive sign. Instead of memorizing the signs, memorize Fig. 3.4. In Fig. 3.4, (a) shows a positive dn/dx (n increases as x increases) and (b) shows a positive dp/dx. In (a), electrons diffuse to the left (toward the lower concentration point). Because electrons carry negative charge, the diffusion current flows to the right. In (b), holes diffuse to the left, too. Because holes are positively charged, the hole current flows to the left, i.e., the current is negative. In general, both drift and diffusion may contribute to the current. Therefore,

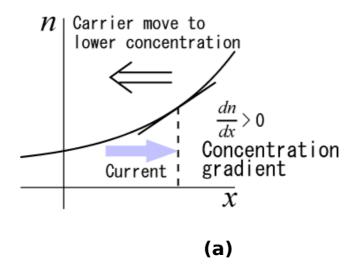
$$j_n = j_{n.drift} + j_{n.diffusion} = qn \, \mu_n E + q \, D_n \frac{dn}{dx}$$
(3.42)

$$j_p = j_{p.drift} + j_{p.diffusion} = qp \,\mu_p E - q \,D_p \frac{dp}{dx}$$
(3.43)

$$j = j_n + j_p$$

(3.44)

Figure 3.3 the thermal motion of an electron or hole[11].



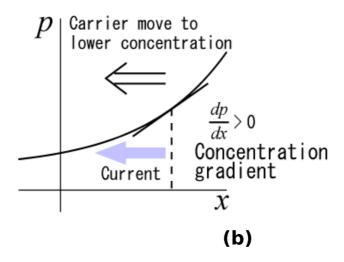


Figure 3.4 A positive slope of carrier concentration produces a positive electron diffusion current (a), but a negative hole diffusion current (b)[11].

Chapter four

4.1 introduction:-

We defined mobility, which relates the average drift velocity of a carrier to the electric field .Electron and hole mobility's are important semiconductor parameters in the characterization of carrier drift. Electron in doped semiconductor that is thermalised at room temperature have a thermal energy which allows them to wonder in any direction. the average drift velocity of carriers per unit electric field in a homogeneous semiconductor , the mobility of electrons is usually different from that of holes[8].

4.2 Electron and hole mobilities:-

The average velocity of the carriers is no longer zero when an electric field is applied to the semiconductor. This nonzero velocity is called the drift velocity. The drift velocity is superimposed on the thermal motion as illustrated in Fig. 4.1. The drift velocity is so much more important than the thermal velocity in semiconductor devices that the term velocity usually means the *drift* velocity. A faster carrier velocity is desirable, for it allows a semiconductor device or circuit to operate at a higher speed. We can develop a

model for the drift velocity using Fig. 4.1. Consider the case for holes. Assume that the mean free time between collisions is τ_{mp} and that the carrier loses its entire drift momentum, m_p v, after each collision. The drift momentum gained between collisions is equal to the force, qE, times the mean free time. Equating the loss to the gain, we can find the steady state drift velocity, v

$$m_p v = qE \tau_{mp}(4.1)$$

$$v = \frac{qE \tau_{mp}}{m_p} \tag{4.2}$$

Where τ_{mp} mean free time between collision fore holes, E is electric field, v is velocity. Eq.(2.2) is usually written as[12]:

$$v = \mu_p E(4.3)$$

Where

$$\mu_p = \frac{q \tau_{mp}}{m_p}$$

(4.4)

Equation (4.3) simply says that the drift velocity is proportional to the electric field. The proportionality constant μ_p is the hole mobility, a metric of how mobile the holes are. Similarly, electron drift velocity and electron mobility are:

$$v = -\mu_n E(4.5)$$

$$\mu_n = \frac{q \, \tau_{mn}}{m_n} (4.6)$$

The negative sign in Eq. (4.5) means that the electrons direction opposite to the field. They do so drift in а the electron is negatively charged. We should because memorize these statements rather than the negative sign. Carrier mobility has the same dimension as v/E, i.e., /V·s. Table 4.1 shows some mobility values. Notice that GaAs has a much higher μ_n than Si (due to a smaller m_n higher-speed transistors can be GaAs. which are typically used in communications has an even higher μ_n , but the equipment. InAs technology of fabricating InAs devices has not yet been fully developed[12].

Table 4.1 Electron and hole mobilities at room temperature of selected lightly doped semiconductors.

Mobilities	Si	Ge	GeAs	InAs
ν.s μ _n (c _{m²/¿¿}	1400	3900	8500	
30,000				
ν.s μρ(c _{m²/ἰί}	470	1900	400	500

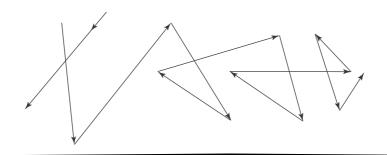


Figure 4.1 An electric field creates a drift velocity that is superimposed on the thermal velocity.

The ratio of the drift velocity to the field strength is referred to as the mobility of carriers:

$$v_d = a\tau = \frac{-qE}{m_o^i}\tau = -\mu E(4.7)$$

where
$$\mu = |v_d|/E$$

Mobility is thus the drift velocity gathered by electrons in the field of unit strength. For copper the relaxation time ${}^{\tau}$ is ${}^{-2\times 10^{-14}s}$ and the electron mobility is $(\mu\approx 3\times 10^{-3}m^2v^{-1}s^{-1}\dot{c})$. The drift velocity in the field having a strength of $10^{-2}V\,m^{-1}is\,v_d=3\times 10^{-5}m\,s^{-1}$, which is many orders of magnitude lower than the random electron velocity(thermal speed) ${}^{V_T}\approx 10^{-6}m\,s^{-1}$ [12].

4.3 Saturated drift velocity:-

Equation(4.3) implies that we can make a carrier go as fast as we like just by increasing the electric field. unfortunately for those of us that work with silicon technology, this is not the case. Figure 4.2 shows how the electron drift velocity saturates at high electric fields. If you look at the drift velocity, v_d , at which the saturation seems to start you can see that it is pretty close to the carrier thermal velocity, v_t , that we calculated

earlier. As we try to increase carrier velocity by increasing the electric field beyond this point, we find that the expected energy increase goes into heating up the lattice, rather than into the electrons own kinetic energy. So you can see that we have a fundamental problem here when it comes to trying to make fast devices. Real devices often have both high-field and low-field regions in their concentration . in the low-field regions the carrier mobility will be a major limitation to device speed, whereas in high-field regions, the limitation is the saturated drift velocity. In this figure the electron drift velocity saturates at high electric fields. As shown below[13].

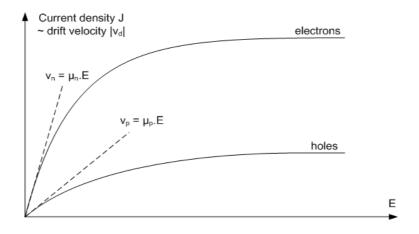


Figure 4.2 saturation drift velocity.

4.4 Mobility variation with temperature:-

If the mobility is made up of several components say $\ ^{\mu_1}$, $\ ^{\mu_2}$, $\ ^{\mu_3}$,then the total mobility is given by:

$$\mu = \left(\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}\right)^{-1}$$

(4.8)

If we consider just two components due to lattice scattering , $\mu_{\scriptscriptstyle L}$, and ionized impurity scattering, $\mu_{\scriptscriptstyle I}$, then the mobility is given by:

$$\mu = \left(\frac{1}{\mu_I} + \frac{1}{\mu_L}\right)^{-1}$$

(4.9)

Where μ_L is lattice scattering, and μ_I ionized impurity scattering. As the lattice warms up, the lattice scattering component will increase, and the mobility will decrease. Experimentally, its found that the mobility decrease often follows a $T^{1.5}$ power law[13]:

$$\mu_L = C_1 \times T^{-3/2}$$

(4.10)

Where C_1 is a constant .

As we cool a piece of semiconductor down, the lattice scattering becomes less important, but now the carriers are more likely to be scattered by ionized impurity, since the carriers are moving more slowly. its found that the ionized impurity scattering often follows a $T^{1.5}$ power law giving :

$$\mu_I = C_2 \times T^{3/2}$$

(4.11)

Figure 4.3 shows the variation in carrier mobility as a function of temperature due to the presence of both ionized impurity

and lattice scattering effects. Note that there is a peak in the mobility curve at a temperature that depends on the number density of ionized impurities. In practice, the number density of ionized impurities is simply the doping –level density. Highly doped samples will therefore cause more scattering, and have a lower carrier mobility, than low-doped samples. This fact is used in high-speed devices called high electron mobility transistors (HEMTs), where electrons are made to move in undoped material, with the resulting high carrier mobilities [13].

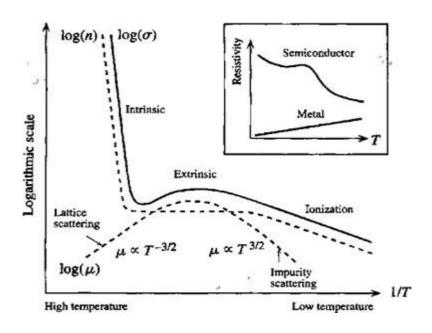


Figure 4.3 mobility as a function of temperature

4.5 Carrier concentration at extremely high and low temperature:-

At very high temperature n_i it is larges and its possible to have $n_i \gg \vee N_d - N_a \vee \iota$. in that case we write the Eq.

$$n = p = n_i(4.12)$$

In other words, the semiconductor becomes "intrinsic" at very high temperatures. At the other extreme of very low temperature, E_F may rise above E, and most of the donor (or acceptor, in the case of P-type material) atoms can remain nonionized. The fifth electrons stay with the donor. This phenomenon is called freeze-out. In this case, if the doping is not heavy enough to form an impurity band, the dopants are not totally ionized. The carrier concentration may be significantly less than the dopant concentration. The exact analysis is complicated, but the result is:

$$n = \left[\frac{N_C N_d}{2}\right]^{1/2} e^{-(E_C - E_d)/2kT} (4.13)$$

Freeze-out is a concern when semiconductor devices are operated at, for example, the liquid-nitrogen temperature (77 K) in order to achieve low noise and high speed. Figure 4.4 summarizes the temperature dependence of majority carrier concentrations. The slope of the curve in the intrinsic regime is $E_g/2k$, and the slope in the freeze-out portion is $(E_C - E_d)/2k$ (according to Eq. (4.13). These facts may be used to determine E_g and $E_C - E_d[14]$.

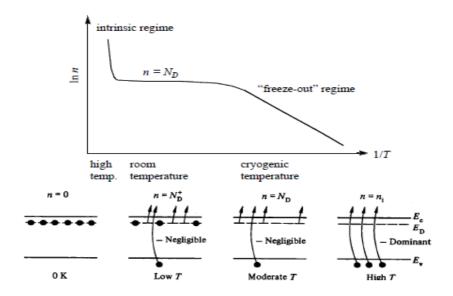


Figure 4.4 variation of carrier concentration in an n-type semiconductor over a wide range of temperature.

4.6 Mechanism of carrier scattering:-

We will now present a more detailed description of carrier collisions and show that $^{\tau_{mn}}$ τ_{mp} can vary and significantly with temperature the and doping concentration. There are two main phonon causes: scattering and ionized impurity scattering. Phonons are the particle representation of the vibration of the atoms in the crystal the same sort of vibration that carries sound, hence the term phonons. Crystal vibration distorts the periodic crystal structure and thus scatters the electron waves. Instead of electron waves and vibration waves, it is more intuitive to think of electron particles scattering phonon particles. The mobility due to phonon scattering alone, $\mu_{phonon} = q^{\tau_{ph}}/m$, is proportional to τ_{ph} , the

mean free time of phonon scattering. Let us use the pinball machine for analogy. In a pinball machine, the mean time of collisions between the ball and the pins is inversely proportional to the density of the pins and the speed of the ball. Similarly, the mean free time of phonon scattering is inversely proportional to the phonon density and the electron speed, which is basically the thermal velocity. In addition, the phonon density is known to be proportional to the absolute temperature, T.

$$\mu_{ph} \propto \tau_{ph} \propto \frac{1}{ph.density \times carrier thermal velocity}$$

$$\propto \frac{1}{T T^{1/2}} \propto T^{\frac{-3}{2}} (4.14)$$

So, the phonon scattering mobility decreases when temperature rises.

The dopant ions are fixed charge in the semiconductor crystal. They can make electrons and holes change the direction of motion through the coulombic force. An electron can be scattered by either a donor (positive) ion or an acceptor (negative) ion as shown in Fig. 4.5. The same is true for a hole[14].

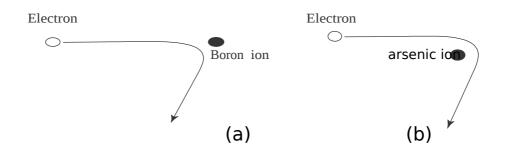


Figure 4.5 An electron can be scattered by an accepter ion(a) and a donor ion(b).

In Fig. 4.5a, the repulsive columbic force between the electron and the negative ion deflects the motion electron. Figure 4.5b shows that an attractive columbic force can induce the same effect on the electron trajectory. The mobility due to impurity scattering is therefore inversely proportional to the sum of the donor and acceptor ion concentrations. It is also proportional to $T^{3/2}$. At a higher temperature, the electron in Fig. 4.6 has a higher thermal velocity and flies by the ion in a shorter time, and its direction of motion is thus less affected by the ion. A sports analogy: a ball carrier that charges by a blocker at high speed gives the blocker less of a chance to stop him.

$$\mu_{impurity} \alpha \frac{T^{3/2}}{N_a + N_d} (4.15)$$

When there is more than one scattering mechanism, the total scattering rate $(1/\tau)$ and therefore the total mobility are determined by the sum of the inverses.

$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}} (4.16)$$

$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}} \quad (4.17)$$

Figure 4.6 shows that the silicon hole mobility is about one-third of the electron mobility. Part of this difference in mobility can be explained by the

difference in the effective mass. The rest is attributable to the difference in the scattering mean free time. The mobilities may be expressed as (with N_a and N_d in per cubic centimeter)

$$\mu_{p} \left(\frac{c \, m^{2}}{V.s} \right) = \frac{420}{1 + \left[\left(N_{a} + N_{d} \right) / 1.6 \times 10^{17} \right]^{0.7}} + 50 \, (4.18)$$

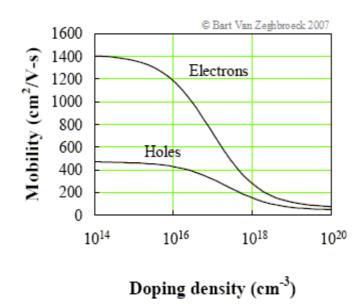


Figure 4.6 The electron and hole mobilities of silicon at 300 K[11].

At low dopant concentration, the electron mobility is dominated by phonon scattering; at high dopant concentration, it is dominated by impurity ion scattering.

$$\mu_n \left(\frac{c \, m^2}{V.s} \right) = \frac{1318}{1 + \left[\left(N_a + N_d \right) / 1 \times 10^{17} \right]^{0.85}} + 92 \left(4.19 \right)$$

You may notice that the inverse proportionality to dopant density (Eq. (4.15)) is not followed in Eqs. (4.18) and (4.19) at the limit of very large $N_{\rm d}$ or $N_{\rm d}$. The reason is

free-carrier screening. When the carrier concentration is large, the carriers can distribute themselves to partially screen out the coulombic field of the dopant ions.

Figure 4.7 shows a schematic plot of the temperature dependence of μ_n . At small dopant concentrations, μ with increasing Т. indicative of decreases the dominance of phonon scattering (Eq.(4.14)). At very high dopant concentration and low temperature, where impurity scattering is expected to dominate, μ indeed increases with increasing T (Eq. (4.15)). The mobility data in Figs. 4.6 and 4.7, do not agree perfectly. This goes to show that it is not easy to measure mobility accurately and that presume the existence of uncertainties in we should experimental data in general[11].

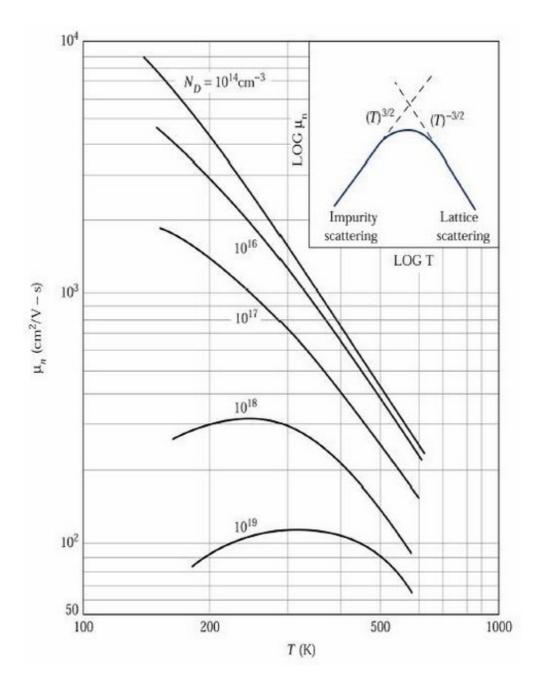


Figure 4.7 Temperature dependence of the electron mobility in Si[11]

4.7 Conclusion

A hole has larger effective mass and lower mobility than the corresponding values for an electron. The charge of a hole is equal and opposite to that of an electron. Electrons move in all different direction. When small electric field is applied to the semiconductor electron will experience force -qE. The electron will accelerated along the field in opposite direction and hole flows in the same direction as electric field. The mobility is influenced by impurity scattering and lattice scattering. The carriers will move from higher concentration region to the lower one.

4.8 Recommendations:-

The worth to be mention here as recommendations after successful finding of this thesis are:

- Electron and hole mobilities are important semiconductor parameters, so that use the materials higher mobility .
- Higher-speed transistors can be made with GaAs which are typically used in communications equipment, because GaAs has a much higher μ_n than Si, thus this materials is more important.
- InAs has an even higher μ_n , but the technology of fabricating devices has not yet been fully developed, so that use materials the technology of fabricating have been fully developed.
- From the best use intrinsic semiconductor materials,
 because the impurity content should be less than 1 part in
 100 million parts of the semiconductor.

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