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Life-time of Resonance Particles on the Basis of Generalized Special Relativity

العمر الزمني للجسيمات الرنينية على ضوء النسبية الخاصة المعممة

A Thesis Submitted For Partial Fulfillment Of The Academic Requirements Of Master Degree In General Physics

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الآيـــــــة:-

قال تعالى:

{ أَوَلَمْ يَرَوْاْ أَنَّا نَأْتِي الأَرْضَ نَنقُصُهَا مِنْ أَطْرَافِهَا وَاللّهُ يَحْكُمُ لاَ مُعَقِّبَ لِحُكْمِهِ وَهُوَ سَرِيعُ الْحِسَابِ{41}}

صدق الله العظيم

سورة الرعد (41)

قال صلى الله عليه وسلم:

{ لا تقومُ الساعةُ حتى يتقاربَ الزمانُ وتكون السنةُ كالشهرِ، والشهرُ كالجمعـــةِ، وتكون الجمعةُ كاليوم، ويكونُ اليومُ كالساعةِ، وتكونُ الساعةُ كالضَّرَمةِ من النار }.

صدق رسول الله صلى الله عليه وسلم

الراوي: أنس بن مالك **المحدث:** الترمذي

المصدر: سنن الترمذي الصفحة أو الرقم: 2332

Dedication:

To my mother & father…

To my aunt Raisa…

To my brothers and sisters…

To all of my family…

Acknowledgement:

The first thanks and foremost to my GOD as without his will nothing is possible.

Thanks my supervisor Prof. **Mubarak Dirar Abd Alla** for your encourage at first and for supervised to this thesis.

Thanks my small family for anything that you make for me.

Special thanks to my aunt **Raisa** whose make me to love sciences and learning, really thank you very much.

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Abstract:

In this work, a very brief study of the most important principles of the theory of special relativity, and the concept of time in both special and generalized special relativity.

By studying the Zeeman Effect on particles and using the equations of generalized special Relativity, the life-time of particles which are moving in a magnetic field was derived.

The effect of magnetic field on spinning and revolving resonance particles shows that the increase of magnetic field intensity decreases lifetime, whereas the mass increase, increases lifetime.

الملخص:

تم في هذا البحث دراسة موجزة جداً عن أهم مبادئ النظرية النسبية الخاصة، ثم مفهوم الزمن في النسبية الخاصة، والخاصة المعممة.

ثم بدراسة تأثير زيمان على الجسيمات وباستخدام معادلات النسبية الخاصة المعممة تم اشتقاق العمر الزمني لتأثير المجال المغنطيسي على الجسيمات المغزلية والجسيمات ذات اللف الدوراني.

وأوضحت نتائج المعادلات للجسيمات ذات اللف المغزلي، والجسيمات ذات اللف الدوراني، والجسيمات ذات اللف المغزلي والدوراني أن العمر الزمني لها يقل بزيادة كثافة المجال المغنطيسي ويزداد بزيادة الكتلة.

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Chapter One

Introduction

(1-1) Time and Relativity:

Yet despite 2,500 years of investigating time, many issues about it are unresolved. One will begin with some existent definitions of the nature of time: René Descartes had a very different answer to "What is Time?" He argued that a material body has the property of spatial extension but no inherent capacity for temporal endurance, and that God by his continual action sustains (or re-creates) the body at each successive instant. Time is a kind of sustenance or re-creation."

In 1686 Newton proposed that "absolute, true and mathematical time, of itself, and from its own nature flows equably without regard to anything external, and by another name is called duration: relative, apparent and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time".

However, in 1908 Mc Taggart said: **"**It would, I suppose, be universally admitted that time involves change…" [But after considering temporal ["A", "B", "C"] "series of events] he concludes "…We cannot explain what is meant by past, present and future." … "Our ground for rejecting time, it may be said, is that time cannot be explained without assuming time".

The real recent concept of time is based on Einstein special relativity (SR) on 1905. Einstein time concept can be understood by studying what he said, he said: "…There is no such thing as an empty space, i.e., a space without field. Space–time does not claim existence on its own, but only as a structural property of the field... It appears... more natural to think of physical reality as a four-dimensional existence, instead of, as hitherto, the evolution of a three-dimensional existence".

Dowden on 2005 defines time as follows "… Time is what clocks measure. We use our concept of time to place events in sequence one after the other, to compare how long an event lasts, and to tell when an event occurs. Those are three key features of time.

The more modern definition of time was made by Peterson and Silberstein on 2009 "This problem stems from two competing notions of time: The first, originally suggested by Heraclitus is the view that only the present is real; both the past and the future are unreal. However, with the advent of relativity, a different stance was translated into the language of relativity of Hermann Minkowski in 1908 to suggest that time and space should be united in a single, four-dimensional manifold. Thus arose the notion of a 4D "block universe" in which the past, present, and future are all equally real" [1].

(1-2) Research Problem:

The problem of this research is related to the fact that there are no studies that shows the effect of magnetic field on lifetime of decaying spinning particles.

(1-3) Aim of the work:

The aim of this thesis is to shed light on what is called **time**, especially as the scientific area does not contain a lot of research on it. The main reason for this is ambiguity and lack of clarity of the concept time.

Everyone can define time by his scientific background and his specialization. For physicists, when one is talking about the time he touch on the theory of relativity, so most of the research are talking about the relativity (special, general or both) as general and does not allocate the term “time”.

This work aims to study the effect of magnetic field on decay time of spinning particles.

(1-4) Presentation of The Thesis:

This thesis consists of three chapters:

Chapter one is the introduction, while chapter two is concerned with the theoretical background. The contribution is in chapter three.

Chapter Two

Relativity

(2-1) The arrow of time:

Thinking of past and future brings us to another problem that has foxed scientists and philosophers: why time should have a direction at all? In every-day life it's pretty apparent that it does. If you look at a movie that's being played backwards, you know it immediately because most things have a distinct time direction attached to them: an arrow of time. For example, eggs can easily turn into omlettes but not the other way around, and milk and coffee mix in your cup but never separate out again.

The most dramatic example is the history of the entire Universe, which, as scientists believe, started with the Big Bang around thirteen billion years ago and has been continually expanding ever since. When we look at that history, which includes our own, it's pretty clear which way the arrow of time is pointing.

But the mystery is that the laws of physics show no preference for forward time or backward time. For example, if you can make an object move one way by applying a force, then, as Newton's second law of motion tells you, you can make it retrace its path by applying the same force in the opposite direction. So when you watch a movie of this process you wouldn't be able to tell if it's being played forwards or backwards, as both are equally possible.

So the problem is how to account for the asymmetry of time in daily life when the laws that govern all the atoms that make up everything around us are symmetric in time. Much has been made of this problem, which affects Einstein's physics just as it did Newton's classical description of the world.

But the answer isn't all that difficult to find. Most processes we feel are irreversible in time are those that (for whatever reason) start out in some very special, highly ordered state, if we use a pack of cards as an example. When you first open up a new pack the cards will be ordered according to suit and numerical value. When you shuffle them for a while they will become disordered, so it seems that, as time passes, things will always move from order to disorder. We might think that this is very strange because there is nothing in the act of shuffling that chooses a direction in time, yet we see a distinct arrow.

However, there is nothing in the laws of physics that prevents the act of shuffling from producing a perfectly ordered set of cards. It's just that the ordered state is only one of many possible states, so the chance that we come across it while shuffling the cards is vanishingly small. So, small that it would never happen even within several lifetimes of shuffling.

So the apparent asymmetry of time is really just an asymmetry of chance. Systems of many components — like a cup full of milk and coffee particles or a bowl full of egg particles evolve from order to disorder not because the reverse is impossible, but because it's highly unlikely. This, in a nutshell, is the second law of thermodynamics, which states that the entropy (a measure of the disorder) in a closed physical system never decreases. It's a statistical principle, rather than a fundamental law describing the behaviour of individual atoms. The apparent arrow of time emerges as a property of the macroscopic system, but it's not there in the laws that govern the individual particle interactions. As the physicist John

Wheeler put it, "If you ask an atom about the arrow of time, it will laugh in your face."

This also applies to the whole Universe. The Universe started out very smooth and expanding uniformly. From a gravitational view point the Big Bang was a low entropy state and the Universe has been increasing its entropy ever since, hence the arrow of time. The question now is why the Universe started in the way it did. Why our Universe went bang in such an ordered state is still a mystery. There is no agreed answer to that, partly because there is no agreed model of cosmology. We all think the Universe began with a Big Bang and we know it's expanding. What we don't know is if the Big Bang is the ultimate origin of time or whether there was a time before that [2].

(2-2) Fundamental principles of special relativity (SR) theory:

The way in which special relativity is taught at an elementary undergraduate level is usually close in spirit to the way it was first understood by physicists. This is an algebraic approach, based on the Lorentz transformation. At this basic level, we learn how to use the Lorentz transformation to convert between one observer's measurements and another's, to verify and understand such remarkable phenomena as time dilation and Lorentz contraction, and to make elementary calculations of the conversion of mass into energy. This purely algebraic point of view began to change, to widen, less than four years after Einstein proposed the theory. Minkowski pointed out that it is very helpful to regard (t,x,y,z) as simply four coordinates in a four-dimensional space which we now call space- time. This was the beginning of the geometrical point of view, which led directly to general relativity. It is this geometrical point of view on special relativity which we must study before all else.

As we shall see, special relativity can be deduced from two fundamental postulates:

1. Principle of relativity (Galileo): No experiment can measure the absolute velocity of an observer; the results of any experiment performed by an observer do not depend on his speed relative to other observers who are not involved in the experiment.
2. Universality of the speed of light (Einstein): The speed of light relative to any unaccelerated observer is $c = 3\times10^8\ ms^{-1}$, regardless of the motion of the light's source relative to the observer. Let us be quite clear about this postulate's meaning: two different unaccelerated observers measuring the speed of the same photon will each find it to be moving at 3×10^8 m.s^{-1} relative to themselves, regardless of their state of motion relative to each other.

As noted above, the principle of relativity is not at all a modern concept; it goes back all the way to Galileo's hypothesis that a body in a state of uniform motion remains in that state unless acted upon by some external agency. It is fully embodied in Newton's second law, which contains only accelerations, not velocities themselves. Newton's laws are, in fact, all invariant under the replacement

$$v(t) \rightarrow v`(t) = v(t) - V, \qquad (2.2.1)$$

Where V is any constant velocity. This equation says that a velocity v(t) relative to one observer becomes v`(t) when measured by a second observer whose velocity relative to the first is V. This is called the **Galilean law of addition of velocities**. By saying that Newton's laws are invariant under the Galilean law of addition of velocities, we are making a statement of a sort we will often make in our study of relativity, so it is well to start by making it very precise. Newton's first law, that a body moves at a constant velocity in the absence of external forces, is unaffected by the replacement above, since if v(t) is really a constant, say v_0, then the new velocity $v_0 - V$ is also a constant. Newton's second law

$$F = ma = mdv/dt, \qquad (2.2.2)$$

is also unaffected, since

$$\alpha` = dv`/\,dt = d(v - V)\,/\,dt = dv\,/dt = \alpha \qquad (2.2.3)$$

Therefore, the second law will be valid according to the measurements of both observers, provided that we add to the Galilean transformation law the statement that F and m are themselves invariant, i.e. the same regardless of which of the two observers measures them. Newton's third law, that the force exerted by one body on another is equal and opposite to that exerted by the second on the first, is clearly unaffected by the change of observers, again because we assume the forces to be invariant, so there is no absolute velocity. Is there an absolute acceleration? Newton argued that there was. Suppose, for example, that I am in a train on a perfectly smooth track eating a bowl of soup in the dining car. Then, if the train moves at constant speed, the soup remains level, thereby offering me no information about what my speed is. But, if the train changes its speed, then the soup climbs up one side of the bowl, and I can tell by looking at it how large and in what direction the acceleration is. Therefore, it is reasonable and useful to single out a class of preferred observers: those who are unaccelerated. They are called inertial observers, and each one has a constant velocity with respect to any other one. These inertial observers are fundamental in special relativity, and when we use the term ‘observer’ from now on we will mean an inertial observer. The postulate of the universality of the speed of light was Einstein’s great and radical contribution to relativity. It smashes the Galilean law of addition of velocities because it says that if v has magnitude c, then so does v`, regardless of V. The earliest direct evidence for this postulate was the Michelson–Morely experiment, although it is not clear whether Einstein himself was influenced by it. The counter-intuitive predictions of special relativity all flow from this postulate, and they are amply confirmed by experiment. In fact it is probably fair to say that special relativity has a firmer experimental basis than any other of our laws of physics, since it is tested every day in all the giant particle accelerators, which send particles nearly to the speed of light.

Although the concept of relativity is old, it is customary to refer to Einstein’s theory simply as ‘relativity’. The adjective ‘special’ is applied in order to distinguish it from Einstein’s theory of gravitation, which acquired the name ‘general relativity’ because it permits us to describe physics from the point of view of both accelerated and inertial observers and is in that respect a more general form of relativity. But the real physical distinction between these two theories is that special relativity (SR) is capable of describing physics only in the absence of gravitational fields, while general relativity (GR) extends SR to describe gravitation itself.

(2-3) Principle of Simultaneity and Synchronicity:

There now arises the question as to whether or not two events taking place at different positions in space can be taken as simultaneous by an observer positioned somewhere else in space. To understand the problem of detecting simultaneous events, consider a thought experiment common to our earth experiences. Let two people be within sight of each other at fixed points A and B. If person A (at point A) flashes a light signal in the direction of person B (at point B) and that light signal is reflected back to A by a mirror at B, we can say that for all practical purposes those events are simultaneous, even if it is not possible to measure accurately the light propagation times involved. Communication between person A and person B has been at the speed of light and over a short distance. In this case, there is no need to define time or ponder the problem of understanding the concept of "time." Now, let us move out in to space where we again consider two points, one with in spaceship A and the other in spaceship B both moving at a uniform velocity v in the same direction, but widely separated from each other at a constant astronomical distance, L, as shown in Figure below:

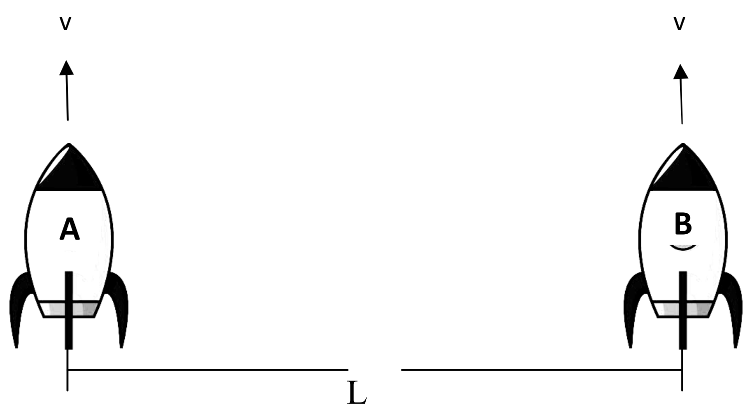

Fig. (2.1)

Let us assume that a light signal is issued from spaceship A at the absolute time t_A as read by a chronometer (clock) in A. Let this light signal arrive at spaceship B and be immediately reflected back in the direction of A at the absolute time t_B as read by a chronometer in B. Then finally assume that the light signal arrives back at spaceship A at the absolute time $\grave{t}_A$ as read by the chronometer in rocket A. Now assert that the time the light signal takes to travel from spaceship A to spaceship B is the same as the time it takes for the signal to travel from spaceship B to spaceship A, hence $t_{AB} = t_{BA}$. Under these assumptions and conditions, the chronometers (clocks) in spaceships A and B are synchronized to each other if, and only if,

$$t_B - t_A = t_A^{`} - t_B = (t_A + 2t_{AB}) - t_B \tag{2.3.1}$$

Again, we have asserted that the transit times $t_{AB} = t_{BA}$ over the fixed distance L. This expression is easily proved by taking $t_A = 0$ giving $t_B = t_{AB}$ as required if clocks A and B are synchronized. From the above and in agreement with Postulate II, we find that:

$$\frac{2L}{t_A^{`} - t_A} = c \tag{2.3.2}$$

Let us pause to reflect on what we have learned. First, it should be evident that the simultaneity of events taking place at separate points in space must be defined in terms of the synchronicity of the clocks at those points. Having established the synchronicity requirements for the clocks in spaceships A and B of Fig. 2.1, it is now possible to determine whether or not events taking place in these spaceships are simultaneous a time element common to both spaceships, e.g., transit times $t_{AB} = t_{BA}$, had to be known. In a sense, this is the definition of time. Take a third point C in space of Fig.2.1 and declare that a clock at point C is synchronous with the clock at A. Since clocks in A and B are synchronized to each other, it follows that the clock at C is synchronized with that at B. What this implies is that point C is moving at a uniform velocity v along with and in the same direction as spaceships A and B. Thus, any number of points in a given inertial reference frame in space can be synchronous with each other if a time element is common between all of them. Conversely, clocks synchronized in system S1 will not appear to be synchronized when observed from another system S2 in motion with respect to S1. Consider again Fig. 2.1, and take the case where spaceship A accelerates, eventually reaching instantaneous velocities approaching the speed of light, c, and let spaceship B remain at uniform velocity relative to spaceship A. Spaceship A must now rely on measurements taken independently of spaceship B since it is a non-inertial frame of reference. Though their clocks may have been synchronized prior to spaceship A's departure from B, as in Fig. 2.1, those clocks are no longer synchronized and simultaneous events cannot be measured. This matter will be discussed in some detail in Chapter 4 where Earth takes the place of spaceship B.

(2-4) Time in Special Relativity:

According to Lorentz transformation, the space is homogenous and there is no preferred reference frame. Consider now a frame $(x`, y`, t`)$ moving with constant speed with respect to another frame. In this case the position (x) and (x`) of any event that takes place at (t) and (t`) are related by:

$$x = \gamma(x` + vt`) \tag{2.4.1}$$
$$x` = \gamma(x + vt) \tag{2.4.2}$$

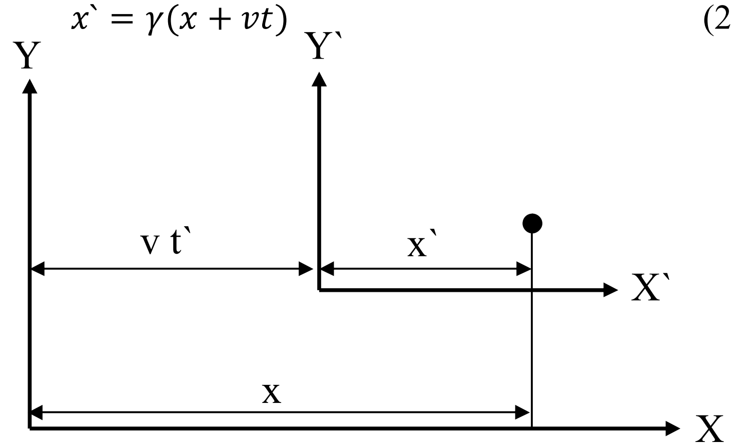

Fig. (2.2): Two frames moving with respect to each other by speed v

Consider now a light pulse emitted at $t = t` = 0$, when the origin of the two frames coincide in this case:

$$x` = ct` \quad , \quad x = ct \tag{2.4.3}$$

Inserting (2.4.3) in (2.4.1) and (2.4.2) yields:

$$ct = \gamma(ct` + vt`) = \gamma(c + v)t` \tag{2.4.4}$$

$$ct` = \gamma(ct - vt) = \gamma(c - v)t \tag{2.4.5}$$

Thus, from (2.4.5):

$$t` = \gamma\left(1 - \frac{v}{c}\right)t \tag{2.4.6}$$

Inserting (2.4.6) in (2.4.4) yields:

$$t = \gamma\left(1 + \frac{v}{c}\right)t` = \gamma^2\left(1 + \frac{v}{c}\right)\left(1 - \frac{v}{c}\right)t$$

$$1 = \gamma^2\left(1 - \frac{v^2}{c^2}\right)$$

$$\gamma = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} = \left(1 - \frac{v^2}{c^2}\right)^{\frac{-1}{2}} \tag{2.4.7}$$

Substitute (2.4.7) in (2.4.6) yields:

$$t` = \left(1 - \frac{v^2}{c^2}\right)^{\frac{-1}{2}}\left(1 - \frac{v}{c}\right)t$$

$$t` = \frac{\left(1 - \frac{v}{c}\right)t}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{2.4.8}$$

(2-5) Time in Generalized Special Relativity:

Lorentz transformation can also be used to derive expression of time in generalized special relativity (GSR). According to this transformation:

$$x = \gamma(x` + v_m t`) \quad , \quad x` = \gamma(x - v_m t) \tag{2.5.1}$$

Assuming that the speed of light is constant one gets:

$$x` = ct` \quad , \quad x = ct \tag{2.5.2}$$

Inserting equation (2.5.2) in (2.5.1) yields:

$$ct = \gamma(ct` + v_m t`) = \ c\gamma\left(1 + \frac{v_m}{c}\right)t`$$

Therefore:

$$t = \gamma\left(1 + \frac{v_m}{c}\right)t` \tag{2.5.3}$$

And:

$$ct` = \gamma(ct - v_m t) = \ c\gamma\left(1 - \frac{v_m}{c}\right)t`$$

$$t` = \gamma\left(1 - \frac{v_m}{c}\right)t \tag{2.5.4}$$

Incorporating relation (2.5.3) in relation (2.5.4) yields:

$$t = \gamma^2\left(1 + \frac{v_m}{c}\right)\left(1 - \frac{v_m}{c}\right)t$$

Thus:

$$1 = \gamma^2\left(1 - \frac{v_m^2}{c^2}\right)$$

Hence:

$$\gamma = \frac{1}{\sqrt{1-\frac{v_m^2}{c^2}}} \tag{2.5.5}$$

If one consider the relation for a particle moving in a field which increases its velocity:

$$v^2 = v_a^2 + 2\varphi$$

And by assuming v and v_a to represent the average values which are related to the maximum values v_x and v_{ax} according to the relations: $= \frac{v_x}{\sqrt{2}}$, $v_a = \frac{v_{ax}}{\sqrt{2}}$. It follows that:

$$v_x^2 = v_{ax}^2 + 4\varphi$$

Hence:

$$v_m^2 = \left(\frac{v_{ax} + v_x}{2}\right)^2 = \left(\frac{v_{ax}^2 + v_x^2 + 2v_{ax}v_x}{4}\right)$$

$$= \left(\frac{2v_x^2 - 4\varphi + 2v_x^2\sqrt{1-\frac{4\varphi}{v_x^2}}}{4}\right) \tag{2.5.6}$$

For weak field: $\left(1 - \frac{4\varphi}{v_x^2}\right)^2 \approx 1 - \frac{4\varphi}{v_x^2}$

Thus:

$$v_m^2 = \frac{2v_x^2 - 4\varphi + 2v_x^2 - 4\varphi}{4} = \frac{4v_x^2 - 8\varphi}{4} = v_x^2 - 2\varphi$$

Hence:

$$\gamma = \frac{1}{\sqrt{1 + \frac{2\varphi}{c^2} - \frac{v_x^2}{c^2}}}$$

This reduces to Einstein Generalized Special Relativity for a weak field. In view of equation (2.5.6) and (2.5.5) it follows that:

$$\gamma = \frac{1}{\sqrt{1 - \frac{2v_x^2 - 4\varphi + 2v_x\sqrt{v_x^2 - 4\varphi}}{4c^2}}} \qquad (2.5.7)$$

It is very striking to find that when no field exists $= 0$, then:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_x^2}{c^2}}} \qquad (2.5.8)$$

Thus, the model reduces to ordinary SR. Inserting (2.5.7) in (2.5.1) it follows that:

$$x = \frac{x` + \left[\frac{v_x^2 - 2\varphi + v_x\sqrt{v_x^2 - 4\varphi}}{2}\right]^{\frac{1}{2}} t`}{\sqrt{1 - \frac{2v_x^2 - 4\varphi + 2v_x\sqrt{v_x^2 - 4\varphi}}{4c^2}}} \qquad (2.5.9)$$

It is clear that the position is dependent on potential as well as on speed v. The expression for $t`$ can be obtained by eliminating $x`$ from the two relations in (2.5.1) to get:

$$t` = \frac{t - \left[\frac{v_x^2 - 2\varphi + v_x\sqrt{v_x^2 - 4\varphi}}{2}\right]^{\frac{1}{2}} x}{\sqrt{1 - \frac{2v_x^2 - 4\varphi + 2v_x\sqrt{v_x^2 - 4\varphi}}{4c^2}}} \qquad (2.5.10)$$

Again time is affected by the field potential.

(2-6) Zeeman Effect:

The Zeeman effect named after the Dutch physicist Pieter Zeeman, is the effect of splitting a spectral line into several components in the presence of a static magnetic field. It is analogous to the Stark effect, the splitting of a spectral line into several components in the presence of an electric field. Also similar to the Stark effect, transitions between different components have, in general, different intensities, with some being entirely forbidden (in the dipole approximation), as governed by the selection rules.

Since the distance between the Zeeman sub-levels is a function of the magnetic field, this effect can be used to measure the magnetic field, e.g. that of the Sun and other stars or in laboratory plasmas. The Zeeman effect is very important in applications such as nuclear magnetic resonance spectroscopy, electron spin resonance spectroscopy, magnetic resonance imaging (MRI) and Mössbauer spectroscopy. It may also be utilized to improve accuracy in atomic absorption spectroscopy. A theory about the magnetic sense of birds assumes that a protein in the retina is changed due to the Zeeman effect.

Historically, one distinguishes between the normal and an anomalous Zeeman effect that appears on transitions where the net spin of the electrons is not 0, the number of Zeeman sub-levels being even instead of odd if there is an uneven number of electrons involved. It was called "anomalous" because the electron spin had not yet been discovered, and so there was no good explanation for it at the time that Zeeman observed the effect.

At higher magnetic fields the effect ceases to be linear. At even higher field strength, when the strength of the external field is comparable to the strength of the atom's internal field, electron coupling is disturbed and the spectral lines rearrange. This is called the Paschen-Back effect.

In the modern scientific literature, these terms are rarely used, with a tendency to use just the "Zeeman effect".

(2-7) Theoretical presentation:

The total Hamiltonian of an atom in a magnetic field is

$$H = H_0 + V_M \tag{2.7.1}$$

where H_0 is the unperturbed Hamiltonian of the atom, and V_M is perturbation due to the magnetic field:

$$V_M = -\vec{\mu} \cdot \vec{B} \tag{2.7.2}$$

where $\vec{\mu}$ is the magnetic moment of the atom. The magnetic moment consists of the electronic and nuclear parts; however, the latter is many orders of magnitude smaller and will be neglected here. Therefore,

$$\vec{\mu} \approx -\frac{\mu_B g \vec{J}}{\hbar} \qquad (2.7.3.)$$

where μ_B is the Bohr magneton, $\vec{J}$ is the total electronic angular momentum, and g is the Landé g-factor. A more accurate approach is to take into account that the operator of the magnetic moment of an electron is a sum of the contributions of the orbital angular momentum $\vec{L}$ and the spin angular momentum $\vec{S}$, with each multiplied by the appropriate gyromagnetic ratio:

$$\vec{\mu} \approx -\frac{\mu_B(g_l\vec{L}+g_s\vec{S})}{\hbar} \quad (2.7.4)$$

where $g_l = 1$ and $g_s \approx 2.0023192$ (the latter is called the anomalous gyromagnetic ratio; the deviation of the value from 2 is due to the effects of quantum electrodynamics). In the case of the LS coupling, one can sum over all electrons in the atom:

$$g\vec{J} = \langle\sum_i(g_l\vec{l_i} + g_s\vec{s_i})\rangle = \langle(g_l\vec{L} + g_s\vec{S})\rangle \quad (2.7.5)$$

where $\vec{L}$ and $\vec{S}$ are the total orbital momentum and spin of the atom, and averaging is done over a state with a given value of the total angular momentum.

If the interaction term V_M is small (less than the fine structure), it can be treated as a perturbation; this is the Zeeman effect proper. In the Paschen-Back effect, described below, V_M exceeds the LS coupling significantly (but is still small compared to H_0). In ultrastrong magnetic fields, the magnetic-field interaction may exceed H_0, in which case the atom can no longer exist in its normal meaning, and one talks about Landau levels instead. There are, of course, intermediate cases which are more complex than these limit cases.

(2-8) Weak field (Zeeman effect):

If the spin-orbit interaction dominates over the effect of the external magnetic field, $\vec{L}$ and $\vec{S}$ are not separately conserved, only the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ is. The spin and orbital angular momentum vectors can be thought of as precessing about the (fixed) total angular momentum vector $\vec{J}$. The (time-)"averaged" spin vector is then the projection of the spin onto the direction of $\vec{J}$:

$$\vec{S}_{avg} = \frac{(\vec{S}.\vec{J})}{J^2}\vec{J} \tag{2.8.1}$$

and for the (time-)"averaged" orbital vector:

$$\vec{L}_{avg} = \frac{(\vec{L}.\vec{J})}{J^2}\vec{J} \tag{2.8.2}$$

Thus,

$$\langle V_M \rangle = \frac{\mu_B}{\hbar}\vec{J}\left(g_l \frac{\vec{L}.\vec{J}}{J^2} + g_s \frac{\vec{S}.\vec{J}}{J^2}\right).\vec{B} \tag{2.8.3}$$

Using $\vec{L} = \vec{J} - \vec{S}$ and squaring both sides, we get

$$\vec{S}.\vec{J} = \frac{1}{2}(J^2 + S^2 - L^2) = \frac{\hbar^2}{2}[j(j+1) - l(l+1) + s(s+1)] \tag{2.8.4}$$

and using $\vec{S} = \vec{J} - \vec{L}$ and squaring both sides, we get:

$$\vec{L}.\vec{J} = \frac{1}{2}(J^2 - S^2 + L^2) = \frac{\hbar^2}{2}[j(j+1) + l(l+1) - s(s+1)] \tag{2.8.5}$$

Combining everything and taking $J_z = \hbar m_j$, we obtain the magnetic potential energy of the atom in the applied external magnetic field,

$$V_M = \mu_B B m_j \left[g_l \frac{j(j+1) + l(l+1) - s(s+1)}{2j(j+1)} + g_s \frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}\right]$$

$$= \mu_B B m_j \left[1 + (g_s - 1)\frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}\right]$$

$$= \mu_B B m_j g_j \tag{2.8.6}$$

where the quantity in square brackets is the Landé g-factor g_J of the atom ($g_l = 1$ and $g_s \approx 2$) and m_j is the z-component of the total angular momentum. For a single electron above filled shells $s = \frac{1}{2}$ and $j = l \pm s$, the Landé g-factor can be simplified into:

$$g_j = 1 \pm \frac{g_s - 1}{2l+1} \qquad (2.8.7)$$

Example: Lyman alpha transition in hydrogen

The Lyman alpha transition in hydrogen in the presence of the spin-orbit interaction involves the transitions

$$2P_{1/2} \rightarrow 1S_{1/2} \text{ and } 2P_{3/2} \rightarrow 1S_{1/2} \qquad (2.8.8)$$

In the presence of an external magnetic field, the weak-field Zeeman effect splits the $1S_{1/2}$ and $2P_{1/2}$ levels into 2 states each ($m_j = \frac{1}{2}, -1/2$) and the $2P_{3/2}$ level into 4 states ($m_j = 3/2, 1/2, -1/2, -3/2$). The Landé g-factors for the three levels are:

$g_l = 2$ for $1S_{1/2}$ (j=1/2, l=0)

$g_l = 2/3$ for $2P_{1/2}$ (j=1/2, l=1)

$g_l = 4/3$ for $2P_{3/2}$ (j=3/2, l=1).

Note in particular that the size of the energy splitting is different for the different orbitals, because the g_J values are different. On the left, fine structure splitting is depicted. This splitting occurs even in the absence of a magnetic field, as it is due to spin-orbit coupling. Depicted on the right is the additional Zeeman splitting, which occurs in the presence of magnetic fields.

(2-9) Strong field (Paschen-Back effect):

The Paschen-Back effect is the splitting of atomic energy levels in the presence of a strong magnetic field. This occurs when an external magnetic field is sufficiently large to disrupt the coupling between orbital ($\vec{L}$) and spin ($\vec{S}$) angular momenta. This effect is the strong-field limit of the Zeeman effect. When $s = 0$, the two effects are equivalent. The effect was named after the German physicists Friedrich Paschen and Ernst E. A. Back.

When the magnetic-field perturbation significantly exceeds the spin-orbit interaction, one can safely assume $[H_0, S] = 0$. This allows the expectation values of L_z and S_z to be easily evaluated for a state $|\psi\rangle$. The energies are simply

$$E_z = \langle\psi \left| H_0 + \frac{B_z\mu_B}{\hbar}(L_z + g_s S_z) \right| \psi\rangle = E_0 + B_z\mu_B(m_l + g_s m_s) \quad (2.9.1)$$

The above may be read as implying that the LS-coupling is completely broken by the external field. However m_l and m_s are still "good" quantum numbers. Together with the selection rules for an electric dipole transition, i.e., $\Delta s = 0,\ \Delta m_s = 0,\ \Delta l = \pm 1,\ \Delta m_l = 0, \pm 1$ this allows to ignore the spin degree of freedom altogether. As a result, only three spectral lines will be visible, corresponding to the $\Delta m_l = 0, \pm 1$ selection rule. The splitting $\Delta E = B\mu_B \Delta m_l$ is *independent* of the unperturbed energies and electronic configurations of the levels being considered. It should be noted that in general (if $s \neq 0$), these three components are actually groups of several transitions each, due to the residual spin-orbit coupling.

In general, one must now add spin-orbit coupling and relativistic corrections (which are of the same order, known as 'fine structure') as a perturbation to these 'unperturbed' levels. First order perturbation theory with these fine-structure corrections yields the following formula for the Hydrogen atom in the Paschen–Back limit:

$$E_{z+fs} = E_z + \frac{\alpha^2}{2n^3}\left\{\frac{3}{4n} - \left[\frac{l(l+1)-m_l m_s}{l\left(l+\frac{1}{2}\right)(l+1)}\right]\right\} \qquad (2.9.2)$$

Intermediate field for j = 1/2

In the magnetic dipole approximation, the Hamiltonian which includes both the hyperfine and Zeeman interactions is

$$H = hA\vec{I}.\vec{J} - \vec{\mu}.\vec{B}$$

$$H = hA\vec{I}.\vec{J} + \mu_B\left(g_J\vec{J} + g_I\vec{I}\right).\vec{B} \qquad (2.9.3)$$

To arrive at the Breit-Rabi formula we will include the hyperfine structure (interaction between the electron's spin and the magnetic moment of the nucleus), which is governed by the quantum number $F \equiv \left|\vec{F}\right| = \left|\vec{J} + \vec{I}\right|$, where $\vec{I}$ is the spin angular momentum operator of the nucleus. Alternatively, the derivation could be done with J only. The constant A is known as the zero field hyperfine constant and is given in units of Hertz. μ_B is the Bohr magneton. $\hbar\vec{J}$ and $\hbar\vec{I}$ are the electron and nuclear angular momentum operators. g_J and g_I can be found via a classical vector coupling model or a more detailed quantum mechanical calculation to be:

$$g_J = g_L\frac{J(J+1)+L(L+1)-S(S+1)}{2J(J+1)} + g_S\frac{J(J+1)-L(L+1)+S(S+1)}{2J(J+1)}$$

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} + g_I \frac{F(F+1) - J(J+1) + I(I+1)}{2F(F+1)}$$

As discussed, in the case of weak magnetic fields, the Zeeman interaction can be treated as a perturbation to the $|F, m_f\rangle$ basis. In the high field regime, the magnetic field becomes so large that the Zeeman effect will dominate, and we must use a more complete basis of $|I, J, m_I, m_J\rangle$ or just $|m_I, m_J\rangle$ since I and J will be constant within a given level.

To get the complete picture, including intermediate field strengths, we must consider eigenstates which are superpositions of the $|F, m_F\rangle$ and $|m_I, m_J\rangle$ basis states. For $J = 1/2$, the Hamiltonian can be solved analytically, resulting in the Breit-Rabi formula. Notably, the electric quadrapole interaction is zero for $L = 0$ ($J = 1/2$), so this formula is fairly accurate.

To solve this system, we note that at all times, the total angular momentum projection $m_F = m_J + m_I$ will be conserved. Furthermore, since $J = 1/2$ between states m_J will change between only $\pm 1/2$. Therefore, we can define a good basis as:

$$|\pm\rangle \equiv |m_J = \pm 1/2, m_I = m_F \mp 1/2\rangle \qquad (2.9.5)$$

We now utilize quantum mechanical ladder operators, which are defined for a general angular momentum operator L as

$$L_{\pm} \equiv L_x \pm i L_y \qquad (2.9.6)$$

These ladder operators have the property

$$L_{\pm}|L, m_L\rangle = \sqrt{(L \mp m_L)(L \pm m_L + 1)}|L, m_L \pm 1\rangle \qquad (2.9.7)$$

as long as m_l lies in the range –L…….L (otherwise, they return zero). Using ladder operators $J_\pm$ and $I_\pm$ We can rewrite the Hamiltonian as

$$H = hAI_zJ_z + \frac{hA}{2}(J_+I_- + J_-I_+) + \mu_B B\big(g_J J_z + g_I I_Z\big) \tag{2.9.8}$$

Now we can determine the matrix elements of the Hamiltonian:

$$\langle \pm|H|\pm\rangle = -\frac{1}{4}A + \mu_B B_{g1} m_F \pm \frac{1}{2}\Big(hAm_F + \mu_B B\big(g_J - g_I\big)\Big)$$

$$\langle \pm|H|\pm\rangle = \frac{1}{2}hA\sqrt{\left(I+\frac{1}{2}\right)^2 - m_F^2} \tag{2.9.9}$$

Solving for the eigenvalues of this matrix, (as can be done by hand, or more easily, with a computer algebra system) we arrive at the energy shifts:

$$\Delta E_{F=I\pm 1/2} = -\frac{h\Delta W}{2(2I+1)} + \mu_B g_I m_F B \pm \frac{h\Delta W}{2}\sqrt{1 + \frac{2m_F x}{I+1/2} + x^2}$$

$$x \equiv \frac{\mu_B B(g_J - g_I)}{h\Delta W} \qquad \Delta W = A(I+1/2) \tag{2.9.10}$$

where ΔW is the splitting (in units of Hz) between two hyperfine sublevels in the absence of magnetic field B,

x is referred to as the 'field strength parameter' (Note: for $m = -(I+\frac{1}{2})$ the square root is an exact square, and should be interpreted as $+(1-x)$). This equation is known as the Breit-Rabi formula and is useful for systems with one valence electron in an s $(J = 1/2)$ level.

Note that index F in $\Delta E_{F=I\pm 1/2}$ should be considered not as total angular momentum of the atom but as *asymptotic total angular momentum*. It is equal to total angular momentum only if $B = 0$ otherwise eigenvectors corresponding different eigenvalues of the Hamiltonian are the superpositions of states with different Fbut equal m_F (the only exceptions are $|F = I + \frac{1}{2}, m_F = \pm F\rangle$).

(2-10) Spin:

In quantum mechanics and particle physics, spin is an intrinsic form of angular momentum carried by elementary particles, composite particles (hadrons), and atomic nuclei.

Spin is one of two types of angular momentum in quantum mechanics, the other being *orbital angular momentum*. Orbital angular momentum is the quantum-mechanical counterpart to the classical notion of angular momentum: it arises when a particle executes a rotating or twisting trajectory (such as when an electron orbits a nucleus). The existence of spin angular momentum is inferred from experiments, such as the Stern–Gerlach experiment, in which particles are observed to possess angular momentum that cannot be accounted for by orbital angular momentum alone.

In some ways, spin is like a vector quantity; it has a definite magnitude, and it has a "direction" (but quantization makes this "direction" different from the direction of an ordinary vector). All elementary particles of a given kind have the same magnitude of spin angular momentum, which is indicated by assigning the particle a *spin quantum number*.

The SI unit of spin is the joule-second, just as with classical angular momentum. In practice, however, it is written as a multiple of the reduced Planck constant $\hbar$, usually in natural units, where the $\hbar$ is omitted, resulting in a unitless number. Spin quantum numbers are unitless numbers by definition.

When combined with the spin-statistics theorem, the spin of electrons results in the Pauli exclusion principle, which in turn underlies the periodic table of chemical elements.

Wolfgang Pauli was the first to propose the concept of spin, but he did not name it. In 1925, Ralph Kronig, George Uhlenbeck and Samuel Goudsmit at Leiden University suggested a physical interpretation of particles spinning around their own axis. The mathematical theory was worked out in depth by Pauli in 1927. When Paul Dirac derived his relativistic quantum mechanics in 1928, electron spin was an essential part of it.

(2-11) Previous Study:

The concept of time in SR shows that it is affected by the speed of the observer only [3]. Lorentz transformation is utilized to derive a useful expression for time in the presence of any field including magnetic field. This expression reduces to that of special relativity and compete with that of the Einstein generalized special relativity [4] and Savickas [5]. In this expressions time is affected by the field as well as by velocity. Unlike the old Einstein generalized special relativity. This new model is not restricted to weak fields only, but holds for all fields including strong fields [6].

Chapter Three

Effect of Magnetic Field on Lifetime of Resonance Particles

(3-1) Introduction:

This chapter is concerned with the derivation of lifetime of spinning particles in the presence of magnetic field. One considers first the effect on particles having intrinsic spin only, beside particles revolving in a certain orbit only, in addition to particles spinning and revolving at the same time.

(3-2) Life time of resonance particles:

According to GSR, the time is given by the equation $\left[t` = \gamma\left(1 - \frac{v_m}{c}\right) t\right]$ to be:

$$t = \frac{t_0}{\sqrt{1 + \frac{2\emptyset}{c^2} + \frac{v^2}{c^2}}} \tag{3.2.1}$$

Where: $\gamma = \left(1 + \frac{2\varphi}{c^2} - \frac{v_x^2}{c^2}\right)^{-\frac{1}{2}}$ for a weak field

For particles at rest:

$$v = 0 \tag{3.2.2}$$

Thus, the time is given by:

$$t = \frac{t_0}{\sqrt{1 + \frac{2\emptyset}{c^2}}} \tag{3.2.3}$$

In a magnetic field (B) according to equation (2.7.2):

$$V_m = -\mu \,.\, B \tag{3.2.4}$$

Where:

$$V_m \equiv magnetic\ potential$$

For spinning particles [see equation (2.7.3):

$$\mu = \frac{-\mu_B g_s \vec{S}}{\hbar} \tag{3.2.5}$$

Where:

$\mu_B \equiv$ Bohr magneton, and it equals $\frac{e\hbar}{2m}$

$\vec{S} \equiv$ spin angular momentum

$g_s \equiv$ appropriate gyromagnetic ratio.

For $g_s = 2$ and $\vec{S} = \frac{1}{2}$, we have:

$$\mu = \frac{-\mu_B}{\hbar} \tag{3.2.6}$$

Hence:

$$\mu = \frac{-e}{2m} \tag{3.2.7}$$

And

$$V_m = \frac{e}{2m} B \tag{3.2.8}$$

In a field we have:

$$\emptyset = \frac{V_m}{m} \tag{3.2.9}$$

Then:

$$\emptyset = \frac{e}{2m^2} B \tag{3.2.10}$$

Finally, the time in the presence of magnetic field for spinning particles is given by:

$$t = \frac{t_0}{\sqrt{1+\frac{e}{m^2 c^2}B}} \tag{3.2.11}$$

For particles that revolve in a certain orbit without having spin, the magnetic moment takes the form:

$$\mu = \frac{-\mu_B g_l \vec{L}}{\hbar} \tag{3.2.12}$$

For: $g_l = 1$, $\vec{L} = 1$

Are has:

$$\mu = \frac{-\mu_B}{\hbar} = \frac{-e}{2m} \tag{3.2.13}$$

Therefore, the magnetic energy is given by:

$$V_m = \frac{e}{2m} B \tag{3.2.14}$$

Hence, the potential per unit mass takes the form:

$$\emptyset = \frac{e}{2m^2} B \tag{3.2.15}$$

Thus, the time for this particle takes the form:

$$t = \frac{t_0}{\sqrt{1+\frac{e}{m^2 c^2}B}} \tag{3.2.16}$$

But, if the particles has spin and revolve in an orbit its magnetic moment is given by:

$$\mu = \frac{-\mu_B\left(g_l \vec{L} + g_s \vec{S}\right)}{\hbar} \tag{3.2.17}$$

If are sets:

$$g_l = 1 \quad , \quad \vec{L} = 1 \quad , \quad g_s = 2 \quad , \quad \vec{S} = \frac{1}{2}$$

It follows that:

$$\mu = \frac{-2\mu_B}{\hbar} = \frac{-e}{m} \tag{3.2.18}$$

Therefore, the magnetic energy is given by:

$$V_m = \frac{e}{m} B \tag{3.2.19}$$

As a result, the potential per unit mass reads:

$$\emptyset = \frac{e}{m^2} B \tag{3.2.20}$$

And the time for this particle become:

$$t = \frac{t_0}{\sqrt{1 + \frac{2e}{m^2 c^2} B}} \tag{3.2.21}$$

(3-3) Discussion and Conclusion:

In chapter two and according to equation (2.4.8) the time depends only on the speed of the observer, and that result is for particles in free space.

In generalized special relativity the time is depends on the speed and the field potential which the particles in it (eq. 2.5.10).

According to equation (3.2.11) the magnetic field increase decreases lifetime for only spinning particles. The same result holds for only revolving particles [see equation (3.2.16)].

For particles revolving and spinning the increase of magnetic field decreases also lifetime. But the mass increase increases lifetime in all cases.

(3-4) Recommendation and future work:

In future work about the time it is important to try to answer some questions:

What time actually is? What is the direction of time? Is it moving a horizontally or vertically? Whether time exists when nothing is changing? What kinds of time travel are possible? Why time has an arrow? Whether the future and past are as real as the present? Whether there was time before the Big Bang event? What neural mechanisms account for our experience of time? Why time is one-dimensional and not two-dimensional?

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