CHAPTER ONE

INTRODUCTION

1.1 General

A servo control system is one of the most important and widely used forms of control system. Any machine or piece of equipment that has rotating parts will contain one or more servo control systems [1]. Automatic systems are common place in our daily life, they can be found in almost any electronic devices and appliances we use daily, starting from air conditioning systems, automatic doors, and automotive cruise control systems to more advanced technologies such as robotic arms, production lines and thousands of industrial and scientific applications. DC servomotors are one of the main components of automatic systems; any automatic system should have an actuator module that makes the system to actually perform its function [2]. The most common actuator used to perform this task is the DC servomotor. Historically, DC servomotors also played a vital role in the development of the computer's disk drive system; which make them one of the most important components in our life that we cannot live without it. Due to their importance, the design of controllers for these systems has been an interesting area for researchers from all over the world. However, even with all of their useful applications and usage, servomotor systems still suffer from several non-linear behaviors and parameters affecting their performance, which may lead for the motor to require more complex controlling schemes, or having higher energy consumption and faulty functions in some cases [7]. For these purposes the controller design of DC servomotor system is an interesting area that still offers multiple topics for research.

1.2 Problem Statement

The servomotor will be consider as a second-order system. The time response of an under damped second-order system to step input for certain terms are used to specify performance such as rise time, steady state error ,peak time maximum over shoot and settling time. The specification of response will be determine for an ideal second-order system by using theoretical laws and outputs of matlab.

1.3 Objective

The time response parameters are determined for investigation theoretically and practically of proportional derivative and integral- control individually and in combination on the closed loop response of servomotor by compared the theoretical results with matlab results .

1.4 Methodology

- Study transient characteristics of a typical second order system and evaluate model or system responses using these specifications.
- Analyze the effects of proportional- derivative- and integral- control individually and in combination on the closed loop response of motor
- Solve a position control problem by calculating PID controller gains analytically and validate the control by monitoring the motor response for different desired trajectories.
- Using mathematical method (Laplace equations) to build system model.
- Using Matlab/Simulink software to simulate the model and determine the time response parameters

1.5 The Layout

This research consists of five chapters: chapter one represents the principles of the work , the reasons and motivation and also discuses the objectives and

outline Methodology of evaluation. chapter two discuses previos works, theoretical back ground of control systems, feedback control, design and compensation of circuits systems, performance specifications, automatic controllers, classifications of industrial controllers. Chapter three represents the system Implementation of DC servo system and transient response specifications. chapter four presents the simulation and results. Finally chapter five is a conclusion and recommendations.

HAPTER TWO

PREVIOS WORKS AND CONTROL SYSTEMS

2.1 Introduction

Dr. Shereen F. Abd-Alkarim is about design and application of a fuzzy logic controller to DC-servomotor is investigated. The proposed strategy is intended to improve the performance of the original control system by use of a Fuzzy Logic Controller (FLC) as the motor load changes. Computer simulation demonstrates that FLC is effective in position control of a DC-servomotor comparing with conventional one [1]. Dong-Seog Bae and Jang-Myung Lee say This paper introduces a high-performance speed control system based on Artificial Neural Networks(ANN) to estimate unknown parameters of a DC servo motor. The goal of this research is to keep the rotor speed of the DC servo motor to follow an arbitrary selected trajectory. In detail, the aim is to obtain accurate trajectory control of the speed, specially when the motor and load parameters are unknown. By using an artificial neural network, we can acquire unknown nonlinear dynamics of the motor and the load. A trained neural network identifier combined with a reference model can be used to achieve the trajectory control. The performance of the identification and the control algorithm are evaluated through the simulation and experiment of nonlinear dynamics of the motor and the load using a typical DC servo motor model [2]. S.Mondi´e, R. Villafuerte and R. Garrido say this paper presents a tuning strategy for Proportional Retarded (PR) control laws in closed loop with second order systems and experimental results on the noise attenuation performance of a DC servomotor. The PR controller is compared with other commonly employed strategies for avoiding the time-derivative measurement in proportional derivative control laws. The experiments show that the PR controller combines good noise attenuation and tracking performance with a simple implementation [3].

2.2 Control Systems

Control theories commonly used are classical control theory (a conventional control theory), modern control theory, and robust control theory [2].

2.2.1 Feedback control

Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and does so on the basis of this difference [3].

2.2.2 Feedback control systems

A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a feedback control system. An example would be a room temperature control system. By measuring the actual room temperature and comparing it with the reference temperature (desired temperature), the thermostat turns the heating or cooling equipment on or off in such a way as to ensure that the room temperature remains at a comfortable level regardless of outside conditions. Feedback control systems are not limited to engineering but can be found in various non engineering fields as well. In fact, feedback performs a vital function: It makes the human body relatively insensitive to external disturbances, thus enabling it to function properly in a changing environment [2].

2.2.3 Open-loop control systems

Those systems in which the output has no effect on the control action are called open-loop control systems. In other words, in an open loop control system the output is neither measured nor feedback for comparison with the input. In any open-loop control system the output is not compared with the reference input. Thus, to each reference input there corresponds a fixed operating condition; as a result, the accuracy of the system depends on calibration. In the presence of disturbances, an open-loop control system will not perform the desired task. Open-loop control can be used, in practice, only if the relationship between the input and output is known and if there are neither internal nor external disturbances. Clearly, such systems are not

feedback control systems. Note that any control system that operates on a time basis is open loop. For instance, traffic control by means of signals operated on a time basis is example of open-loop control [2].

2.2.4 Closed-loop control systems

Feedback control systems are often referred to as closed-loop control systems. In practice, the terms feedback control and closed-loop control are used interchangeably. In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal (which may be the output signal itself or a function of the output signal and its derivatives and/or integrals), is fed to the controller so as to reduce the error and bring the output of the system to a desired value. The term closed-loop control always implies the use of feedback control action in order to reduce system error [2].

2.2.5 Closed-loop versus open-loop control systems

An advantage of the closed loop control system is the fact that the use of feedback makes the system response relatively insensitive to external disturbances and internal variations in system parameters. It is thus possible to use relatively in accurate and inexpensive components to obtain the accurate control of a given plant, whereas doing so is impossible in the open-loop case. From the point of view of stability, the open-loop control system is easier to build because system stability is not a major problem. On the other hand, stability is a major problem in the closed-loop control system, which may tend to overcorrect errors and there by can cause oscillations of constant or changing amplitude. It should be emphasized that for systems in which the inputs are known ahead of time and in which there are no disturbances it is advisable to use open-loop control. Closed-loop control systems have advantages only when unpredictable disturbances and/or unpredictable

variations in system components are present. Note that the output power rating partially determines the cost, weight, and size of a control system. The number of components used in a closed-loop control system is more than that for a corresponding open-loop control system. Thus, the closed-loop control system is generally higher in cost and power. To decrease the required power of a system, open loop control may be used where applicable. A proper combination of open-loop and closed-loop controls is usually less expensive and will give satisfactory overall system performance. Therefore, it is worthwhile to summarize the advantages and disadvantages of using open-loop control systems.

The major advantages of open-loop control systems are as follows [2]:

1. Simple construction and ease of maintenance.

2. Less expensive than a corresponding closed-loop system.

3. There is no stability problem.

4. Convenient when output is hard to measure or measuring the output precisely is economically not feasible. (For example, in the washer system, it would be quite expensive to provide a device to measure the quality of the washer's output, cleanliness of the clothes.)

The major disadvantages of open-loop control systems are as follows:

1. Disturbances and changes in calibration cause errors, and the output may be different from what is desired.

2. To maintain the required quality in the output, recalibration is necessary from time to time.

2.3 Design And Compensation of Circuits Systems

Compensation is the modification of the system dynamics to satisfy the given specifications. The approaches to control system design and compensation used in this research is The PID-based compensational approach to control systems design [2].

2.3.1 Performance specifications

Control systems are designed to perform specific tasks. The requirements imposed on the control system are usually spelled out as performance specifications. The specifications may be given in terms of transient response requirements (such as the maximum overshoot and settling time in step response) and of steady-state requirements (such as steady-state error in following ramp input). The specifications of a control system must be given before the design process begins. For routine design problems, the performance specifications (which relate to accuracy, relative stability, and speed of response) may be given in terms of precise numerical values. In other cases they may be given partially in terms of precise numerical values and partially in terms of qualitative statements. Generally, the performance specifications should not be more stringent than necessary to perform the given task. If the accuracy at steady-state operation is of prime importance in a given control system, then we should not require unnecessarily rigid performance specifications on the transient response, since such specifications will require expensive components. Remember that the most important part of control system design is to state the performance specifications precisely so that they will yield an optimal control system for the given purpose [2].

2.3.2 System compensation

Setting the gain is the first step in adjusting the system for satisfactory performance. In many practical cases, however, the adjustment of the gain alone may not provide sufficient alteration of the system behavior to meet the given specifications. As is frequently the case, increasing the gain value will improve the steady-state behavior but will result in poor stability or even instability. It is then necessary to redesign the system (by modifying the structure or by incorporating additional devices or components) to alter the

overall behavior so that the system will behave as desired. Such a redesign or addition of a suitable device is called compensation. A device inserted into the system for the purpose of satisfying the specifications is called a compensator .The compensator compensates for deficient performance of the original system [2].

2.3.3 Design procedure

The most time-consuming part of the work is the checking of the system performance by analysis with each adjustment of the parameters. The designer can use MATLAB or other available computer package to avoid much of the numerical drudgery necessary for this checking. Once a satisfactory mathematical model has been obtained, the designer must construct a prototype and test the open-loop system. If absolute stability of the closed loop is assured, the designer closes the loop and tests the performance of the resulting closed loop system. Because of the neglected loading effects among the components, nonlinearities, distributed parameters, and so on, which were not taken into consideration in the original design work, the actual performance of the prototype system will probably differ from the theoretical predictions. Thus the first design may not satisfy all the requirements on performance. The designer must adjust system parameters and make changes in the prototype until the system meets the specifications. In doing this, he or she must analyze each trial, and the results of the analysis must be incorporated into the next trial. The designer must see that the final system meets the performance specifications and, at the same time, is reliable and economical [2].

2.4 Automatic Controllers

An automatic controller compares the actual value of the plant output with the reference input (desired value), determines the deviation, and produces a control signal that will reduce the deviation to zero or to a small value. The

manner in which the automatic controller produces the control signal is called the control action. Figure 2.1 is a block diagram of an industrial control system, which consists of an automatic controller, an actuator, a plant, and a sensor (measuring element) [2].
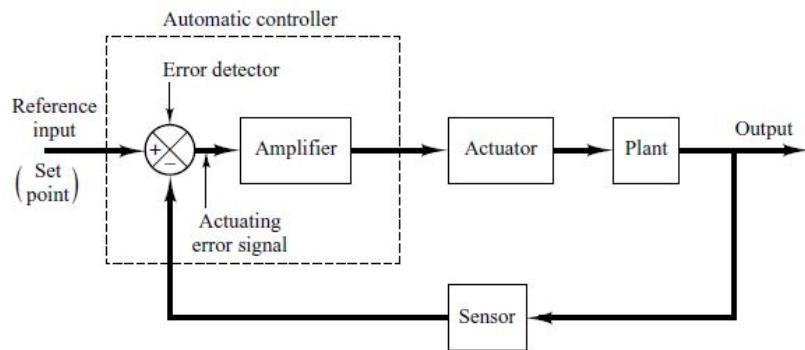
Figure 2.1: An automatic controller

The controller detects the actuating error signal, which is usually at a very low power level, and amplifies it to a sufficiently high level. The output of an automatic controller is fed to an actuator, such as an electric motor, a hydraulic motor, or a pneumatic motor or valve. (The actuator is a power device that produces the input to the plant according to the control signal so that the output signal will approach the reference input signal). The sensor or measuring element is a device that converts the output variable into another suitable variable, such as a displacement, pressure, voltage, etc., that can be used to compare the output to the reference input signal. This element is in the feedback path of the closed-loop system. The set point of the controller must be converted to a reference input with the same units as the feedback signal from the sensor or measuring element.

2.4.1 Classifications of industrial controllers

Most industrial controllers may be classified according to their control actions as:

1. Two-position or on–off controllers

2. Proportional controllers

3. Integral controllers

4. Proportional-plus-integral controllers

5. Proportional-plus-derivative controllers

6. Proportional-plus-integral-plus-derivative controllers

Most industrial controllers use electricity or pressurized fluid such as oil or air as power sources. Consequently, controllers may also be classified according to the kind of power employed in the operation, such as pneumatic controllers, hydraulic controllers, or electronic controllers .What kind of controller to use must be decided based on the nature of the plant and the operating conditions, including such considerations as safety, cost, availability, reliability, accuracy, weight, and size[4].

2.4.2 Two-position or on–off control action

In a two-position control system, the actuating element has only two fixed positions, which are, in many cases, simply on and off. Two-position or on–off control is relatively simple and inexpensive and, for this reason, is very widely used in both industrial and domestic control systems. Let the output signal from the controller be u(t) and the actuating error signal be e(t).

In two-position control, the signal u(t) remains at either a maximum or minimum value, depending on whether the actuating error signal is positive or negative, so that:

$$U(t) = U_1, \text{ for } e(t) > 0 \tag{2.1}$$

$$= U_2, \text{ for } e(t) < 0 \tag{2.2}$$

where U_1 and U_2 are constants. The minimum value U_2 is usually either zero or $-U_1$. Two-position controllers are generally electrical devices, and an electric solenoid-operated valve is widely used in such controllers. Pneumatic proportional controllers with very high gains act as two-position controllers and are sometimes called pneumatic two position controllers.

2.4.3 Proportional control action

For a controller with proportional control action, the relationship between the output of the controller u(t) and the actuating error signal e(t) is:

$$u(t) = k_p e(t) \qquad (2.3)$$

Or in Laplace-transformed quantities,

$$\frac{U(S)}{E(S)} = K_P \qquad (2.4)$$

where K_p is termed the proportional gain. Whatever the actual mechanism may be and whatever the form of the operating power, the proportional controller is essentially an amplifier with an adjustable gain.

2.4.4 Integral control action

In a controller with integral control action, the value of the controller output u(t) is changed at a rate proportional to the actuating error signal e(t). That is:

$$\frac{du(t)}{dt} = k_i e(t) \qquad (2.5)$$

Or:

$$u(t) = k_i \int_0^t e(t)dt \tag{2.6}$$

Where K_i is an adjustable constant .The transfer function of the integral controller is

$$\frac{U(S)}{E(S)} = \frac{K_i}{s} \tag{2.7}$$

2.4.5 Derivative control action

A derivative controller adds a differential gain. This type of controller will now act as a virtual damper connected between your actual system and the imaginary ideal system that exerts corrective damping force on the actual system to maintain the desired trajectory at the velocity level.

$$u(t) = k_d \frac{de(t)}{dt} \tag{2.8}$$

where K_d is an adjustable constant. The transfer function of the integral controller is:

$$\frac{U(S)}{E(S)} = K_d S \tag{2.9}$$

2.4.6 Proportional-plus-integral control action

The control action of a proportional plus- integral controller is defined by:

$$u(t) = k_p e(t) + \frac{k_p}{T_i} \int_0^t e(t)\, dt \qquad\qquad (2.10)$$

Or the transfer function of the controller is

$$\frac{U(S)}{E(S)} = k_p \left(1 + \frac{1}{T_i S}\right) \qquad\qquad (2.11)$$

where T_i is called the integral time.

2.4.7 Proportional- plus-derivative control action

The control action of a proportional plus- derivative controller is defined by:

$$u(t) = k_p e(t) + K_P T_d \frac{de(t)}{dt} \qquad\qquad (2.12)$$

And the transfer function is:

$$\frac{U(S)}{E(S)} = K_P(1 + T_d S) \qquad\qquad (2.13)$$

Where T_d is called the derivative time.

2.4.8 Proportional plus integral plus derivative control action

The combination of proportional control action, integral control action, and derivative control action termed proportional-plus-integral-plus-derivative control action. It has the advantages of each of the three individual control actions. The equation of a controller with this combined action is given by:

$$u(t) = k_p e(t) + \frac{k_p}{T_i} \int_0^t e(t)\, dt + K_P T_d \frac{de(t)}{dt} \qquad\qquad (2.14)$$

Or the transfer function is:

$$\frac{U(S)}{E(S)} = K_P\left(1 + \frac{1}{T_i S} + T_d S\right) \tag{2.15}$$

where K_p is the proportional gain, T_i is the integral time, and T_d is the derivative time. The block diagram of a Proportional-plus-Integral plus-Derivative(PID) controller is shown in Figure 2.2.
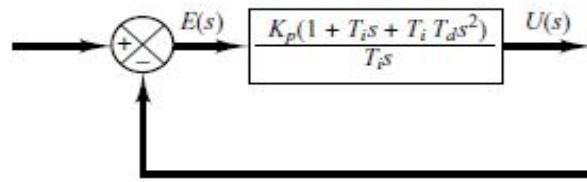
Figure 2.2: Block diagram of PID controller

2.5 Transient And Steady-State Response Analyze

The time response of a control system consists of two parts: the transient response and the steady-state response. By transient response, we mean that which goes from the initial state to the final state. By steady-state response, we mean the manner in which the system output behaves as t approaches infinity. Thus the system response c(t) may be written as [5]:

$$c(t) = c_{tr} + c_{ss} \tag{2.16}$$

where the first term on the right-hand side of the equation is the transient response and the second term is the steady-state response.

2.5.1 Typical test signals

The commonly used test input signals are step functions, ramp functions, acceleration functions, impulse functions, sinusoidal functions, and white noise. In this research we use test signal step. With this test signal, mathematical and experimental analyses of control systems can be carried out easily, since the signal is very simple functions of time. Which of this typical input signal to use for analyzing system characteristics may be determined by the form of the input that the system will be subjected to most frequently under normal operation. If a system is subjected to sudden disturbances a step function of time may be a good test signal. Once a control system is designed on the basis of test signals, the performance of the system in response to actual inputs is generally satisfactory. The use of such test signal enables one to compare the performance of many systems on the same basis.

2.5.2 Stability and steady state error

In designing a control system, we must be able to predict the dynamic behavior of the system from a knowledge of the components. The most important characteristic of the dynamic behavior of a control system is absolute stability—that is, whether the system is stable or unstable. A control system is in equilibrium if, in the absence of any disturbance or input, the output stays in the same state. A linear time-invariant control system is stable if the output eventually comes back to its equilibrium state when the system is subjected to an initial condition. A linear time-invariant control system is critically stable if oscillations of the output continue forever. It is unstable if the output diverges without bound from its equilibrium state when the system is subjected to an initial condition. Actually, the output of a physical system may increase to a certain extent but may be limited by mechanical "stops" or the system may break down or become nonlinear after the output exceeds a certain magnitude so that the linear differential equations no longer apply. Important system behavior (other than absolute stability) to which we must

give careful consideration includes relative stability and steady-state error. Since a physical control system involves energy storage, the output of the system, when subjected to an input, cannot follow the input immediately but exhibits a transient response before a steady state can be reached. The transient response of a practical control system often exhibits damped oscillations before reaching a steady state. If the output of a system at steady state does not exactly agree with the input, the system is said to have steady state error. This error is indicative of the accuracy of the system. In analyzing a control system, we must examine transient-response behavior and steady-state behavior.

2.6 Steady-State Errors in Unity Feedback Control Systems

Errors in a control system can be attributed to many factors. Changes in the reference input will cause unavoidable errors during transient periods and may also cause steady state errors. Imperfections in the system components, such as static friction, backlash, and amplifier drift, as well as aging or deterioration, will cause errors at steady state. We shall investigate a type of steady-state error that is caused by the incapability of a system to follow particular types of inputs. Any physical control system inherently suffers steady-state error in response to certain types of inputs. A system may have no steady-state error to a step input, but the same system may exhibit nonzero steady-state error to a ramp input. The only way we may be able to eliminate this error is to modify the system structure. Whether a given system will exhibit steady-state error for a given type of input depends on the type of open-loop transfer function of the system, to be discussed in what follows [6].

2.6.1 Classification of control systems

Control systems may be classified according to their ability to follow step inputs, ramp inputs, parabolic inputs, and so on. This is a reasonable classification scheme, because actual inputs may frequently be considered combinations of such inputs. The magnitudes of the steady-state errors due to these individual inputs are indicative of the goodness of the system. Consider the unity-feedback control system with the following open loop transfer function $G(s)$ [7]:

$$G(S) = \frac{K(T_aS+1)(T_b+1)...(T_mS+1)}{S^N(T_1S+1)(T_2S+1)...(T_pS+1)} \tag{2.17}$$

It involves the term S^N in the denominator, representing a pole of multiplicity N at the origin. The present classification scheme is based on the number of integrations indicated by the open-loop transfer function. A system is called type 0, type 1, type 2,..., if N=0, N=1, N=2, ..., respectively. Note that this classification is different from that of the order of a system. As the type number is increased, accuracy is improved; however, increasing the type number aggravates the stability problem. A compromise between steady-state accuracy and relative stability is always necessary. We shall see later that, if $G(s)$ is written so that each term in the numerator and denominator, except the term S^N, approaches unity as s approaches zero, then the open loop gain K is directly related to the steady-state error.
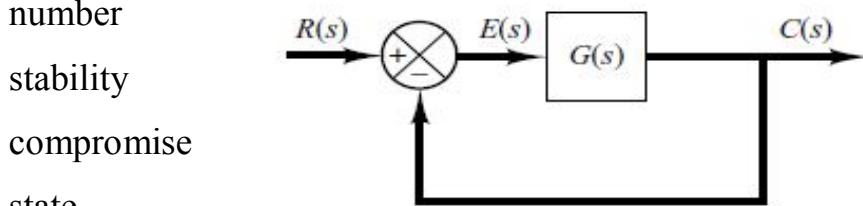
Figure 2.3: Unity control system

2.6.2 Steady state errors

Consider the system shown in Figure 2.3. The closed-loop transfer function is:

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)} \tag{2.18}$$

The transfer function between the error signal $e(t)$ and the input signal $r(t)$ is:

$$\frac{E(S)}{R(S)} = 1 - \frac{C(S)}{R(S)} = \frac{1}{1+G(S)} \tag{2.19}$$

where the error $e(t)$ is the difference between the input signal and the output signal. The final-value theorem provides a convenient way to find the steady-state performance of a stable system. Since $E(s)$ is:

$$E(S) = \frac{1}{1+G(S)} R(S) \tag{2.20}$$

The steady-state error is:

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{S \to 0} sE(s) = \lim_{S \to 0} \left(\frac{SR(S)}{1+G(S)} \right) \tag{2.21}$$

The static error constants defined in the following are figures of merit of control systems . The higher the constants, the smaller the steady-state error. In a given system, the output may be the position, velocity, pressure,

temperature, or the like. The physical form of the output, however, is immaterial to the present analysis. Therefore, in what follows, we shall call the output "position" the rate of change of the output "velocity" and so on. This means that in a temperature control system "position" represents the output temperature, "velocity" represents the rate of change of the output temperature, and so on.

2.6.3 Static position error constant

The steady-state error of the system for a unit-step input is:

$$e_{ss} = \lim_{S \to 0} \frac{S}{1+G(S)} \frac{1}{s} = \frac{1}{1+G(0)} \tag{2.22}$$

The static position error constant K_p is defined by:

$$k_p = \lim_{S \to 0} G(S) = G(0) \tag{2.23}$$

Thus, the steady-state error in terms of the static position error constant K_p is given by:

$$e_{ss} = \frac{1}{1+K_P} \tag{2.24}$$

For a type 0 system we have:

$$k_p = \lim_{S \to 0} \frac{K(T_a S+1)(T_b+1)\dots}{(T_1 S+1)(T_2 S+1)\dots} = K \tag{2.25}$$

For a type 1 or higher system we set:

$$k_p = \lim_{S \to 0} \frac{K(T_a S+1)(T_b+1)\dots}{S^N(T_1 S+1)(T_2 S+1)\dots} = \infty \text{ , for } N \geq 1 \tag{2.26}$$

Hence, for a type 0 system, the static position error constant is finite, while for a type 1 or higher system, K_p is infinite. For a unit-step input, the steady-state error e_{ss} may be summarized as follows:

$$e_{ss} = \frac{1}{1+K} \quad , \text{ for type 0 systems} \tag{2.27}$$

$$e_{ss} = 0 \quad , \text{ for type 1 or higher systems} \tag{2.28}$$

From the foregoing analysis, it is seen that the response of a feedback control system to a step input involves a steady-state error if there is no integration in the feed forward path. If small errors for step inputs can be tolerated, then a type 0 system may be permissible, provided that the gain K is sufficiently large. If the gain K is too large, however, it is difficult to obtain reasonable relative stability. If zero steady-state error for a step input is desired, the type of the system must be one or higher.

CHAPTER THREE
SYSTEM IMPLEMENTATION

3.1 Aservo System

Consider the servo system shown in Figure 3.1. The motor shown is a servomotor, a DC motor designed specifically to be used in a control system. The operation of this system is as follows: A pair of potentiometers acts as an error-measuring device. They convert the input and output positions into proportional electric signals. The command input signal determines the angular position r of the wiper arm of the input potentiometer. The angular

position r is the reference input to the system, and the electric potential of the arm is proportional to the angular position of the arm. The output shaft position determines the angular position c of the wiper arm of the output potentiometer. The difference between the input angular position r and the output angular position c is the error signal e, or:

$$e = r - c \tag{3.1}$$

The potential difference $e_r - e_c = e_v$ is the error voltage, where e_r is proportional to r and e_c is proportional to c; that is $e_r = k_0 r$, and $e_c = k_0 c$ where K_0 is a proportionality constant . The error voltage that appears at the potentiometer terminals is amplified by the amplifier whose gain constant is K_1. The output voltage of this amplifier is applied to the armature circuit of the DC motor. A fixed voltage is applied to the field winding. If an error exists, the motor develops a torque to rotate the output load in such a way as to reduce the error to zero. For constant field current, the torque developed by the motor is:

$$T = K_2 \, i_a \tag{3.2}$$

where K_2 is the motor torque constant and i_a is the armature current. When the armature is rotating, a voltage proportional to the product of the flux and angular velocity is induced in the armature. For a constant flux, the induced voltage e_b is directly proportional to the angular velocity or:

$$e_b = K_3 \frac{d\theta}{dt} \tag{3.3}$$

where e_b is the back emf, K_3 is the back emf constant of the motor, and θ is the angular displacement of the motor shaft [2].
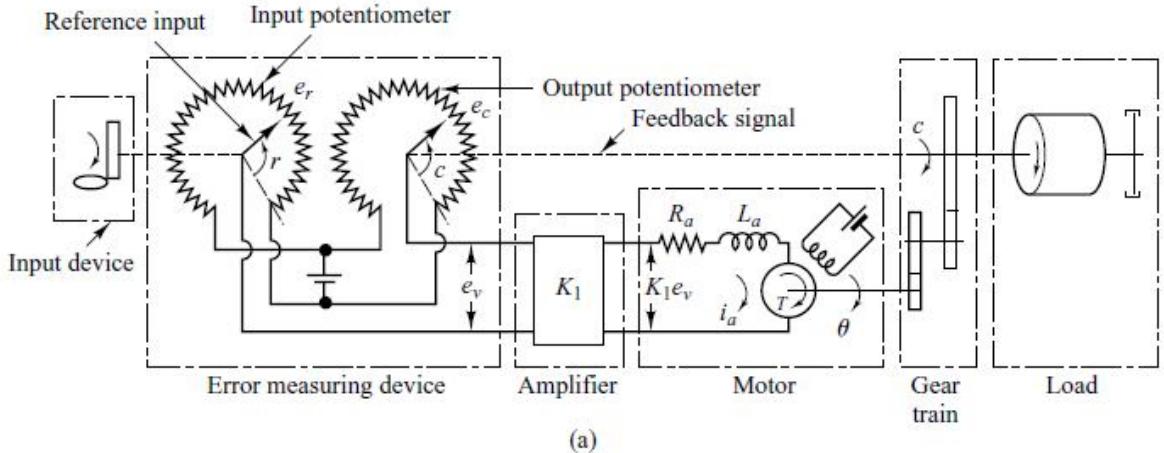
Figure 3.1: Schematic diagram of servo system

We can obtain the transfer function between the motor shaft angular displacement θ and the error voltage e_v. Obtain also a block diagram for this system and a simplified block diagram when L_a is negligible. The speed of an armature-controlled dc servomotor is controlled by the armature voltage e_a. The armature voltage $e_a = K_1 e_v$ is the output of the amplifier. The differential equation for the armature circuit is [2]:

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a \qquad (3.4)$$

By substitution equation (3.3) in to (3.4) for e_a, we set:

$$L_a \frac{di_a}{dt} + R_a i_a + K_3 \frac{d\theta}{dt} = K_1 e_v \qquad (3.5)$$

The equation for torque equilibrium is:

$$J_0 \frac{d^2\theta}{dt^2} + b_0 \frac{d\theta}{dt} = T = K_2 i_a \qquad (3.6)$$

where J_0 is the inertia of the combination of the motor, load, and gear train referred to the motor shaft and b_0 is the viscous-friction coefficient of the combination of the motor, load, and gear train referred to the motor shaft. By eliminating i_a from Equations (3.5) and (3.6), we obtain:

$$\frac{\theta(S)}{E_V(S)} = \frac{K_1 K_2}{S(L_a S + R_a)(J_0 S + b_0) + K_2 K_3 S} \qquad (3.7)$$

We assume that the gear ratio of the gear train is such that the output shaft rotates n times for each revolution of the motor shaft. Thus,

$$C(s) = n\theta(s) \qquad (3.8)$$

The relationship among $E_v(s)$, $R(s)$, and $C(s)$ is:

$$E_v(s) = K_0[R(s) - C(s)] = K_0 E(s) \qquad (3.9)$$

The block diagram of this system can be constructed from Equations (3.7), (3.8), and (3.9), as shown in Figure 3.2(a). The transfer function in the feed forward path of this system is:

$$G(S) = \frac{C(S)\theta(S)E_V(S)}{\theta(S)E_V(S)E(S)} = \frac{K_0 K_1 K_2 n}{S[(L_a S + R_a)(J_0 S + b_0) + K_2 K_3]} \qquad (3.10)$$

When L_a is small, it can be neglected, and the transfer function G(s) in the feed forward path becomes as follows:

$$G(S) = \frac{K_0 K_1 K_2 n}{S[R_a(J_0 S + b_a) + K_2 K_3]} = \frac{K_0 K_1 K_2 n / R_a}{J_0 S^2 + \left(b_0 + \frac{K_2 K_3}{R_a}\right)S} \qquad (3.11)$$

The term $[+[b_0 + (K_2 K_3/R_a)]s$ indicates that the back emf of the motor effectively increases the viscous friction of the system. The inertia J_0 and viscous friction coefficient are referred to the motor shaft. When J_0 and

$b_0 + (K_2 K_3/R_a)$ are multiplied by $1/n^2$, the inertia and viscous-friction coefficient are expressed in terms of the output shaft . Introducing new parameters defined by [2]:

$J = J_0/n^2$ = Moment of inertia referred to the output shaft.

$B = \left[b_0 + \left(\frac{K_2 K_3}{R_a}\right) \right]/n^2$ =Viscous-friction coefficient referred to the output shaft.

$K = K_0 K_1 K_2/nR_a$

The transfer function G(s) given by Equation (3.11) can be simplified, yielding:

$$G(S) = \frac{K}{JS^2 + BS} \qquad\qquad (3.12)$$

Or:

$$G(S) = \frac{K_m}{(T_m S + 1)} \qquad\qquad (3.13)$$

Where:

$$K_m = \frac{K}{B}, \ T_m = \frac{J}{B} = \frac{R_a J_0}{(R_a b_0 + K_2 K_3)} \qquad\qquad (3.14)$$

The block diagram of the system shown in Figure 3.2(a) can thus be simplified as shown in Figure 3.2(b).
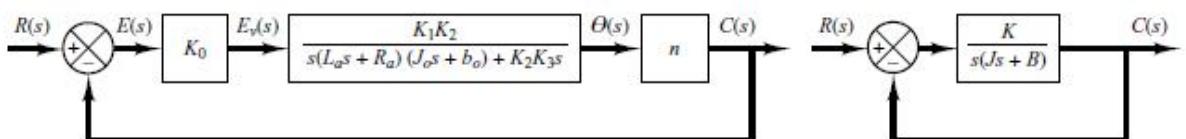
(a) block diagram for the system   (b) simplified block diagram

Figure 3.2: block diagram system and simplified

Most important among the characteristics of servo motor is maximum acceleration obtainable. For a given available torque,the rotor moment of

inertia be minimum. Since the servo motor operates under continuously varing conditions ,acceleration and decelaration of the rotor occur from time to time. The servo motor must be able to obsorb mechanical energy as well as to generate it. The performance of the servo when used as brake should be satisfactory [2].

Let J_m and $Ь_m$ be respectively, the moment on inertia and viscous- friction coefficient of the rotor , and let J_L and $Ь_L$ be, respectively, the moment on inertia and viscous- friction coefficient of gear train are either negligible or included in J_L and $Ь_L$ be, respectively. Then, the equivalent of inertia Jeq referred to the motor shaft and equivalent viscous- friction coefficient b_{eq} referred to the motor shaft can be written as [2]:

$$J_{eq} = J_m + n^2 J_L \qquad (3.15)$$

$$b_{eq} = b_m + n^2 b_L \qquad (3.16)$$

Where n(n<1) is the gear ratio between the motor and load . If the ratio n is small and $J_m \gg n^2 J_L$ then the moment of inertia of the load referred to motor shaft is neglible with respect to the rotor moment of inertia. Asimilar argument applies to the load friction. In general ,when the gear ratio n is small, the transfer function of the electrical servo motor may be obtained without taking in to account the load moment of inertia and friction. If neither J_m nor $n^2 J_L$ is negligibly small compared with other; however, then the equivalent moment of inertia J_{eq} must be used for evaluating the transfer function of the motor –load combination [2].

3.2 Second Order Systems

In this section, we shall obtain the response of a typical second-order control system to a step input. Here we consider a servo system as an example of a second-order system [2].

3.2.1 Servo as second order systems

The servo system shown in Figure3.1 consists of a proportional controller and load elements (inertia and viscous-friction elements). Suppose that we wish to control the output position c in accordance with the input position r. The equation for the load elements is:

$$J\ddot{C} + B\dot{C} = T \tag{3.17}$$

where T is the torque produced by the proportional controller whose gain is K. By taking Laplace transforms of both sides of this last equation, assuming the zero initial conditions, we obtain:

$$J\,S^2 C(S) + BSC(S) = T(S) \tag{3.18}$$

So the transfer function between $C(s)$ and $T(s)$ is:

$$\frac{C(S)}{T(S)} = \frac{1}{S(JS + B)} \tag{3.19}$$

By using this transfer function, Figure 3.1 can be redrawn as in Figure 3.2(a), which can be modified to that shown in Figure 3.2(b). The closed-loop transfer function is then obtained as:

$$\frac{C(S)}{R(S)} = \frac{K}{JS^2 + BS + K} = \frac{K/J}{S^2 + \left(\frac{B}{J}\right)S + \frac{K}{J}} \tag{3.20}$$

Such a system where the closed-loop transfer function possesses two poles is called a second-order system. Some second-order systems may involve one or two zeros [2].
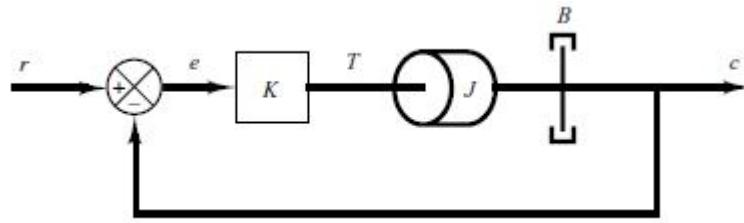
(a) Servo system
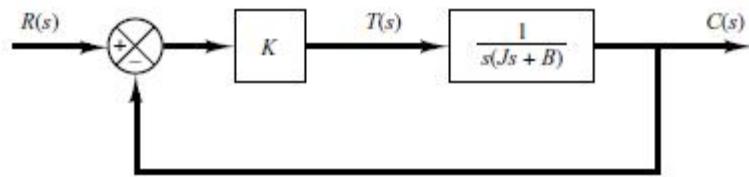
(b) Block diagram
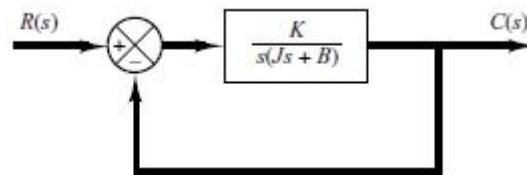
(c) Simplified block

Figure 3.3: A Servo as second order system

3.2.2 Step response of second-order system

The closed-loop transfer function of the system shown in Figure 3.3(c) is:

$$\frac{C(S)}{R(S)} = \frac{K}{JS^2 + BS + K} \qquad (3.21)$$

This can be rewritten as follows:

$$\frac{C(S)}{R(S)} = \frac{\frac{K}{J}}{\left[S + \frac{B}{2J} + \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]\left[S + \frac{B}{2J} - \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]} \qquad (3.22)$$

The closed-loop poles are complex conjugates if $B^2 - 4JK < 0$ and they are real if $B^2 - 4JK \geq 0$. In the transient-response analysis, it is convenient to write:

$$\frac{K}{J} = \omega_n^2, \quad \frac{B}{J} = 2\xi\omega_n = 2\sigma \tag{3.23}$$

where σ is called the attenuation, ω_n is the un damped natural frequency, and ξ the damping ratio of the system. The damping ratio ξ is the ratio of the actual damping β to the critical damping $B_C = 2\sqrt{JK}$ or:

$$\xi = \frac{B}{B_C} = \frac{B}{2\sqrt{JK}} \tag{3.24}$$
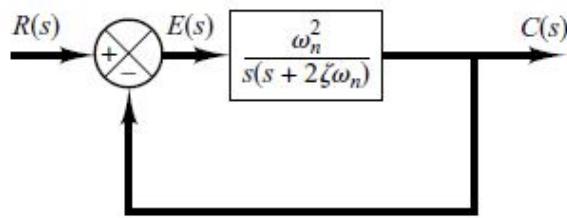
Figure 3.4: Second-order system

In terms of ξ and ω_n, the system shown in Figure 3.3(c) can be modified to that shown in Figure 3.4, and the closed-loop transfer function C(s)/R(s) given by Equation (3.23) can be written as follows:

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2} \tag{3.25}$$

This form is called the standard form of the second-order system. The dynamic behavior of the second-order system can then be described in terms of two parameters ξ and ω_n. If $0 < \xi < 1$, the closed-loop poles are complex conjugates and lie in the left-half s plane. The system is then called under damped, and the transient response is oscillatory. If $\xi = 0$, the transient response does not die out. If $\xi = 1$, the system is called critically damped. Over

damped systems correspond to $\xi > 1$. We shall now solve for the response of the system shown in Figure 3.3 to a unit-step input. We shall consider three different cases: the under damped $(0 < \xi < 1)$, critically damped $(\xi = 1)$, and over damped $(\xi > 1)$ cases.

(1) Underdamped case $(0 < \xi < 1)$: In this case, C(s)/R(s) can be written as follows [2]:

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{(S + \xi\omega_n + j\omega_d)(S + \xi\omega_n - j\omega_d)} \tag{3.26}$$

where $\omega_d = \omega_n\sqrt{1 - \xi^2}$. The frequency ω_d is called the damped natural frequency. For a unit-step input, C(s) can be written as:

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{(S^2 + 2\xi\omega_n S + \omega_n^2)S} \tag{3.27}$$

The inverse Laplace transform of Equation (3.27) can be obtained easily if $C(s)$ is written in the following form:

$$C(S) = \frac{1}{S} - \frac{S + 2\xi\omega_n}{S^2 + 2\xi\omega_n S + \omega_n^2} = \frac{1}{S} - \frac{S + \xi\omega_n}{(S + \xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n}{(S + \xi\omega_n)^2 + \omega_d^2} \tag{3.28}$$

Referring to the Laplace transform table , it can be shown that:

$$\left[\frac{S + \xi\omega_n}{(S + \xi\omega_n)^2 + \omega_d^2} \right] = e^{-\xi\omega_n t} \cos \omega_d t \tag{3.29}$$

$$\left[\frac{\omega_d}{(S + \xi\omega_n)^2 + \omega_d^2} \right] = e^{-\xi\omega_n t} \sin \omega_d t \tag{3.30}$$

Hence the inverse Laplace transform of Equation (3.27) is obtained as:

$$c(t) = 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t +$$

$$\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}) \quad , \text{ for } t \geq 0 \qquad\qquad (3.31)$$

From Equation (3.31), it can be seen that the frequency of transient oscillation is the damped natural frequency ω_d and thus varies with the damping ratio ξ. The error signal for this system is the difference between the input and output and is [2]:

$$C(t) = 1 - \cos\omega_n t , \quad \text{for } t \geq 0 \qquad\qquad (3.32)$$

$$e(t) = r(t) - c(t) = e^{-\xi\omega_n t}\left(\cos\omega_n t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right),$$

$$\text{for } t \geq 0 \qquad\qquad (3.33)$$

This error signal exhibits a damped sinusoidal oscillation. At steady state, or at t= ∞, no error exists between the input and output. If the damping ratio ξ is equal to zero, the response becomes un damped and oscillations continue indefinitely. The response c(t) for the zero damping case may be obtained by substituting ξ=0 in Equation (3.31), yielding:

$$C(t) = 1 - \cos\omega_n t , \quad \text{for } t \geq 0 \qquad\qquad (3.34)$$

Thus, from Equation (3.34), we see that ω_n represents the un damped natural frequency of the system .That is, ω_n is that frequency at which the system output would oscillate if the damping were decreased to zero. If the linear system has any amount of damping, the un damped natural frequency cannot be observed experimentally. The frequency that may be observed is the

damped natural frequency ω_d, which is equal to $\omega_n\sqrt{1-\xi^2}$. This frequency is always lower than the un damped natural frequency. An increase in ξ would reduce the damped natural frequency ω_d. If ξ is increased beyond unity, the response becomes over damped and will not oscillate [2].

(2) Critically damped case ($\xi=1$): If the two poles of C(s)/R(s) are equal, the system is said to be a critically damped one. For a unit-step input, R(s)=1/s and C(s) can be written as:

$$C(s) = \frac{\omega_n^2}{(S+\omega_n)^2 S} \qquad (3.35)$$

The inverse Laplace transform of Equation (3.٣٥) may be found as

$$c(t) = e^{-\omega_n t}(1 + \omega_n t), \text{ for } t \geq 0 \qquad (3.36)$$

This result can also be obtained by letting ξ approach unity in Equation (3.31) and by using the following limit:

$$\lim_{\xi \to 0} \frac{\sin \omega_d t}{\sqrt{1-\xi^2}} = \lim_{\xi \to 0} \frac{\sin \omega_n\sqrt{1-\xi^2} t}{\sqrt{1-\xi^2}} = \omega_n t \qquad (3.37)$$

(3) Over damped case ($\xi>1$): In this case, the two poles of C(s)/R(s) are negative real and unequal. For a unit-step input, R(s)=1/s and C(s) can be written as:

$$C(s) = \frac{\omega_n^2}{\left(S+\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right)\left(S+\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right) S} \qquad (3.38)$$

The inverse Laplace transform of Equation (3.38) is:

$$C(t) =$$

$$1 + \frac{1}{2\sqrt{\xi^2-1}\left(\xi+\sqrt{\xi^2-1}\right)} \; e^{-\left(\xi-\sqrt{\xi^2-1}\right)\omega_n t} -$$

$$\frac{1}{2\sqrt{\xi^2-1}\left(\xi-\sqrt{\xi^2-1}\right)} \; e^{-\left(\xi-\sqrt{\xi^2-1}\right)\omega_n t} = 1 + \frac{\omega_n}{2\sqrt{\xi^2-1}}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right), \text{for } t \geq$$

$$0 \hspace{8cm} (3.39)$$

Where $S_1 = \left(\xi+\sqrt{\xi^2-1}\right)\omega_n$ and $S_2 = \left(\xi-\sqrt{\xi^2-1}\right)\omega_n$

Thus , the response c(t) includes two decaying exponential terms. When ξ is appreciably greater than unity, one of the two decaying exponentials decreases much faster than the other, so the faster-decaying exponential term (which corresponds to a smaller time constant) may be neglected . That is, if $^-s_2$ is located very much closer to the jω axis than $^-s_1$ (which means $\left|s_2\right| \ll \left|s_1\right|$), then for an approximate solution we may neglect $^-s_1$. This is permissible because the effect of $^-s_1$ on the response is much smaller than that of $^-s_2$, since the term involving s_1 in Equation (3.39) decays much faster than the term involving s_2 . Once the faster-decaying exponential term has disappeared, the response is similar to that of a first-order system, and C(s)/R(s) may be approximated by:

$$\frac{C(S)}{R(S)} = \frac{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}{S+\xi\omega_n - \omega_n\sqrt{\xi^2-1}} = \frac{S_2}{S+S_2} \hspace{4cm} (3.40)$$

This approximate form is a direct consequence of the fact that the initial values and final values of both the original C(s)/R(s) and the approximate one agree with each other. With the approximate transfer function C(s)/R(s), the unit-step response can be obtained as:

$$C(S) = \frac{\xi\omega_n - \omega_n \sqrt{\xi^2-1}}{\left(S + \xi\omega_n - \omega_n \sqrt{\xi^2-1}\right)S}$$

(3.41)

The time response c(t) is then:

$$c(t) = 1 - e^{-\left(\xi - \sqrt{\xi^2-1}\right)\omega_n t}, \quad \text{for } t \geq 0$$

(3.42)

This gives an approximate unit-step response when one of the poles of C(s)/R(s) can be neglected.

3.3 The Transient-Response Specifications

Frequently, the performance characteristics of a control system are specified in terms of the transient response to a unit-step input, since it is easy to generate and is sufficiently drastic. If the response to a step input is known, it is mathematically possible to compute the response to any input. The transient response of a system to a unit-step input depends on the initial conditions.

For convenience in comparing transient responses of various systems, it is a common practice to use the standard initial condition that the system is at rest initially with the output and all time derivatives thereof zero. Then the response characteristics of many systems can be easily compared. The transient response of a practical control system often exhibits damped oscillations before reaching steady state. In specifying the transient-response characteristics of a control system to a unit-step input, it is common to specify the following:

1. Delay time, t_d.

2. Rise time, t_r.

3. Peak time, t_p.

4. Maximum overshoot, M_p.

5. Settling time, t_s.

These specifications are defined in what follows and are shown graphically in Figure 3.4.

1. Delay time: The delay time is the time required for the response to reach half the final value the very first time.

2. Rise time: The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value. For under damped second order systems, the 0% to 100% rise time is normally used. For over damped systems, the 10% to 90% rise time is commonly used.

3. Peak time: The peak time is the time required for the response to reach the first peak of the overshoot.

4. Maximum (percent) overshoot: The maximum overshoot is the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by:

$$M_P = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\% \qquad (3.43)$$

The amount of the maximum (percent) overshoot directly indicates the relative stability of the system.

5. Settling time: The settling time is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or 5%). The settling time is related to the largest time constant of the control system. Which percentage error criterion to use may be determined from the objectives of the system design in question. The time-domain specifications just given are quite important, since most control systems are time-domain systems; that is, they must exhibit acceptable time responses. This means that, the control system must be modified until the transient response is satisfactory [2].
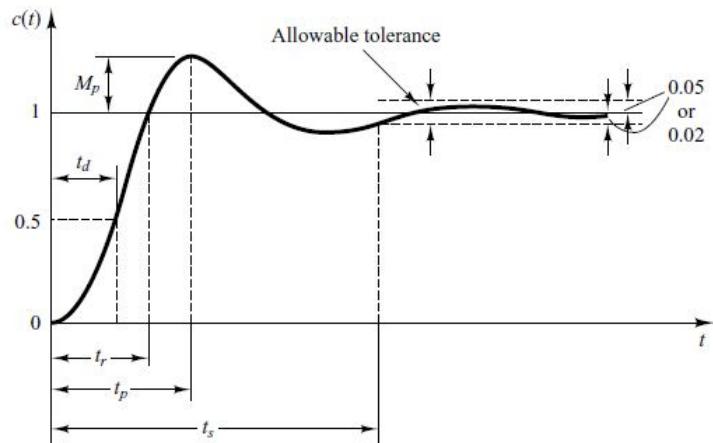
Figure 3.5: Unit-step response curve

Unit-step response curve showing t_d, t_r, t_p, M_p, and t_s. Note that not all these specifications necessarily apply to any given case. For example, for an over damped system, the terms peak time and maximum overshoot do not apply. For systems that yield steady-state errors for step inputs, this error must be kept within a specified percentage level. **3.3.1 A few comments**

Except for certain applications where oscillations cannot be tolerated, it is desirable that the transient response be sufficiently fast and be sufficiently damped .Thus, for a desirable transient response of a second-order system, the damping ratio must be between 0.4 and 0.8. Small values of ξ (that is, ξ<0.4) yield excessive overshoot in the transient response, and a system with a large value of ξ(that is, ξ>0.8) responds sluggishly. We shall see later that the maximum overshoot and the rise time conflict with each other. In other words, both the maximum overshoot and the rise time cannot be made smaller simultaneously. If one of them is made smaller, the other necessarily becomes larger [2].

3.3.2 Second-order systems specifications

In the following, we shall obtain the rise time, peak time, maximum overshoot, and settling time of the second-order system given by Equation (3.20). These values will be obtained in terms of ξ and ω_n.

The system is assumed to be under damped.

(1) Rise time: Referring to Equation (3.25), we obtain the rise time by letting $c(t_r) = 1$, we have:

$$c(t_r) = 1 = 1 - e^{-\xi \omega_n t_r} \left(\cos \omega_d t_r + \frac{\xi}{\sqrt{1-\xi^2}} \sin \omega_d t_r\right) \qquad (3.44)$$

Since $e^{-\xi \omega_n t_r} \neq 0$, we obtain from Equation (3.44) the following equation:

$$\cos \omega_d t_r + \frac{\xi}{\sqrt{1-\xi^2}} \sin \omega_d t_r = 0 \qquad (3.45)$$

Since $\omega_n \sqrt{1 - \xi^2} = \omega_d$ and $\xi \omega_n = $, we have:

$$\tan \omega_d t_r = \frac{\sqrt{1-\xi^2}}{\xi} = -\frac{\omega_d}{\sigma} \qquad (3.46)$$

Thus, the rise time is:

$$t_r = \frac{1}{\omega_d} \tan^{-1}\left(\frac{\omega_d}{-\sigma}\right) = \frac{\pi - \beta}{\omega_d} \qquad (3.47)$$

Where angle β is defined in Figure 3.6 Clearly, for a small value of t_r, ω_d must be large.
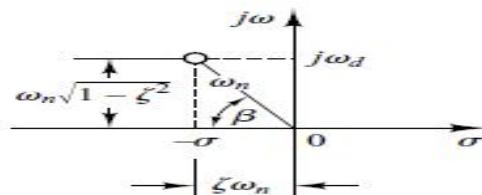
Figure 3.6: Definition of the angle β

(2) Peak time: Referring to Equation (3.25), we may obtain the peak time by differentiating c(t) with respect to time and letting this derivative equal zero.

$$\frac{dc}{dt} = \xi\omega_n e^{-\xi\omega_n t}\left(\cos\omega_n t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right) + e^{-\xi\omega_n t}\left(\cos\omega_n t - \right.$$

$$\left.\frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right) \tag{3.48}$$

Since and the cosine terms in this last equation cancel each other, dc/dt, evaluated at $t=t_p$, can be simplified to:

$$\left.\frac{dc}{dt}\right| t = t_p = \left(\sin\omega_d t_p\right)\frac{\omega_n}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t_p} = 0 \tag{3.49}$$

This last equation yields the following equation:

$$\sin\omega_d t_p = 0 \tag{3.50}$$

Or:

$$\omega_d t_p = 0\,,\pi\,,2\pi\,,3\pi\,,\dots \tag{3.51}$$

Since the peak time corresponds to the first peak overshoot $\omega_d t_p = \pi$. Hence:

$$t_p = \frac{\pi}{\omega_d} \tag{3.52}$$

The peak time corresponds to one-half cycle of the frequency of damped oscillation.

(3) Maximum overshoot: The maximum overshoot occurs at the peak time or at $t=t_p=\pi/\omega_d$. Assuming that the final value of the output is unity, M_p is obtained from Equation (3.25) as:

$$M_P = c(t_p) - 1 = e^{-\xi\omega_n\left(\frac{\pi}{\omega_d}\right)}\left(\cos\pi + \frac{\xi}{\sqrt{1-\xi^2}}\sin\pi\right) = e^{-\left(\frac{\sigma}{\omega_d}\right)} =$$

$$e^{-\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\pi}$$

(3.53)

The maximum percent overshoot is $e^{-\left(\frac{\sigma}{\omega_d}\right)\pi} \times 100\%$. If the final value $c(\infty)$ of the output is not unity, then we need to use the following equation:

$$M_P = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

(3.54)

(4) Settling time: For an under damped second-order system, the transient response is obtained from Equation (3.25) as:

$$c(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right), \text{ for } t \geq 0$$

(3.55)

For convenience in comparing the responses of systems, we commonly define the settling time to be:

$$t_s = 4T = \frac{4}{\sigma} = \frac{4}{\xi\omega_n} \text{ (2\% criterion)}$$

(3.56)

$$t_s = 3T = \frac{3}{\sigma} = \frac{3}{\xi\omega_n} \text{ (5\% criterion)}$$

(3.57)

Note that the settling time is inversely proportional to the product of the damping ratio and the un damped natural frequency of the system. Since the value of ξ is usually determined from the requirement of permissible maximum overshoot, the settling time is determined primarily by the un damped natural frequency ω_n. This means that the duration of the transient period may be varied, without changing the maximum overshoot, by adjusting the un damped natural frequency ω_n. From the preceding analysis, it

is evident that for rapid response ω_n must be large. To limit the maximum overshoot Mp and to make the settling time small, the damping ratio ξ should not be too small [2].

Table 3.1: Transient response specifications

Parameter	Symbol (unit)	Formula
Rise Time	$t_r(s)$	$\dfrac{\pi - \beta}{\omega_d}$, where $\beta = tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)$ $\omega_d = \omega\sqrt{1-\zeta^2}$
Maximum Overshoot	$M_p(\%)$	$e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$
Delay Time	$t_d(s)$	$\dfrac{1 + 0.7\zeta}{\omega_n}$
Settling Time	$t_s(s)$	$\dfrac{4}{\zeta\omega_n}$ (2 % setting time) $\dfrac{3}{\zeta\omega_n}$ (5 % settling time)
Peak Time	$t_p(s)$	$\dfrac{\pi}{\omega_d}$
Steady State Error	e_{ss}	$[\lim_t \to \infty\, y(t)] - y_{des}^{ss}$

3.4 Servo System with Velocity Feedback

The derivative of the output signal can be used to improve system performance. In obtaining the derivative of the output position signal, it is desirable to use a tachometer instead of physically differentiating the output signal. Note that the differentiation amplifies noise effects. In fact, if discontinuous noises are present, differentiation amplifies the discontinuous noises more than the useful signal. For example, the output of a potentiometer is a discontinuous voltage signal because, as the potentiometer

brush is moving on the windings, voltages are induced in the switchover turns and thus generate transients. The output of the potentiometer therefore should not be followed by a differentiating element [1].
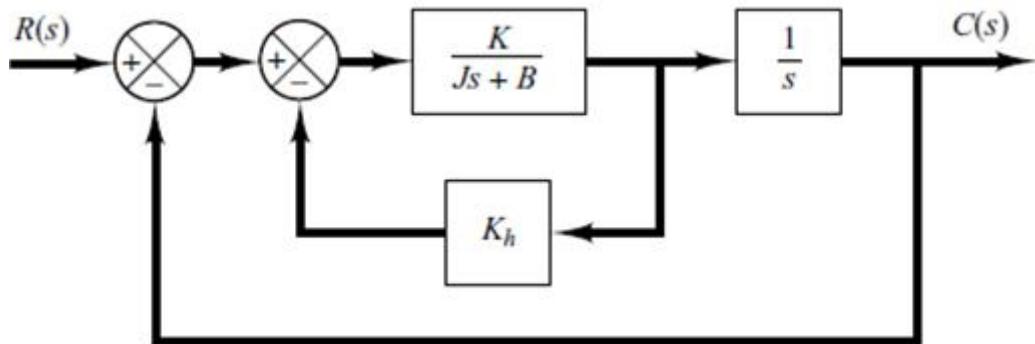
(a)Block diagram of the servo system
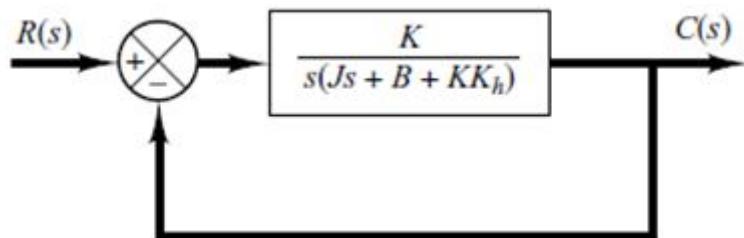
(b) simplified block diagram

Figure 3.7: Block diagram of the servo system and simplified

The tachometer, a special dc generator, is frequently used to measure velocity without differentiation process. The output of a tachometer is proportional to the angular velocity of the motor. Consider the servo system shown in Figure 3.7(a). In this device, the velocity signal, together with the positional signal, is fed back to the input to produce the actuating error signal. In any servo system, such a velocity signal can be easily generated by a tachometer. The block diagram shown in Figure 3.7(a) can be simplified, as shown in Figure 3.7(b), giving:

$$\frac{C(S)}{R(S)} = \frac{K}{JS^2 + (B + KK_h)S + K} \qquad (3.58)$$

Comparing Equation (3.58) with Equation (3.25), notice that the velocity feedback has the effect of increasing damping. The damping ratio ξ becomes:

$$\xi = \frac{B + kk_h}{2\sqrt{kJ}} \qquad (3.59)$$

The un damped natural frequency $\omega_n = \sqrt{K/J}$ is not affected by velocity feedback. Noting that the maximum overshoot for a unit-step input can be controlled by controlling the value of the damping ratio ξ, we can reduce the maximum overshoot by adjusting the velocity-feedback constant K_h so that ξ is between 0.4 and 0.7. It is important to remember that velocity feedback has the effect of increasing the damping ratio without affecting the un damped natural frequency of the system.

CHAPTER FOUR

SIMULATION AND RESULTS

4.1 DC Servomotor Model

Recalling the DC servomotor diagram from Figure 3.1, the closed – loop transfer function of the DC servomotor for the position servo system can be shown below. Assume that the input and output of the system are the input shaft position and output shaft position, respectively. Assume the following numerical values for system constants are shown in Table 4.1.

Table 4.1 DC servomotor parameter values

r	Angular displacement of the reference input shaft, radians
c	Angular displacement of output shaft, radians
θ	Angular displacement of motor shaft, radians
K_0	Gain of the potentiometric error detector = $24/\pi$ v/rad
K_1	Amplifier gain = 10 v/v
e_a	Armature voltage, v
e_b	Back emf, v
R_a	Armature –winding resistance =0.2Ω
L_a	Armature –winding inductance = negligible
K_3	Back emf constant =5.5×10^{-2} V-sec/rad
K_2	Motor torque constant =6×10^{-5} N –m/A
J_m	Moment of inertia of motor referred to the motor shaft =10^{-5} kg-m^2
b_m	Viscous-friction coefficient of the motor referred to the motor shaft = negligible
J_L	Moment of inertia of the load referred to the output shaft = 4.4×10^{-3} kg-m^2
b_L	Viscous-friction coefficient of the load referred to the output shaft= 4×10^{-2} N. m /rad/sec
n	Gear ratio N1/N2 =1/10
i_a	Armature –winding current, A

To solve this problem we shall obtain the equivalent moment of inertia J0 and equivalent viscous coefficient b0 referred to the motor shaft are, respectively

$J_0 = J_m + n^2 J_L = 10^{-5} + 4.4 \times 10\text{-}5 = 5.4 \times 10^{-5}$

$b_0 = b_m + n^2 b_L = 4 \times 10\text{-}4$

Referring to the equation (3.11) the transfer function can be written as follows:

$$\frac{C(S)}{E(S)} = K_m / s(T_m s + 1) \qquad (4.1)$$

Where:

$K_m = \dfrac{K_0 K_1 K_2 n}{R_a b_0 + K_2 K_3} = 7.64 \times 10 \times 61 \times 10^{-5} \times 0.1/(0.2 \times 4 \times 10^{-4} + 6 \times 5.5 \times 10^{-7}) \quad = $

5.5

$T_m = R_a J_0 / (R_a b_0 + K_2 K_3) = 0.2 \times 5.5 \times 10^{-5} / (0.2 \times 4 \times 10^{-4} + 6 \times 5.5 \times 10^{-7}) = $

0.13

Thus,

$C(S)/E(S) = 5.5/S(0.13S+1) \qquad (4.2)$

By using equation (4.2), we can draw the block diagram of the system shown in figure below.
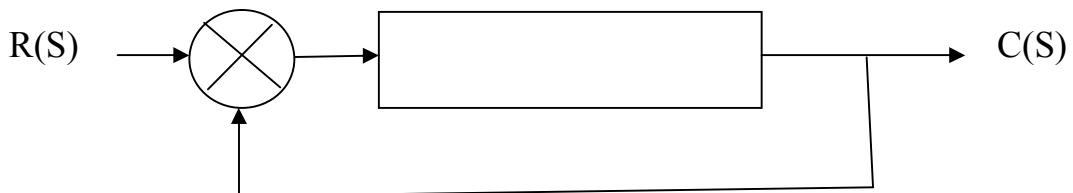
Figure 4.1: Block diagram of the system

Then the closed loop transfer function as follows:

$$\frac{C(S)}{R(S)} = G(S) = \frac{42.3}{S^2+7.69S+42.3} \qquad (4.3)$$

By comparing the equation (4.3) with the general form of the second order equation (3.25) we obtain:

$\omega_n^2 = 42.3$,   thus $\omega_n = 6.5$.

$2\xi\omega_n = 7.69$,   thus $\xi = 0.6$.

Then the system specifications are:

$t_r = \frac{\pi-\beta}{\omega d}$,   where $\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 53.1 = 0.93$ rad/sec.

$\omega d = \omega_n\sqrt{1-\xi^2} = 5.2$.

Then $t_r = 0.425$ sec.

$t_p = \frac{\pi}{\omega d} = 0.604$ sec.

$t_s = \frac{4}{\xi\omega n} = 1.026$ sec, for (2% error).

$t_s = \frac{3}{\xi\omega n} = 0.769$ sec, for (5% error).

$M_p = e^{-\pi(\frac{\xi}{\sqrt{1-\xi^2}})} = 0.095 \times 100 = 9.5$ %.

$e_{ss} = \lim_{S\to0}(\frac{SR(S)}{1+G(S)}) = 0.023$.

Table 4.2 shows the time response parameters for system without controller.

Table 4.2: Time Response Results without controller

Characteristics	By calculation	Directly From MATLAB
t_r	0.425	0.282
t_s	1.026	0.911
t_p	0.604	0.599

M_p	9.5%	10%
e_{ss}	0.023	0

Figure 4.2 shows the system time response, while Figure 4.3 shows the system simulation.
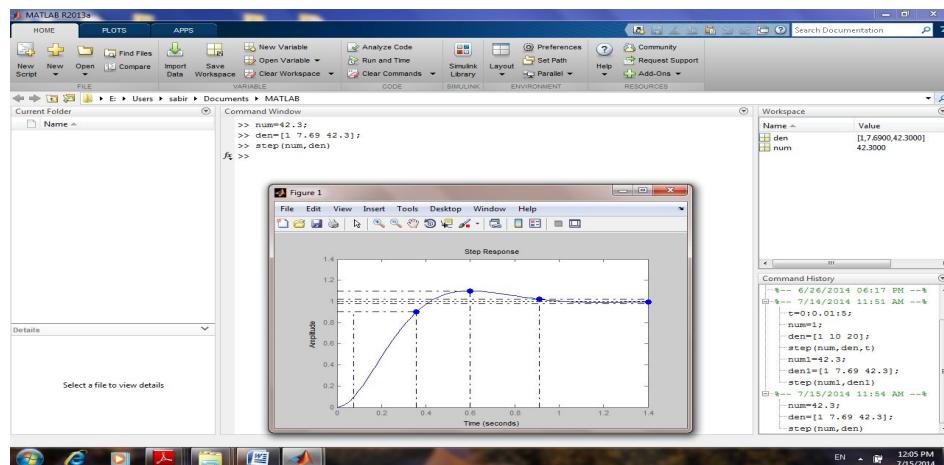
Figure 4.2: System response without controller
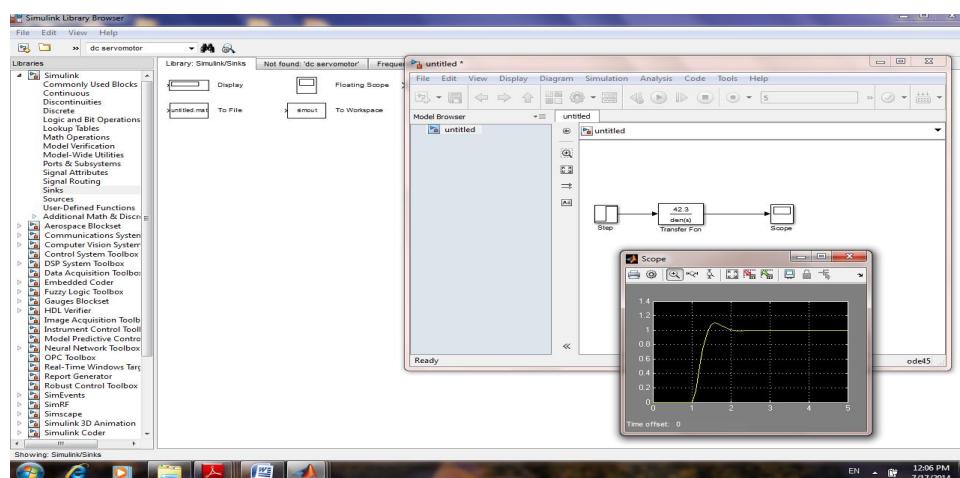
Figure 4.3 System simulation without controller

4.1.1 Proportional control

By using Proportional Control (K_p) the system can be shown below:
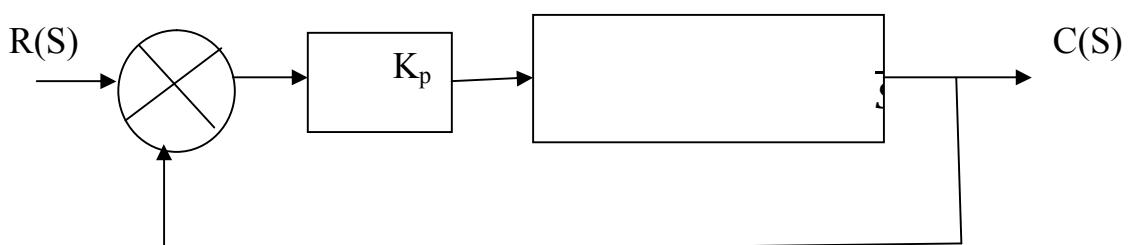
Figure 4.4: block diagram of the system with P controller

Then the closed loop transfer function as follows:

$$C(S)/R(S) = \frac{42.3kp}{S^2+7.69S+(42.3+kp)}$$ (4.4)

For $k_p = 2$ and by comparing the Equation (4.3) with the general form of the second order Equation (3.25) we obtain:

$\omega^2{}_n = 44.3$, thus $\omega n = 6.7$.

$2\xi\omega_n = 7.69$, thus $\xi = 0.6$.

Then the system specifications are:

$t_r = \frac{\pi-\beta}{\omega d}$ where $\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 53.1 = 0.93$ rad/sec.

$\omega_d = \omega_n\sqrt{1-\xi^2} = 5.4$

Then:

$t_r = 0.410$ sec.

$t_p = \frac{\pi}{\omega d} = 0.581$ sec.

$t_s = \frac{4}{\xi\omega n} = 0.995$ sec, for (2% error).

$t_s = \frac{3}{\xi\omega n} = 0.746$ sec, for (5% error).

$M_p = e^{-\pi(\frac{\xi}{\sqrt{1-\xi^2}})} = 0.095 \times 100 = 9.5$ %.

$e_{ss} = \lim_{S\to0}(\frac{SR(S)}{1+G(S)}) = 0.008$.

For $k_p = 5$ and by comparing the Equation (4.4) with the general form of the second order equation (3.25) we obtain:

$\omega^2{}_n = 47.3$, thus $\omega_n = 6.9$.

$2\xi\omega_n = 7.69$, thus $\xi = 0.6$.

Then the system specifications are:

$t_r = \frac{\pi-\beta}{\omega d}$ where $\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 53.1 = 0.93$ rad/sec

$\omega_d = \omega_n\sqrt{1 - \xi^2} = 5.5$

Then:

$t_r = 0.402$ sec.

$t_p = \dfrac{\pi}{\omega d} = 0.571$ sec.

$t_s = \dfrac{4}{\xi\omega n} = 0.966$ sec, for (2% error).

$t_s = \dfrac{3}{\xi\omega n} = 0.725$ sec, for (5% error).

$M_p = e^{-\pi\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)} = 0.095 \times 100 = 9.5$ %.

$e_{ss} = \lim_{S\to0}\left(\dfrac{SR(S)}{1+G(S)}\right) = 0.004$.

For $k_p = 10$ and by comparing the Equation (4.4) with the general form of the second order Equation (3.25) we obtain:

$\omega^2{}_n = 52.3$, thus $\omega_n = 7.2$.

$2\xi\omega_n = 7.69$, thus $\xi = 0.6$.

Then the system specifications are:

$t_r = \dfrac{\pi-\beta}{\omega d}$ where $\beta = \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi} = 53.1 = 0.93$ rad/sec.

$\omega_d = \omega_n\sqrt{1 - \xi^2} = 5.8$.

Then:

$t_r = 0.381$ sec.

$t_p = \dfrac{\pi}{\omega d} = 0.542$ sec.

$t_s = \dfrac{4}{\xi\omega n} = 0.926$ sec, for (2% error).

$t_s = \dfrac{3}{\xi\omega n} = 0.694$ sec, for (5% error).

$M_p = e^{-\pi\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)} = 0.095 \times 100 = 9.5$ %.

$e_{ss} = \lim_{S\to0}\left(\dfrac{SR(S)}{1+G(S)}\right) = 0.002$.

Figure 4.5 shows the system response when k_p equal to two.
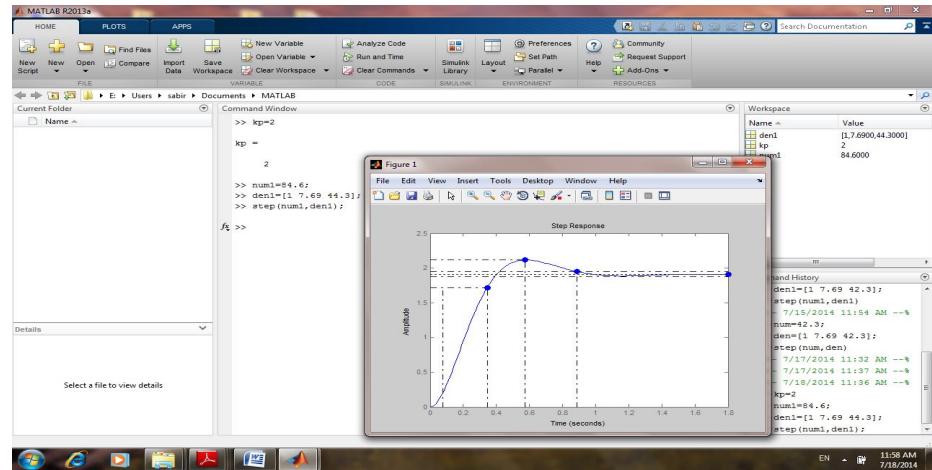
Figure 4.5: System response for $k_p=2$

Figure 4.6 shows the system response when k_p equal to five while figure 4.7 for k_p equal to ten .
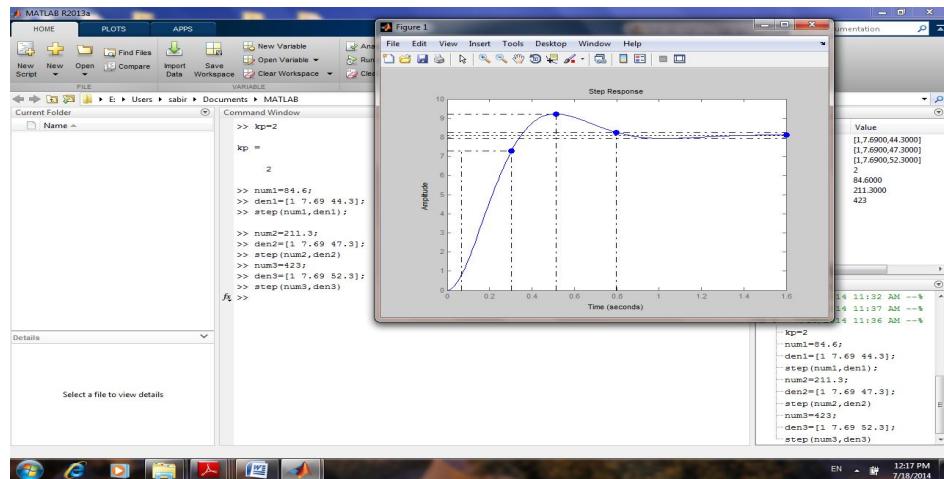

Figure 4.6: System response for $k_p=10$
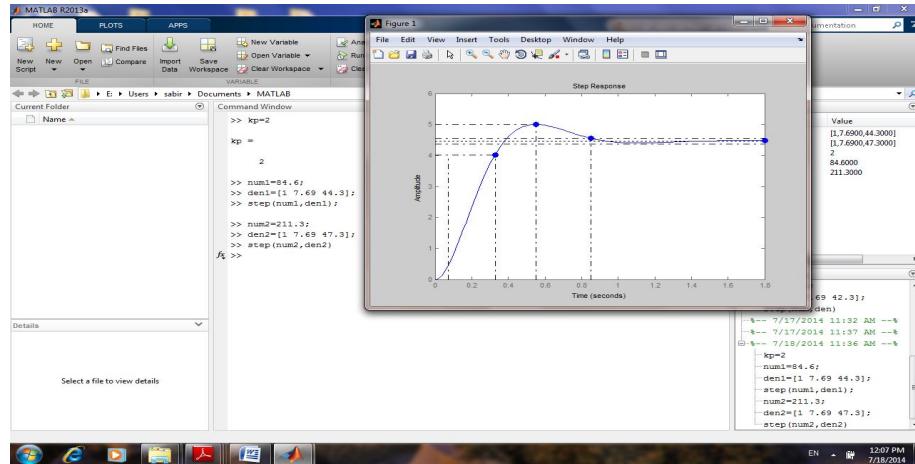
Figure 4.7: System response for $k_p=10$

Table 4.3 shows the values of performance criteria for different values of k_p.

Table 4.3: Time response results with proportional controller

Parammeters	$K_P = 2$		$K_P = 5$		$K_P = 10$	
	Calculation	MATLAB	Calculation	MATLAB	Calculation	MATLAB
t_r	0.410	0.271	0.402	0.256	0.381	0.235
t_s	0.965	0.886	0.966	0.851	0.926	0.8
t_p	0.581	0.575	0.571	0.551	0.542	0515
M_p	9.5%	10.8%	9.5%	12%	9.5%	13%
e_{ss}	0.008	0.91	0.004	3.47	0.002	7.09

Figure 4.8 shows the system simulation for k_p equal to two.
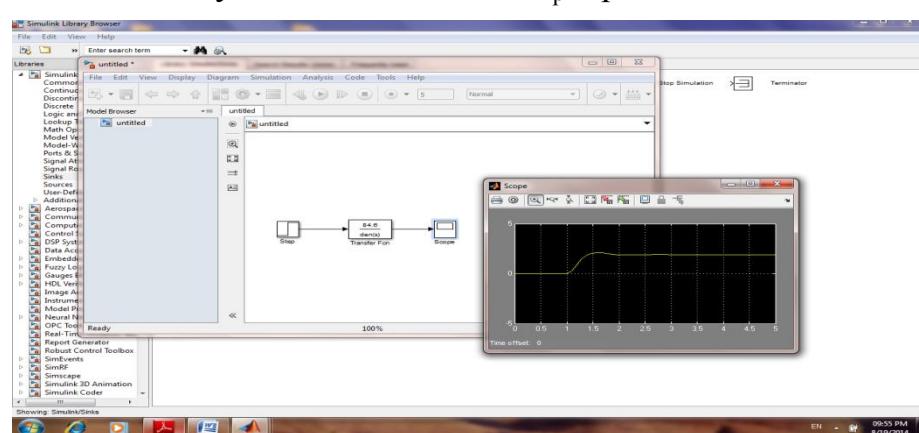
Figure 4.8: System simulation for $k_p = 2$

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Figure 4.9 shows the system simulation when k_p equal to five, while figure 4.10 for k_p equal to ten .
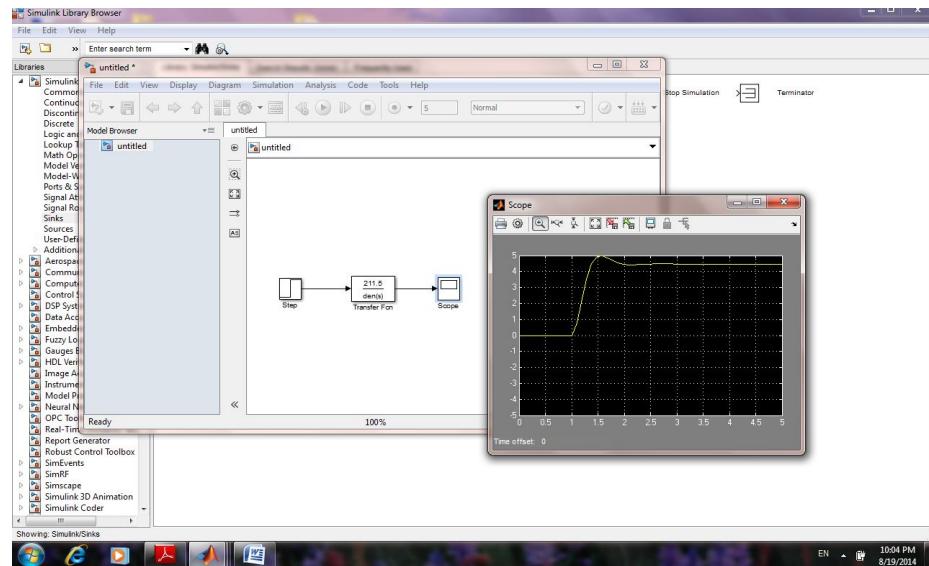
Figure 4.9: System simulation for $k_p = 5$
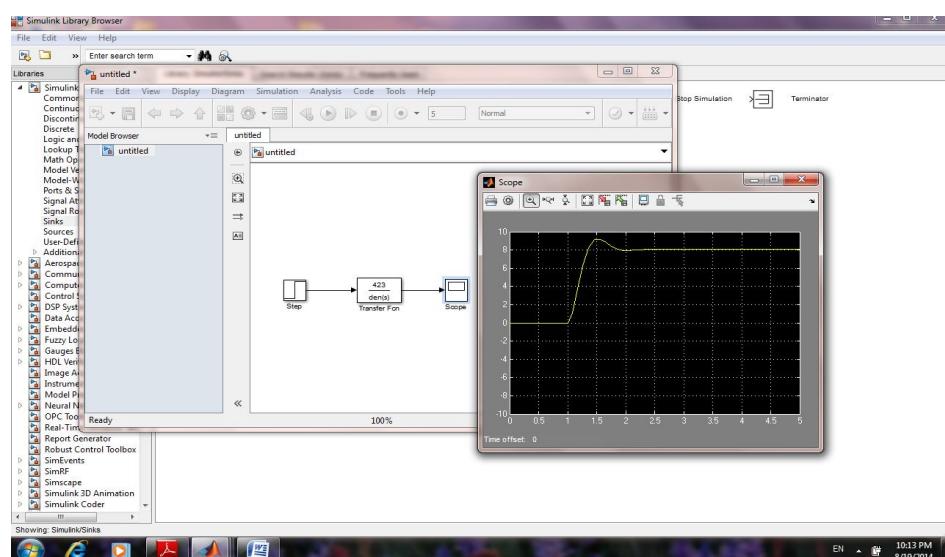
Figure 4.10: System simulation for kp = 10

4.1.2 PD control implementation

By using PD Control Implementation the system can be shown below.
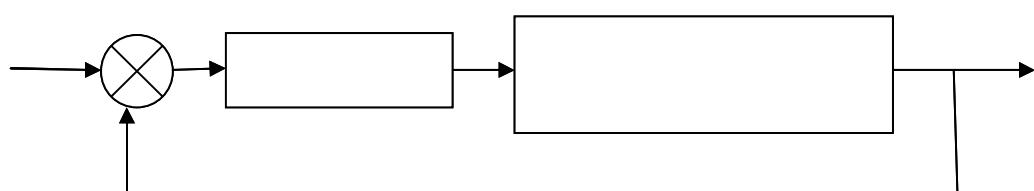
Figure 4.11: Block diagram of the system with PD controller

Then the closed loop transfer function as follows:

$$C(S)/R(S) = \frac{42.3K_dS+42.3K_P}{S^2+(7.69+42.3K_d)S+(42.3+42.3K_P)} \qquad (4.5)$$

For k_d = 0.1 and K_P =0.2 and by comparing the equation (4.5) with the general form of the second order equation (3.25) we obtain

$\omega^2_n = 42.3$, thus $\omega_n =$ 7.1.

$2\xi\omega_n = 11.92$, thus $\xi =$ 0.8.

Then the system specifications are:

$t_r = \dfrac{\pi-\beta}{\omega d}$ where $\beta = \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi} = 36.8 = 0.64$ rad/sec.

$\omega d = \omega_n\sqrt{1-\xi^2} = 4.3$.

Then:

$t_r = 0.582$ sec.

$t_p = \dfrac{\pi}{\omega d} = 0.731$ sec.

$t_s = \dfrac{4}{\xi\omega n} = 0.704$ sec , for (2% error).

$t_s = \dfrac{3}{\xi\omega n} = 0.528$ sec, for (5% error).

$M_p = e^{-\pi(\frac{\xi}{\sqrt{1-\xi^2}})} = 0.015 \times 100 = 1.5$ %.

$e_{ss} = \lim_{S\to0}(\dfrac{SR(S)}{1+G(S)}) = 0.167$.

Figure 4.12 shows the system response for k_d equal to 0.1 and k_p equal to 0.2.
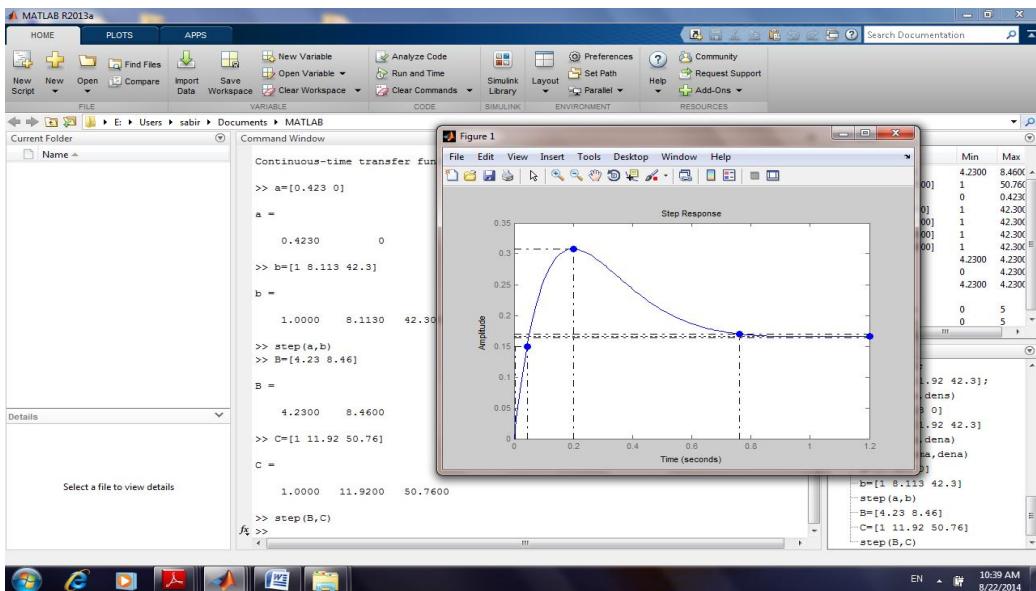
Figure 4.12: System response for k_d= 0.1and K_P =0.2

Figure 4.13 shows the system simulation for k_d equal to 0.1 and k_p equal to 0.2.
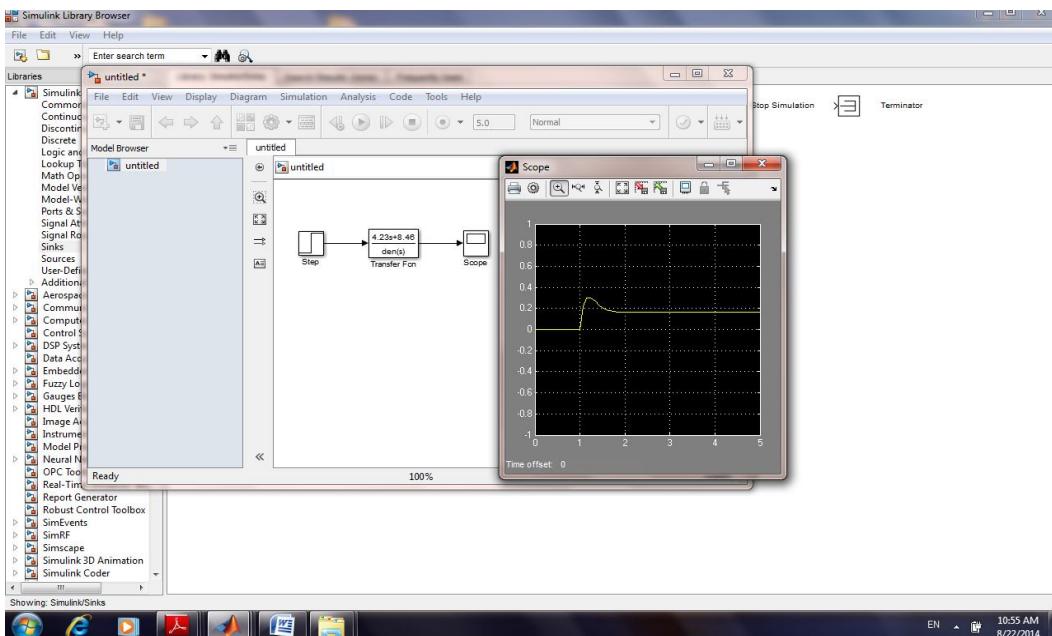
Figure 4.13: System simulation for k_d = 0.1and K_P =0.2

For k_d = 0.15 and K_P =0.25 and by comparing the Equation (4.5) with the general form of the second order Equation (3.25) we obtain:

ω^2_n = 52.875,   thus ω_n =   7.3.

$2\xi\omega_n$ =14.035 , thus ξ =   0.9.

Then the system specifications are:

$t_r = \dfrac{\pi-\beta}{\omega d}$, where $\beta = \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi} = 25.8 = 0.451$ rad/sec.

$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.2.$

Then:

$t_r = 0.841$ sec.

$t_p = \dfrac{\pi}{\omega d} = 0.982$ sec.

$t_s = \dfrac{4}{\xi\omega n} = 0.609$ sec,   for (2% error).

$t_s = \dfrac{3}{\xi\omega n} = 0.457$ sec,   for (5% error).

$M_p = e^{\dfrac{-\pi(\frac{\xi}{\sqrt{1-\xi^2}})}{}} = 0.002 \times 100 = 0.2$ %.

$e_{ss} = \lim_{S\to 0}(\dfrac{SR(S)}{1+G(S)}) = 0.2.$

Figure 4.14 shows the system response for k_d equal to 0.15 and k_p equal to 0.25, while Figure 4.15 shows the system simulation for k_d equal to 0.15 and k_p equal to 0.25.
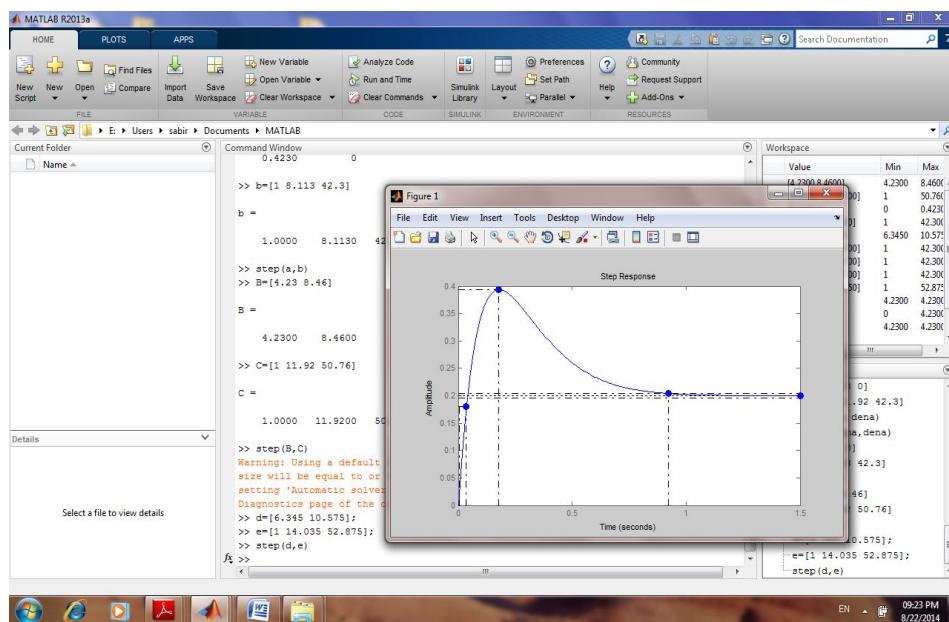
Figure 4.14: System response for k_d = 0.15 and K_P =0.25
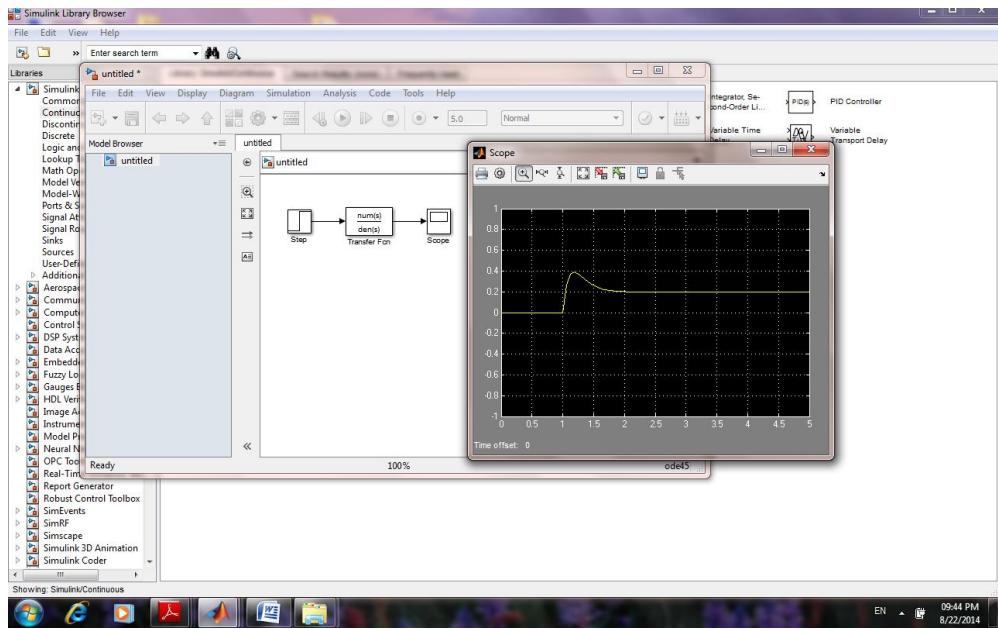
Figure 4.15 System simulation for $k_d = 0.15$ and $K_P = 0.25$

Table 4.4 shows the values of the performance criteria for different values of k_d and k_p.

Table 4.4: Time response results with PD controller

Parammeters	Kd =0.1, KP=0.2		Kd =0.15, KP=0.25	
	Calculation	MATLAB	Calculation	MATLAB
t_r	0.582	0.0402	0.841	0.032
t_s	0.704	0.76	0.609	0.922
t_p	0.731	0.201	0.982	0.177
M_p	1.5%	84.7%	0.2%	96.7%
e_{ss}	0.167	0.833	0.2	0.8

4.1.3 PI control implementation

By using PI control implementation the system can be shown below.
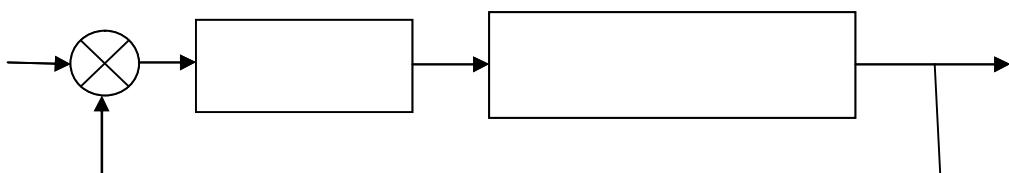
Figure 4.16: Block diagram of the system with PI controller

Then the closed loop transfer function as follows:

$$C(S)/R(S) = \frac{42.3K_P S + 42.3K_I}{S^3 + 7.69S^2 + (42.3 + 42.3K_P)S + 42.3K_I} \qquad (4.6)$$

For kI = 70 and KP =30 and there is no comparing the Equation (4.6) with the general form of the second order Equation (3.25), thus the third order.

Figure 4.17 shows the system response for k_I equal to 70 and k_p equal to 30, while Figure 4.18 shows the system simulation for k_I equal to 70 and k_p equal to 30.
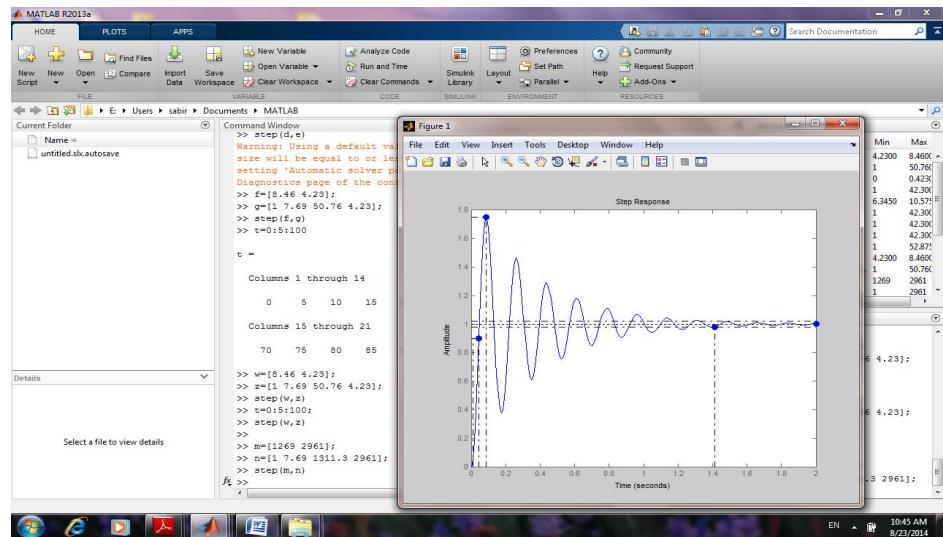

Figure 4.17: System response for k_I = 70 and K_P =30
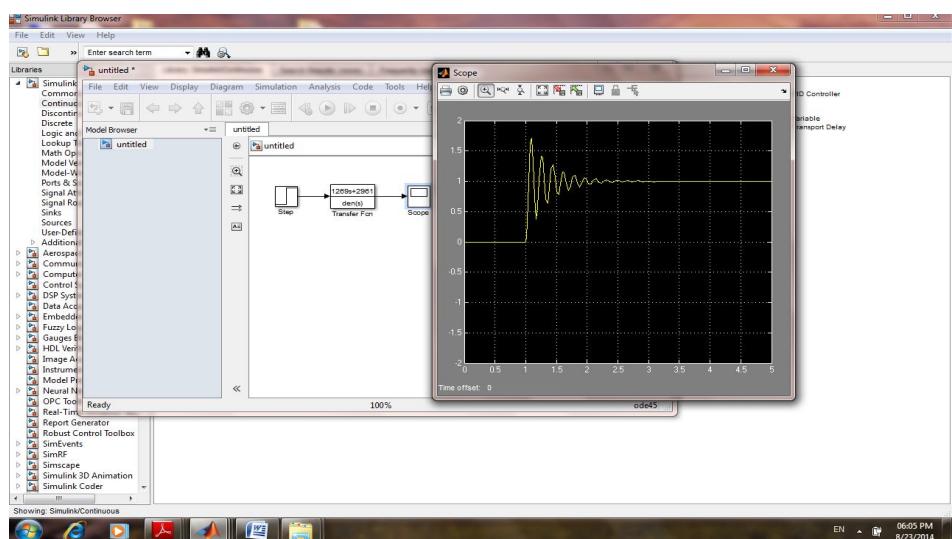
Figure 4.18: System simulation for k_I = 70 and K_P =30

Figure 4.19 shows the system response for k_I equal to 30 and k_p equal to 70, while Figure 4.20 shows the system simulation for k_I equal to 30 and k_p equal to 70.
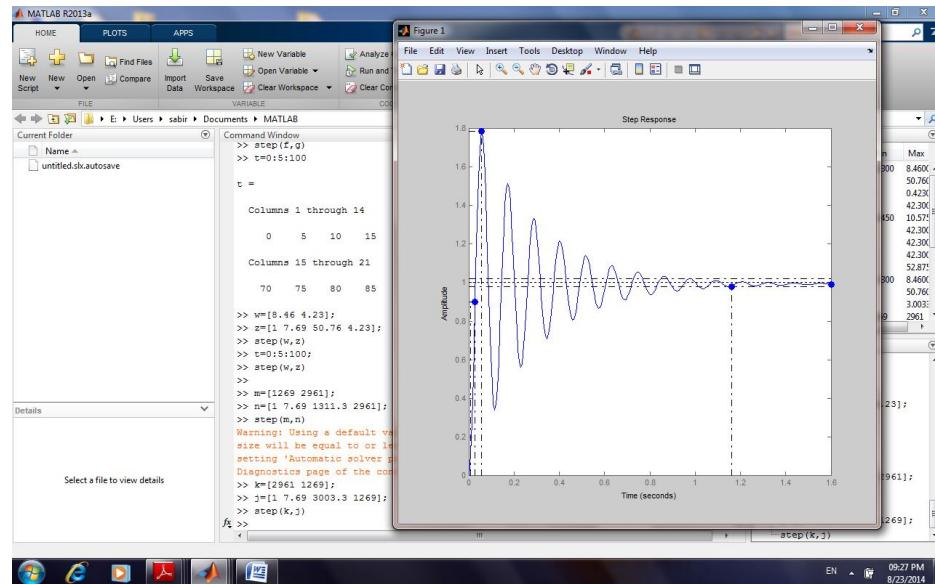
Figure 4.19: System response for k_I = 30 and K_P =70
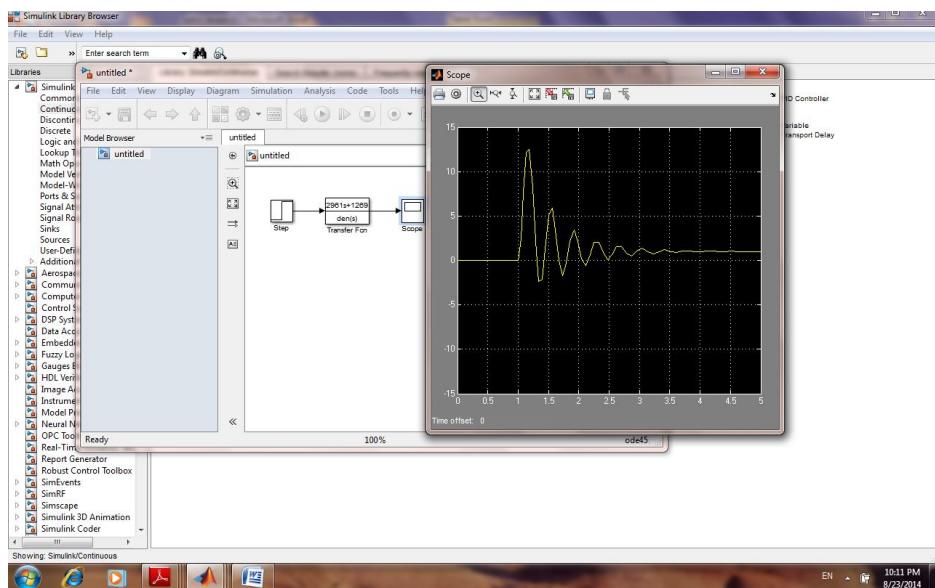
Figure 4.20: System simulation for k_I = 30 and K_P =70

Table 4.5 shows the values of the performance criteria for different values of k_I and k_p.

Table 4.5: Time response results with PI controller

Parammeters	$K_I = 70, K_P = 30$	$K_I = 30, K_P = 70$
	MATLAB	MATLAB
t_r	0.0311	0.202
t_s	1.41	1.16
t_p	0.0872	0.0574
M_p	75.2%	78.8%
e_{ss}	0	0

4.1.4 PID- control implementation

Similar to PD control, PI, and PID control shall be implemented by combining proportional, derivative and integral control elements. Block representation of PID control is given in Figure 4.21 below.
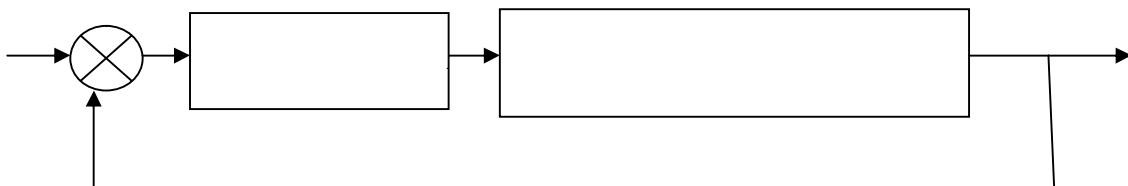

Figure 4.21: Block diagram of the system with PID controller

Then the closed loop transfer function as follows:

$$C(S)/R(S) = \frac{42.3K_d S^2 + 42.3K_P S + 42.3K_I}{S^3 + (7.69 + 42.3K_d)S^2 + (42.3 + 42.3K_P)S + 42.3K_I} \qquad (4.7)$$

For $k_I = 30$ and $K_P = 70$ and $K_d = 20$ there is no comparing the Equation (4.7) with the general form of the second order Equation (3.25),thus the third order.

Figure 4.22 shows the system response for k_I equal to 30, k_p equal to 70, and kd equal to 20, while Figure 4.23 shows the system simulation for k_I equal to 30, k_p equal to 70 and k_d equal to 20.
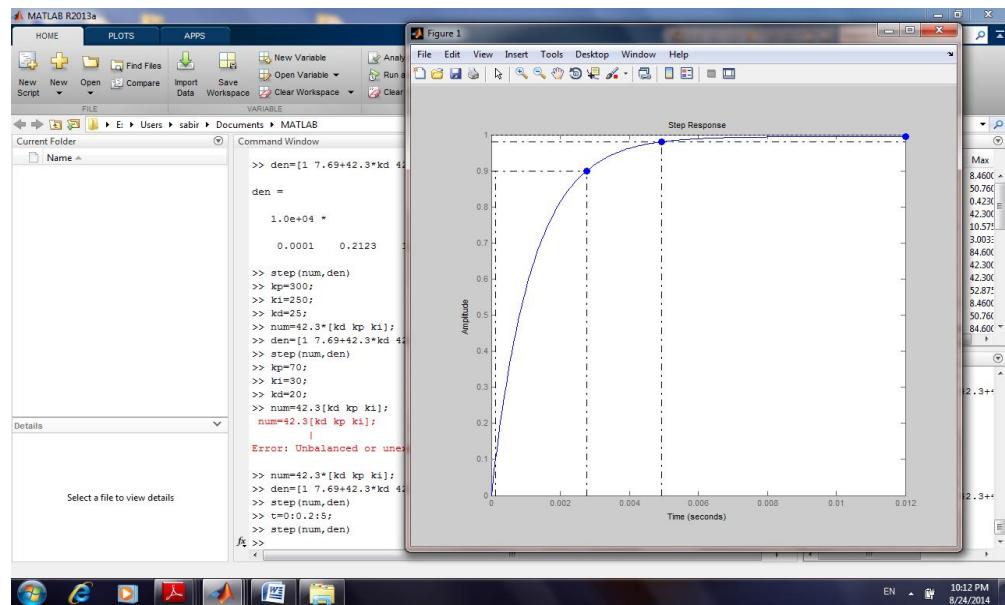
Figure4.22: System response result for k_I=30, K_P=70 and k_d=20
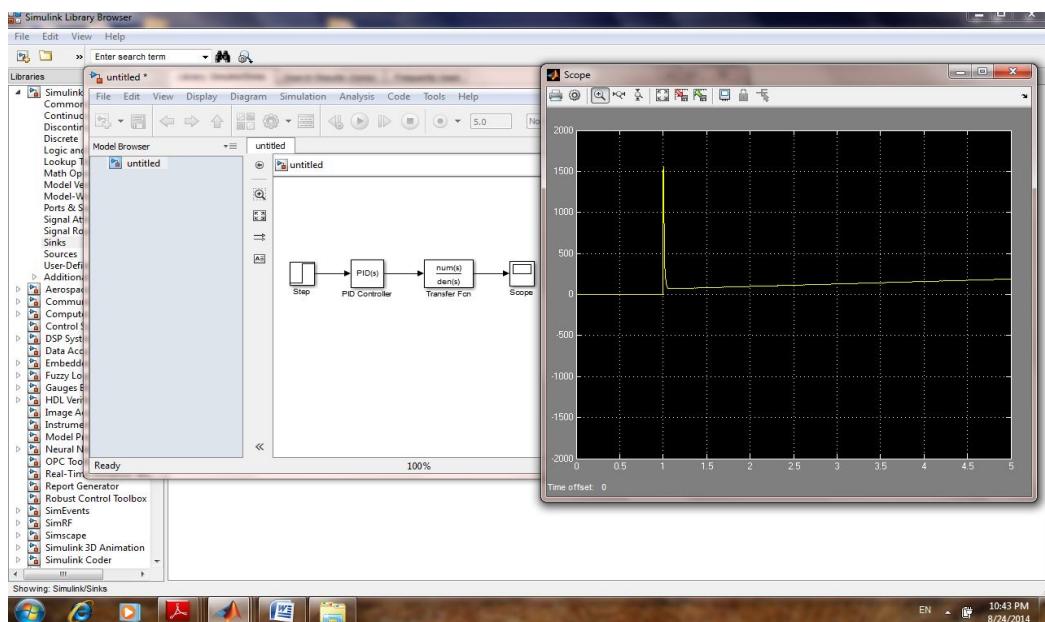

Figure 4.23: System simulation for k_I = 30, K_P =70, and K_d=20

Figure 4.24 shows the system response for k_I equal to 300, k_p equal to 350, and k_d equal to 50, while Figure 4.25 shows the system simulation for k_I equal to 300, k_p equal to 350 and k_d equal to 50.
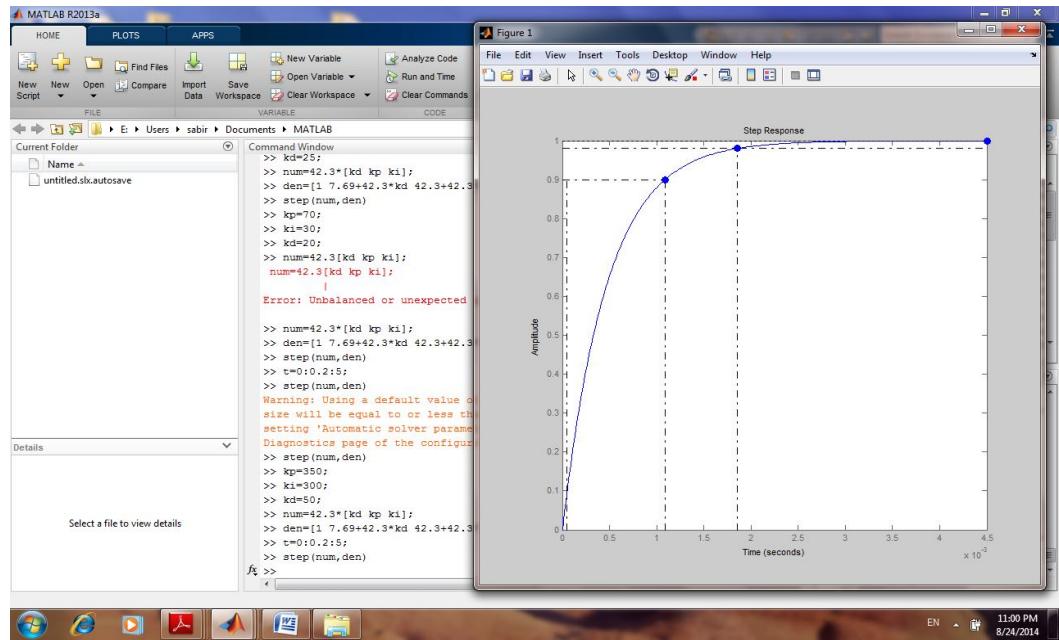
Figure 4.24 System response for $k_I = 300$, $K_P = 350$ and $K_d = 50$
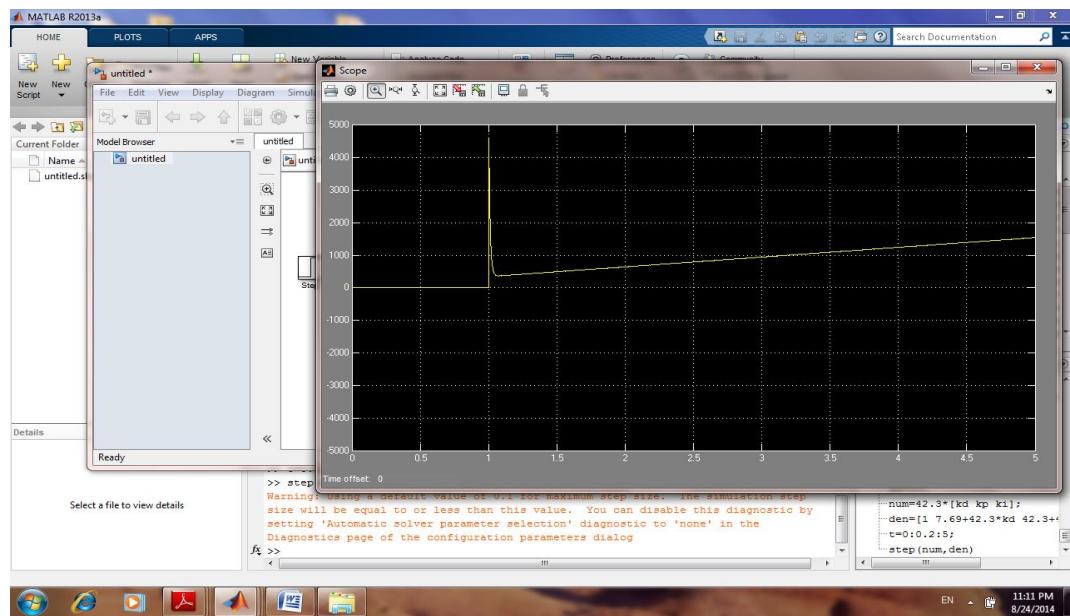
Figure 4.25: System Simulation for $k_I = 300$, $K_P = 350$ and $K_d = 50$

Figure 4.26 shows the system response for k_I equal to 350, k_p equal to 400, and k_d equal to 50, while Figure 4.27 shows the system simulation for k_I equal to 350, k_p equal to 400 and k_d equal to 50.
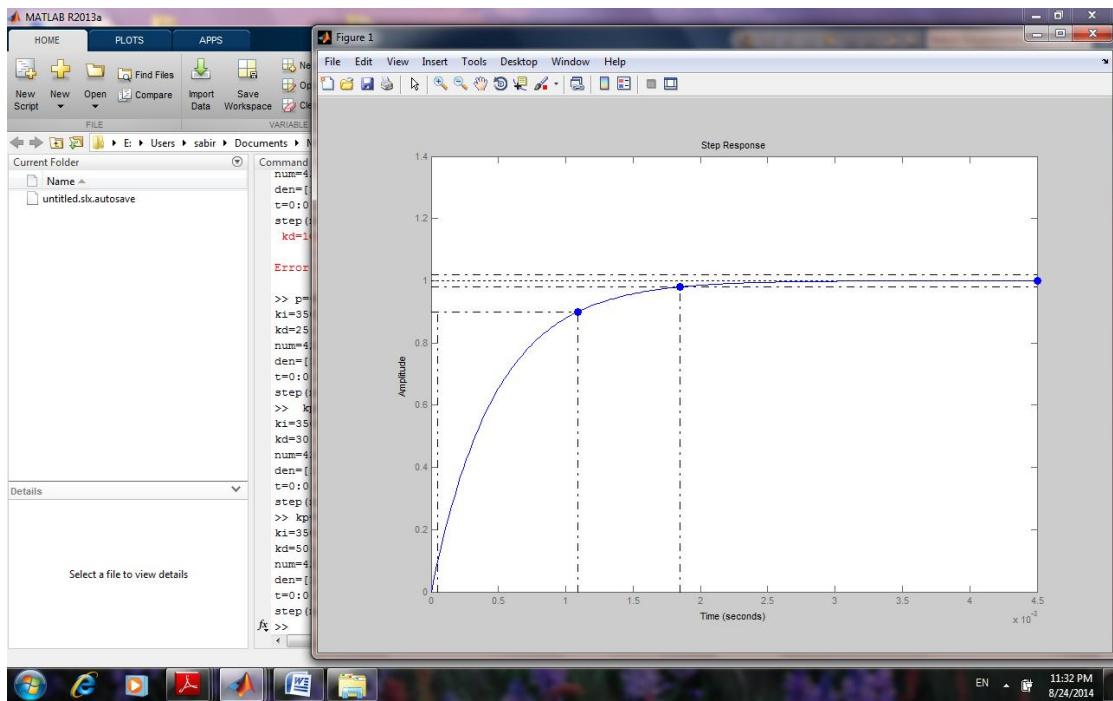
Figure 4.26: System response for $k_I = 350$, $K_P = 400$ and $K_d = 50$
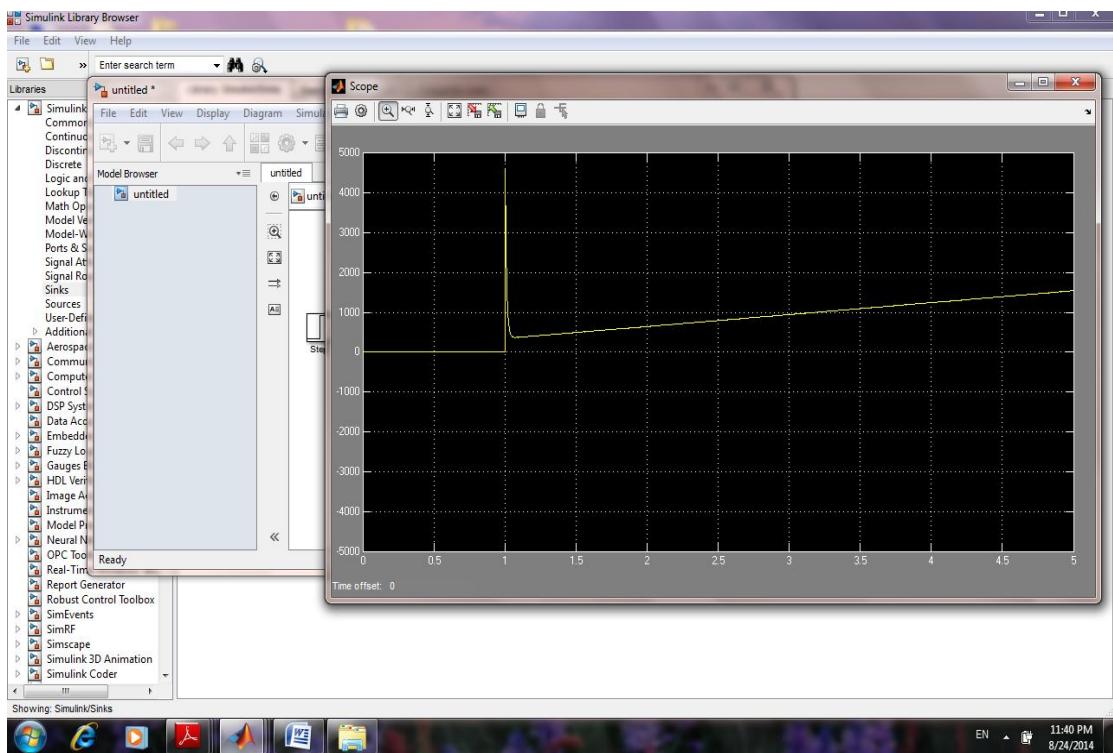

Figure 4.27: System Simulation for $k_I = 350$, $K_P = 400$ and $K_d = 50$

Table 4.6 shows the values of the performance criteria for different values of k_I, k_p, and k_d.

Table 4.6:Time response results with PID controller

Para mme ters	K_I = 30, K_P = 70 , k_d=20	K_I = 300, K_P=350 , k_d =50	K_I =350, K_P=400 , k_d =50
	MATLAB	MATLAB	MATLAB
t_r	0.00264	0.00104	0.00104
t_s	0.00493	0.00186	0.00185
t_p	0.012	0.0045	0.0045
M_p	0%	0%	0.00493%
e_{ss}	0	0	0

4.2 Final Results

For a given desired transient specification, calculate a PID control gain based on step input response . Given a second order system response, these parameters can be calculated and responses for different inputs can be compared with directly from MATLAB. So once we give the values of t_r, t_s, t_d, t_p and M_p, then the transient response represented on Figure 4.2 - 4.27 can be completely specified. Nevertheless, in most real applications, desired values of these parameters would be given and the objective will be to design controllers that can meet the requirements. Some desirable characteristics in addition of requiring a dynamic system to be stable, the system should possess:

- Faster and "instantaneous" response.
- Minimal overshoot above the desired value (i.e., relatively stable).
- A bility to reach and remain close to the desired reference value in the minimum time possible.

We will use these parameters to analyze the DC motor system under different form of controls and optimize the controller gains to achieve desired performance by end of this session. For implementation(K_P=2, K_P =5,and

K_P =10) we give the values of t_r, t_s, t_d, t_p and M_p shows in Table 4.3 that compared with calculated values but we note that when the value of k_p increasing the difference of values of specifications values calculated and directly from MATLAB are increasing then the best value of k_p is as k_p=2 and (k_p< 2).

And for implementation PD and for (k_d =0.1 ,k_p =0.2 and k_d =0.15,k_p =0.25) we give the values of t_r, t_s, t_d, t_p and M_p show in Table 4.4 that compared with calculated values, but we note that when the values of kd and kp increasing the difference of values of specifications calculated and Directly From MATLAB are increasing then the best values of k_d and k_p are (k_d=0.1, k_d < 0.1) and (k_p = 0.2, k_p< 0.2).

From two cases above the values directly from MATLAB cannot be typically to the values calculated because the MATLAB programe is very a ccurated and sensitive to the numbers intered for it but the calculated values depend on the approximation estimated, also we note that the relative increasing and deceasing for the values from MATLAB not constant but randomly compared with the calculated values to add constant factor to the equations of specifications to be basic law. For K_I and PID there is no comparing is possible because the system becomes third order equation.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIOMS

٥.1 Conclusion

The PID controller performance was consistent with old trials of controlling this type of motors, the change in system's parameters does not yield any change in the technique used to tune the PID controller for, but changes the performance of the PID. The PID controller cannot be improved further, since the tuning results were the best to get the output shown on Figure 4.22 up to 4.27. The tuning results for the PID controller were best match for the system performance and the ability to build such a controller. The PID controller can be used with servomotors that are not components of very efficient systems or time critical systems, since they will require high power to operate them and may lead to failure in their function due to the high power used by the controller. The PID controller has been tuned to get the best response possible from the system, the values obtained for the PID parameters values are: KP = 350. KI = 300 and KD = 50. From these values we need to build a controller that consumes more power due to the proportional parameter value. Even though, this controller can be built, due to high gain value this controller may not be the best solution to our system, taking into consideration the power ratings of the motor; which may not be able to withstand this value of input voltage.

٥.٢ Recommendations

No further can be done with the PID controller, since the tuning of the controller parameters resulted in the best match of performance and real implementation. While how to compare the higher order systems with the second order systems to find the performance specifications.

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