



بسم الله الرحمن الرحيم



**Sudan University of Science and Technology**

**College of Graduate Studies**

**Derivation of Zeeman Effect Equation On the Basis Of Magnetic  
Flux Density**

إستنباط معادلات تأثير زيمان في ضوء مفهوم كثافة الفيض المغناطيسي

**A thesis Submitted for Partial Fulfillment of the Requirements for  
the Degree of M. Sc of Science In General Physics**

**By**

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

# الآية

قال تعالى:

(يَا أَيُّهَا الَّذِينَ آمَنُوا إِذَا قِيلَ لَكُمْ تَفَسَّحُوا فِي الْمَجَالِسِ فَافْسَحُوا يَفْسَحِ اللَّهُ لَكُمْ وَإِذَا قِيلَ انشُرُوا فَانشُرُوا يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنْكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ دَرَجَاتٍ وَاللَّهُ بِمَا تَعْمَلُونَ خَبِيرٌ)

صدق الله العظيم

سورة المجادلة الآية (11)

# **Dedication**

**This research work is dedication to:**

**My Parents, Brother, and Sisters**

**My Teachers**

**My Friend**

**And**

**All Those**

**Whom I Love**

**Hleima**

# **Acknowledgment**

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**Firstly and finally, thank and praise is to Allah almighty**

# **Abstract**

**Zeeman Effect is one of the most important physics phenomena that have a wide variety of application.**

**This effect is described usually by using the concept of magnetic moment which is complex and ambiguous as shown by some researchers. Thus there is need for simple treatment.**

**In this work the notion of magnetic flux density is used to find the Zeeman energy shift simple relation. And also use current. This new relation is the same as the ordinary one which agrees with experiment.**

## مستخلص البحث

تأثير زيمان هو احدى الظواهر الفيزيائية الأكثر أهمية التى لديها مجموعة واسعة من التطبيقات. يوصف هذا التأثير عادة باستخدام مفهوم العزم المغناطيسى، وهو معقد وغامض ، كما هو موضح من قبل بعض الباحثين وبالتالي يحتاج لعلاج بسيط.

فى هذا العمل استخدمت فكرة كثافة الفيض المغناطيسى لإيجاد علاقة بسيطة لإزاحة طاقة زيمان وكذلك استخدم التيار. هذه العلاقة الجديدة هى نفس العلاقة العادية التى تتوافق مع التجربة.

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# Chapter one

## (1.1) Introduction

It is well known that an atom can be characterized by a unique set of discrete energy states. When excited through heating or electron bombardment in a discharge tube, the atom makes transitions between these quantized energy states and emits light. The emitted light forms a discrete spectrum, reflecting the quantized nature of the energy states or energy levels. In the presence of a magnetic field, these energy levels can shift. This effect is known as the Zeeman Effect. In 1896, p. Zeeman observed that the spectral lines of atoms split in the presence of an external magnetic field. [1, 2].

The origin of Zeeman Effect is the following. In an atomic energy state, an electron orbits around the nucleus of the atom and has a magnetic dipole moment associated with its angular momentum. In a magnetic field, it acquires an additional energy just as a bar magnet does and consequently the original energy level is shifted. The energy shift may be positive, zero, or even negative, depending on the angle between the electron magnetic dipole moment and the field.

Due to Zeeman Effect, some degenerate energy levels will split into several non- degenerate energy levels with different energies. This allows for new transitions which can be observed as new spectral lines in the atomic spectrum. [1]

## **(2.1) Zeeman Effect**

Is applied widely in many areas. Its applied in magnetic resonance imaging (MRI) which is used in medical diagnosis. It is also used to indentify magnetic properties of matter.

## **(1.3) Research Problem:**

The research problem is related to the fact that the derivation of Zeeman energy is based on the concept of magnetic moment which is complex and ambiguous

## **(1.4) Literature Review:**

Different attempts were made to explain magnetic atomic phenomena by using magnetic flux density instead of magnetic moment. But no attempt was made to explain Zeeman Effect by using the concept of magnetic flux density.

## **(1.5) Aim of the work:**

The aim of this work is to use the concept of magnetic flux density to derive Zeeman Effect equation.

## **(1.6) presentation of the thesis:**

The thesis consists of three chapter .Chapter one is introduction. Chapter tow is devoted for theretiae back ground. The contribution is in chapter three.

## Chapter tow

### Magnetic Moment, Flux Density and Zeeman Effect

#### (2.1) Introduction:

This chapter is concerned with definition of orbital angular momentum and magnetic moment. It is also concerned with the derivation of Zeeman Effect equation.

#### (2.2) Orbital Angular Momentum:

The orbital angular momentum is associated with the orbital motion of particles. If a particle of mass ( $m$ ) move with speed ( $v$ ) in a circular orbit of radius ( $r$ ), its orbital angular momentum is given by:

$$L = mvr \quad (2.2.1)$$

The motion of particle in a circular orbit is a sort of simple harmonic motion.

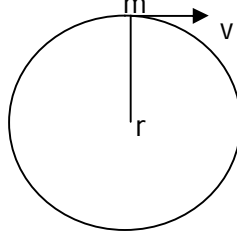


Figure (2.2.1) a particle moving in a circular orbit

Period of motion  $T$  is related to the particle speed and orbit radius  $r$  according to the relation

$$T = \frac{2\pi r}{v} \quad (2.2.2)$$

The frequency ( $f$ ) of motion is given by:

$$f = \frac{1}{T} = \frac{v}{2\pi r} \quad (2.2.3)$$

The angular frequency  $\omega$  is given by:

$$\omega = 2\pi f = \frac{v}{r} \quad (2.2.4)$$

### (2.3): Magnetic Moment

The magnetic moment of a magnet is a quantity that determines the torque it will experience in an external magnetic field. A loop of electric, a bar magnet, an electron (revolving around a nucleus), a molecule, and a planet all have magnetic moments.

The magnetic moment may be considered to be a vector having a magnitude and direction. The direction of the magnetic moment points from the south to North Pole of the magnet. The magnetic field produced by the magnet is proportional to its magnetic moment. The dipole component of an object's magnetic field is symmetric about the direction of its magnetic dipole moment, and decreases as the inverse cube of the distance from the object. [3]

The magnetic moment is defined to be in the form

$$\vec{\mu} = i\vec{A} \quad (2.3.1)$$

Where

$\mu$  = Magnetic moment of the infinitesimal current loop

$A$  = area enclosed by the orbit of radius  $r$

$i$  = current associated with the particle due to its motion. For particles revolving in a circular orbit.

$$i = \frac{dq}{dt} = \frac{q}{T} \quad (2.3.2)$$

Here  $T$  is the orbital period

For a planar loop encircling an [area  $A$ ] this magnetic moment magnitude is given by:

$$\mu = iA \quad (2.3.3)$$

In circular linear, velocity associated with angular velocity with radius, with form:  $v = \omega r$  but,  $\omega = \frac{2\pi}{T} = 2\pi$  (2.3.4)

$$v = \frac{2\pi}{T} r$$

$$T = \frac{2\pi r}{v}$$
(2.3.5)

Substituted equation (2.3.5) in (2.3.2):

$$i = \frac{qv}{2\pi r}$$
(2.3.6)

$$\mu = iA = \frac{qv}{2\pi r} A$$
(2.3.7)

This equation represents the magnetic field for particles moving in a circular orbit where A is given by: [4.5]

$$A = \int_{\circ}^A dA = \pi r^2$$
(2.3.8)

Substituted equation (2.3.8) in equation (2.3.7):

$$\mu = iA = \frac{qv}{2\pi r} \pi r^2 = \frac{qvr}{2}$$
(2.3.9)

For a single electron:

$$q = -e$$

$$\mu = -\frac{evr}{2}$$
(2.3.10)

## (2.4) Quantization of Angular Momentum:

Is possible to justify the observed allowed energy level of one electron atoms by assuming that the angular momentum of the electron is quantized. The allowed values for the angular momentum according to Bohr's theory where given by the equation:

$$L = n\hbar$$
(2.4.1)

This relation was derived by Bohr for the particular case of circular orbit, the orbital angular momentum of an electron is a according to Schrodinger equation given by:

$$L = \sqrt{l(l+1)}\hbar$$
(2.4.2)

Where  $L = 0, 1, 2, 3, \dots$  is a positive integer. Equations (2.4.1) agree with equation (2.4.2):

$$L = n\hbar \quad (2.4.3)$$

Where  $n = 0, 1, 2, 3, \dots$

When  $L$  is very large because in this case we may neglect 1 in comparison with  $L$  and write:

$$\begin{aligned} L &\approx L\hbar \\ L &= L\hbar \end{aligned} \quad (2.4.4)$$

$$L = 0, 1, 2, 3, \dots, (n-1), (n: \text{value}) \quad (2.4.5)$$

The orientation of the angular momentum  $L$  relative to a given axis is limited to certain direction. The given axis as  $Z$ , the allowed values of the  $Z$ -component of  $L$  are:

$$L_z = m_l \hbar \quad (2.4.6)$$

Where  $m_l$  is positive or negative integer, having the values of:

$$m_l = 0, \pm 1, \pm 2, \dots, \pm (L-1), \pm L \quad (2.4.7)$$

The upper value of  $m_l$  is  $\pm L$ , because  $L_z$  cannot be large, then:

$|L|$ . For  $L = 0$ , only  $m_l = 0$ , is possible.

An orbiting electron has a magnetic dipole moment which, according to equation (2.4.1) is given by: [7].

$$M_L = \frac{-e}{2m} L \quad (2.4.8)$$

## (2.5): Normal Zeeman Effect:

For singlet states, the spin is zero and the total angular momentum  $J$  is equal to the orbital angular momentum  $L$ . When placed in an external magnetic field, the energy of the atom changes because of the energy of its magnetic moment in the field, which is given by:

$$\Delta E = -\mu \cdot B = -\mu_z B \quad (2.5.1)$$

Where the magnetic moment is given by:

$$\mu_z = -m_l \mu_B = -m_l \left( \frac{e\hbar}{2me} \right) \quad (2.5.2)$$

And

$$\Delta E = +m_l \frac{e\hbar}{2m_l} B = m_l \mu_B B \quad (2.5.3)$$

Since there are  $2L + 1$  values of  $m_l$  each energy level splits into  $2L + 1$  levels. Figure shows the splitting of the levels for the case of a transition between a state with  $L=2$  and one with  $L=1$ . The selection rule  $\Delta m_l = \pm 1$

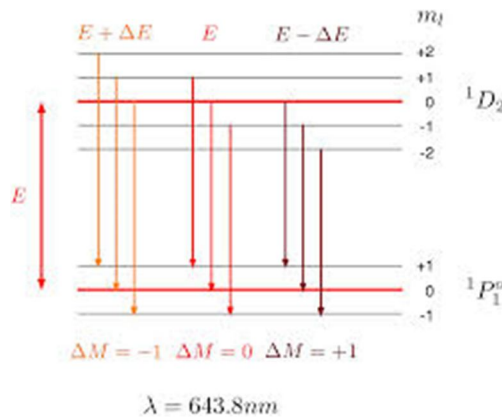


Figure (2.5.1): Energy level splitting in the normal Zeeman Effect for single level

To study the normal Zeeman Effect one can consider a calcium arc placed between the pole pieces of strong electro magnet, one capable of producing a flux density of about 30,000 gauss. The light may be viewed in a direction perpendicular to the direction of the magnetic field or parallel to the magnetic field; to make the latter possible a small hole may be drilled along the axis of the pole pieces of the magnet and the light coming through it sent through a spectroscope, or small mirror may be inserted near one of the pole pieces to reflect such light into spectroscope.

When there is no magnetic field present, the spectral line can be sharply focused and will appear at a certain position in the spectroscope. Let the strong frequency of this line,  $f$ . When no



magnetic field is present. Suppose now that a strong magnetic field of flux density  $B$  is applied to the source of light and that the light emitted in direction transverse to the magnetic field is viewed in the spectroscope. The single line will be found to be split into three lines or three components.

One of these components will be in the same position as the original line and will thus have the same frequency  $f_0$ ; the other two components of frequencies  $f_1$  and  $f_2$  will be seen on either side of the original line an equally displaced from the central component  $f_0$  when this experiment is repeated with the light that is emitted parallel to the direction of  $B$ , it will be observed that the original line splits into two component only and that these two components have the frequencies  $f_1$  and  $f_2$ .

Although the light emitted by the source is unpolarized, the three components seen when the light is viewed perpendicular to the magnetic field are linearly polarized.

The two components of the spectral line that are produced when the light is emitted parallel to the direction of the magnetic field are found to be circularly polarized in opposite direction.

The classical explanation of the normal Zeeman Effect is based on Lorentz's electron theory. Assume that an electron in the some central force  $F_0$ , then from Newton's second law of motion

$$F_0 = \frac{mv_0^2}{r} = m\omega_0^2 r \quad \text{of motion:} \quad (2.5.4)$$

Where  $v_0$  is its linear velocity in the orbital and  $\omega_0$  is its angular velocity. If an external magnetic is applied perpendicular to the plane of the orbital of the electron, two effects will be produced. During the time that the magnetic field is being established, there

will be an electric field tangent to the orbit because of the emf produced by the changing magnetic flux through it.

Suppose that the velocity of the electron has been increased to  $v_1$  by the application of the magnetic field of flux density B; then the force  $F_B$  due to the magnetic field is:

$$F_B = Be v_1 = Be \omega_1 r \quad (2.5.5)$$

Where:

$$v_1 = \omega_1 r \quad (2.5.6)$$

The conversion factor c must be used and the equation will read:

$$F_B = B \frac{e}{c} v_1 = B \frac{e}{c} \omega_1 r \quad (2.5.7)$$

The total force acting radially is given by:

$$F_c + F_B = m \omega_1^2 r \quad (2.5.8)$$

Substituting the values for  $F_c$  and  $F_B$  we get:

$$m \omega_c^2 r + Be \omega_1 r = m \omega_1^2 r \quad (2.5.9)$$

Solving equation (2.5.8) for  $\omega_1$ , we get: [6]

$$\omega_1 = \frac{eB/m \pm \sqrt{(eB/m)^2 + 4\omega_c^2}}{2} \quad (2.5.10)$$

It can be shown that:

$$\left(\frac{eB}{m}\right)^2 \ll 4\omega_c^2 \quad (2.5.11)$$

Therefore we can write:

$$\omega_1 = \omega_0 + \frac{eB}{2m} \quad (2.5.12)$$

Only the positive sign is retained, since the effect of the magnetic field is small and can produce only a slight change in the magnitude of the angular velocity. If the charge should be rotating in the opposite direction, its angular velocity will be:

$$\omega = \omega_0 \pm \frac{eB}{2m} \quad (2.5.13)$$

$$\begin{aligned} \omega &= 2\pi f \\ \omega_0 &= 2\pi f_0 \end{aligned} \quad (2.5.14)$$

Where  $f$  is the frequency corresponding to the angular velocity  $\omega$ . Equation (2.5.12) then becomes:

$$f = f_0 \pm \frac{eB}{4\pi m} \quad (2.5.15)$$

The quantity  $\frac{eB}{4\pi m}$ , where  $e$  is in flux density  $B$ . the quantity  $\frac{e}{m}$  can thus be determined from a measurement of a normal Zeeman separation of a single spectrum line [8].

$$\frac{e}{m} = 1.759 \times 10^2 \text{ emu/gm} \quad (2.5.16)$$

## (2.6): Anomalous Zeeman Effect:

The anomalous Zeeman Effect occurs when the spin of either the initial or the final states, or both, is nonzero. The calculation of the energy-level splitting is complicated a bit by the fact that the magnetic moment due to spin is 1 rather than Bohr magneton, and as a result the total magnetic moment is not parallel to the total angular momentum. Consider an atom with orbital angular momentum  $L$  and spin  $S$ . Its total angular momentum is:

$$J = L + S \quad (2.6.1)$$

Where the total magnetic moment is:

$$\mu = -g_e \mu_B \frac{L}{\hbar} - g_s \mu_B \frac{S}{\hbar} \quad (2.6.2)$$

Since:

$$g_L = 1, g_s = 2 \quad (2.6.3)$$

Thus one has:

$$\mu = -\frac{\mu_B}{\hbar}(L + 2S) \quad (2.6.4)$$

Each energy level is split into  $2j + 1$  level, corresponding to the possible values of  $m_j$ . For the usual laboratory magnetic fields, which are weak compared with the internal magnetic field associated with the spin-orbit effect, the level splitting is small compared with the fine-structure splitting. Unlike the case of the singlet levels in the normal effect, the Zeeman splitting of these levels depends on  $j$ ,  $l$ , and  $s$ , and in general there are more than three different transition energies due to the fact that the upper and lower states are split by different amounts. The level splitting, that is, the energy shift relative to the position of the no-field energy level, can be written [8.9]

$$\Delta E = g m_j \left( \frac{e\hbar B}{2m_e} \right) = g m_j \mu_B B \quad (2.6.5)$$

Where  $g$ , called the Lande  $g$  factor, is given by

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \quad (2.6.6)$$

Also when the light from sodium flame or arc, which has been placed in a magnetic field about 30,000 gauss, is examined with aid of a spectroscope of high resolving power, it is found that the lines of the principal exhibit the following

anomalous Zeeman effect pattern: the longer wave length component  $3^2S_{1/2} \rightarrow 2^2P_{1/2}$ , splits into four lines.

If the external magnetic field is sufficiently large, the Zeeman splitting is greater than the fine-structure splitting. If B is large enough so that we can neglect the fine- structure splitting, the Zeeman splitting is given by:

$$\Delta E = (m_\ell + m_s) \left( \frac{e\hbar B}{2m_s} \right) = (m_\ell + m_s) \mu_B B \quad (2.6.7)$$

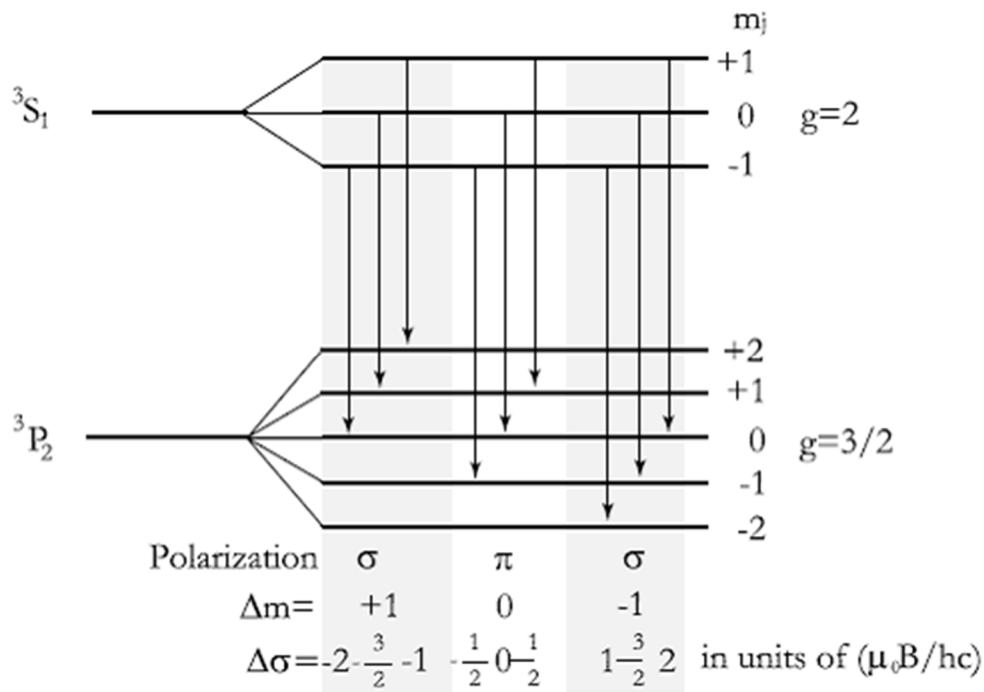


Figure (2.6.1): anomalous Zeeman Effect.

The splitting is then similar to the normal Zeeman Effect and only three lines are observed. This behavior in large magnetic fields is called the Paschen-Back effect after its discoverers, F. Paschen and E. Back. Transition of the splitting of the levels from the anomalous Zeeman Effect to the Paschen-Back effect as the magnitude of B increases. The basic reason for the change in the appearance of the anomalous effect as B increases is that the external magnetic field overpowers the spin-orbit effect and decouples L and S so that they precess about B nearly

independently; thus, the projections of  $L$  behave as if  $S \approx 0$ , and the effect reduces to three lines, each of which is a closely spaced doublet.

The magnetic field of the Sun and stars can be determined by measuring the Zeeman-effect splitting of spectral lines.

Suppose that the sodium D1 line emitted in a particular region of the solar disk is observed to be split into the four-component Zeeman Effect (Figure 2.6.1). What is the strength of the solar magnetic field  $B$  in that region if the wavelength difference  $\Delta\lambda$  between the shortest and the longest wavelengths is 0.022 nm (The wavelength of the D1 line is 589.8 nm).

**Solution:**

The D1 line is emitted in the  $3^2 P_{1/2} \rightarrow 3^2 S_{1/2}$ , from equation (2.6.6) we compute the Landé  $g$  factors to use in computing the  $\Delta E$  values from Equation (2.6.5) as follows:

For the  $3^2 P_{1/2}$  level:

$$g = 1 + \frac{\frac{1}{2}(\frac{1}{2}+1) + \frac{1}{2}(\frac{1}{2}+1) - (1+1)}{2(\frac{1}{2})(\frac{1}{2}+1)} = \frac{2}{3} \quad (2.6.8)$$

For the  $3^2 S_{1/2}$  level:

$$g = 1 + \frac{\frac{1}{2}(\frac{1}{2}+1) + \frac{1}{2}(\frac{1}{2}+1) - 0}{2(\frac{1}{2})(\frac{1}{2}+1)} = 2 \quad (2.6.9)$$

And from equation 2.6.5,

For the  $3^2 P_{1/2}$  level:

$$\Delta E = \left(\frac{2}{3}\right) \left(\pm \frac{1}{2}\right) \left(5.79 \times 10^{-9} \text{ eV/gauss} \right) B \quad (2.6.10)$$

For the  $3^2S_{1/2}$  level:

$$\Delta E = (2) \left( \pm \frac{1}{2} \right) \left( 5.79 \times 10^{-9} \text{ ev} / \text{gauss} \right) B \quad (2.6.11)$$

The longest-wavelength line ( $m_j = -\frac{1}{2} \rightarrow m_j = +\frac{1}{2}$ ) will have undergone a net energy shift of

$$-1.93 \times 10^{-9} B - 5.79 \times 10^{-9} B = -7.72 \times 10^{-9} \text{ BeV} \quad (2.6.12)$$

The shortest-wavelength line ( $m_j = +\frac{1}{2} \rightarrow m_j = -\frac{1}{2}$ ) will have undergone a net energy shift of

$$1.93 \times 10^{-9} B - 5.79 \times 10^{-9} B = 7.72 \times 10^{-9} \text{ BeV} \quad (2.6.13)$$

The total energy difference between these two photons is

$$\Delta E = -1.54 \times 10^{-8} \text{ BeV} \quad (2.6.14)$$

Since  $\lambda = c/f = \frac{hc}{E}$ , then  $\Delta\lambda = -\left(\frac{hc}{E^2}\right)\Delta E = 0.022 \text{ nm}$ .

We then have that:

$$\Delta E = -0.022 \text{ nm} \left( \frac{E^2}{hc} \right) = -1.54 \times 10^{-8} B \quad (2.6.15)$$

Where  $E = \frac{hc}{\lambda} = hc / (589.9) \text{ nm}$ .

Finally, we have

$$B = \frac{(0.022 \times 10^{-9} \text{ nm}) hc}{(589.8 \times 10^{-9} \text{ nm})^2 (1.54 \times 10^{-8} \text{ ev} / T) (1.60 \times 10^{-19} \text{ J} / \text{ev})} \quad (2.6.16)$$

$$B=0.51T=5100 \text{ gauss}$$

For comparison, the Earth's magnetic field averages about 0.5 gauss [8,10]

## Chapter three

### (3.1) Introduction

This chapter is concerned with derivation of Zeeman Effect by using the concept of magnetic flux density and current instead of magnetic moment

### (3.2) New Derivation of Zeeman Effect on the Basis of magnetic Flux Density

Consider an electron moving in a magnetic field. The force on it is given by

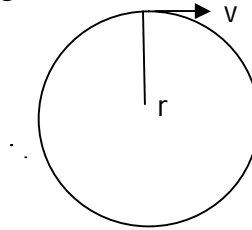


Figure: (3.2.1) Electron moving in a circular orbit.

$$F = Bev\sin\theta \quad (3.2.1)$$

F=Force on the electron

Where:

V= velocity

$v$  = Velocity,

B = magnetic flux density

$\theta$  = the Angle between  $v$  and B



Where

$$v = \omega r, \omega = \frac{v}{r} \quad (3.2.2)$$

For circular orbit

$$\begin{aligned} \underline{r} &= r[\cos \omega t \hat{i} + \sin \omega t \hat{j}] \\ v &= \frac{dr}{dt} = r(-\omega \sin \omega t \hat{i} + \omega \cos \omega t \hat{j}) \\ v &= \omega r(-\sin \omega t \hat{i} + \cos \omega t \hat{j}) \\ v &= v_x \hat{i} + v_y \hat{j} \\ v_x &= -\omega r \sin \omega t, v_y = \omega r \cos \omega t \end{aligned} \quad (3.2.3)$$

The magnitude of v is given by

$$\begin{aligned} V &= |V| = \sqrt{v_x^2 + v_y^2} = \omega r \sqrt{\sin^2 \omega t + \cos^2 \omega t} \\ V &= |V| = \omega r \end{aligned} \quad (3.2.4)$$

Inserting (3.2.4) in (3.2.1) yields

$$F = Be\omega r \sin \theta \quad (3.2.5)$$

Thus the magnetic energy  $V_m$

$$V_m = \iint F d\theta \cdot dr \quad (3.2.6)$$

Substitute (3.2.1) in (3.2.6)

$$V_m = \iint Be\omega r \sin \theta d\theta \cdot dr \quad (3.2.7)$$

$$\begin{aligned} V_m &= Be\omega \int r dr \int \sin \theta d\theta \\ V_m &= -Be\omega \frac{r^2}{2} \cos \theta \end{aligned} \quad (3.2.8)$$

From (3.2.4) and (3.2.5)

$$\begin{aligned}
L &= mvr \\
L &= m\omega r^2
\end{aligned}
\tag{3.2.9}$$

Substituted (3.2.9) in (3.2.8)

$$V_m = -\frac{Be}{2m} \omega r^2 \cos \theta \dots \dots \dots (3.2.10)$$

$$V_m = -\frac{Be}{2m} m\omega r^2 \cos \theta \tag{3.2.11}$$

Thus the magnetic energy is given by

$$V_m = -\frac{Bel}{2m} \tag{3.2.12}$$

This expression is typical to the ordinary one in equation (2.5.3), where

$$\Delta E = V_m = -\frac{Be m_l}{2m} \tag{3.2.13}$$

Where one replaces  $L$  by  $m_l$  where the orbital angular momentum is given by

$$L = \hbar \sqrt{L(L+1)} \tag{3.2.14}$$

Thus equation (3.2.12) become

$$V_m = -\frac{Be}{2m} \hbar \sqrt{L(L+1)} \tag{3.2.15}$$

### (3.3) Zeeman Effect

Explanation Using Electric Current:

When the electron revolve in a circular orbit in external magnetic field of flux density B, the force on it F, and the current I, produced by it, are given by

$$F = BiL \sin \theta \tag{3.3.1}$$

The current is given by:

$$i = ef = \frac{e\omega}{2\pi} \quad (3.3.2)$$

The magnetic energy is given by

$$V_m = \iint f dr \cdot d\theta \quad (3.3.3)$$

In view of equation (3.3.1) the length of the electron orbit is:

$$L = 2\pi r \quad (3.3.4)$$

Substituted equation (3.3.1), (3.3.2), (3.3.4), in (3.3.3) yields:

$$V_m = \int BiL \sin \theta d\theta \cdot dr \quad (3.3.5)$$

$$V_m = \int Bef(2\pi r) \sin \theta d\theta \cdot dr \quad (3.3.6)$$

$$V_m = \iint B \frac{e\omega}{2\pi} (2\pi r) \sin \theta d\theta \cdot dr$$

$$V_m = \frac{Be\omega}{2\pi} \int 2\pi r dr \int \sin \theta d\theta \quad (3.3.7)$$

$$V_m = \frac{-Be\omega r^2 \cos \theta}{2} \quad (3.3.8)$$

$$V_m = -\frac{Bem\omega r^2 \cos \theta}{2m} \quad (3.3.9)$$

Since:

$$L = m\omega r^2 \quad (3.3.10)$$

It follows that

$$\begin{aligned} V_m &= -\frac{Bem\omega r^2 \cos \theta}{2m} \\ V_m &= -\frac{BeL}{2m} \end{aligned} \quad (3.3.11)$$

Which is again similar to the expression.

### **(3.4) Discussion:**

According to equation (3.2.5) up to (3.2.12) the energy shift in the Zeeman effect can be explained by using the expression of magnetic flux density and the expression of the force acting on a certain charge.

The Zeeman energy shift can also be explained by using the expression of the acting on a current as shown by equation (3.2.13).

The two derivations show that the concept of magnetic moment is not important to explain Zeeman Effect.

### **(3.5) Conclusion:**

The derivation of Zeeman Effect equation by using the concept of magnetic flux density is simple and straight forward compared to that derived by using the concept of magnetic moment.

### **(3.6): References:**

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