Dedication

To my dear lovely parents..

Acknowledgement

My gratitude to my supervisor

Prof. Shawgy Hussein Abdalla

for this ingenious advice and hard efforts

Special thanks are extended to Sudan University of Science & Technology for giving me this chance for higher studies.

Last my heart is full thanks to my brother, sisters who helped in preparing this project and all dear friends for their help.

ABSTRACT

We begin using some geometric methods to study The isometric problem in general real Banach spaces. Moreover we study the "sharp corner points" in many classical Banach spaces and solve isometric extension problem . We show the existence of such families on the cardinal number and study their properties. We show that the Banach spaces are large subspaces of the essential Hilbert spaces. The Banach space with a countable infinite number of complex structures was considered.

المستخلص

بدأنا باستخدام بعض الطرق الهندسية لدراسة مسألة الأيزومترك متساوية المقياس في فضاءات باناخ الحقيقية العامة بتوسيع تمت دراسة " نقاط الركن القاطعة" في فضاءات باناخ التقليدية المتعددة وحل مسألة التمديد الايزومترية متساوية. تم إيضاح وجود مثل هذه العائلات على العدد الرئيسي ودراسة خواصها. تم إيضاح أن فضاءات باناخ هي فضاءات جزئية كبيرة من فضاءات هلبرت الأساسية. اعتبرنا فضاء باناخ مع العدد اللانهائي القابل للعد من التشييدات المركبة.

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