بِسمِ اللهِ الرحَّمن الرَّحِيم

وَلِلهِ مَا فيِ السَّمَواَتِ وَمَا فيِ الأَرضِ وَكَانَ اللهُ بِكُل شَيءٍ مُّحِيطاً

صدق الله العظيم"

الآية (١٢٥) من سورة النساء .

DEDICATION

I wish to dedicate this thesis to my parents, to my family, to my wife, to my child, to my friends.

ABSTRACT

Elevations are used for many purposes such as contour maps so as design of canals, drains, railways, roads and agricultural projects.

These elevations can be determined by a practical means, which is a tedious job and time and money consuming.

The interpolation is one of the ways to estimate mathematically the height of a point using the heights of neighbor points.

The aim of this research is to perform methods of interpolation in both geodesy and Gis and to compare the results.

The main conclusion is that the use of GIS in interpolations is better than using equations for interpolation as in geodesy.

الملخص

نحتاج للإرتفاعات لأغراض كثيرة مثل الخرائط الكنتورية ،تصميم القنوات والمصارف ،

السكك الحديديه،الطرق والمشاريع الزراعية . يمكن تحديد هذه الإرتفاعات بواسطة وسائل عملية، والتي هي مملة ومضيعة للوقت والمال .

الإستكمال هو احدي الطرق الحسابية لتقدير إرتفاع نقطة بإستخدام إرتفاعات نقاط مجاورة.

الهدف من هذا البحث تكوين طرق الإستكمال في كل من نظم المعلومات الجغرافية والجيوديسيا

ومن ثم مقارنة النتائج.

خلصت هذة الدراسة الي أن:-

إستخدام نظم المعلومات الجغرافية في الإستكمال هو أفضل من إستخدام المعادلات الجيوديسية في الإستكمال.

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CHAPTER ONE

INTRODUCTION

1.1 Introduction

Elevations are needed for many purposes such as mapping contours , design of canals , drains , railways , roads and agricultural projects.

These elevations can be determined by a practical means, which is tedious and time and money consuming.

There are many mathematical models used for interpolation of different physical quantities in geodesy and surveying e.g inverse distance weighting (IDW), polynomial and least squares. In recent years a computer based programs such as GIS is widely used with interpolation functions build inside e.g.: IDW, Kriging and natural neighbor.

1.2 **Objectives of the research**

This research is directed towards the investigation of the interpolation results obtained from the methods in geodesy and geographical information systems which are used for different purposes.

The objectives of the research work can be summarized as follows:-

i) To study the methods of interpolation in geodesy.

ii) To study the methods of interpolation in GIS.

iii) To compare the results from geodesy methods and GIS methods.

1.3 Thesis layout

The thesis consists of sex chapters, including this introductory chapter. The content of the next five chapters are follows. Chapter two outlines the interpolation methods in geodesy. Chapter three presents interpolation methods in GIS .Chapter four discusses the tests area and data collection, Chapter five result and analysis and in Chapter sex the main conclusions are outlines together with the suggestions for future works.

CHAPTER TWO

INTERPOLATION METHODS IN GEODESY

2.1 Introduction

The interpolation is the methods. It can be used for different purposes in geodesy. One of the important applications of interpolation in geodesy and surveying is to predict a height of a point by using known similar heights of neighboring points.

2.2 Classification of interpolation methods

Interpolation techniques can be classified according to different criterias.

There are many classes of interpolation methods, depending on the area of interpolation. We find point based or area based. Considering the exactness of the surface there are exact fitting or best fitting. Considering smoothness of the surface we find linear or nonlinear surface. According to the continuity of the surface. There are steps, continuous, with respect to the preciseness of the surface. We find approximate, considering the domain of interest, there are spatial, spectal regarding the complexity of the phenomenon we find analytical, numerical iteration.

2.3 Interpolation Techniques used in geodesy

Different methods in geodesy are used for interpolation. The current in use are the following:-

2.3.1 The Inverse Distance Weighting

The technique estimates the Z value at a point by weighting the influence of nearby data points according to their distance from the interpolation point, using the following relation :-

$$Z(p) = \frac{\sum_{i=1}^{n} \left(\frac{Z_i}{d_{ip}^{\gamma}} \right)}{\sum_{i=1}^{n} \left(\frac{1}{d_{ip}^{\gamma}} \right)} \tag{2.1}$$

Where:-

d_{ip} is the plan metric distance between the reference point *(i)* and the interpolation point *(p)*

γ is an exponent depending on the nature of the field, and term $\frac{1}{d_{pi}^{\gamma}}$ known as the weight (W).

$$d_{ip} = \sqrt{\left(x_i - x_p\right)^2 + \left(y_i - y_p\right)^2} \tag{2.2}$$

2.3.2 The polynomial

This method is the most widely used surface fitting procedure .The modeling of a surface using this technique follows the assumption that, the height of a point(Z) is a function of its coordinates (x, y) ,or

$$z = f(x, y) \tag{2.3}$$

A mathematical function has to be used. The general mathematical expression of a surface with the n[th] order degree polynomial is

$$z_i = a_0 + a_1x + a_2y + a_3xy + \cdots + a_nxy \tag{2.4}$$

Where:-

a_0, a_1, a_2 . . . etc., are the polynomial coefficients.

Each individual term of the general polynomial function has its own characteristics. to make a correct selection of the terms that represent the best model the surveyor must keep in mind the shape produced by each term. In order to determine quadratic and cubic surface equations, the minimum number of the reference points is six and nine points respectively. For redundant reference points, the unknown polynomial coefficients ($\hat{x}$) can be determined by the least squares method according to the equation

$$\hat{x} = (A^tA)^{-1}A^tb \tag{2.5}$$

Where:-

A: is the coefficient matrix

b: is the vector of the observation, showing height values belongs to reference points.

for simplification, the four terms bilinear polynomial in the form of the following expression

$$Z_i = a_0 + a_1 x + a_2 y + a_3 xy \quad (2.6)$$

Generally used with many problems.

2.3.3 Least Squares Collocation

We may speak of least squares prediction when time is the variable involved, and therefore we estimate (or predict) what would happen in the future in the basis of the past occurrences (history). On the other hand, many of the applications in photogrammetry, geodesy and surveying often involve location instead of time, and hence least squares interpolation is used. The task of prediction is to estimate (interpolate) at locations other than those for which observed data are given. For geodetic applications it has been a special case of what is known as 'least squares collocation'.

This technique is the most powerful in today's applications where various types of data are available for determination of a single quantity .It permits to combine all relevant information together. The procedure is little complicated and require some prior information about the given data.

2.3.3.1 Basic Equations of Least Squares Collocation

In the case of multiple values, is talking about the so-called Wight matrix (w).It is a matrix with dimensions n*n where n is the numbers of observed values. This matrix is symmetric.

and rely on the accuracy of the observed values, greater weight means the accuracy of values is high. Elements on the diagonal of this matrix represent with inverse of the verance and elements outside the diagonal represent with

the extent depended of the observed value (I) at the observed value (j). In this case that the observed values do not depend on each other so the weight matrix becomes diagonally matrix.

Theory of least squares is the most common way to adjust observed, estimate unknowns and they give us the following:-

1- The most likely estimate statistics estimated values.
2- Give us accuracy estimate the output of the process least squared.
3- Give us accuracy calculated values of the unknown estimated.
   In this way we write the basis of the theory of least squared as follows: - for the observed equation

$$A\hat{x} = b + \hat{v} \qquad (2\text{-}7)$$

Where:-

b= number of observed.

$\hat{v}$ =matrix of residual errors .

w= weight matrix.

$\hat{x}$= matrix of unknown.

A=matrix of unknowns coefficients.

The equation in (2-7) solved by French scientist lagransh. and he get the most probable values of the (unknowns and the residual errors). We must create a theoretical model that fits the empirical covariance functions, and applying least squares adjustment. The parameters describing the model can be determined. Two theoretical models are current in use:-

i) Straight line model, in the form

$$C\left(d_{ij}\right)=c_0+bd_{ij} \tag{2.8}$$

Where C_0 and b are the parameters describing the covariance model and dij is the specified distance.

ii) Exponential model(Gaussian) as follows

$$C\left(i,j\right)=C_0e^{-bd_{ij}} \tag{2.9}$$

CHAPTER THREE

INTERPOLATION METHODS IN GIS

3.1 Introduction

Interpolation is the process of estimating the height of surface in area where no original data points exist. More and more phenomena can be measured and might be involved in the spatial analysis. Among others, we can mention the precipitation, temperature, soil parameters, ground water characteristics, pollution sources, vegetation data. We are not able to measure the values of the particular phenomenon in all points of the sphere, but only in sample points. The interpolation gives us values in such points where we have no measurements.

The goodness of interpolation can be characterized by the discrepancy of the interpolated value from the true value. Because the true value is not known in general. We can select some measured points for testing the interpolation procedure. In the stage of data production we can calculate the values of a particular phenomenon in predefined spots using interpolation procedures. for example if we want to deliver the data in a regular grid, but the samples are measured in scattered points we have to calculate the values of grid points from the samples using interpolation procedures. In the phase of analysis we face another problem. We can have the different attributive data in different (regular) spots. To perform the interaction computations we should transform the data to a common grid (to a common coordinate system) with the same discrete steps.

Seemingly similar tasks of interpolation are used for the creation of area objects representing a particular interval of the data in question. The construction of iso-lines or iso-surfaces in three dimensional case however aims to create and visualize aesthetic features and not too accurate ones (because of the generalization of individual function values into interval membership, that is because of the artificial degradation of resolution the high accuracy in interpolation has no reason).

Discussing the role of interpolation we should pay our attention to the global and local subdivision of interpolation approaches. even if we take into the consideration that the global approaches turn into local ones by numerical solutions we have to realize that the GIS requires more local (or less global) methods, while the data production more global (less local) procedures we have also to notice that the GIS interpolation computations should be fast and desirably automatic without of setting a large amount of parameters.

3.2 Interpolation methods

To create a surface grid in ArcGIS, the spatial analyst extension employs one of several interpolation tools. Interpolation is a procedure used to predict the values of cells at location that lack sampled points .It is based on the principle of spatial autocorrelation or spatial dependence, which measures degree of relationship\dependence between near and distant objects. Spatial autocorrelation determines if values are interrelated. If values are interrelated, it determines if there is a spatial pattern. This correlation is used to measure similarity of objects within an area. The degree to which a spatial phenomenon is correlated to itself in space the level of interdependence between the variables nature and strength of the interdependence different interpolation methods will almost always produce different results. There are different ways of classifying the interpolation methods. Probably approach does the wide community of people using GIS.

We will discuss the following not satisfying strict mathematical requirements, but attempts to clarify the issue for groups of methods:

- Statistical methods based on weighted average.
- Methods using basis functions.

3.2.1 Statistical methods

The statistical methods interpolate the function value in the unknown point using weighted average of the known values in the sample points:-

$$F(h_0) = \sum_{i=1}^{n} W_{(i,0)} F(h_i) \tag{3-1}$$

Where:-

$F(h_0)$ is the interpolated value .

$F(h_i)$ is the value y of the measurement in the sample point h_i.

i= 1,2,3,……, n,.

$W_{i,0}$ are the weights.

3.2.1.1 Natural neighbor

Natural neighbor interpolation finds the closest subset of input samples to a query point and applies weight to them based on proportionate areas in order to interpolate a value. Its basic properties are that it's local, using only a subset of samples that surround a query point, and that interpolated heights are guaranteed to be within the range of the samples used.It does not infer trends and will not produce peaks, pits, ridges represented by the input samples.Based on the atural neighbor coordinates,(Robin Sibson) developed a weight average interpolation technique that he named natural neighbor interpolation (Sibson,1980,1981).The points used to estimate the value of an attribute at location x are the natural neighbor of x,and the weight of each neighbor is equal to the natural neighbor coordinate of x with respect to this neighbor.If we consider that each data point in sample has an attribute a_i (a scalar value), the natural neighbor interpolation is

$$F(x)=\sum_{i=1}^{n}W_i(x_i)a_i \qquad (3\text{-}2)$$

Where:-

F(x) is the interpolated value.

(x_i) is the value of the measurement in the sample point x_i.

Natural neighbor interpolation has many positive features that can be used for both interpolation and extrapolation, and generally works well with clustered scatter points. Another weighted –average method, the basic equation used in natural neighbor interpolation is identical to the one used in inverse distance weightng interpolation. This method can efficiently handle large input point datasets.

3.2.1.2 The inverse distance weighting

The (IDW) interpolation determines cell values using a linearly weighted combination of a set sample points. The weight is a function of inverse distance. the surface being interpolation should be that of a location ally dependent variable. The IDW function should be used when the set of points is dense enough to capture the extent of local surface variation needed for analysis.IDW determines cell values using a linear-weighted combination set of sample points. The weight assigned is a function of the distance of an input point from the output cell location.

The greater the distance, the less influence the cell has one the output value. The power option of IDW lets you control the significance of known points on the interpolation values, based on their distance from the output point. It is a positive, real number. The default value is 2.By defining the higher (power) option, more emphasis can be put on the nearest points .Thus, nearby data will have more detail (be less smooth).

As the power increases, the interpolated values being to approach the values of the nearest sample point. Specifying a lower values for power will provide a bit more influence to surrounding points a little farther away. Since the IDW formula is not linked to any real physical process, there is no way to determine that a particular power values is too large as a general guideline, a power of 30 would be consider extremely large, and thus of questionable use. IDW relies mainly on the inverse of the distance raised to the power. If the distances are large, or the power values is large, the result may be incorrect. The characteristics of the interpolated surface can also be controlled by limiting the input points for calculating each interpolated point. The input can be limited by the number of sample points to be use or by a radius within which there is points to be used in the calculation of the interpolated points. Generally speaking, things that are closer together tend to be more alike than things that farther apart. This is a fundamental geographic principle. Suppose you are a town planner and you need to build a scenic park in your town. you have several candidate sites and you may want to model their view shed at each location. This will require a more detailed elevation surface dataset for your study area. Suppose you have pre-existing elevation data for 1,000 locations throughout the town .You can use this to build a new elevation surface. Throughout the town .You can use this to build a new elevation surface sample.

Values closest to the prediction location will be similar. But how many sample location should you consider? Should at of the sample values be considered equally? As you move farther away from the prediction location, the influence of the points will decrease. Considering a point too far away may actually be detrimental because the point may be located an area that is dramatically different from the prediction location.

One solution is to consider enough point to give a good prediction, but few enough points to be practical. The number will vary with the amount and distribution of the sample points and the character of the surface. If the elevation samples are relatively evenly distributed and the surface characteristics do not change significantly across your landscape, you can predict surface values from nearby points with reasonable accuracy. To account for the distance relationship, the values of closer points are usually weighted more heavily than those fathers away. This is the basis for the

(IDW) interpolation technique. As its name implies, the weight of a value decreases as the distance increases from the prediction location.

The (IDW) form in GIS like the form in geodesy. moreover the main disadvantage of this method is in the arbitrary definition of the interpolation subset since the method itself does not generate the points to be involved in the interpolation.

3.2.1.3 The kriging

The kriging estimates the unknown values with minimum variances if the measured data fulfill some conditions of stationary (first order stationary, second order stationary, intrinsic stationary). In the cases when no stationary hypotheses can be stated the universal kriging can be used. This method however, has not a unique solution and needs a large amount of interactive input and subjective assessment during the processing. Kriging is an advanced geostatistical procedure that generates an estimated surface from a scattered set of point with z-values. Unlike other interpolation Methods supported by ArcGIS spatial analyst, kriging involves an interactive Investigation of the spatial behavior of the phenomenon represented by the Z-values before you select the best estimation method for generating the output surface. Kriging assumes that the distance or direction between sample points reflects a spatial correlation that can be used to explain variation in the surface.kriging fits a mathematical function to a specified number of points, or all points within a specified radius, to determine the output value for each location.Kriging is a multistep process. It includes exploratory statistical analysis of the data. A powerful statistical interpolation method use for diverse applications such as elevation , gravity, and height. Kriging assumes that the distance or direction between sample points reflects a spatial correlation that can be use to explain variation in the surface. It fits a function to a specified number of points or all points within a specified radius to determine the output value for each location .Kriging is most appropriate when a spatially correlated distance or directional bias in the data is known. The predicted values are derived from the measure of relationship in samples using sophisticated weight average technique .It uses a search radius that can be fixed or variable. The generated cell values can exceed value range of sample, and the surface does not pass through samples. There are several types of Kriging.Ordinary Kriging, the most common method assumes that there is no constant mean for the data over an area mean (i.e., no rend). Universal Kriging dose assume that an overriding trend exists in the data and that it can be modeled. The general formula for both interpolations is formed as a weighted sum of data:-

$$z(s_0) = \sum_{i=1}^{n} \lambda_i z(s_i) \tag{3-3}$$

Where:

$Z(s_i)$ = the measured value at the i^{th} location.

λ_i = an unknown weight for the measured value at the i^{th} location.

s_0 = the prediction location.

n = the number of measured values.

3.2.2 Methods using basis functions

There are several methods that reconstruct the function using a linear combination of a set of basic functions with the closest fit to the samples.

Depending on the type of the basic functions we can distinguish among others polynomial interpolation.

3.2.2.1 Trend Surface Analysis

Trend is statistical method that finds the surface that fits the sample points using a least-square regression fit. It fits one polynomial equation to the entire Surface. This results in a surface that minimized surface variance in relation to Input values. The surface is constructed so that for every input point, the total of the differences between the actual values and the estimated values (i.e., the variance) will be as small as possible .it is an inexact interpolator, and the resulting surface rarely passes through the input points. However, this method detects trends in the sample data and is similar to natural phenomena that typically vary smoothly.

There are two basic types of trend interpolation, linear and logistic.

3.2.2.1.1 Linear Trend interpolation

The linear trend surface interpolated creates a floating –point raster. It uses a Polynomial regression to fit a least-squares surface to the input points. The linear option allows you to control the order of the polynomial used to fit the Surface .to understand the linear option oftrend; consider a first-order Polynomial. A first –order linear trend surface interpolation performs a least- Squares fit of a plan to the set of input points. Trend surface interpolation creates smooth surface. The surface generated will Seldom pass through the original data points, since it perform a best fit for the entire surface. When a polynomial (order) higher than one is used, the interpolator may generate a raster whose minimum and maximum exceed the minimum and maximum of the input file of the input feature data.

3.2.2.1.2 logistic Trend interpolation

The logistic option for generating a trend surface is appropriate for Prediction of the presence or absence of certain phenomena (in the form of probability) for a given set of location (x,y) in space .the z-value is a categorized random variable with only two possible outcomes--- for example, the existence of an endangered species or the lack of existence of that species. these two z-values can be coded as one and zero, respectively. the logistic option creates a continuous probability grid with cell values between one and zero.

The trend function uses the following formula for the surface interpolation :

$$h_i = a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 x^3 + a_7 y^3 + a_8 x^2 y + a_9 xy^2 \qquad (3\text{-}4)$$

3.2.2.2 Spline interpolation

The spline estimates values using a mathematical function that minimizes overall surface curvature .This result in a smooth surface that passes exactly through the input points conceptually, it is like bending a sheet of rubber so that it passes through the points while minimizing the total curvature of the surface it can predict ridges and valleys in the data and is the best method for representing the smoothly varying surfaces of phenomena such as elevation.

There are two variations of spline-regularized and tension.

A regularized spline incorporates the first derivative (slope) second derivative (rate of change in slope) ,and third derivative (rate of change In the second derivative) into its minimization calculation. Although a tension spline use only first and second derivatives, it includes more points in the spline calculations, which usually creates smoother surfaces but increases computation time The basic form of the minimum curvature spline interpolation imposes the following two conditions on the interpolation:-

The surface must pass exactly through the data points. The surface must have minimum curvature-the cumulative sum of the squares of the second derivative terms of the surface taken over each point on the Surface must be a minimum. The basic minimum curvature technique is also referred to as thin plate Interpolation. It ensures a smooth (continuous and differentiable) surface, together with continuous first-derivative surface.

Rapid changes in gradient or slope (the first derivative) can occur in the vicinity of the data points; hence this model is not suitable for estimating second derivative (curvature) the basic interpolation technique can be applied by using a value of zero for the (weight) argument to the spline function.

The spline function uses the following formula for the surface interpolation:

$$S(x,y) = T(x,y) + \sum_{i=1}^{n} \lambda_j R(r_j) \qquad (3\text{-}5)$$

Where :

j =1, 2,……..n.

n is the number of points.

λ_j are coefficients found by the solution of a system of linear equations.

r_j is the distance from the point (x,y) to the j^{th} point.

T(x,y) and R(r) are defined differently, depending on the selected option.

3.2.2.3 Topo to Raster interpolation

By interpolating elevation values for a raster, the topo to raster method imposes constraints that ensure a hydro logically correct digital elevation model that Contains a connected drainage structure and correctly represents ridges and streams from input contour data.

It uses an iterative finite difference interpolation technique that optimizes the computational efficiency of local interpolation without losing the surface continuity of global interpolation.

CHAPTER FOUR

TEST AREA AND DATA COLLECTION

4.1 Test Areas

Two test areas were selected the first area in almanagil locality in gazira state with dimension of 2*1.5 km was chosen 30 points 10 points data point and 20 points check points .The second area in white Nile with dimension of 2.3*1.8 km was chosen 30 points 10 points data points and 20 points check points.

4.2 Data Collection

The coordinate were collected using GPS and total station, height were collected using automatic level.

Mainly two tests were carried out on each test area.The first one by applying methods used in geodesy (IDW, polynomial and least squares) . Those applied in GIS (IDW,kriging and natural neighbor).

4.3 Information About Test Areas

Table (4.1) shows information about test areas.

Test Areas	Dimensions (k m)	Differences in height(m)	Locations
1	2*1.5	2.897	Almanagel Locality in Gazira State
2	2.3*1.8	4	In white Nile State

Table (4.1). The tests areas.

Figure (4.1) shows information about white nile area read points data points and black points check points

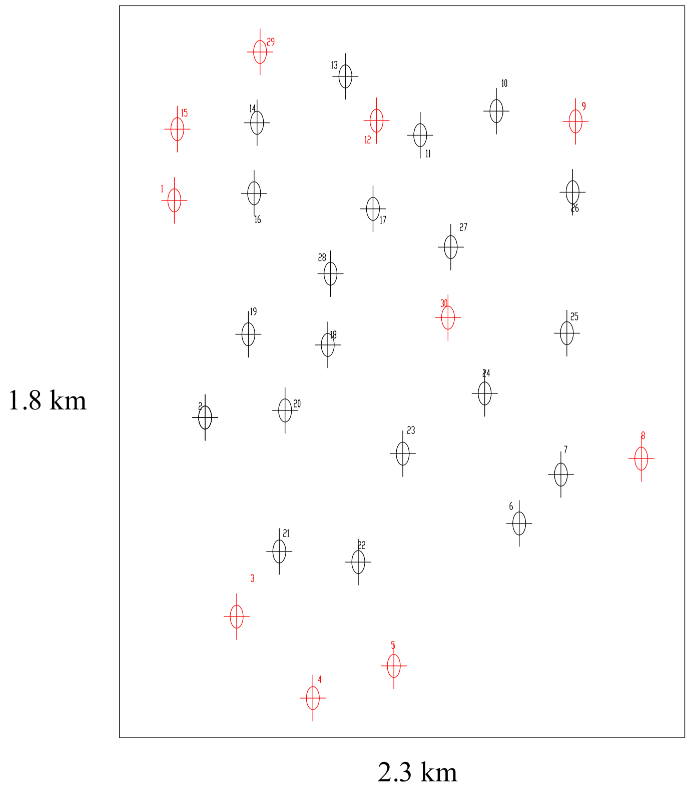

Figure (4.1). The data points and check points

of white nile area

Figure (4.2) shows information about Almanagel area read points data points and black points check points

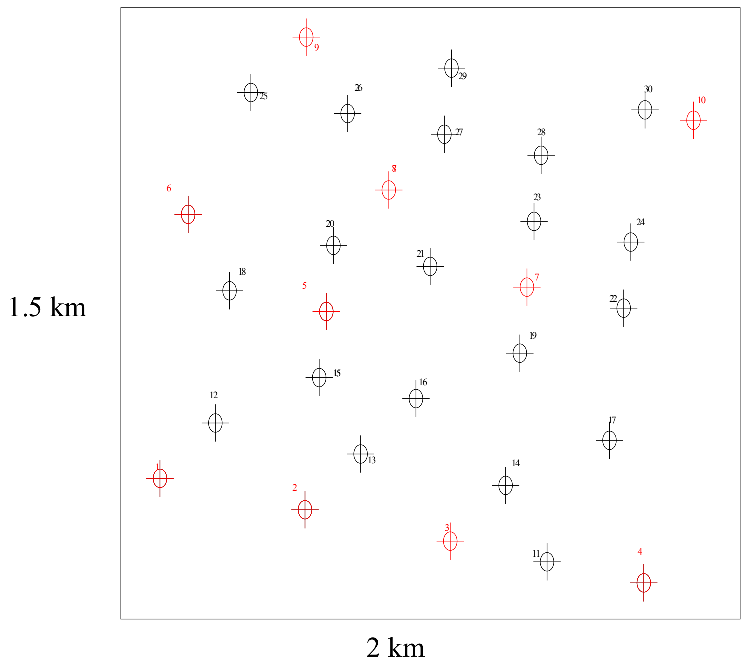

Figure (4.2). The data points and check points

Of Almanagel area

CHAPTER FIVE

RESULT AND ANALYSIS

5.1 Tests Results using equations applied in Geodesy

In this technique γ is an exponent depending on the field

5.1.1 Inverse distance weighting with test area1

Table (5.1) shows the result obtained for test area 1. Equation (2.1) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
11	419.601	419.325	**+ 0.276**
12	419.430	419.450	**- 0.020**
13	419.52	419.327	**+0.193**
14	419.500	419.298	**+0.202**
15	419.261	418.986	**-0.275**
16	419.371	418.988	**+0.383**
17	419.033	418.996	**+0.037**
18	418.840	419.691	**-0.851**
19	418.651	418.561	**+0.090**
20	418.472	418.583	**-0.111**
21	418.480	418.391	**-0.089**
22	418.461	418.479	**-0.018**
23	417.911	418.419	**-0.508**
24	417.920	418.222	**-0.302**
25	417.861	417.849	**+0.012**
26	417.800	417.863	**-0.063**
27	417.851	418.127	**-0.276**
28	417.87	418.300	**-0.430**
29	417.55	418.108	**-0.558**
30	417.42	417.626	**-0.206**

Table (5.1). The inverse distance weighting result.

5.1.2 Inverse distance weighting with test area2

In this technique use **γ=2**

Table (5.2) shows the result obtained for test area 2. Equation (2.1) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
2	377.342	378.053	-0.711
6	377.54	378.011	-0.471
7	377.64	377.999	-0.359
10	377.612	378.131	-0.519
11	377.542	379.337	-1.795
13	377.406	379.405	-1.999
14	377.747	378.289	-0.542
16	377.758	378.037	-0.279
17	377.024	379.180	-2.156
18	377.694	378.295	-0.601
19	378.566	378.017	+0.549
20	377.359	378.149	-0.79
21	377.589	378.229	-0.64
22	377.432	377.895	-0.463
23	378.012	378.109	-0.097
24	378.132	378.157	-0.025
25	377.736	377.988	-0.252
26	377.586	377.575	+0.011
27	378.095	378.302	-0. 207
28	377.703	377.703	0.000

Table (5.2). The inverse distance weighting result.

5.1.3 Polynomial interpolation with test area1

In this technique use $h_i = a_1 + a_2 x + a_3 y$

Table (5.3) shows the result obtained for Test area 1

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
11	419.601	420.282	**-0.681**
12	419.430	419.883	**-0.453**
13	419.52	419.902	**-0.382**
14	419.500	420.092	**-0.592**
15	419.261	419.905	**-0.644**
16	419.371	420.175	**-0.804**
17	419.033	419.465	**-0.432**
18	418.840	419.225	**-0.385**
19	418.651	419.771	**-1.12**
20	418.472	418.848	**-0.376**
21	418.480	417.984	**+0.496**
22	418.461	419.135	**-0.674**
23	417.911	418.282	**-0.371**
24	417.920	418.449	**-0.529**
25	417.861	417.869	**-0.008**
26	417.800	417.726	**+0.074**
27	417.851	418.21	**-0.359**
28	417.87	418.452	**-0.582**
29	417.55	417.585	**-0.035**
30	417.42	417.824	**-0.404**

Table (5.3). The Polynomial result.

5.1.4 Polynomial interpolation with test area2

In this technique use $h_i = a_1 + a_2 x + a_3 y$

Table (5.4) shows the result obtained for Test area 2

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
2	377.342	378.282	**-0.940**
6	377.54	378.165	**-0.625**
7	377.64	378.167	**-0.527**
10	377.612	378.429	**-0.817**
11	377.542	378.431	**-0.889**
13	377.406	378.516	**-1.110**
14	377.747	378.343	**-0.596**
16	377.758	378.290	**-0.532**
17	377.024	378.570	**-1.546**
18	377.694	378.213	**-0.519**
19	378.566	377.881	**+0.685**
20	377.359	378.290	**-0.931**
21	377.589	378.101	**-0.512**
22	377.432	378.162	**-0.730**
23	378.012	378.023	**-0.011**
24	378.132	378.030	**+0.102**
25	377.736	378.232	**-0.496**
26	377.586	378.391	**-0.805**
27	378.095	378.145	**-0.050**
28	377.703	378.261	**-0.558**

Table (5.4). The Polynomial result.

5 .1.5 least squares interpolated with test area1

5.1.5.1 Least squares uses straight line interpolated

Table (5.5) shows the result obtained for test area 1. Equation (2.8) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
11	419.601	421.539	-1.938
12	419.430	421.975	-2.545
13	419.520	422.042	-2.522
14	419.500	421.026	-1.526
15	419.261	420.623	-1.362
16	419.371	420.707	-1.336
17	419.033	420.041	-1.008
18	418.840	419.969	-1.129
19	418.651	420.535	-1.884
20	418.472	419.964	-1.492
21	418.480	419.069	-0.589
22	418.461	420.445	-1.984
23	417.911	420.023	-2.112
24	417.920	420.385	-2.465
25	417.861	419.397	-1.536
26	417.800	419.194	-1.394
27	417.851	419.958	-2.107
28	417.870	419.866	-1.996
29	417.550	418.482	-0.932
30	417.420	418.74	-1.320

Table (5.5). Least square uses straight line result.

5.1.5.2 Least squares uses exponential interpolated

Table (5.6) shows the result obtained for test area 1. Equation (2.9) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
11	419.601	418.335	+1.266
12	419.430	417.557	+1.873
13	419.52	417.671	+1.849
14	419.500	418.233	+1.267
15	419.261	417.996	+1.264
16	419.371	418.106	+1.265
17	419.033	417.769	+1.264
18	418.840	417.577	+1.263
19	418.651	417.391	+1.260
20	418.472	417.210	+1.262
21	418.480	417.217	+1.263
22	418.461	417.199	+1.262
23	417.911	416.650	+1.261
24	417.920	416.66	+1.260
25	417.861	416.600	+1.261
26	417.800	416.538	+1.262
27	417.851	416.591	+1.260
28	417.87	416.609	+1.261
29	417.55	416.692	+0.858
30	417.42	416.162	+1.258

Table (5.6). Least square uses exponential result.

5.1.6 Least squares interpolated with test area2

5.1.6 .1 least squares use straight line interpolated

Table (5.7) shows the result obtained for test area 2. Equation (2.8) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
2	377.342	379.280	-1.938
6	377.54	380.085	-2.545
7	377.64	380.162	-2.522
10	377.612	379.138	-1.526
11	377.542	378.904	-1.362
13	377.406	378.742	-1.1336
14	377.747	378.755	-1.008
16	377.758	378.887	-1.129
17	377.024	378.908	-1.884
18	377.694	379.186	-1.492
19	378.566	379.155	-0.589
20	377.359	379.343	-1.984
21	377.589	379.701	-2.112
22	377.432	379.897	-2.465
23	378.012	379.548	-1.536
24	378.132	379.526	-1.394
25	377.736	379.843	-2.107
26	377.586	379.582	-1.996
27	378.095	379.027	-0.932
28	377.703	379.023	-1.320

Table (5.7). least squares uses straight line result.

5.1.6 .2 least squares use exponential interpolated

Table (5.8) shows the result obtained for test area 2. Equation (2.9) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
2	377.342	375.997	+1.345
6	377.540	375.589	+1.951
7	377.640	375.740	+1.900
10	377.612	376.277	+1.335
11	377.542	376.197	+1.345
13	377.406	376.006	+1.400
14	377.747	376.397	+1.350
16	377.758	376.158	+1.600
17	377.024	375.624	+1.400
18	377.694	376.144	+1.550
19	378.566	377.23	+1.336
20	377.359	375.809	+1.550
21	377.589	376.139	+1.450
22	377.432	375.932	+1.500
23	378.012	376.562	+1.450
24	378.132	376.692	+1.440
25	377.736	376.276	+1.460
26	377.586	376.251	+1.335
27	378.095	377.145	+0.950
28	377.703	376.408	+1.295

Table (5.8). Least square uses exponential result.

5.2 Tests Results Using Equations applied in GIS

5.2.1 Inverse distance weighting with test area1

Table (5.9) shows the result obtained for test area 1. Equation (2.1) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference in(ht) between exist and interpolated data(m)
11	419.601	419.325	**+0.276**
12	419.430	419.450	**-0.02**
13	419.52	419.327	**+0.193**
14	419.500	419.298	**+0.202**
15	419.261	418.986	**+0.275**
16	419.371	418.988	**+0.383**
17	419.033	418.996	**+0.037**
18	418.840	419.691	**-0.851**
19	418.651	418.561	**+0.09**
20	418.472	418.583	**-0.111**
21	418.480	418.391	**+0.089**
22	418.461	418.479	**-0.018**
23	417.911	418.419	**-0.508**
24	417.920	418.222	**-0.302**
25	417.861	417.849	**+0.012**
26	417.800	417.863	**-0.063**
27	417.851	418.127	**-0.276**
28	417.87	418.300	**-0.43**
29	417.55	418.108	**-0.558**
30	417.42	417.626	**-0.206**

Table (5.9). The inverse distance weighting result.

5.2.2 Inverse distance weighting with test area2

Table (5.10) shows the result obtained for test area 2. Equation (2.1) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
2	377.342	378.053	-0.711
6	377.54	378.011	-0.471
7	377.64	377.999	-0.359
10	377.612	378.131	-0.519
11	377.542	379.337	-1.795
13	377.406	379.405	-1.999
14	377.747	378.289	-0.542
16	377.758	378.037	-0.279
17	377.024	379.180	-2.156
18	377.694	378.295	-0.601
19	378.566	378.017	-0.549
20	377.359	378.149	-0.79
21	377.589	378.229	-0.64
22	377.432	377.895	-0.463
23	378.012	378.109	-0.097
24	378.132	378.157	-0.025
25	377.736	377.988	-0.252
26	377.586	377.575	-0.011
27	378.095	378.302	-0. 207
28	377.703	377.703	0.000

Table (5.10). Inverse distance weighting result.

5.2. 3 kriging interpolated contours with test area1

Table (5.11) shows the result obtained for test area 1. Equation (3.5) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
11	419.601	419.567	**+0.034**
12	419.430	419.653	**-0.223**
13	419.52	419.505	**+0.015**
14	419.500	419.412	**+0.088**
15	419.261	419.259	**+0.002**
16	419.371	419.183	**+0.188**
17	419.033	419.058	**-0.025**
18	418.840	418.868	**-0.028**
19	418.651	418.783	**-0.132**
20	418.472	418.410	**+0.062**
21	418.480	418.400	**+0.080**
22	418.461	418.380	**+0.081**
23	417.911	418.057	**-0.146**
24	417.920	418.029	**-0.109**
25	417.861	417.613	**+0.248**
26	417.800	417.513	**+0.287**
27	417.851	417.643	**+0.208**
28	417.87	417.734	**+0.136**
29	417.55	417.412	**+0.138**
30	417.42	417.459	**-0.039**

Table (5.11) .The kriging result.

5.2.4 **kriging interpolated contours with test area2**

Table (5.12) shows the result obtained for test area 2. Equation (3.5) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
2	377.342	378.148	-0.806
6	377.540	378.131	-0.591
7	377.640	378.083	-0.443
10	377.612	378.042	-0.43
11	377.542	379.212	-1.67
13	377.406	379.358	-1.952
14	377.747	379.269	-1.522
16	377.758	378.048	-0.290
17	377.024	379.101	-2.077
18	377.694	378.169	-0.475
19	378.566	378.072	-0.494
20	377.359	378.156	-0.797
21	377.589	378.281	-0.692
22	377.432	377.951	-0.483
23	378.012	378.106	-0.094
24	378.132	378.140	-0.008
25	377.736	378.063	-0.327
26	377.586	377.676	-0.090
27	378.095	378.386	-0.291
28	377.703	378.440	-0.737

Table (5.12) .The kriging result.

5.2.5 **Natural neighbor interpolated contours with test area1**

Table (5.13) shows the result obtained for test area 1. Equation (3.2) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
11	419.601	419,606	-0.005
12	419.430	419.580	-0.150
13	419.52	419.445	+0.075
14	419.500	419.342	+0.158
15	419.261	419.196	+0.065
16	419.371	419.110	+0.261
17	419.033	418.982	+0.051
18	418.840	418.850	-0.010
19	418.651	418.739	-0.088
20	418.472	418.427	+0.045
21	418.480	418.400	+0.080
22	418.461	418.362	+0.990
23	417.911	418.065	-0.154
24	417.920	418.058	-0.138
25	417.861	417.601	+0.260
26	417.800	417.522	+0.278
27	417.851	417.589	+0.262
28	417.87	417.675	+0.195
29	417.55	417.223	+0.327
30	417.42	417.384	+0.036

Table (5.13). The natural neighbor result

5.2.6 Natural **neighbor interpolated contours with test area2**

Table (5.14) shows the result obtained for test area 2. Equation (3.2) was used for the test.

points	Exist elevations(m)	Interpolated Elevations(m)	Difference In(ht) between Exist and interpolated data(m)
2	377.342	377.951	**-0.609**
6	377.54	377.831	**-0.273**
7	377.64	377.892	**-0.252**
10	377.612	377.907	**-0.295**
11	377.542	379.381	**-1.839**
13	377.406	378.413	**-1.007**
14	377.747	378.420	**-0.673**
16	377.758	378.258	**-0.500**
17	377.024	379.067	**-2.043**
18	377.694	378.394	**-0.700**
19	378.566	378.110	**+0.456**
20	377.359	378.226	**-0.867**
21	377.589	378.241	**-0.652**
22	377.432	377.983	**-0.551**
23	378.012	378.055	**-0.043**
24	378.132	378.026	**+0.106**
25	377.736	377.868	**-0.132**
26	377.586	377.550	**+0.036**
27	378.095	378.457	**-0.362**
28	377.703	378.585	**-0.882**

Table (5.14). The natural neighbor result

5.3 **Comparisons between the Tested Interpolation Methods in Geodesy and GIS for test 1**

method	Mean(m)	Sd(m)	Remark
IDW	-0.089	0.460	In geodesy
polynomial	-0.413	0.348	"
straight-line	-1.659	0.550	"
Exponential	+1.302	0.211	"
IDW	-0.089	0.460	In GIS
kriging	+0.043	0.138	"
natural neighbor	+0.082	0.148	"

Table (5.15). The mean and standard deviation for the methods in geodesy and GIS for test1

5.4 Comparisons between the Tested Interpolation Methods In GIS and geodesy for test 2

method	Mean (m)	Sd (m)	Remark
IDW	-0.608	0.675	In geodesy
polynomial	-0.570	0.480	"
straight-line	-1.660	0.590	"
Exponential	1.447	0.215	"
IDW	-0.608	0.675	In GIS
kriging	-0.664	0.666	"
natural neighbor	-0.554	0.600	"

Table (5.16). The mean and standard deviation for the methods in GIS and geodesy for test 2

Mean= $$\sum_{I=1}^{n} \frac{x_i}{n}$$ (5.1)

Sd= $$\sqrt{\frac{\sum_{I=1}^{n}(x_i - mean)^2}{n-1}}$$ (5.2)

Where:

xi = Observed Value

n = 1,2,3,…………

CHAPTER SEX

CONCLUSIONS AND SUGGESIONS FOR FUTURE

6.1 Conclusions

From the results and analysis presented in Chapter five, the following can be

Concluded:-

- Inverse distance gives the same result in geodesy and GIS. However, it is better to use GIS because it is more simple and can be added to other map.
- The polynomial and least square methods do not exist in GIS.
- The selection of a suitable method depend on the topography of the area rather than other factors.
- In using least squares it is better to use exponential equation with these types of area.
- The contour maps produced from interpolated data are matched as the difference in height is less.
- The inverse distance method, in GIS can be change the (γ) easily.

6.2 Suggestions for future work

- Testing the possibility of adding the polynomial to a third analyst tool.
- Testing the effect of increasing known points.
- Testing the effect of the distribution of known points.
- Testing the effect of the (spline, trend, and topo raster) methods interpolated in GIS.
- Testing the effect of the universal kriging method.

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APPENDICES

APPENDIX A: Contour Maps of test

Exist data for test area 1 interpolation data with Kriging

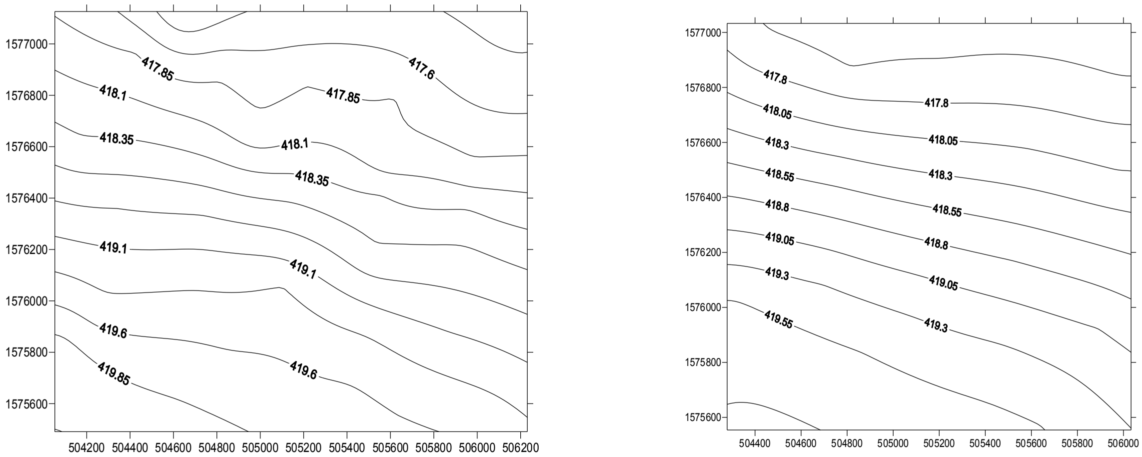

Figure (4.1): Contour map for test area 1.

Exist data for test area 2 interpolation data with polynomial

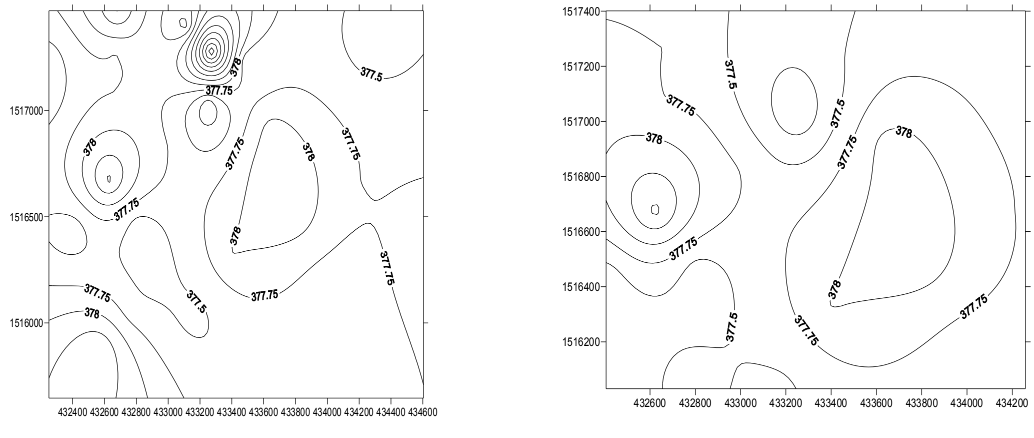

Figure (4.2): Contour map for test area 2.

APPENDIX B: Table of Data in Test Area1 .

Table (B) shows the information data for test area 1

Point	N(m)	E(m)	h(m)
1	504053	1575804	420.000
2	504645	1575710	419.761
3	505238	1575616	419.663
4	506028	1575491	419.561
5	504732	1576304	418.912
6	504168	1576595	418.413
7	505551	1576377	418.434
8	504987	1576668	417.902
9	504650	1577126	417.103
10	506231	1576877	417.400
11 *	505633	1575554	419.601
12 *	504279	1575970	419.430
13 *	504872	1575877	419.52
14 *	505464	1575783	419.500
15 *	504703	1576106	419.261
16 *	505098	1576043	419.371
17 *	505888	1575918	419.033
18 *	504337	1576366	418.840
19 *	505522	1576179	418.651
20 *	504761	1576502	418.472
21 *	505156	1576439	418.480
22 *	505946	1576314	418.461
23 *	505580	1576574	417.911
24 *	505975	1576512	417.920
25 *	504424	1576960	417.861
26 *	504819	1576897	417,800
27 *	505214	1576835	417.851
28 *	505609	1576772	417.87
29 *	505243	1577033	417.55
30 *	506033	1576908	417.42

Table (B) height (h) and coordinates (E, N) for test area1.

Points with star check points

APPENDIX C: Table of Data in Test Area2

Table(C) shows the information data for test area 2

Point	N(m)	E(m)	h(m)
1	432251	1517051	377.376
2 *	432406	1516438	377.342
3	432565	1515876	378.51
4	432949	1515646	377.909
5	433357	1515737	377.542
6 *	433989	1516139	377.54
7 *	434199	1516277	377.64
8	434604	1516322	377.938
9	434273	1517274	377.388
10 *	433874	1517303	377.612
11 *	433490	1517235	377.542
12	433270	1517276	380.000
13 *	433113	1517401	377.406
14 *	432668	1517270	377.747
15	432266	1517252	377.397
16 *	432653	1517071	377.758
17 *	433252	1517027	377.024
18 *	433024	1516643	377.694
19 *	432624	1516673	378.566
20 *	432809	1516458	377.359
21 *	432780	1516060	377.589
22 *	433178	1516030	377.432
23 *	433402	1516336	378.012
24 *	433815	1516506	378.132
25 *	434229	1516676	377.736
26 *	434258	1517074	377.586
27 *	433644	1516919	378.095
28 *	433038	1516844	377.703
29	432683	1517470	378.518
30	433630	1516720	378.186

Table (C) height (h) and coordinates (E, N) of test area2

Points with star check points.