

Dedication

I dedicate this effort to my mother,

To my father,

To my brothers and sisters,

To all my family,

To my friends and colleagues

Acknowledgement

At first I would like to thank Allah who gives me the power to complete this work.

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Abstract

In this research we consider the problem of the characterization of chaotic dynamics, for this we discuss and analysis the standard maps by using the perturbative method to study their induced orbits. We discuss the stationary solution and the periodic orbits of dynamical system, and we investigate the Floquet theorem. We also discuss the local Bifurcations in terms of dependent parameter of ordinary differential equations and the dependent parameter maps with some applications. Also we illustrate the characterization of chaotic dynamics briefly by discussing the symbolic dynamics and the strange attractors with some remarks and applications.

الخلاصة

في هذا البحث اعتبرنا مسألة تمييز الديناميكا المشوشة، لهذا ناقشنا وحللنا الرواسم الاساسيه بواسطة إستخدام طريقة الإضطراب لدراسة مداراتها المُحدثه. ناقشنا الحل الثابت والمدارات الدورية للنظام الديناميكي، وبحثنا مبرهنة فلوكوينت (Floquet). أيضاً ناقشنا التفرعات الموضعية بدلالات الوسيط المستقل للمعادلات التفاضلية العادية ورواسم الوسيط المستقل مع بعض التطبيقات.

ايضاً وضحنا تمييز الديناميكا المشوشه بايجاز بواسطة مناقشة الديناميكا الرمزية والجاذبات الغريبة مع بعض الملاحظات والتطبيقات.

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Introduction

The last 30 years have witnessed a renewed interest in dynamical systems, partly due to the discovery of chaotic behavior, and on going research has brought many new insights in their behavior. This research aims to study the dynamical systems and their geometrical theorems. And it is organized as follows:

In chapter one, we require the notion of invariant sets, linear systems, and the stable manifold theorem. Also we discuss the standard map which describes the motion of rotator. We use the perturbative method to analysis some iterated maps and then we study their induced orbits, with some famous examples.

In chapter two we discuss orbit and flows. We present Peano-Cauchy theorem, Picard-Lindelöf theorem with some applications and examples. Also we discuss the stationary solution of dynamical systems. We present the concepts of the unstable, stable, and center subspaces of fixed point with some applications. We discuss the stable manifold theorem in terms of hyperbolic equilibrium point of a given dynamical system. Also we discuss the periodic orbits of flows and Poincare sections, and then we present the Floquet theorem, with applications.

In chapter three, we discuss some general concepts of the center manifold that associates with an equilibrium point of the dynamical systems with some examples and applications. We consider especial parameter dependent ordinary differential equations to discuss the Bifurcations of ordinary differential equations, and then we discuss some famous Bifurcations as applications. Also we consider a special parameter dependent iterated maps to discuss their Bifurcations, and as applications we present some famous Bifurcations of maps in dynamical systems.

In chapter four we discuss only a few selected topics to show how similar properties of chaotic dynamics can be proven to exist for more realistic systems, so we discuss the symbolic dynamics with some examples and applications. Also we discuss the attracting sets and attractors with some examples and remarks, and then we present the strange attractor and its orbits. We discuss Benedicks, Carleson theorem as an application.