Sudan University of Science and Technology

College of Science

Physics department

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Neutrino-Electron and Antineutrino-Electron Scattering

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الآية

قال الله تعالى في محكم تنزيله:

(يَا أَيُّهَا الَّذِينَ آمَنُوا إِذَا قِيلَ لَكُمْ تَفَسَّحُوا فِي الْمَجَالِسِ فَافْسَحُوا يَفْسَحِ اللَّهُ لَكُمْ ۖ وَإِذَا قِيلَ انْشُزُوا فَانْشُزُوا يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنْكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ دَرَجَاتٍ ۚ وَاللَّهُ بِمَا تَعْمَلُونَ خَبِيرٌ)

[المجادلة: 11]

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Abstract

In this project the cross-section of neutrino electron and antineutrino-electron scattering is calculated using two models: Fermi effective theory and the standard model in particle physics, that unify between the strong and the electroweak forces. A substantial difference between the two models is found that is the cross-section in Fermi theory is proportional to the center of mass energy (S), while in the Standard model the cross section is inversely proportional to (S). Consequently at high energies (S> m_w) the Fermi theory break down and we cannot trust calculations of the cross-section using this theory, therefore, it is the most to use the theory of standard model when dealing with higher energies. Furthermore we find that the cross section of anti-neutrino is 1/3 of that of neutrino cross-section for both models.

ملخص

في هذا البحث تم حساب مساحه مقطع التفاعل لجسيم النيوترينو وضديده في نموذجي نظريه فيرمي والنظريه العياريه للجسيمات الاوليه التي توحد القوي القويه والكهروضعيفه . حيث تم الحصول علي نتائج مختلفه في هذين النوذجيين إذ أنه باستخدام نظريه فيرمي فان مساحه مقطع التفاعل للنترينو تتناسب طرديا مع الطاقه لمركز الكتل(S) , اما النظريه العياريه فقد اعطت مساحه مقطع التفاعل للنترينو تتناسب عكسيا مع طاقه لمركز الكتل وعليه عند طاقات عاليه ($S> m_w$) فان نظريه فيرمي تكون غير صالحه للاستعمال ويجب استخدام النظريه العياريه لتوحيد القوي في الطبيعه لحساب مساحه مقطع التفاعل عند طاقات عاليه. في هدا البحث ايضا تم الحصول علي علاقه بين مساحه مقطع التفاعل لضديد النيوترينو والنيوترينو،حيث وجدنا ان مساحة مقطع التشتت لضديد النيوترينو يساوي 1/3 من مساحة مقطع التشتت للنيوترينو في كلا النظريتين.

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Chapter one

INTRODUCTION

(1-1) Outline of the project:

In this project the neutrino-electron and antineutrino-electron scattering is the main task, and the project is structured as follows: In chapter one we introduced a brief introduction; in chapter two we introduced the standard model; chapter three was dedicated to carryout the calculations and the scattering cross-section was been calculated; we present in chapter four the numerical results. And in chapter five we discussed and concluded out results.

(1-2)The importance of studying neutrino scattering in theoretical physics:

Neutrino-electron scattering are fundamental electroweak processes which play important roles in neutrino oscillation studies and in probing the electroweak parameters of the Standard Model (SM) as well as playing such important role in the studies of neutrino properties like: electromagnetic moments and charge radius. It is important also whether there is any deviation from the measured of cross-section which can be expected with weak interaction. Therefore, the aforementioned Neutrino scattering provides a likely window to new physics.

(1-3)The main objectives of the project:

A deeper understanding of nuclear effects induced by neutrinos and considerably more accurate measurements of neutrino exclusive cross-sections is crucial for minimizing systematic of neutrino oscillation experiments. In addition, the knowledge gained by neutrino scattering experiments is important to the Nuclear Physics and Astor-Particle physics communities as well. A review of where we expect to be in the study of neutrino scattering physics starts from the Super Neutrino Beam Facility as well as the types of beams and detectors needed. Such a facility will be discussed.

The objectives of project are:

- To study neutrino and antineutrino scattering.

- Determin the cross-section for neutrino and anti neutrino scattering.
- Make a compersion between two models fermi theory and Standard model theory in study neoutrino and anti-neutrino scattering.
- Cheack the limit of theoretical method (fermi theory and standard model) in determin scettering cross-section.

(1-4)Problem of the project:

Cross-sections neutrino-electron scattering are extremely small and consequently very difficult to measure. But theoreticaly we need to know at which energies we must have to set the accelerators, Therefore studying neutrino-electron scattering and determining the cross-section at high accuracy is of a higher importance.

Chapter two

Introduction to Standard Model

(2-1) introduction:

In this chapter we introduced the structure of the standard model and its mathematical formlation, then we talked about the higgs mechanism to see how particles obtian their masses.

(2-2) what is the standard model (SM):

The **Standard Model** of particle physics is a theory concerning the electromagnetic, weak, and strong nuclear interactions, which mediate the dynamics of the known subatomic particles. It was developed throughout the latter half of the 20th century, as a collaborative effort of scientists around the world. The current formulation was finalized in the mid-1970s upon experimental confirmation of the existence of quarks Since then, discoveries of the top quark (1995), the tau neutrino (2000), and more recently the Higgs boson (2013), have given further credence to the Standard Model. Because of its success in explaining a wide variety of experimental results, the Standard Model is sometimes regarded as a "theory of almost everything". Mathematically, the standard model is a quantized Yang–Mills theory[1].

The Standard Model falls short of being a complete theory of fundamental interactions because it makes certain simplifying assumptions. It does not incorporate the full theory of gravitation as described by general relativity, or predict the accelerating expansion of the universe (as possibly described by dark energy). The theory does not contain any viable dark matter particle that possesses all of the required properties deduced from observational cosmology. It also does not correctly account for neutrino oscillations (and their non-zero masses). Although the Standard Model is believed to be theoretically self-consistent and has demonstrated huge and continued successes in providing experimental predictions, it does leave some phenomena unexplained[1],[2].

The development of the Standard Model was driven by theoretical and experimental particle physicists alike. For theorists, the Standard Model is a paradigm of a quantum field theory, which exhibits a wide range of physics including spontaneous symmetry breaking, anomalies, non-perturbative behavior, etc. It is used as a basis for building more that incorporate hypothetical particles, extra dimensions, and elaborate symmetries (such as supersymmetry) in an attempt to explain experimental results at variance with the Standard Model, such as the existence of dark matter and neutrino oscillations [3],[4].

(2-2-1) Fermions

The Standard Model includes 12 elementary particles of spin-1/2 known as fermions According to the spin-statistic theorem, fermions respect the Pauli Exclusion Principle Each fermion has a corresponding antiparticle[1].

The fermions of the Standard Model are classified according to how they interact (or equivalently, by what charges they carry). There are six quarks (up, down, charm, strange, top, bottom.), and six leptons (electron, electron neutrino, muon, muon neutrino, tau, tau neutrino). Pairs from each classification are grouped together to form a generation, with corresponding particles exhibiting similar physical behavior [2], [3] and [4].

The defining property of the quarks is that they carry color charge, and hence, interact via the strong interaction. A phenomenon called color confinement results in quarks being perpetually (or at least since very soon after the start of the big bang) bound to one another, forming color-neutral composite particles (hadrons) containing either a quark and an antiquark (mesons) or three quarks (baryons). The familiar proton and the neutron are the two baryons having the smallest mass. Quarks also carry electric charge and weak isospin Hence they interact with other fermions both electromagnetically and via the weak interaction [5] and [6].

The remaining six fermions do not carry color charge and are called leptons. The three neutrinos do not carry electric charge either, so their motion is directly influenced only by the weak nuclear force, which makes them notoriously difficult to detect. However, by virtue of carrying an electric charge, the electron, muon, and tau all interact electromagnetically [3] and [4].

Each member of a generation has greater mass than the corresponding particles of lower generations. The first generation charged particles do not decay; hence all ordinary (baryonic) matter is made of such particles. Specifically, all atoms consist of electrons orbiting atomic nuclei ultimately constituted of up and down quarks. Second and third generations charged particles, on the other hand, decay with very short half lives, and are observed only in very high-energy environments. Neutrinos of all generations also do not decay, and pervade the universe, but rarely interact with baryonic matter [5].

(2-2-2) Gauge bosons

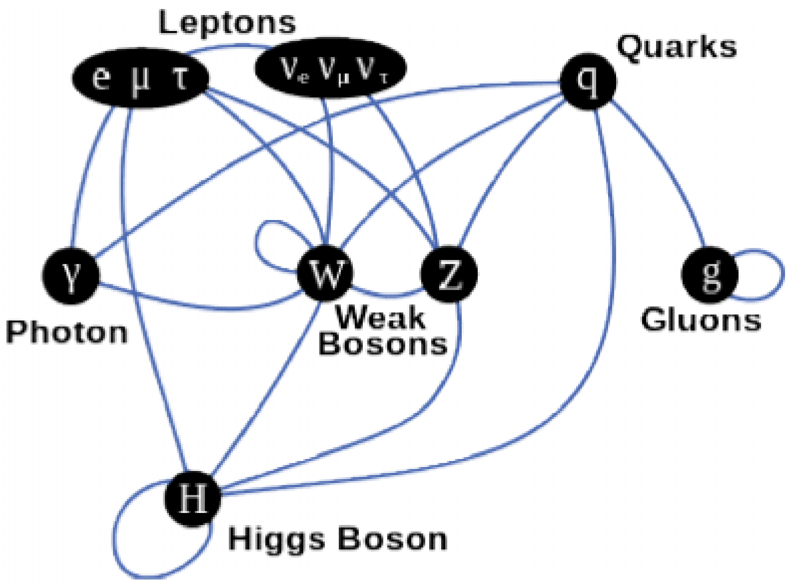

Fig2.1 Summary of interactions between particles described by the Standard Model.

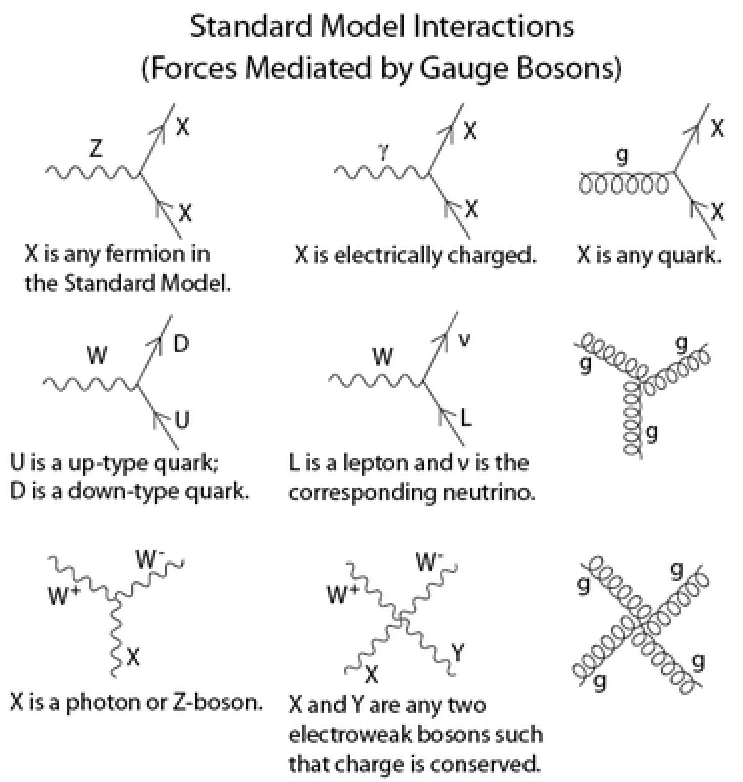

Fig2.2 forces mediated by gauge bosons

The above interactions form the basis of the standard model. Feynman diagrams in the standard model are built from these vertices. Modifications involving Higgs boson interactions and neutrino oscillations are omitted. The charge of the W bosons is dictated by the fermions they interact with; the conjugate of each listed vertex (i.e. reversing the direction of arrows) is also allowed.

In the Standard Model, gauge bosons are defined as force carriers that mediate the strong, weak, and electromagnetic fundamental interactions.

Interactions in physics are the ways that particles influence other particles. At a macroscopic level, electromagnetism allows particles to interact with one another via electric and magnetic fields, and gravitation allows particles with mass to attract one another in accordance with Einstein's theory of general relativity. The Standard Model explains such forces as resulting from matter particles exchanging other particles, known as *force mediating particles* (strictly speaking, this is only so if interpreting literally what is actually an *approximation method* known as perturbation theory). When a force-mediating particle is exchanged, at a macroscopic level the effect is equivalent to a force influencing both of them, and the particle is therefore said to have *mediated* (i.e., been the agent of) that force. The Feynman diagram calculations, which are a graphical representation of the perturbation theory approximation, invoke "force mediating particles", and when applied to analyze high-energy scattering experiment are in reasonable agreement with the data. However, perturbation theory (and with it the concept of a "force-mediating particle") fails in other situations. These include low-energy quantum chromo dynamic, bound state, and solutions [3], [4] and [5].

The gauge bosons of the Standard Model all have spin (as do matter particles). The value of the spin is 1, making them bosons. As a result, they do not follow the Pauli Exclusion Principle that constrains fermions: thus bosons (e.g. photons) do not have a theoretical limit on their spatial density (number per volume). The different types of gauge bosons are described below [7].

- Photons mediate the electromagnetic force between electrically charged particles. The photon is massless and is well-described by the theory of quantum electrodynamic.

- The W+, W-, and Z gauge bosons mediate the weak interaction between particles of different flavors (all quarks and leptons). They are massive, with the Z being more massive than the W±. The weak interactions involving the W± exclusively act on *left-handed* particles and *right-handed* antiparticles only. Furthermore, the W± carries an electric charge of +1 and −1 and couples to the electromagnetic interaction. The electrically neutral Z boson interacts with both left-handed particles and antiparticles. These three gauge bosons along with the photons are grouped together, as collectively mediating the electroweak interaction [1], [2], [3] and [4].

- The eight gluons mediate the strong interaction between color charged particles (the quarks). Gluons are massless. The eightfold multiplicity of gluons is labeled by a combination of color and anticolor charge (e.g. red–antigreen). Because the gluons have an effective color charge, they can also interact among themselves. The gluons and their interactions are described by the theory of quantum chromodynamics.

(2-3) The standard model lagrangian:

It has become conventional to formulate SM of particle physics giving the lagrangian. By using the rules of quantum field theory, all observable can in principle be calculated. The lagrangian defines the theory. It is written in terms of the elementary particles of the theory. Any comosite objects should appear as bound states that arise as soluation of the theory. For electrodynamic the photon is the quantum of the electromagnetic field , it is represented by the vector potential field A^{μ}. The electron represented by the fermion field ψ[2],[3]and[5].

The lagrangian contains the fundamental interaction of the theory. For electrodynamic that is the conventional $\vec{J}\cdot\vec{A}$ interaction hamiltonian, while becomes $J_{\mu}A^{\mu}$ relativistically. More precisely, it is the potential energy part of the lagrangian that specify the theory. The kinetic energy parts are general and only depend on the spins of the particles. The potential energy parts specify the forces; we will often call them the interaction lagrangian[2].

One of the main resons why SM is formulated in terms of the lagrangian is the $\mathcal{L}$ is a single function determains the dynamics, and $\mathcal{L}$ must be a scalar in every relevent space,invariant under transformations, since the action is invariant. Making the lagrangian invariant under lorentz transformation we can guarantees that all predictions of the theory are lorentz invariant[8].

Thus we will write the theory in terms of a lagrangian.in a full SM the lagrangian would be used is a variety of ways for us will only serve a guide in writing the basic interactions, from which will read off the feynman rules of the theory[9].

In order to describe the particles and interactions known today, three internal symmetries are needed. At the present time all experiments are consistent with the notion that three symmetries are necessary and sufficient to describe the interactions of the known particles. It is easiest to describe how these symmetries act in the language of group theory [2] and [4].

The three symmetries that used in SM are U(1),and SU(2), and SU(3). All particles appear to have U(1) invariance, it is like the U(1) invariance or phase invariance that related to electromagnetic interaction. The gauge boson required by the invariance of the theory under the U(1) transformations will be called B^{μ}. The index μ is present sinceB^{μ} must transform under spatial rotations the same way the ordinary derivative ∂^{μ} does, these guaranteeing the associated particle has spin one. We will reseve A^{μ} for the name of the photon field[8].

All particles appear to have second internal invariance under a set of transformation that form an SU(2) group , called the electroweak SU(2) invariance. The associated gauge bosons necessary to maintain the invariance to maintain the invariance of the theory are called W_i^{μ} . the index μ again is erquired to have space-time transformations that are the same as ordinary derivative, so the W bosons have spin one. There is one boson for each of the three generator of SU(2) transformation so i = 1,2 or 3, these are called weak isospin transformation[5]and[6].

All particles appear to have a third internal invariance, under a set or transformations that form an SU(3) group, give us additional independent non-abelian invariance. The associated gauge bosons are labeled G_a^{μ} , where now a = 1,2,...,8 since there is one spin-one boson for each of the eight generators of SU(3). The bosons are called gluons, and the theory of particle interaction via gluon exchange is called Quantum Chromodynamics (QCD)[3] and [8].

The standard model is based on the SU(3) ⊗ SU(2) ⊗ U(1):

So SM lagrangian can be written as:

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{higgs} + \mathcal{L}_{yuk} \quad (2.1)$$

It defines the fundamental interaction of fermions, gauge bosons and higgs bosons. We will go through the details on next sections, then we will write down the full SM lagrangian.

(2-3-1) The fermion sector:

The weak interactions are known to violate parity.parity non-invariant interactions for fermions can be constructed by giving different interactions to the "left-handed" and "right-handed" components defined in eq (2.2). Thus, in writing down the standard model, we will treat the left-handed and right-handed parts separately[1] and [2].

A dirac field, ψ, representing a fermions, can be expressed as the sum of a left-handed part ψ_L, And a right-handed part,ψ_R,[10]

$$\psi = \psi_L + \psi_R \qquad (2.2)$$

Where:

$$\psi_L = P_L\psi \quad with\ \ P_L = \frac{1}{2}(1-\gamma^5) \qquad (2.3)$$

$$\psi_R = P_R\psi \quad with\ P_R = \frac{1}{2}(1+\gamma^5) \qquad (2.4)$$

P_L And P_R are projection operators, i.e.

$P_LP_L = P_L\,, P_RP_R = P_R$ And$P_RP_L = 0 = P_LP_R$.

In the original formulation of the standard model the massless left-handed neutrinos had no right-handed partners. Recently observed neutrino oscillations suggest right-handed neutrinos but we shall stick to the original formulation in the following:

Leptons	*T(isospin)*	*Y(hypercharg)*	T_3	$Q = T_3 + Y$
Left $\begin{pmatrix} \upsilon_e \\ e \end{pmatrix}_L \begin{pmatrix} \upsilon_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \upsilon_\tau \\ \tau \end{pmatrix}_L$	$1/2$	$-1/2$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
Right $e_R \quad \mu_R \quad \tau_R$	*0*	-1	-1	*0*

Table.1 left-handed and right handed leptons.

Quarks	*T(isospin)*	*Y(hypercharg)*	T_3	$Q = T_3 + Y$
Left $\begin{pmatrix} u \\ \acute{d} \end{pmatrix}_L \begin{pmatrix} c \\ \acute{s} \end{pmatrix}_L \begin{pmatrix} t \\ \acute{b} \end{pmatrix}_L$	$1/2$	$1/6$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
Right $u_R \quad c_R \quad t_R$ $\acute{d}_R \quad \acute{s}_R \quad \acute{b}_R$	0 0	$2/3$ $-1/3$	0 0	$2/3$ $-1/3$

Table.2 1 left-handed and right handed quarks.

The reason for introducing the primed quarks field $\acute{q}$ will become clear below.

We can write the fermions lagrangian as [10]:

$$\mathcal{L}_{\substack{\text{fermions} \\ \text{coloris} \\ \text{flavours}}} = \sum\left\{\overline{\psi}_L^{cf} i\gamma^\mu \mathcal{D}_\mu \psi_L^{cf} + \overline{\psi}_R^{cf} i\gamma^\mu \mathcal{D}_\mu \psi_R^{cf}\right\} \qquad (2.5)$$

Where we have to distinguish the covariant derivative acting on the left fields

$$\mathcal{D}_\mu = \partial_\mu + igt_i\overrightarrow{W}_\mu + i\acute{g}YB_\mu \quad (2.6)$$

From the one acting on the right fields, as later do not couple to $SU(2)_L$ gauge bosons. ($igt_i\overrightarrow{W}_\mu = 0$)

$$\widetilde{\mathcal{D}}_\mu = \partial_\mu + i\acute{g}YB_\mu \quad (2.7)$$

The lagrangian related to the first family:

$$\mathcal{L}^{(I)} = \bar{\psi}_L^{(l)} i\gamma^\mu \left(\partial_\mu - \mathrm{igt_i} W_\mu^i + \mathrm{i}\frac{1}{2}\acute{g}B_\mu\right)\psi_L^{(l)} + \bar{e}_R i\gamma^\mu(\partial_\mu + \mathrm{i}\acute{g}B_\mu)e_R + \bar{\psi}_L^{(q)} i\gamma^\mu(\partial_\mu - \mathrm{igt_i} W_\mu^i - \mathrm{i}\frac{1}{6}\acute{g}B_\mu)\psi_L^{(q)} + \bar{U}_R i\gamma^\mu\left(\partial_\mu - \mathrm{i}\frac{2}{3}\acute{g}B_\mu\right)U_R + \bar{d}_R i\gamma^\mu(\partial_\mu + \mathrm{i}\frac{1}{3}\acute{g}B_\mu)d_R. \quad (2.8)$$

Generalized to 3- family case:

$$\mathcal{L}_F = \mathcal{L}_F^{(I)} + \mathcal{L}_F^{(II)} + \mathcal{L}_F^{(III)} \quad (2.9)$$

(2-3-2) Gauge boson sector:

The gauge boson and the scalar lagrangians give rise to the free lagrangian for the photon, W, Z. and the higgs boson .The standard model gauge boson lagrangian (gauge fields) is [10]:

$$\mathcal{L}_G = \frac{-1}{4} G_{\mu v}^a G^{a\mu v} - \frac{1}{4} W_{\mu v}^i W^{i\mu v} - \frac{1}{4} B_{\mu v} B^{\mu v} \qquad (2.10)$$

Where

$$G_{\mu v}^a = \partial_\mu G_v^a - \partial_v G_\mu^a + g_s f_{abc} G_\mu^b G_v^c \qquad (2.11)$$

$$W_{\mu v}^i = \partial_\mu W_v^i - \partial_v W_\mu^i + g\epsilon_{ijk} W_\mu^i W_v^i \quad (2.12)$$

$$B_{\mu v} = \partial_\mu B_v - \partial_v B_\mu \qquad (2.13)$$

$B_{\mu v}$ Is the hypercharge field strength, $W_{\mu v}^i$ contains the SU (2) field strength, so 'i' runs from one to three (over the three vector bosons of SU (2)), and $G_{\mu v}^a$ is the gluon kinetic term, so a = 1 . . . . 8.

(2-4) Higgs mechanism:

In particle physics, the **Higgs mechanism** is essential to explain the generation mechanism of the property "mass". This is a most-important property of almost all elementary particles [11].

In the Standard Model, quarks, hadrons, leptons and the three weak bosons gain mass through the Higgs mechanism by interacting with the Higgs field that permeates all space. Normally bosons are massless, but the W^+, W^-, and Z bosons have values around 80 GeV/c^2. In gauge theory, the Higgs field induces a spontaneous symmetry breaking, where instead of the usual transverse Nambu–Goldstone boson, the longitudinal Higgs boson appears [11].

The simplest implementation of the mechanism adds an extra Higgs field to the gauge theory. The specific spontaneous symmetry breaking of the underlying local symmetry, which is similar to that one appearing in the theory of superconductivity, triggers conversion of the longitudinal field component to the Higgs boson, which interacts with itself and (at least a part of) the other fields in the theory, so that produce mass terms for the three gauge bosons, and also to fermions [11]

The Higgs doublet Lagrangian should contain a "spontaneous symmetry breaking" potential which will give the Higgs a "VEV" and self-interactions, and kinetic terms which will generate the gauge boson masses and interactions between the Higgs and the gauge bosons [1] and [11].

Add new particle ϕ

$$\mathcal{L}_{higgs} = \frac{1}{2}\left(D_\mu\phi\right)^\dagger (D^\mu\phi) - V(\phi) \qquad (2.14)$$

$$V(\phi) = \frac{\mu^2}{2}\phi^*\phi + \frac{\lambda}{4}\phi^4 \qquad (2.15)$$

Therefore equation (2.14) becomes

$$\mathcal{L}_{higgs} = \frac{1}{2}\left(D_\mu\phi\right)^\dagger (D^\mu\phi) - \frac{\mu^2}{2}\phi^*\phi + \frac{\lambda}{4}\phi^4 \qquad (2.16)$$

$\boldsymbol{\lambda} \equiv$ Higgs self coupling.

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix} \qquad (2.17)$$

By minimize$\boldsymbol{V(\phi)}$:

$$\frac{\partial V}{\partial \phi} = 0 \qquad (2.18)$$

$$\frac{\partial V}{\partial \phi} = -\mu^2\phi + \lambda\phi^3 \qquad (2.19)$$

$$\phi(\mu^2 + \lambda\phi^2) = 0 \qquad (2.20)$$

$$\phi = 0\ (trivial\ solution)\ or\ (\mu^2 + \lambda\phi^2) = 0$$

$$\langle\phi^2\rangle = \frac{-\mu^2}{\lambda} \qquad (2.21)$$

$$\langle\phi\rangle = \pm\sqrt{\frac{-\mu^2}{\lambda}} = \nu \qquad (2.22)$$

Where ν is known as the vacuum expectation value (VEV), $\boldsymbol{\nu = 246\ GeV.}$

$$\left({\phi_1}^2 + {\phi_2}^2 + {\phi_3}^2 + {\phi_4}^2\right) = \nu^2 \qquad (2.23)$$

let us investigate more about our potential V(ϕ): as a first example we give both parameters μ^2, $\boldsymbol{\lambda}$ a positive value. We then know the form of the potential from calculus you should have had some time in the past at school: a quartic curve, with a minimum where the field is equal to zero. It is shown in the plot below, on the left diagram.

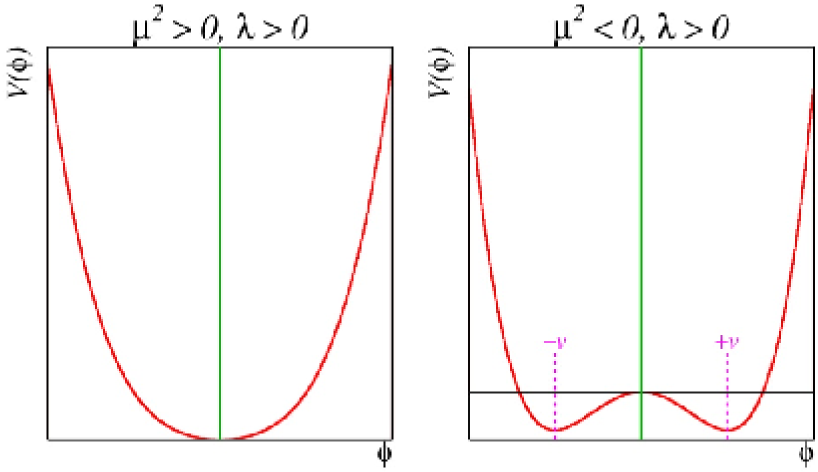

Fig 2.1 higgs field potential.

Imagine a particle sitting at the point $\phi(x) = 0$. It is at the minimum value of the potential. If you move it about the minimum, it produces small oscillations – perturbations of its physical state – and we can compute the physics of the field using something called "perturbation series": the potential difference introduced by the perturbation is small, so its effect is a small modification to the motion of the particle. By grouping together modifications of the same order of magnitude and summing them we can determine the dynamics. Crucially, the particle has a positive mass, corresponding to the resistence it opposes to any attempts at displacing it from the point at $\phi = 0$: you may well call it inertia [2] and [11]

Much more interesting is the case arising if we instead take $\mu^2 < 0$. We then get the form of potential shown on the right diagram in the figure above. The potential term with a negative value of $\mu^2$is at odds with what you would have learned by browsing the first few chapters of a quantum field theory book: it appears to represent a particle with imaginary mass. It is easy to see why it is so: it gives a "negative resistance" to any attempts of moving it from the origin, where the field is zero. The potential decreases in both directions, so it is energetically favorable for our field to roll down to one of the two saddle points. These lie at the value $\phi = \pm v = \pm \sqrt{-\mu^2/\lambda}$, as is easy to realize by inspecting the form of V – or, if you know better, by just setting to zero the derivative of V with respect to the field [2], [3] and [11].

$$\phi = \frac{1}{\sqrt{2}}\binom{0}{v} \qquad (2.24)$$

Next we will see how to use this technique to give bosons and fermions a mass.

(2-4-1) Gauge bosons mass:

To obtain the masses for the gauge bosons we will only need to study the scalar part of the lagrangian [12]:

$$\mathcal{L} = \frac{1}{2}\left(D_\mu \phi\right)^\dagger (D^\mu \phi) - V(\phi)$$

Where D_μ is the covariant derivative.

$$D_\mu = (\partial_\mu + ig\tau^a W_\mu^a + i\acute{g}\frac{Y_\phi}{2}\boldsymbol{B}_\mu) \quad (2.25)$$

$$D_\mu = \left|\partial_\mu + ig\begin{pmatrix} W_3 & W^- \\ W^+ & -W_3 \end{pmatrix} + i\acute{g}\frac{Y_\phi}{2}\boldsymbol{B}_\mu\right| \quad (2.26)$$

$$D_\mu\phi = \left|\partial_\mu\phi + ig\begin{pmatrix} W_3 & W^- \\ W^+ & -W_3 \end{pmatrix}\phi + i\acute{g}\frac{Y_\phi}{2}\boldsymbol{B}_\mu\phi\right| \quad (2.27)$$

$$\square = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \mathrm{v} \end{pmatrix} \quad (2.28)$$

Using (1.27) and (1.28):

$$D_\mu\phi = \frac{ig}{\sqrt{2}}\begin{pmatrix} W_3 & W^- \\ W^+ & -W_3 \end{pmatrix}\begin{pmatrix} 0 \\ \mathrm{v} \end{pmatrix} + \frac{i\acute{g}Y_\phi B_\mu}{\sqrt{2}}\begin{pmatrix} 0 \\ \mathrm{v} \end{pmatrix} \quad (2.29)$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} igW_3 & igW^- \\ igW^+ & -igW_3 \end{pmatrix}\begin{pmatrix} 0 \\ \mathrm{v} \end{pmatrix} + \frac{i\acute{g}}{\sqrt{2}}\begin{pmatrix} 0 \\ \boldsymbol{B}_\mu \mathrm{v} \end{pmatrix} \quad (2.30)$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{igv}w^- \\ -\mathrm{igvw}_3 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ i\acute{g}\boldsymbol{B}_\mu \mathrm{v} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{igv}w^- \\ -\mathrm{igvw}_3 + i\acute{g}v\boldsymbol{B}_\mu \end{pmatrix} \quad (2.31)$$

$$\Rightarrow D_\mu\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{igv}w^- \\ -\mathrm{igvw}_3 + i\acute{g}v\boldsymbol{B}_\mu \end{pmatrix} \quad (2.32)$$

Since $\left(D_\mu\phi\right)^\dagger$ is the complex conjugate of $D_\mu\phi$:

$$\left(D_\mu\phi\right)^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} -\mathrm{igv}w^- & \mathrm{igvw}_3 - i\acute{g}v\boldsymbol{B}_\mu \end{pmatrix} \quad (2.33)$$

$$\left(D_\mu\phi\right)^\dagger (D^\mu\phi) = \frac{1}{\sqrt{2}}\begin{pmatrix} -\mathrm{igv}w^- & \mathrm{igvw}_3 - i\acute{g}v\boldsymbol{B}_\mu \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{igv}w^- \\ -\mathrm{igvw}_3 + i\acute{g}v\boldsymbol{B}_\mu \end{pmatrix} \quad (2.34)$$

$$= \frac{1}{2}\left[g^2v^2w^+w^- + v^2(gw_3 - \acute{g}B_\mu)^2\right] \quad (2.35)$$

$$\frac{1}{2}\left(D_\mu\phi\right)^\dagger (D^\mu\phi) = \frac{1}{4}g^2v^2w^+w^- + \frac{1}{4}v^2\left(gw_3 - \acute{g}B_\mu\right)^2 \quad (2.36)$$

From above equation (1.36) we obtain that:

$$m_w^2 = \frac{1}{4} g^2 v^2 \qquad (2.37)$$

$$\boldsymbol{m_w = \frac{1}{2} v g} \qquad \textbf{(2.38)}$$

For Z boson, we have:

$$z_\mu = \frac{g w_3 - \acute{g} B_\mu}{\sqrt{g^2 + \acute{g}^2}} = (\cos\theta_w w_3 - \sin\theta_w \; B_\mu) \qquad (2.39)$$

And the photon:

$$A_\mu = \frac{1}{\sqrt{g^2 + \acute{g}^2}} (\acute{g} w_3 + g B_\mu) \qquad (2.40)$$

By using a rotation transformation:

$$\begin{pmatrix} z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} w_3 \\ B_\mu \end{pmatrix} \qquad (2.41)$$

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + \acute{g}^2}} \quad and \quad \sin\theta_w = \frac{\acute{g}}{\sqrt{g^2 + \acute{g}^2}} \qquad (2.42)$$

Multiply the second part of equation (2.36) by $\frac{\sqrt{g^2 + \acute{g}^2}}{\sqrt{g^2 + \acute{g}^2}}$:

$$\frac{1}{4} v^2 \left(g w_3 - \acute{g} B_\mu\right)^2 \cdot \frac{\sqrt{g^2 + \acute{g}^2}}{\sqrt{g^2 + \acute{g}^2}} = \frac{1}{4} v^2 (\sqrt{g^2 + \acute{g}^2})^2 z_\mu z^\mu \qquad (2.43)$$

$$m_{z=}^2 \frac{1}{4} v^2 (g^2 + \acute{g}^2)^2 \qquad (2.44)$$

$$\boldsymbol{m_z = \frac{1}{2} v \sqrt{g^2 + \acute{g}^2}} \qquad \textbf{(2.45)}$$

Although since g and $\acute{g}$ are free parameters. The SM makes no absolute predictions for M_w andM_z, it has been possible to set a lower limit before the W-and Z-boson were discovered. The measured values are $\boldsymbol{M_w = 80.4\ GeV}$ and $\boldsymbol{M_z = 91.2\ GeV}$ [12].

(2-4-2) Fermions mass and Yukawa interaction:

The Yukawa interaction is uniquely fixed by the dynamic of the system [10] and [12].

$$\mathcal{L}_{yukawa} = Y_d \bar{q}_L \phi d_R + Y_U \bar{q}_L \phi^* U_R \qquad (2.46)$$

$$\mathcal{L}_{yukawa} = Y_d (\bar{U}_L \quad \bar{d}_L) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} d_R + Y_u (\bar{U}_L \quad \bar{d}_L) \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} U_R + Y_e (\bar{v}_L \quad \bar{e}_l) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} e_R \qquad (2.47)$$

$$\mathcal{L}_{yukawa} = \frac{Y_d}{\sqrt{2}} (\bar{U}_L \quad \bar{d}_L) \begin{pmatrix} 0 \\ v \end{pmatrix} d_R + \frac{Y_u}{\sqrt{2}} (\bar{U}_L \quad \bar{d}_L) \begin{pmatrix} v \\ 0 \end{pmatrix} U_R + \frac{Y_e}{\sqrt{2}} (\bar{v}_L \quad \bar{e}_l) \begin{pmatrix} 0 \\ v \end{pmatrix} e_R \qquad (2.48)$$

$$\mathcal{L}_{yukawa} = \frac{Y_d}{\sqrt{2}} v \bar{d}_L d_R + \frac{Y_u}{\sqrt{2}} v \bar{U}_L U_R + \frac{Y_e}{\sqrt{2}} v \bar{e}_l e_R \qquad (2.49)$$

From last equation and analog to previous section we find that [3], [4], [5] and [6]:

$$\boldsymbol{m_d} = \frac{Y_d}{\sqrt{2}}\boldsymbol{\nu} \quad \textbf{(2.50)}$$

$$\boldsymbol{m_u} = \frac{Y_u}{\sqrt{2}}\boldsymbol{\nu} \quad \textbf{(2.51)}$$

$$\boldsymbol{m_e} = \frac{Y_e}{\sqrt{2}}\boldsymbol{\nu} \quad \textbf{(2.52)}$$

Where **Y** is Yukawa coupling.

(2-5) Full SM lagrangian:

To summarize the standard model we gather together all the ingredients of the lagrangian. The complete (full) lagrangian is[3]:

$$\mathcal{L} = -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$ — $W^{\pm}$, Z, γ kinetic energies and self-interaction.

$$+\bar{L}\gamma^{\mu}\left(i\partial_{\mu} - g\frac{1}{4}\tau W_{\mu} - \acute{g}\frac{Y}{2}B_{\mu}\right)L + \bar{R}\gamma^{\mu}(i\partial_{\mu} - \acute{g}\frac{Y}{2}B_{\mu})R$$ — Lepton and quark Kinetic energies and Their interaction with $W^{\pm}$, Z, γ

(2.53)

$$+\left|(i\partial_{\mu} - g\frac{1}{4}\tau W_{\mu} - \acute{g}\frac{Y}{2}B_{\mu})\phi\right|^2 - V(\phi)$$ — $W^{\pm}$, Z, γ and higgs masses couplings

$$-(G_1\bar{L}\phi R + G_2\bar{L}\phi_c R + hermition\, conjugate)$$ — Yukawa interactions which give fermion masses

L denotes a left-handed fermion (lepton or quark) doublet, and R a right-handed fermion singlet.

Chapter three

SCATTERING CROSS-SECTION

(3-1) Introduction:

In this chapter we will derive the cross section formula for neutrino and antineutrino-electron scattering, using two methods, that is Fermi effective Theory and the Standard model of Particle Physics[3].

(3-2) The Cross Section:

The cross section is the effective area that governs the probability of some scattering or absorption event. Together with particle density and path length, it can be used to predict the total scattering probability via the Beer–Lambert law

In nuclear and particle physics, the concept of a cross section is used to express the likelihood of interaction between particles [2], [4] and [5].

When particles in a beam are thrown against a foil made of a certain substance, the cross section σ is a hypothetical area measure around the target particles of the substance (usually its atoms) that represents a surface. If a particle of the beam crosses this surface, there will be some kind of interaction.

The term is derived from the purely classical picture of (a large number of) point-like projectiles directed to an area that includes a solid target. Assuming that an interaction will occur (with 100% probability) if the projectile hits the solid, and not at all (0% probability) if it misses, the total interaction probability for the single projectile will be the ratio of the area of the section of the solid (the cross section, represented by σ) to the total targeted area.

This basic concept is then extended to the cases where the interaction probability in the targeted area assumes intermediate values - because the target itself is not homogeneous, or because the interaction is mediated by a non-uniform field. A particular case is scattering [2], [3] and [8].

(3-3) Neutrino-electron scattering

In this section we will derive the cross section formula that we used in our calculation in two scenarios.

(3-3-1) Fermi approach:

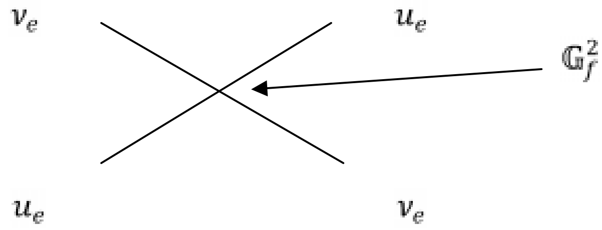

Fig3.1 feynman diagram of neutrino-electron scattering in Fermi theory

The cross section of two scattering particle can be written as [8]:

$$\frac{d\sigma}{d\Omega}(A+B \to C+D) = |\mathcal{M}| \int \frac{d^3P_C}{(2\pi)^3E_C}\frac{d^3P_D}{(2\pi)^3E_D} \times (2\pi)^4 \frac{\delta^4(P_A+P_B-P_C-P_D)}{4\sqrt{(P_AP_B)^2-m_A^2m_B^2}} \quad (3.1)$$

In the center of mass energy:

$$|P_i| = |P_A| = |P_B|$$

And

$$E = E_A + E_B$$

Therefore

$$|P_B|E_B + |P_A|E_A = \sqrt{(P_AP_B)^2 - m_A^2m_B^2}$$

$$|P_i|(E_A + E_B) = \sqrt{(P_AP_B)^2 - m_A^2m_B^2}$$

$$|P_i|E = \sqrt{(P_AP_B)^2 - m_A^2m_B^2} \quad (3.2)$$

Thus

$$\frac{d\sigma}{d\Omega} = \frac{1}{4P_iE}|\mathcal{M}|^2 d\phi \quad (3.4)$$

Where $$d\phi = \int \frac{(2\pi)^4}{4(2\pi)^6}\delta^4(P_A + P_B - P_c - P_D)\frac{d^3P_cd^3P_D}{E_cE_D} \quad (3.5)$$

$$d\phi = \frac{1}{16\pi^2}\int \frac{P_f^2dP_f}{E_cE_D}\delta(w - E_c - E_D) \quad (3.6)$$

$P_f = |P_C| - |P_D|$In the center of mass (CM): W is total energy

$$dw = \frac{P_fdP_f}{E_cE_D}w \quad (3.7)$$

$$d\phi = \frac{1}{16\pi^2}\frac{P_f}{w} = \frac{1}{16\pi^2}\frac{P_f}{\sqrt{s}} \quad (3.8)$$

$$E = \sqrt{s} = w \quad (3.9)$$

Finally we get the differential cross section as follow[3]:

$$\frac{d\sigma}{d\Omega} = |\mathcal{M}|^2 \frac{1}{64\pi^2}\frac{1}{s}\frac{P_f}{P_i} \quad (3.10)$$

In Fermi Theory the amplitude is given by[3].

$$\mathcal{M} = \frac{\mathbb{G}_f^2}{2}\left[\bar{u}(\acute{k},\acute{t})\gamma^\mu(1+\gamma_5)u(\acute{p},\acute{s}) \times \bar{v}_e\gamma_\mu(1+\gamma_5)u\right] * \left[\bar{u}_{\nu_e}\gamma^\nu(1+\gamma_5)u_{\nu_e} \times \bar{u}_{\nu_e}\gamma^\nu(1+\gamma_5)u_{\nu_e}\right] \quad (3.11)$$

To calculate the amplitude we use the so called trace technology to reduce the gamma matrices [13]

$$\sum\left[\bar{u}\gamma^\mu(1+\gamma_5)u \times \bar{u}\gamma_\mu(1+\gamma_5)u\right] \quad (3.12)$$

$$= 2Tr\left[\gamma^\mu\, \mathbb{K}\, \gamma^\nu(1+\gamma_5)\acute{\mathbb{K}}\right] \quad (3.13)$$

$$[\gamma^\mu, \gamma^5] = 0 \quad \gamma_5^2 = 1 \quad (3.14)$$

$$= 2k_\mu \acute{k}_\nu Tr[\gamma^\mu\gamma^\nu\gamma^\mu\gamma^\nu(1-\gamma^5)] \quad (3.15)$$

$$Tr(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu} \quad (3.16)$$

$$Tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4[g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}] \quad (3.17)$$

Therefore the first bracket gives us

$$= 8\left[k^\mu\acute{k}^\nu - g^{\mu\nu}(k\cdot\acute{k}) + (k^\mu\acute{k}^\nu) - \varepsilon^{\mu\nu\rho\sigma}k_\rho\acute{k}_\sigma\right] \quad (3.18)$$

$$\sum\left[\bar{u}\gamma_\mu(1+\gamma_5)u \times \bar{u}\gamma_\mu(1+\gamma_5)u\right]$$

$$= Tr\left[\gamma_\mu(1+\gamma_5)\,\mathbb{P}\,\gamma_\nu(1+\gamma_5)\acute{\mathbb{P}}\right] \quad (3.19)$$

$$= 8\left[p_\mu\acute{p}_\nu + p_\nu\acute{p}_\mu - g_{\mu\nu}(p\cdot\acute{p}) - i\varepsilon_{\nu\mu\rho\sigma}p^\rho\acute{p}^\sigma\right] \quad (3.20)$$

As such, the square of our amplitude in Fermi theory is given by:

$$|\mathcal{M}|^2 = \left[64(p\cdot k)(\acute{p}\cdot\acute{k}) + (\acute{p}\cdot k)(p\cdot\acute{k}) + (p\cdot k)(\acute{p}\cdot\acute{k}) - (\acute{p}\cdot k)(p\cdot\acute{k})\right] \quad (3.21)$$

$$|\mathcal{M}|^2 = 128\left[(p\cdot k)(\acute{p}\cdot\acute{k})\right] \quad (3.22)$$

In center of mass:

$$p + k = \acute{p} + \acute{k} \quad (3.22)$$

$$s = (p+k)^2 = (\acute{p} + \acute{k})^2 \quad (3.23)$$

$$k^2 = 0\ ;\ \acute{k}^2 = 0 \quad (3.24)$$

$$p \cdot k = \frac{1}{2}(s - m_e)^2 = \frac{1}{2}s^2 m_e \to 0 \quad (3.25)$$

$$\frac{1}{2}|\mathcal{M}|^2 = \frac{128}{4}(s - m_e)^2 \frac{G_f^2}{2} \quad (3.26)$$

Then the differential cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{128}{8}G_f^2(s - m_e)^2 \quad (3.27)$$

$$= \frac{1}{64\pi^2}\frac{1}{s}s^2\frac{128}{8}G_f^2 \quad (3.28)$$

$$\boldsymbol{\frac{d\sigma}{d\Omega} = \frac{G_f^2}{4\pi^2} \cdot s} \quad \mathbf{(3.29)}$$

To find the cross section, we need to integrate the differential cross section to get:

$$d\sigma = \left(\frac{G_f}{2\pi}\right)^2 \cdot s\, d\Omega \quad (3.30)$$

$$\sigma = \frac{G_f^2}{4\pi^2} \cdot s \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\ d\theta \quad (3.31)$$

$$= \frac{G_f^2}{4\pi^2} \cdot s\, 2\pi[-\cos\theta]_0^{\pi} \quad (3.32)$$

$$\boldsymbol{\sigma(\nu_e.\, e \to \nu_e.\, e) = \frac{G_f^2}{\pi} \cdot s} \quad \mathbf{(3.33)}$$

(3-3-2) Standard model approach:

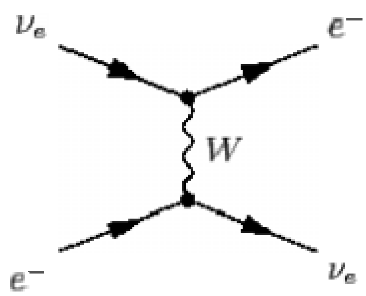

Fig. 3.2 Feynman diagram for neutrino-electron scattering in SM theory.

By using the Feynman rule, we can write down the matrix element $\mathcal{M}$as fallow[13] :

$$\mathcal{M} = \frac{(-ig)^2}{8}\bar{u}_{\nu_e}(\acute{k},\acute{t})\gamma^{\mu}(1+\gamma_5)u_{\nu_e}(\acute{p},\acute{s})\left[\frac{g_{\mu\nu} - (p+k)_{\mu}(k+p)_{\nu}}{(k+p)^2 - m_w^2}\right|\bar{u}_e(\acute{p},\acute{s})\gamma^{\nu}u_e(p,s) \quad (3.34)$$

$$\frac{1}{2}\sum|\mathcal{M}|^2 = \frac{g^4}{128}\left[\frac{g_{\mu\nu}-(p+k)_\mu(k+p)_\nu}{(k+p)^2-m_W^2}\right] \times \left[\frac{g^{\rho\sigma}-(p+k)_\rho(k+p)_\sigma}{(k+p)^2-m_W^2}\right| \times Tr\left[(\mathbb{P} + m_e)\gamma^\mu(1 + \gamma_5)\mathbb{K}\gamma^\rho(1 + \gamma_5)\right] \times Tr\left[\acute{\mathbb{K}}\gamma^\nu(1 + \gamma_5)(\mathbb{P} + m_e)\gamma^\sigma(1 + \gamma_5)\right| \quad (3.35)$$

In center of mass:

$$\acute{p}^2 = m_e^2 = 0 \qquad (3.36)$$

$$(k + p)_\mu Tr[\mathbb{P}\gamma^\mu(1 + \gamma_5)\mathbb{K}\gamma^\rho(1 + \gamma_5)]$$
$$= Tr\left[\acute{p}^2(1 + \gamma_5)\mathbb{K}\gamma^\mu(1 + \gamma_5) - \acute{\mathbb{P}}(1 - \gamma_5)k^2\gamma^\rho(1 + \gamma_5)\right| \qquad (3.37)$$

$$= m_e^2[tr(1 + \gamma_5)\mathbb{K}\gamma^\mu(1 + \gamma_5) - (1 - \gamma_5)k^2\gamma^\rho(1 + \gamma_5)] = 0 \qquad (3.38)$$

$$\frac{1}{2}\sum|\mathcal{M}|^2 = \frac{4g^4}{128}Tr[\mathbb{P}\gamma^\mu(1 + \gamma_5)\mathbb{K}\gamma^\nu(1 + \gamma_5)] \times Tr[\mathbb{K}\gamma_\nu(1 + \gamma_5)\mathbb{P}\gamma_\nu(1 + \gamma_5)] \qquad (3.39)$$

$$\frac{1}{2}\sum|\mathcal{M}|^2 = \frac{4g^4}{128}\frac{64[(p\cdot k)(\dot{p}\cdot\acute{k})]}{[(k+p)^2-m_W^2]^2} = \frac{2g^4[(p\cdot k)(\dot{p}\cdot\acute{k})]}{[(k+p)^2-m_W^2]^2} \qquad (3.40)$$

We have:

$$s = (s + k)^2 k^2 = \acute{k}^2 = 0 \qquad (3.41)$$
$$k = \frac{1}{2}(s - m_e) = \dot{p}\cdot\acute{k} \qquad (3.42)$$

Thus:

$$\frac{1}{2}\sum|\mathcal{M}|^2 = \frac{2g^4\frac{1}{2}(s - m_e^2)\frac{1}{2}(s - m_e^2)}{(s - m_W^2)^2} \qquad (3.43)$$
$$\frac{1}{2}\sum|\mathcal{M}|^2 = \frac{g^4 s^2}{2(s - m_W^2)^2} \qquad (3.44)$$

In which the differential cross section become:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}\frac{1}{2}\sum|\mathcal{M}|^2 = \frac{1}{64\pi^2}\frac{1}{s}\left[\frac{g^4 s^2}{2(s - m_W^2)^2}\right| \qquad (3.45)$$

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{128}\frac{s}{(s - m_W^2)^2} \qquad (3.46)$$

Integrate the above equation, we obtain:

$$\sigma = \frac{g^4}{128\pi^2}\frac{s}{(s - m_W^2)^2}\, 2\pi\int_0^\pi \sin\theta\, d\theta \qquad (3.47)$$

$$\boldsymbol{\sigma(\nu_{e'}\, e \rightarrow \nu_{e'}\, e) = \frac{g^4}{32\pi}\frac{s}{(s - m_W^2)^2}} \qquad \textbf{(3.48)}$$

(3-4) Antineutrino scattering:

In previous section we did calculate the cross section for the neutrino in two scenarios. In this section we will compute the cross section of antineutrino in both scenarios by following the same procedure for neutrino scattering.

(3-4-1) Fermi approach :

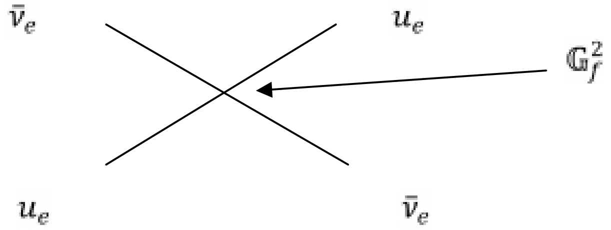

Fig3.3 feynman diagram of antineutrino-electron scattering in fermi theory.

$$❯\ \bar{v}_e(k,t)\gamma_\mu(1+\gamma_5)u_e(p,s)\bar{u}_e(p,s)\gamma_\nu(1+\gamma_5)u_{v_e}(k,t)$$

$$= Tr\big(\gamma_\nu(1+\gamma_5)\big)(p+m_e)\gamma_\nu(1+\gamma_5)k) \quad (3.49)$$

$$= 2p^\mu k_\nu Tr(\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\sigma(1-\gamma_5))$$

$$= 8\big[p_\mu k_\nu - g_{\mu\nu}(p\cdot k) + p_\nu k_\mu + i\epsilon_{\mu\nu\rho\sigma}p^\rho k^\sigma\big] \quad (3.50)$$

$$\sum \bar{u}_e(\acute{p},\acute{s})\gamma^\mu(1+\gamma_5)v_e(\acute{k},\acute{t})\gamma^\rho(1+\gamma_5)u_e(\acute{p},\acute{s}) \quad (3.51)$$

$$= Tr\big[\gamma^\mu(1+\gamma_5)\acute{k}\gamma^\nu(1+\gamma_5)(p+m_e)\big]$$

$$= 2\acute{k}_\mu\acute{p}_\nu Tr(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma(1-\gamma_5)) \quad (3.52)$$

$$= 8\big[\acute{k}^\mu\acute{p}^\nu - g^{\mu\nu}(\acute{p},\acute{k}) + \acute{k}^\nu\acute{p}^\mu - i\varepsilon^{\mu\nu\rho\sigma}\acute{k}_\rho\acute{p}_\sigma\big] \quad (3.53)$$

$$□\ \frac{1}{2}\sum\|M\|^2 = 128(\acute{k}\cdot p)(\acute{p}\cdot k) \quad (3.54)$$

<u>In center of mass(C.M)</u>,[3]:

$$p + k = \acute{p} + \acute{k} = 0\ ;\ s = (P+K)^2 = (\acute{p}+\acute{k})^2$$

$$\acute{k}^2 = k^2 = 0$$

$$\acute{k}p = \acute{k}(\acute{p}+\acute{k}-k) = \acute{k}\acute{p} - \acute{k}k$$

$$\acute{p}k = (p+k-\acute{k})k = p\cdot k - \acute{k}k$$

$$p\cdot k = k_\circ\acute{k}_\circ - k\acute{k} = k_\circ\acute{k}_\circ(1-\cos\theta) \quad (3.55)$$

$$(\acute{k}\cdot p)(\acute{p}\cdot k) = \Big[\frac{1}{2}(s-m_e^2)^2 - k_\circ\acute{k}(1-\cos\theta)\Big]^2 \quad (3.56)$$

$$k_\circ = \sqrt{s} - \sqrt{p^2 + m_e^2} = \sqrt{s} - \sqrt{k^2 + m_e^2}$$

$$k_\circ = \left(2\sqrt{s}\right)^{-1}(s - m_e^2) = \acute{k}_\circ \quad (3.57)$$

$$(\acute{k} \cdot p)(\acute{p} \cdot k) = \left[\frac{1}{2}(s - m_e^2)^2 - k_\circ \acute{k}(1 - \cos\theta)\right]^2 \quad (3.58)$$

$$= \frac{1}{4}(s - m_e^2)^2 \left|1 - \frac{1}{2}(1 - \frac{m_e^2}{s})(1 - \cos\theta)\right]^2 \quad (3.59)$$

$$\square \ \frac{1}{2}\sum \| M \|^2 = \frac{128}{4}\frac{\mathbb{G}_f^2}{2}(s - m_e^2)^2 \left|1 - \frac{1}{2}(1 - \frac{m_e^2}{s})(1 - \cos\theta)\right]^2 \quad (3.60)$$

Limit $m_e^2 \to 0$

$$\frac{d\sigma}{d\Omega} = \frac{64}{64}\mathbb{G}_f^2 s^2 \frac{1}{4}(1 + \cos\theta)^2 = \frac{(1+\cos\theta)^2}{4}\frac{\mathbb{G}_f^2}{4\pi^1}S \quad (3.61)$$

Therefor, the differential cross section of antineutrino can be related to the differential cross section of the neutrino as follow:

$$\frac{d\sigma}{d\Omega} = \frac{(1+\cos\theta)^2}{4}\frac{d\sigma}{d\Omega}(\nu_e . e \to \nu_e . e) \quad (3.62)$$

$$d\sigma = \frac{(1+\cos\theta)^2}{4}\frac{\mathbb{G}_f^2}{4\pi^1}S \, d\Omega \quad (3.63)$$

$$\sigma(\acute{\nu}_e e \to \acute{\nu}_e e) = \frac{\mathbb{G}_f^2}{8\pi^2} S \int_0^{2\pi} d\Phi \int_0^{\pi} 1 + (\cos\theta)^2 \sin\theta \, d\theta \quad (3.64)$$

$$\sigma = \frac{\mathbb{G}_f^2}{8\pi^2} S \frac{2\pi}{2} \int_0^{\pi}(1 + (\cos\theta)^2 + 2\cos\theta)\sin\theta \, d\theta \quad (3.65)$$

But

$$\square \ \int_0^{\pi}(1 + (\cos\theta)^2 + 2\cos\theta)\sin\theta \, d\theta = \int_0^{\pi}\sin\theta \, d\theta + \int_0^{\pi} d\theta \sin\theta (\cos\theta)^2 + 2\int_0^{\pi}\cos\theta \sin\theta \, d\theta \quad (3.66)$$

$$= -\cos\theta\Big|_0^{\pi} - \int_0^{\pi}(\cos\theta)^2 d\cos\theta = 2 + \frac{2}{3} = \frac{8}{3} \quad (3.67)$$

Substation equation (3.67) in (3.65):

$$\boldsymbol{\sigma(\acute{\nu} . e \to \acute{\nu} . e) = \frac{\mathbb{G}_f^2}{8\pi} S \cdot \frac{8}{3} = \frac{1}{3}\frac{\mathbb{G}_f^2}{\pi} S = \frac{1}{3}\sigma(\nu_e . e \to \nu_e . e)} \quad \textbf{(3.68)}$$

(3-4-2) Standard model approaches:

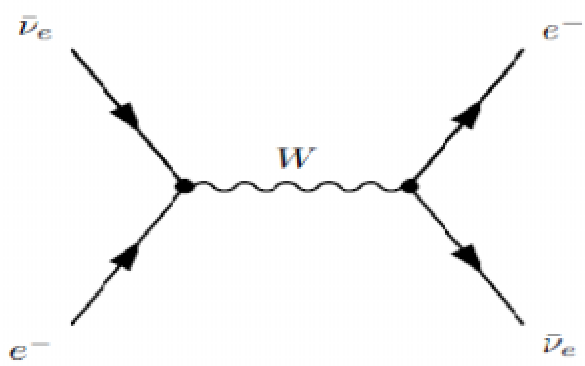

Fig3.4 Feynman diagram for antineutrino-electron scattering in SM theory.

From above Feynman diagram, the amplitude is given by[3]:

$$\mathcal{M} = -i\frac{g^2}{8}\bar{v}(k,t)i\gamma^\mu(1+\gamma_5)u_e(p,s)\left(g_{\mu\nu} - \frac{(p-k)_\mu(p-k)_\nu}{m_w^2}\right)$$

$$\times\frac{1}{((p-k)^2-m_w^2)^2}\bar{u}_e(p,s)i\gamma^\nu(1+\gamma_5)v_e(k,t) \qquad (3.69)$$

$$\frac{1}{2}\sum \|\mathcal{M}\|^2 = \frac{g^4}{128}\left(g_{\mu\nu} - \frac{(p-k)_\mu(p-k)_\nu}{m_w^2}\right)\left(g^{\rho\sigma} - \frac{(p-k)^\rho(p-k)^\sigma}{m_w^2}\right)$$
$$\times\frac{1}{((p-k)^2-m_w^2)^2}Tr\left(\gamma_\mu(1+\gamma_5)(\square+m_e)\gamma_\nu(1+\gamma_5)k\right)Tr(\gamma^\nu(1$$
$$+\gamma_5)\acute{k}\gamma^\nu(1+\gamma_5)(\acute{p}+m_e))$$

(3.70)

$$= \frac{2g^4}{((p-k)^2-m_w^2)^2}\left[(\acute{k}\cdot p)(\acute{p}\cdot k)\right] \quad (3.71)$$

We have $\acute{p}\cdot k = \frac{1}{4}s(1+\cos\theta)$

$$\acute{k}\cdot p = \acute{p}\cdot k = \frac{1}{16}s^2(1+\cos\theta)^2 \quad (3.72)$$

$$\frac{1}{2}\sum\|\mathcal{M}\|^2 = \frac{2g^4}{16(s-m_w^2)^2}s^2(1+\cos\theta)^2 \quad (3.73)$$

$$\frac{d\sigma}{d\Omega}(\acute{\nu}\,e \to \acute{\nu}\,e) = \frac{1}{64\pi^2 s}\frac{2g^4}{16(s-m_w^2)^2}s^2(1+\cos\theta)^2 \quad (3.74)$$

$$= \frac{g^4}{128\pi^2}\frac{s}{(s-m_w^2)^2}\frac{(1+\cos\theta)^2}{4} \quad (3.75)$$

$$= \frac{(1+\cos\theta)^2}{4}\frac{d\sigma}{d\Omega}(\nu_e.\, e \rightarrow \nu_e.\, e) \qquad (3.76)$$

$$\boldsymbol{\sigma(\nu'.\, e \rightarrow \nu'.\, e) = \frac{1}{3}\frac{g^4}{32\pi^2}\frac{s}{(s-m_W^2)^2} = \frac{1}{3}\sigma(\nu_e.\, e \rightarrow \nu_e.\, e)} \qquad \textbf{(3.77)}$$

Chapter four

RESULTS AND CALCULATION

(4-1) Introducion:

In this chapter we shall calculate the scattering cross section for $\sigma(\nu_e . e \rightarrow \nu_e . e)$ **and** $\sigma(\acute{\nu} . e \rightarrow \acute{\nu} . e)$ By using the two approaches Fermi theory and standard model theory.

(4-2) Neutrino-electron cross section:

We shall use the formulas that we had derived in the previous chapter for neutrino-electron scattering cross-section.

(4-2-1) calculation using Fermi formula:

$$\sigma(\nu_e . e \rightarrow \nu_e . e) = \frac{G_f^2}{\pi} \cdot s \quad (4.1)$$

By setting S as in table we calculate by substitute the value of Fermi constant G_f^2 = 1.166 × 10^{-5} GeV^{-2} in the above formula, we get different values of the cross-section, which are written down in table below:

(The main reason behind why we start from 20/Gev^2 is that we do not want S to be equal to the mass of the W- gauge boson, so it's a boundary condition, since M_w = 80.4 GeV).

s/Gev^2	$\sigma(\nu_e . e \rightarrow \nu_e . e)$ $/\text{GeV}^{-2}$	$\sigma(\nu_e . e \rightarrow \nu_e . e)/cm^2$
20	8.866 × 10^{-10}	0.34 × 10^{-36}
40	1.713 × 10^{-9}	0.67 × 10^{-36}
60	2.569 × 10^{-9}	1.01 × 10^{-36}
80	3.426 × 10^{-9}	1.35 × 10^{-36}
100	4.283 × 10^{-9}	1.69 × 10^{-36}
120	5.139 × 10^{-9}	2.03 × 10^{-36}
140	5.996 × 10^{-9}	2.36 × 10^{-36}
160	6.853 × 10^{-9}	2.70 × 10^{-36}
180	7.709 × 10^{-9}	3.04 × 10^{-36}
200	8.566 × 10^{-9}	3.38 × 10^{-36}

Table4.1 neutrino-electron cross-section calculation in Fermi theory.

(4-2-2) Calculation using SM formula:

SM formula is given by:

$$\sigma(\nu_{e}.e \to \nu_{e}.e) = \frac{g^4}{32\pi}\frac{S}{(S - m_w^2)^2} \quad (4.2)$$

This formula we substitute the values of **W boson mass** m_w^2 and the coupling constant g^4**:**

$$g^4 = 8.413 \square 10^{-3}$$

$$m_w^2 = (80.4)^2/MeV^2$$

Putting S as in table in **the** above formula we get results which are written in the table below:

s/Gev^2	$\sigma(\nu_{e}.e \to \nu_{e}.e)$ $/\text{Gev}^{-2}$	$\sigma(\nu_{e}.e \to \nu_{e}.e)/cm^2$
20	4.112×10^{-11}	0.162145×10^{-37}
40	8.275×10^{-11}	0.326301×10^{-37}
60	1.249×10^{-10}	0.492508×10^{-37}
80	1.676×10^{-10}	0.660883×10^{-37}
100	2.108×10^{-10}	0.831230×10^{-37}
120	2.546×10^{-10}	1.003943×10^{-37}
140	2.989×10^{-10}	1.178628×10^{-37}
160	3.438×10^{-10}	1.355678×10^{-37}
180	3.893×10^{-10}	1.535094×10^{-37}
200	4.354×10^{-10}	1.716877×10^{-37}

Table4.3 neutrino-electron cross-section calculation in SM theory.

(4-3) Antineutrino-electron cross section:

We will use the formulas (3.68) and (3.77) which we had derived in the previous chapter for antineutrino-electron scattering cross-section.

(4-3-1)Calculation using Fermi formula:

Fermi formula is given by:

$$\sigma(\acute{\nu}.e \to \acute{\nu}.e) = \frac{\mathbb{G}_f^2}{8\pi}S \cdot \frac{8}{3} = \frac{1}{3}\frac{\mathbb{G}_f^2}{\pi}S = \frac{1}{3}\sigma(\nu_{e}.e \to \nu_{e}.e) \quad (4.3)$$

From this formula we use the same result of neutrino-electron cross section and multiply it by 1/3 to get:

s/Gev^2	$\sigma(\acute{\nu}.e \to \acute{\nu}.e)$ $/\mathrm{Gev}^{-2}$	$\sigma(\acute{\nu}.e \to \acute{\nu}.e)/cm^2$
20	2.955×10^{-10}	0.12×10^{-36}
40	5.71×10^{-10}	0.23×10^{-36}
60	8.563×10^{-10}	0.34×10^{-36}
80	1.142×10^{-9}	0.45×10^{-36}
100	1.427×10^{-9}	0.56×10^{-36}
120	1.713×10^{-9}	0.68×10^{-36}
140	1.998×10^{-9}	0.79×10^{-36}
160	2.284×10^{-9}	0.90×10^{-36}
180	2.569×10^{-9}	1.01×10^{-36}
200	2.855×10^{-9}	1.13×10^{-36}

Table4.2 antineutrino-electron cross-section calculation in Fermi theory.

(4-3-2)Calculation using SM formula:

$$\frac{1}{3}\frac{g^4}{32\pi^2}\frac{s}{(s-m_w^2)^2} = \frac{1}{3}\sigma(\nu_{e'}\, e \to \nu_{e'}\, e) \quad (4.4)$$

Using the result of neutrino-electron cross-section in SM calculation and multiply them by 1/3 we get:

s/GeV	$\sigma(\acute{\nu}.e \to \acute{\nu}.e)$ $/\mathrm{Gev}^{-1}$	$\sigma(\acute{\nu}.e \to \acute{\nu}.e)/cm^{-1}$
20	1.371×10^{-11}	0.054062×10^{-37}
40	2.758×10^{-11}	0.187539×10^{-37}
60	4.163×10^{-11}	0.164156×10^{-37}
80	5.486×10^{-11}	0.216325×10^{-37}
100	7.026×10^{-11}	0.277050×10^{-37}
120	8.486×10^{-11}	0.334621×10^{-37}
140	9.963×10^{-11}	0.392863×10^{-37}
160	1.146×10^{-10}	0.451893×10^{-37}
180	1.297×10^{-10}	0.511435×10^{-37}
200	1.451×10^{-10}	0.572161×10^{-37}

Table4.3 anti-neutrino-electron cross-section calculation in SM theory.

(4-4) plotting results:

For plotting the result in the above tables we used scientific plotting mathematica software.

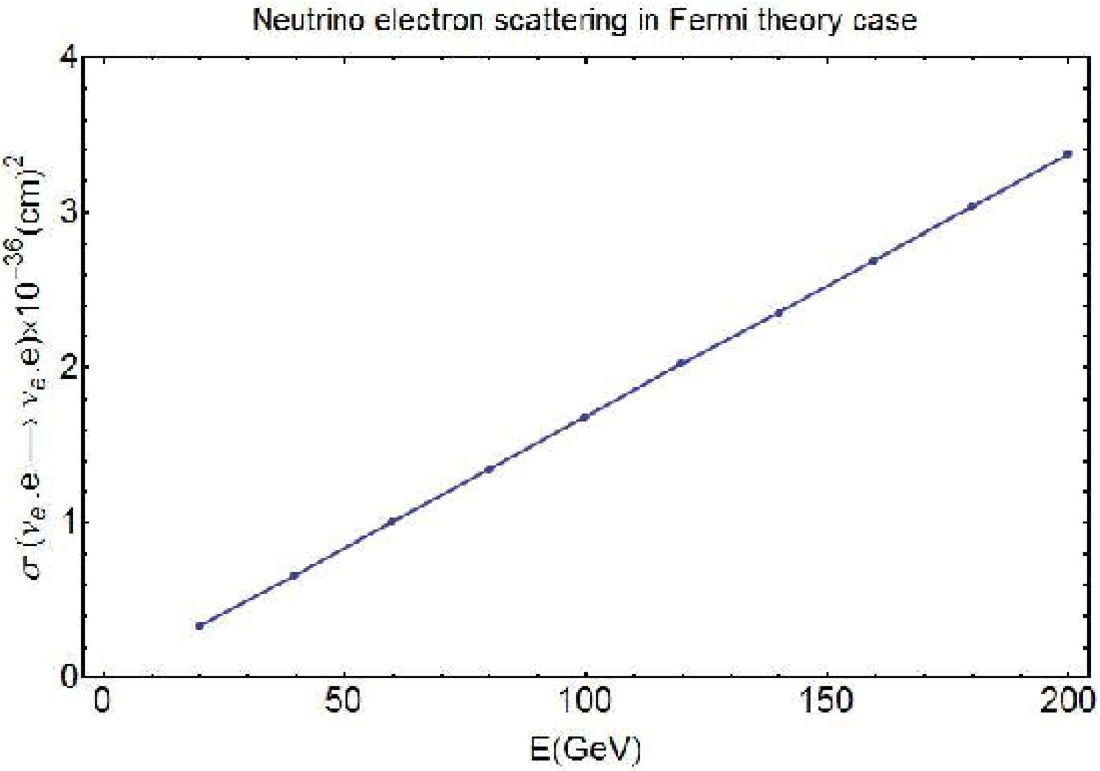

Fig.4.1 Cross section of neutrino-electron scattering in Fermi theory

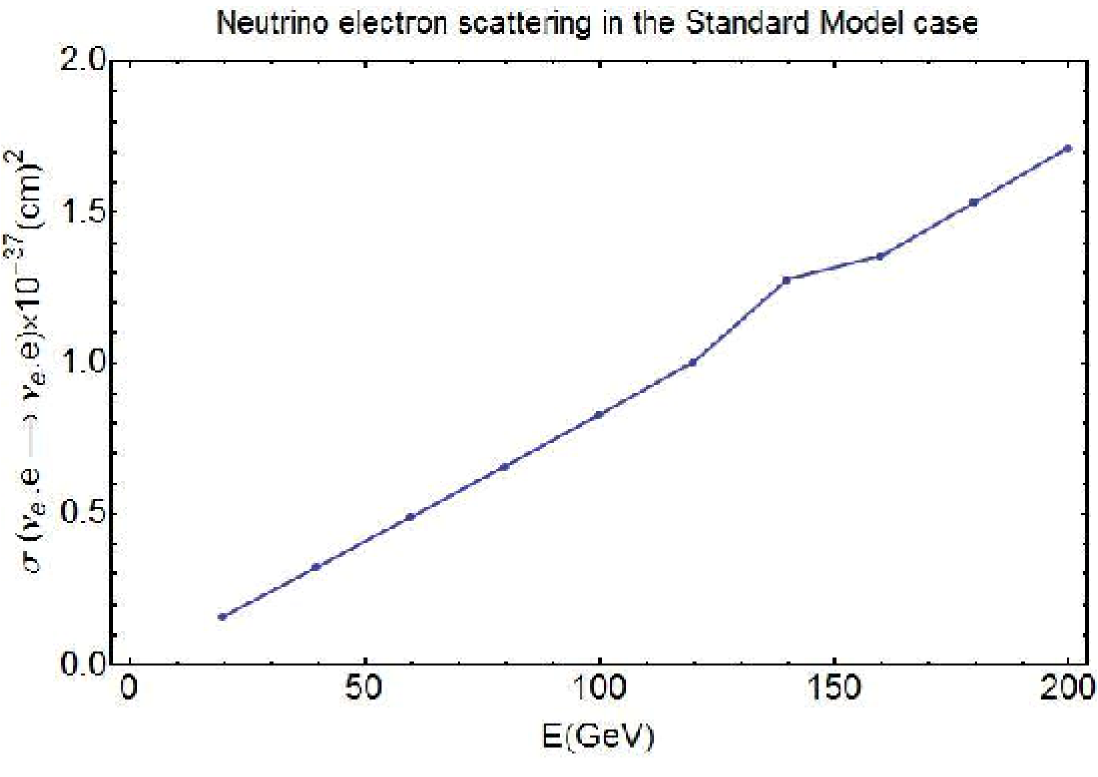

Fig4.2 Cross section of neutrino-electron in SM theory

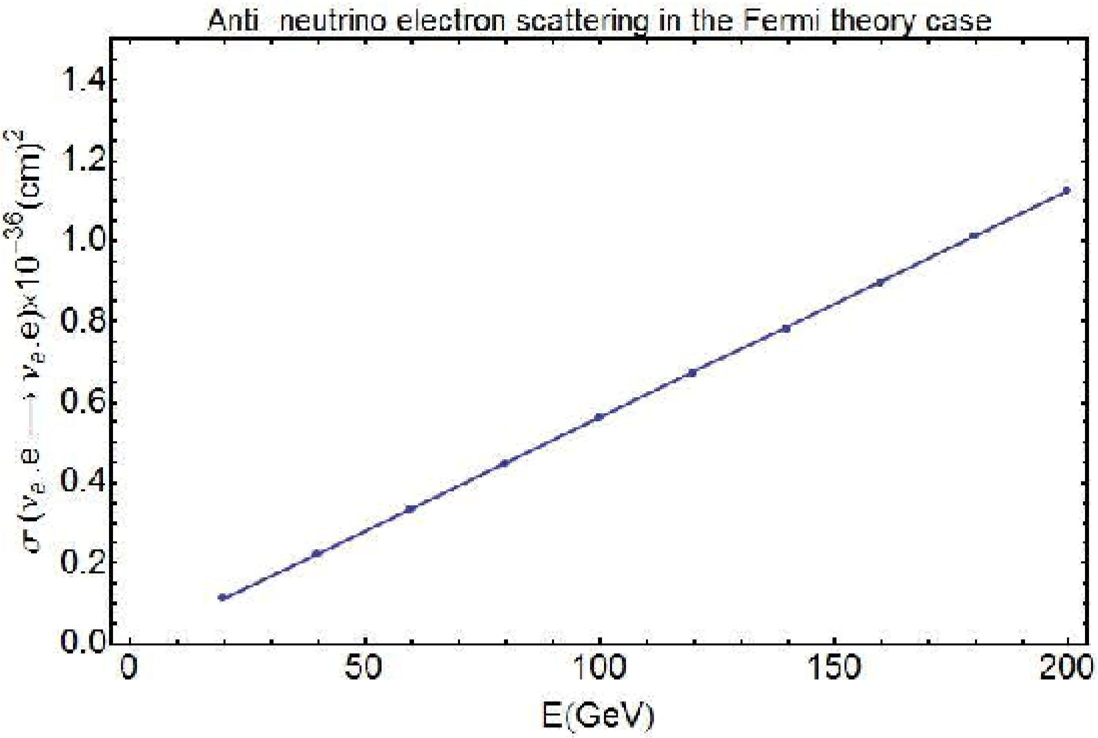

Fig4.3 Cross section of antineutrino-electron in Fermi theory

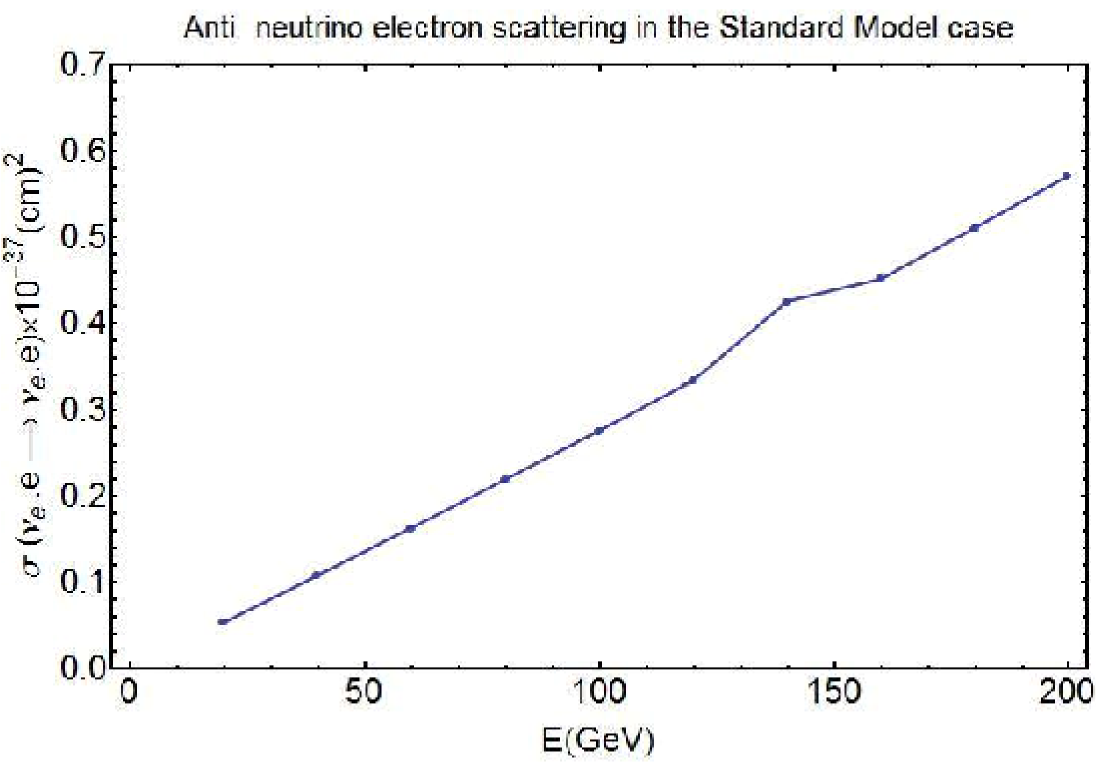

Fig4.4 Cross section of antineutrino-electron in SM theory

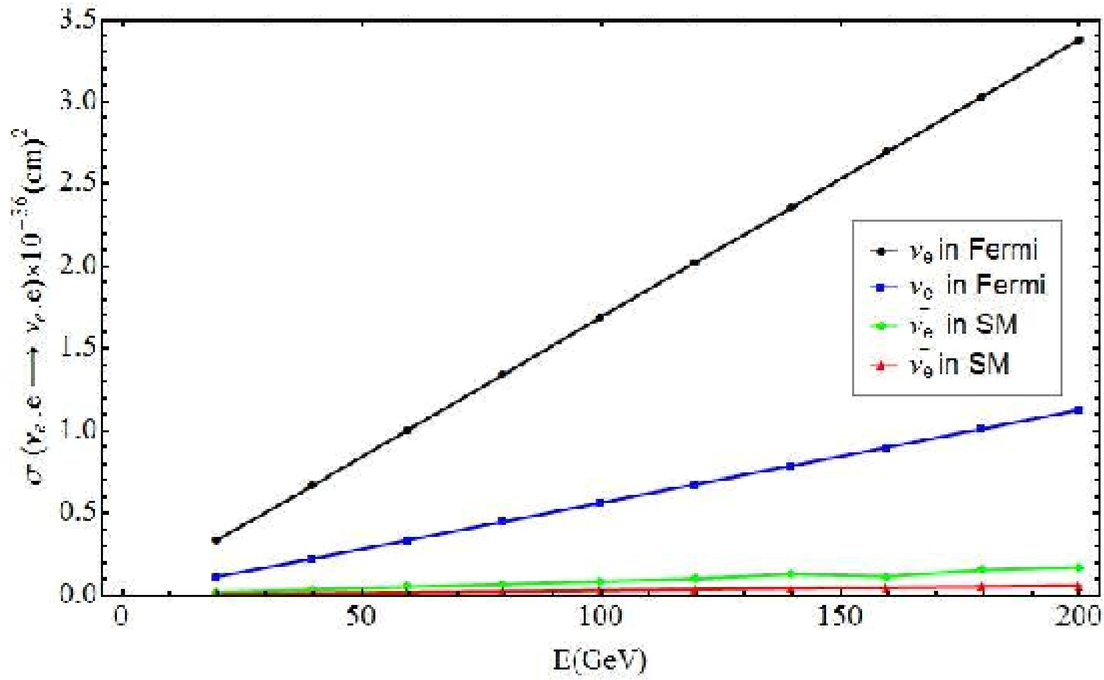

Fig4.5 Cross section of neutrino anti neutrino in both cases Fermi theory and Standard model, black line represent the neutrino cross section in Fermi theory, and blue line represent the antineutrino cross section. The dark red line and the green line represent the neutrino and antineutrino scattering cross section in SM theory respectively.

Chapter five

Discussion and Conclusion

Discussion:

As can be seen from our calculation, that the scattering cross-section of the anti-neutrino is always equal to $\frac{1}{3}$ the scattering cross-section of neutrino. The main reson behind that is: in neutrino electron scattering the initial state has spin projection $J_z = 0$. Because the incoming neutrino and electron are both left handed. They can emerge in any direction the outgoing neutrino and electron has $J_z = 0$, no problem. However, in antineutrino electron scattering the situation is different. The incoming antineutrino is right handed, so the initial state has projection $J_z = 1$ for backward scattering and the outgoing antineutrino and electron combine to give $J_z = -1$, therefore scattering at cosine theta =1 is forbidden by angular momentum conservation, this reflect the nature of parity violation in weak interaction.
As depicated in the figures above figure.1 and figure.2 the cross-section of neutrino-electron scattering and anti-neutrino electron scattering cross-section by using Fermi effective theory, from the equation (3.33) and (3.68) increase as energy increases and from the plot we find that the total cross-section is directly proportional to the center of mass energy S, that means at high energy the cross-section become so large that the probability of an interaction exceeds unity. It requires inclusion in the amplitude of the massive-boson exchange between the currents to restrain this rise in cross-section and that what we found in the standard model theory SM equation (3.48) and (3.77).

Conclusion:

In conclusion, we had calculated the cross section of the neutrino electron and anti-neutrino scattering in Fermi theory and the Standard Model. We found that at High energy the Fermi theory is break down which means we cannot trust our calculation in this theory unlike the Standard Model.
We find also the cross-section of anti-neutrino is less than that of neutrino cross-section by factor equal to 1\3 in both models that we studied.

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