

# *Dedication*

First of all we thank Allah for giving us the power and ability to establish this work.

To our dear parents and their love and care.

To our lovely friends who share us happiness and sorrow.

To all teachers who light up our ways by their knowledge.

And to all who participate in this work we dedicate this search.

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# ***Abstract***

*We study the rings and ideals in linear algebra and homomorphism and given some theorems and examples, and we define the quotient rings, algebra of ideals . The ideal in a linear algebra .*

*We also study the iterated algebra extension in finite fields. We defined also Gaussian integers ,irreducible integers and a monic polynomial .*

*Finally we study the Galois group, the splitting field and separable field normal extension.*

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