

ABSTRACT

Geodetic data processing relies on proper functional models relating observations to unknowns. Such models are usually adequately formulated. In addition, stochastic modelling of observations is of utmost importance. In both models covariance functions play a key role in the computation of geodetic quantities and their relevant precision.

This research is directed towards:

- (1) the development of empirical covariance functions from real and simulated data using different models.
- (2) investigation of various mathematical models for the prediction of heights as an example of deterministic quantities.
- (3) The development of criterion matrices using suitable covariance functions and their use in analytical design of levelling networks.

Empirical covariance functions for real and simulated levelling networks are developed and their use for prediction and analytical design is tested.

The main conclusions are:

- i) Covariance functions for deterministic quantities take the form of a straight line function. However, for small areas (i.e. less than 9 km^2) it was found that a negative gradient straight line is adequate for levelling networks.
- ii) The method of least squares prediction is found to be the best model for data densification in levelling networks.

- iii) Covariance functions describing the behavior of errors in levelling networks can be fully described by straight line functions. However, the exponential models are suitable for use with two dimensional networks.

- iv) The method of least squares used for the design of levelling networks gives different solutions when using the criterion matrix or its inverse. However, both solutions are equivalent as far as the decision of rejecting the observation(s) with the least contribution to the precision of the network.

المخلص

معالجة البيانات الجيوديسية يعتمد على نماذج دوال سليمة تربط الإرسادات بالقيم المجهولة. هذه النماذج عادة ما تصاغ بشكل مناسب. فضلاً عن أهمية صياغة النماذج الإحصائية و في كلا النموذجين نجد أن دوال التغيرات تلعب دوراً رئيسياً في حساب المقادير الجيوديسية و دقتها.

هدفت هذه الدراسة إلى:

- (1) عمل دوال التغيرات التجريبية من بيانات حقيقية و محاكاة و ذلك بإستخدام نماذج رياضية مختلفة.
- (2) التحقق من إستخدام النماذج أعلاه في الإستكمال البيني للإرتفاعات كمثال للكميات القطعية.
- (3) عمل المصفوفات المعيارية بإستخدام دوال تغيرات مناسبة لإستخدامها في التصميم التحليلي لشبكات الميزانية.

تم إعداد دوال تغيرات تجريبية لشبكات حقيقية و محاكاة و من ثم تم إختبار إستخدامها في مسائل الإستكمال البيني و تصميم الشبكات.

خلصت هذه الدراسة إلى أن:

- (1) دوال التغيرات للكميات القطعية تأخذ شكل دوال خط مستقيم. على كل بالنسبة للمساحات الصغيرة (أقل من 9 كلم مربع) وجد أن إستخدام دوال تتبع شكل الخط المستقيم ذات إنحدار سالب يعتبر كافياً.
- (2) طريقة الإستكمال البيني بإستخدام دوال التغيرات من أفضل الطرق لتكثيف البيانات في شبكات الميزانية.
- (3) دوال التغيرات التي تصف سلوك الأخطاء في شبكات الميزانية يمكن وصفها تماماً بدوال الخط المستقيم. بينما نجد أن الدوال الاسية مناسبة مع الشبكات ثنائية الابعاد.
- (4) طريقة أقل التربيعات التي تستخدم في تصميم شبكات الميزانية تعطى حلولاً مختلفة عند إستخدام المصفوفة المعيارية أو معكوسها غير أن كلا الحلين متطابقين في إتخاذ قرار إستبعاد إرسادات ذات تأثير أقل على دقة الشبكة.

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