

## ملخص

في هذا البحث تم تطوير برنامج حاسوب (NUSAP) بطريقة العناصر المحددة لتحليل المنشآت الوحية (مسائل الإجهاد/الانفعال المستوي). المعرضة للأحمال الاستاتيكية.

تم ربط النظام بمعالجين مرنين: لإدخال البيانات (معالج الإدخال) ولعرض المخرجات (معالج الإخراج) اللذين أعدا خصيصاً لهذه الدراسة وذلك لتوفير طريقة سهلة لإدخال البيانات للبرنامج عن طريق قوائم تحاور مباشر وقوالب إدخال، ولعرض المخرجات للمستخدم بيانياً وفي شكل جداول مصاغة بطريقة سلسلة.

في هذه الدراسة تم التركيز علي التحليل الخطي والتحليل اللاخطي للمنشآت الغشائية المستوية (مسائل الإجهاد/الانفعال المستوي) تم استخدام عنصر ذو أربع عقد (QPM4) وعنصر ذو ثمان عقد (QPM8).

تم تطوير التقنين اللاخطي هندسياً (بناءً علي تقنين لاقرانج الكلي) باستخدام انفعال قرين (GNGRS) والانفعال الهندسي (GNGES) والانفعال اللوغريثمي (GNLGS) وطبقت هذه الصيغ في برنامج الحاسوب (NUSAP). تم حل مجموعة المعادلات اللاخطية باستخدام طريقة نيوتن- رابسون

تم التحقق من دقة نتائج البرنامج باستخدام أمثلة عديدة حيث أظهرت النتائج المتحصل عليها موافقة مع النتائج المنشورة في عدة أوراق علمية.

وتم الحصول علي الاجهادات الناتجة من التحليل الاخطي من الصيغ المختلفة للانفعال وتأكيد دقتها بمقارنتها مع النتائج المنشورة.

ومن ثم تم الحصول علي الاجهاد الحقيقي المرافق للانفعال اللوغريثمي وذلك لأحمال تؤدي إلي إزاحات كبيرة جداً.

## ***ABSTRACT***

In this research, a general-purpose finite element system (NUSAP) is developed for the analysis of plate structures (plane stress/strain problems) subjected to static loading.

The program is linked to two processors: Pre-Processor, developed specially to supply the program with the necessary data through an interactive menu and dialogue boxes, and Post-processor for presenting the output results in formatted sense.

This study focused on the linear and nonlinear analysis of plane stress/strain structures using 4-node (QPM4) and 8-node (QPM8) plane stress/strain elements.

A geometric nonlinear formulation based on the total Lagrangian formulation and using Green's strain (GNGRS), geometric strains (Engineering strains) (GNEGS) and Logarithmic strains (GNLGS) was adopted. The formulations were implemented into the nonlinear finite element program (NUSAP). The solution of nonlinear equations was obtained by the Newton-Raphson method.

The accuracy of the results of the formulation is demonstrated by using four numerical examples and the results are in agreement with other available published solutions.

The non-linear stresses from the different strain formulations were obtained. A comparison was made between the obtained strains and published results

The true stress associated with the logarithmic strain was thus obtained for loads that result in very large displacements.

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# LIST OF SYMBOLS

|                 |  |
|-----------------|--|
| $a$             | Set of discretization parameters   |
| $a_n$           | converged solution at a previous load level or time step                   |
| $a$             | vector of nodal displacements  |
| $A$             | matrix containing displacements derivatives w.r.t. Cartesian displacements |
| $B_o$           | matrix contains shape function derivatives w.r.t $\xi, \eta, \zeta$        |
| $B_L$           | matrix of nonlinear displacement components                                |
| $D$             | Elastic matrix   |
| $E$             | young's modulus  |
| $g_x, g_y$      | displacement gradient vector   |
| $G$             | matrix containing shape function derivatives                               |
| $G$             | shear modulus  |
| $h$             | thickness  |
| $i, j, k$       | unit vectors in direction of global coordinates                            |
| $j$             | jacobian matrix  |
| $K$             | stiffness matrices   |
| $K_T$           | tangent stiffness matrix due to green strains                              |
| $K_\sigma$      | additional geometric stiffness matrix                                      |
| $K_\sigma^*$    | additional geometric stiffness matrix (geometric strains)                  |
| $K_\sigma^{**}$ | additional geometric stiffness matrix (logarithmic strains)                |

|  |  |
|--|--|
| $\mathbf{K}_{GT}^*$  | tangent stiffness matrix due to geometric strains                            |
| $\mathbf{K}_{TL}^*$  | tangent stiffness matrix due to logarithmic strains                          |
| $l, m, n$  | components of unit vectors (direction cosines)                               |
| $\mathbf{M}$   | additional initial stress matrix   |
| $N_i$  | shape function at node $i$   |
| $\mathbf{P}$   | additional initial   |
| $r, s, t$  | natural coordinates  |
| $\mathbf{T}$   | transformation matrix  |
| $\mathbf{u}$   | nodal displacement vector  |
| $u, v, w$  | displacement in global Cartesian coordinates                                 |
| $X, Y, Z$  | global Cartesian coordinates   |
| $\theta$   | vector containing displacement's derivatives w.r.t.<br>Cartesian coordinates |
| $\sigma$   | 2 <sup>nd</sup> piola - kirchhoff stress vector                              |
| $\sigma'_1, \sigma'_2$   | vector containing engineering stress   |
| $\Psi$   | residual forces  |
| $\gamma_{xz}, \gamma_{yz}$                                     | green strains (shear component)  |
| $\gamma'_{xz}, \gamma'_{yz}$                                   | geometric strains (shear component)  |
| $\delta\epsilon, \delta\mathbf{A}, \dots, \text{etc.}$         | variation of $\epsilon, \dots, \text{etc.}$                                  |
| $\alpha_i, \beta_i$  | nodal rotation   |
| $\gamma', \gamma$  | shear angle  |
| $\xi, \eta, \zeta$   | element curvilinear  |
| $\epsilon_x, \epsilon_y \text{ and } \gamma_{xy}$              | green strains  |
| $\epsilon'_x, \epsilon'_y \text{ and } \gamma'_{xy}$           | geometric strains  |
| $\epsilon_{.xlx}, \epsilon_{.ylx} \text{ and } \gamma_{.xylx}$ | logarithmic strains  |

# LIST OF ABBREVIATIONS

|                |  |
|----------------|--|
| <b>CBF</b>     | <b><u>C</u>onstant <u>B</u>ody <u>F</u>orce</b>  |
| <b>CL</b>      | <b><u>C</u>oncentrated <u>L</u>oad</b>   |
| <b>DLNORM</b>  | Limit of <b><u>D</u>isp<u>L</u>acement <u>N</u>orm</b>   |
| <b>GNGRS</b>   | <b><u>G</u>eometric <u>N</u>onlinear <u>G</u>reen's <u>S</u>train</b>  |
| <b>GNGES</b>   | <b><u>G</u>eometric <u>N</u>onlinear <u>G</u>eometric <u>S</u>train</b>                                      |
| <b>GNLGS</b>   | <b><u>G</u>eometric <u>N</u>onlinear <u>L</u>ogarithmic <u>S</u>train</b>                                    |
| <b>INCOUT</b>  | <b><u>I</u>NCrement interval for <u>O</u>UTput of results</b>  |
| <b>ISOFLEX</b> | <b><u>I</u>SOtropic <u>F</u>LEXural Element</b>  |
| <b>MAR</b>     | Limit of <b><u>M</u>aximum <u>A</u>bsolute <u>R</u>esidual (RMAXAL)</b>                                      |
| <b>NALPS</b>   | Maximum no. of conjugate Newton iteration  |
| <b>NIT</b>     | Maximum <b><u>N</u></b> o. of <b><u>I</u></b> teration for each load increment                               |
| <b>NITOUT</b>  | <b><u>N</u></b> o. of <b><u>I</u></b> teration interval for <b><u>O</u></b> utput of results                 |
| <b>NRIT</b>    | Maximum no. of <b><u>N</u></b> ewton- <b><u>R</u></b> aphson <b><u>I</u></b> teration                        |
| <b>NUSAP</b>   | <b><u>N</u></b> Uha <b><u>S</u></b> tructural <b><u>A</u></b> nalysis <b><u>P</u></b> rogram                 |
| <b>QAX4</b>    | <b><u>Q</u></b> uadrilateral <b><u>A</u></b> Xi-symmetric element with <b><u>4</u></b> node                  |
| <b>QAX8</b>    | <b><u>Q</u></b> uadrilateral <b><u>A</u></b> Xi-symmetric element with <b><u>8</u></b> node                  |
| <b>QF4</b>     | <b><u>Q</u></b> uadrilateral plate <b><u>F</u></b> lexural element with <b><u>4</u></b> nodes                |
| <b>QF8</b>     | <b><u>Q</u></b> uadrilateral plate <b><u>F</u></b> lexural element with <b><u>8</u></b> nodes                |
| <b>QF9</b>     | <b><u>Q</u></b> uadrilateral plate <b><u>F</u></b> lexural element with <b><u>9</u></b> nodes                |
| <b>QPM4</b>    | <b><u>Q</u></b> uadrilateral <b><u>P</u></b> lane <b><u>M</u></b> embrane element with <b><u>4</u></b> nodes |
| <b>QPM8</b>    | <b><u>Q</u></b> uadrilateral <b><u>P</u></b> lane <b><u>M</u></b> embrane element with <b><u>8</u></b> nodes |
| <b>QPM9</b>    | <b><u>Q</u></b> uadrilateral <b><u>P</u></b> lane <b><u>M</u></b> embrane element with <b><u>9</u></b> nodes |
| <b>RLNORM</b>  | limit of <b><u>R</u></b> esidual <b><u>N</u></b> ORM   |

|             |   |
|-------------|---|
| <b>RMS</b>  | Limit for <b><u>R</u>oot <u>M</u>ean <u>S</u>quare of residual (RNORAL)</b>       |
| <b>TAX3</b> | <b><u>T</u>riangular <u>A</u>Xi-symmetric element with <u>3</u> nodes</b>         |
| <b>TAX6</b> | <b><u>T</u>riangular <u>A</u>Xi-symmetric element with <u>6</u> nodes</b>         |
| <b>TF3</b>  | <b><u>T</u>riangular Plate <u>F</u>lexural element with <u>3</u> nodes</b>        |
| <b>TF6</b>  | <b><u>T</u>riangular <u>P</u>late <u>F</u>lexural element with <u>6</u> nodes</b> |
| <b>TPM3</b> | <b><u>T</u>riangular <u>P</u>lane <u>M</u>embrane element with <u>3</u> nodes</b> |
| <b>TPM6</b> | <b><u>T</u>riangular <u>P</u>lane <u>M</u>embrane element with <u>6</u> nodes</b> |
| <b>UDL</b>  | <b><u>U</u>niformly <u>D</u>istributed <u>L</u>oad</b>                            |