

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

# الاستهلال

قال تعالى :-

اللَّهُ نُورُ السَّمَاوَاتِ وَالْأَرْضِ مَثَلُ نُورِهِ كَمِشْكَاةٍ فِيهَا  
مِصْبَاحٌ الْمِصْبَاحُ فِي رُجَاحَةِ الْرُّجَاحَةِ كَأَنَّهَا كَوْكُبٌ دُرْرِيٌّ  
يُوقَدُ مِنْ شَجَرَةِ مُبَارَكَةِ زَيْتُونَةٍ لَا شَرْقِيَّةٍ وَلَا غَرْبِيَّةٍ  
يَكَادُ زَيْتُهَا يُضِيءُ وَلَوْ لَمْ تَمْسَسْهُ تَلْرُ نُورٌ عَلَى نُورٍ  
يَهْدِي اللَّهُ لِنُورِهِ مَنْ يَشَاءُ وَيَصْرِبُ اللَّهُ الْأَمْتَالَ لِلنَّاسِ  
وَاللَّهُ بِكُلِّ شَيْءٍ عَلِيمٌ

صدق الله العظيم

سورة النور ( الآية 35 )

## Dedication

This thesis is dedicated to my wonderful parents, who encouraged me to work hard and to respect others.

They help me in every stage in my study through good and bad times.

Thanks to all brothers, sisters and friends for their assistance.

I really appreciate your role in helping me to succeed and in installing confidence in me that I became capable of doing anything in my mind.

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## **Abstract**

This research presents an analytical study of all variables that affect hydraulic fracture width calculation .

The differential equations That govern hydraulic fracturing in an integral form suitable for finite element analysis were formulated. Then a finite element program using MatLab was developed, implemented and checked. The program was then applied to analyse specific case studies .

The results obtained using the program were compared with the results from the application of (An Adaptive Finite Element Scheme for Hydraulic Fracturing with Proppant Transport). The comparison shows good agreement between the two sets of results.

## المستخلص

هذا البحث يقدم دراسة تحليلية لجميع المتغيرات التي تؤثر في حساب عرض الشق في عمليات التشقيق الهيدروليكي. وصيغت في المعادلات التفاضلية التي تحكم التشقيق الهيدروليكي في شكل متكامل يتناسب للتحليل با ستخدام العناصر المحدودة. ثم تم تطوير برونا مج

العناصر المحددة باستخدام برنامج الماتلاب وتنفيذها والتحقق منها. وقد تم تطبيق البرنامج لتحليل دراسات حالة محددة . وقورنت النتائج التي تم الحصول عليها باستخدام البرنامج مع النتائج من تطبيق (عذر صر محدود على التكيف لمخطط التسقيف الهيدروليكي مع نقل الوسائل ) ، وقد اكملت المقارنة توافق جيد بين مجموعتين من النتائج .

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