Dedication

To my father, mother, brothers, sisters, friends and teachers

Acknowledgment

First of all I would like to express my deep gratitude to my Supervisor: Prof.Dr. Shawgy Hussein Abdalla for many helpful and suggestion. All thanks to mathematical department in Omdurman Islamic University. And a full thanks to Sudan University of Science and Technology, college of Science, Department of Mathematic Finally my deep thanks to my family for their full support.

Abstract

We show the solutions of certain integral equations, by means of operators of arbitrary order, and fractional differintegral equations. Complete analysis of bounded variation penalty methods is shown for ill-posed problems with more considerations to the autoconvolution equations, total variation constraints and exact determination of the density function by its autoconvolution coefficients. The free multiplicities in branching problems are obtained with unitary highest modules. Existence of compact quotients of homogenous spaces, measurably proper actions, and decay of matrix coefficients with tempered actions are established. In addition, conformal geometry and branching laws for representations concerned with minimal nilpotent orbits are investigated. We give explicit solutions of fractional integral, fractional differential equations and differential equations involving Erdélyi-Kober operators, with high applications on the generalized Gronwall inequality. We also find explicit bounds for weakly singular integral inequalities with applications to fractional differential and integral equations. We study the integro-differential equations of first order with autoconvolution integral structure and show the theory and present the general class of autoconvoluation equations of the third kind. We construct the composition formulas in the Weyl calculus.

الخلاصة

أوضحنا حلول معادلات التكامل المعينة بواسطة متوسطات مؤثرات الرتبة الإختيارية ومعادلات التفاضل التكاملية الكسرية. تم توضيح التحليل التام للطرق ناعمة التغيير المحدود للمسائل المعقدة الطرح مع الإعتبارات الأكثر إلي معادلات الإلتفاف التلقائي وقيود التغيير الكلية والتحديد التام لدالة الكثاقة بواسطة معاملات الإلتفاف التلقائي. تم إعطاء المضاريب الحرة في مسائل التفرع مع المقاسات العليا الواحدية. تم تأسيس وجود حواصل القسمة المتراصة للفضاءات المتجانسة والأفعال المناسبة القابلة للقياس واضمحلال معاملات المصفوفة مع الأفعال المتساوية. اضافة تم مناقشة الهندسة الحافظة للزوايا وقوانين التفرع للتمثيلات المعينة مع المدارات متلاشية القوي الأصغرية. أعطينا حلولاً صريحة للتكامل الكسري والمعادلات التفاضلية الكسرية والمعادلات التفاضلية المتضمنة مؤثرات أردلي- كوبر مع التطبيقات العليا علي متباينة قرونول المعممة. أيضاً أوجدنا الحدود الصريحة لمتباينات التكامل الشاذة الضعيفة مع التطبيقات إلي التفاضل الكسري والمعادلات التكاملية. درسنا معادلات التكامل- التفاضلية من الرتبة الأولي مع بناء تكامل الإلتفاف التلقائي وأوضحنا نظرية والعائلة العامة لمعادلات الإلتفاف التلقائي من الرتبة الثانية. تم علي نحو واسع تشييد الصيغ المركبة في حسبان ويل.

Contents

Introduction

The solution of the Volterra differintegral equation using the Riemann-Liouville definition of fractional calculus is expressed interms of the Mittag-Leffler functions which allow us to extend the range of parameters to a wider range.

We present an abstract analysis of bounded variation methods for ill-posed aperator equations. In addition, convergence results are obtained which apply when the perturbations vanish and the regularization parameter is chosen appropriately. We present a discrete constrained least squares approach for the $x * x = y$ and show its convergence in $L^p(0,1)$, $1 \leq p < \infty$, where the regularization is based on a prescribed bound for the total variation of admissible solutions. This approach includes the case of non-smooth solutions possessing jumps. Moreover, an adaption to the Sobolev space $H^1(0,1)$ and some remarks on monotone functions are added. A numerical case study concerning the determination of non-monotone smooth and non-smooth functions x from the autoconvolution equation with noisy data y is considered. We deal with a modification of the well-known ill-posed autoconvolution equation $x * x = y$ on a finite interval, we focus on solutions which are probability density functions and assume to have data of the autoconvolution coefficient k of the density function x, which we define as the quotient of the autoconvolution function $x * x$ and x itself. The corresponding inverse problem leads to the nonlinear integral equation $kx - x * x = 0$ of the third kind.

For a unitary highest weight module of a reductive Lie group, a reductive symmetric pair induces a holomorphic embedding of Hermitian symmetric spaces. We show that the multiplicity of irreducible representations occurring in the restriction is uniformly bounded. Furthermore, we show that the multiplicity is free for a one dimensional minimal type. the method here also establishes an analogous result for the tensor product of unitary highest weight modules, and also for finite dimensional representations of compact groups. Finally, we give an explicit branching formula of a holomorphic discrete series representation with respect to a semisimple symmetric pair. A new approach for constructing results of homogenous spaces with no compact quotients is show. For the unitary representation attached to the minimal nilpotent coadjoint orbit, we explicitly calculate its restriction to certain natural dual pairs in the unitary representation. We furthermore show how the results are compatible with the orbit method, in particular when viewing the minimal nilpotent orbit as belonging to the limit set of semisimple orbits.

By means of fractional calculus techniques we find explicit solutions of Volterra integral equations of second kind and fractional differential equations, involving Erdélyi-Kober fractional integrals or derivatives. We present a generalized Gronwall inequality with singularity. Weakly singular integral inequalities of Gronwall–Bellman type are established, which generalized some known weakly singular inequalities and can be used in the analysis of various

problems in the theory of certain classes of differential equations, integral equations and evolution equations.

Two classes of first order integro-differential equations with autoconvolution integral are studied generalizing an equation from the turbulence theory. General existence and stability theorems in a finite interval are show and the asymptotic behavior of the solutions at infinity is discussed. Two existence theorems for a general class of a model integral equation of autoconvolution type of the third kind are derived. Existence results are generalized to two types of autoconvolution equations of the third kind having free terms with nonzero values at $x = 0$ like the well-known Bernstein–Doetsch equation for the Jacobian theta zero functions.

The composition formula in pseudodifferential analysis extends that valid for differential operators. We develop the decomposition of the symboles functions as an integral superposition of homogenous one of degree lying on the complex line.