$$\|C_C(f)\|_{B_p(f)} \le \left(\frac{1+|\phi(0)|}{1-|\phi(0)|}\right)^{\frac{1}{p}} \|f\|_{B_p(f)}$$

remark (2-2-9)[1] : We have included the proof , al thought it is very elementary, because the change of variable at the right moment can improve the estimate of the norm

[where $\|C_\phi\| \le (B_p, B_p)$ is estimated by $\left(\frac{1+|\phi(0)|}{1-|\phi(0)|}\right)^{\frac{2}{p}}$] .

we are mainly concerned with analyzing when hankel aperators improve the condition of integrableility. To this purpose we need the following notion

Given $\phi D \to D$ analytic, let us consider the following image measure $A_\phi(B) = \int_{\phi^{-1}(B)} dA(z)$ for any Borel set $B \subset D \to (15)$

Theorem (2-2-10) [1] : Let $0 < p < 1$ and $\phi D \to D$ analytic. The following are equivalent

(a) $C_\phi : B_p(D) \to B_1(D)$

(b) A is a $\frac{z}{p}$ .Carleson measurer.

Proof :-

Since $\theta(t) = t \in b_{1+t}$ for any $t > 0$, we can apply theorem (2.1.12) to this case and get that (I) measure

$$\int_0 \frac{dA(w)}{|1-\phi(w)z|^{\frac{2+\sigma}{p}}} = O\left(\frac{1}{(1-|z|)^{\frac{t}{p}}}\right) ,$$

which in terms of the image measure A_ϕ, says

$$\int_0 \frac{dA_\phi(w)}{|1-wz|^{\frac{2+\sigma}{p}}} = O\left(\frac{1}{(1-|z|)^{\frac{t}{p}}}\right) . \qquad (16)$$

Alook at lemma (2.2.2) shows that (10) is equivalent to the fact that A_ϕ is a [illegible] . Car lesson measure .

Remark : 2.2.11)[1]: Observe that (16) for $t = 2$ and $p = 1$ gives the following interesting characterization for the norm of C_ϕ as an operator on $B_1(D)$

$$\|C_\phi\|_{\xi(B_1,B_1)} \approx \sup_{|z|<1} \int_0 |\Phi'_2(w)|^2 \, dA_\phi(w) \qquad (17)$$

where $\phi_2(w) = \frac{w-z}{1-\bar{z}w}$.

As before our results can used to describe composition operators acting on H_p spaces for $0 < p < 1$. Given $Q : D \to D$ analytic , write Q^* for the “function” (defined a.e.) consisting of its boundary limits. Let us consider the following image measure m_Q on $\bar{D}$ defined by

$$m_\phi(B) = m((\phi^*)^{-1}(B)) \qquad (18)$$

(m stands for the normalized lebesque measure on the unite circle)

Arguing as in theorem (2-2-10) it is easy to get the next result.

Theorem (2-2-12)[1]: Let be Dini weight such that $f \in b_1$, and $\phi D \to D$ - analytic $C_\phi : B_1(p) \to H^1$ if and only if m_ϕ is a $tp(t)-$ car leson measure.

Remark (2-2-13) [1] :-

It was pointed out that $C_\phi : H^p \to H^1$ is equivalent to $C_\phi : H^1 \to H^{1/p}$.