Dedication

To my family,,,

Acknowledgements

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Abstract

We show a discussion of the following questions: for a given real –valued function $f \in L^{\infty}(R^n)$ there can not be

(i) bi-Lipschitz homeomorphism $\varphi: R^n \to R^n$ such that the Jacobian determinant, $\det D\varphi = f$,

(ii) a Lipschitz or quasiconformal vector field with $\operatorname{div} u = f$,

(iii) for a given separated net $y \subset R^n$ a bi-Lipschitz map $\varphi: y \to Z^n$ except for $n = 1$ or if the Lipschitz condition is relaxed to a Hölder condition.

We show also an extent to certain metric measure spaces, a generalization of the theorem of Rademacher which asserts that : a real-valued Lipschitz function on $\mathbb{R}^n$ is differentiable almost everywhere with respect to Lebesgue measure and the blow ups of a real-valued Lipschitz function converge to a unique linear function.

الخـــــــــــلاصة

اوضحنا مناقشة الاسئلة التالية:

معطى دالة قيم حقيقية $f \in L^{\infty}(R^n)$ لا يوجد:

(i) هوميومورفيزم $\varphi: R^n \to R^n$ له ثنائية ليبتشز حيث ان محددة الجاكوبيات $D\varphi = f$ det.

(ii) حقل متجه ليبتشز او حافظ الزوايا المساوي له $div\ u = f$.

(iii) لأجل شبكة منفصلة معطى $y \subset R^n$ تطبيق ثنائية ليبتشز $\varphi: y \to \mathbb{Z}^n$ ، عدا عندما $n = 1$ او اذا كان شرط ليبتشز اقل شروطاً الى شرط هولدر.

اوضحنا ايضا التمدد الى فضاءات القياس المترية الخاصة. وتعميم مبرهنة رادماشر التي تقول: ان دالة القيمة الحقيقية لليبتشز على $\mathbb{R}$ تفاضلية في أي مكان على الاكثر بالنسبة لقياس لبيق. واظهار دالة القيمة الحقيقية لليبتشز تتقارب الى دالة خطية وحيدة.

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