

Dedication

To my family,,

Acknowledgements

I would like to express my deep sense of gratitude, and thanks to my supervisor Dr. Shawgy Hussein Abd Alla for useful comments, suggestions and constructive criticism.

My thanks are also due to my teachers: Mohammed Arafa ,Dr. Bakri Mirghani Ahmed and Dr. Mansour Elshiek Hassan for their help, and encouragement.

I am grateful to Sudan University of Science and Technology.

Finally I am grateful to my friends for their helpful remarks and full thanks to Tarig Rabbah for his excellent typing.

Abstract

We show a discussion of the following questions: for a given real n -valued function $f \in L^\infty(\mathbb{R}^n)$ there can not be

- (i) a bi-Lipschitz homeomorphism $\varphi: \mathbb{R}^n \rightarrow \mathbb{R}^n$ such that the Jacobian determinant, $\det D\varphi$,
- (ii) a Lipschitz or quasiconformal vector field with $\operatorname{div} u = f$,
- (iii) for a given separated net $y \subset \mathbb{R}^n$ a bi-Lipschitz map $\varphi: y \rightarrow \mathbb{Z}^n$ except for $n=1$ or if the Lipschitz condition is relaxed to a Hölder condition.

We show also an extent to certain metric measure spaces, a generalization of the theorem of Rademacher which asserts that: a real-valued Lipschitz function on \mathbb{R}^n is differentiable almost everywhere with respect to Lebesgue measure and the blow ups of a real-valued Lipschitz function converge to a unique linear function.

الخلاصة

اوضحنا مناقشة الاسئلة التالية:

معطى دالة قيم حقيقية $f \in L^1(\mathbb{R}^n)$ لا يوجد:

(i) هوميومورفيزم $\varphi: \mathbb{R}^n \rightarrow \mathbb{R}^n$ له ثنائية ليبتشز حيث ان محددة

الجاكوبيات $\det D\varphi = 1$.

(ii) حقل متجه ليبتشز او حافظ الزوايا المساوي له $\operatorname{div} u = f$.

(iii) لأجل شبكة منفصلة معطى $y \in \mathbb{R}^n$ تطبيق ثنائية ليبتشز $\varphi: \mathbb{R}^n \rightarrow \mathbb{R}^n$

، عدا عندما $n=1$ او اذا كان شرط ليبتشز اقل شروطاً الى شرط

هولدر.

اوضحنا ايضا التمدد الى فضاءات القياس المترية الخاصة. وتعميم مبرهنة رادماشر التي تقول: ان دالة القيمة الحقيقية لليبتشز على \mathbb{R}^n تفاضلية في

أي مكان على الاكثر بالنسبة لقياس ليبيق. واطهار دالة القيمة الحقيقية

لليبتشز تتقارب الى دالة خطية وحيدة.

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