## Dedication

To my father , mother , brothers and sisters

# Acknowledgement

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## ABSTRACT

**In wide class of weighted Bergman spaces , we construct inevitable non-cyclic elements ,these are then used to produce z- invariant sub spaces of index higher than one generated non trivial bilaterally invariant subspaces in a nti symmetricallyweighted Hilbert Spaces of sequences .We also study the boundary behavior of the Berezin transform on C*-algebra generated analytic Toeplitz operators on the Bergaman Spaces.**

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# Large Bergman Spaces and The Berezin Transform on the Toeplitz Algebra

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# List of symbols

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|---|---|---|---|
| **B(z)** | **:** | **Blashcke product** | 1 |
| $H^p$ | **:** | **Hardy Space** | 1 |
| $L^p$ | **:** | **Space of Lebesgue integrable functions** | 1 |
| | **:** | **Unit disc** | 1 |
| $M_{H^\infty}$ | **:** | **The maximal ideal space** | 1 |
| $H^\infty$ | **:** | **Commutative Banach algebra** | 1 |
| | **:** | **dual Banach Space** | 1 |
| | **:** | **Gelfand-transform** | 1 |
| $L_a^2$ | **:** | **The Bergman Space** | 2 |
| $B(L_a^2)$ | **:** | **The set of bounded operator on** $L_a^2$ | 2 |
| | **:** | **The Toeplitz algebra** | 2 |
| $B^p(D)$ | **:** | **The Bergman Space** | 3 |
| $\tau(X, \mathcal{A})$ | **:** | **The soft topology space** | 3 |
| **min** | **:** | **Minimum** | 4 |
| | **:** | **The Spectrum** | 6 |
| **exp** | **:** | **exponential** | 6 |
| **arg** | **:** | **argument** | |
| **sup** | **:** | **Supremum** | |
| **max** | **:** | **maximum** | |

# □□□□□□□

فى عائلة واسعة من فضاءات بيرجمان ذات الوزن تم بناء معكوس عناصرغير دوارة استخدمت في انتاج فضاءات جزئية ثابتة –z لها دليل اكبر من وا حد. ا ضافة ا لي ان هذه العناصر ولدت فضاءات جزئية ثابتة ثنائية الجانب وغ ير بديهي ة في ف ضاءات هـ لبرت ذات الوزن للمتتاليات قطرية التناظر .أيضا تمت درا سة حدود ية ت حو ل ب يرزن عـ لي جبر C* _ المولد بمؤثرات التبولتز التحليلية علي فضاءات بيرجمان .