

# Dedication

To My Family

## ***Acknowledgments***

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## **Abstract**

It is shown that Banach space of certain density contains an unconditional sequence of cardinality and a separable super reflexive Banach space admits a continuous surjection on each of its subspaces.

We introduce a new class of Banach spaces that are called sub B-convex and show that any separable sub B-convex Banach space may be almost isometrically embedded in a separable Banach space of the same cotype.

We show a method for producing Banach spaces with the Tsirelson property and a sub symmetric basis which enjoys the hereditarily approximative property.

We obtained a result on the existence of an uncomplemented subspace isomorphic to the Hilbert space and proved that for any pair of symmetric spaces there exists an intermediate space that is reflexive or weakly sequentially complete. We study the isometric version of the amalgamation property in Banach spaces.

## الخلاصة

لقد أوضحنا أن فضاء باناخ له كثافة خاصة ويحتوي متتالية غير مشروطة لها أساس، انعكاسي علوي وقابل للفصل يفسح مجالاً لاستمرار غامر لكل من فضاءاته الجزئية.

لقد وجدنا عائلة جديدة من فضاءات باناخ يقال لها محدب  $B$  الجزئي، وفضاء باناخ محدب  $B$  الجزئي قابل للفصل وربما يكون على الأكثر له تمديد حافظ للمسافة في فضاء باناخ القابل للفصل من نوع مصاحب مساوي.

لقد أوضحنا طريقة إنتاج فضاء باناخ له خاصية تسلسون و أساس متماثل جزئي يتمتع بخاصية التقريب الموروثة.

لقد اعطينا نتيجة وجود فضاء جزئي ليس له متمم حافظاً للمسافة على فضاء هلبرت، وأثبتنا لأي زوج من فضاءات متماثلة يوجد فضاء متوسط أي انعكاسي أو تام بتتابع ضعيف. ودرسنا نهج حفظ المسافة لخاصية المزج في فضاءات باناخ.

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