Sudan University of Science and Technology

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Numerical Solution of The Solitons Propagation IN Optical Fibers Using Sptit-Step Fourier method

الحل العددي لا نتشار الموجة في المنفردة في الألياف البصرية باستخدام طريقة الخطوة المنفصلة فورير

A Thesis Submitted in Partial Fulfillment of the Requirements for Degree of M. Sc. in Physics

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قال الله تعالي:

﴿وأنزلنا من السماء ماء بقدر فأسكناه في

الأرض وعلي ذهاب به لقادرونَ﴾

سورة المؤمنون ، ألآية: (81)

DEDICTION

I dedicate this work to:

- My parents who supported and encouraged me all of time.

-My wife and kids

- My brothers and sisters for their patience and continues support.
- My friends
- Staff at Sudan University of Science and

Technology

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Abstract

Despite the vast research in the notions of Maxwell's equations, the propagation equation of an electromagnetic wave and the notion of soliton allowed us to derive a model known in optics as the Nonlinear Schrodinger Equation (NLSE) which will take into consideration the dispersion and non-linearity effects. That most of the systems in this universe qualify to be nonlinear so that, the immediate objective of this research project is to study some phenomena that occur in optical fibers during the propagation of an ultra-short pulse "Electromagnetic wave", which are nonlinear.

One factor which has led to a numerical method is used in the analytical solution of such equation is difficult and sometimes impossible. As a result, the most appropriate tool to solve this type of problem, called the Split Step Fourier Method (SSFM). To this end, this process leads to the formation of optical solitons which retain shape during propagation, the compression mechanism fundamentally due to high order solution.

This a numerical simulation increases our understanding to describe the evolution of a pulse in an optical fiber while revealing the advantage of the coexistence of the two phenomena "dispersion and non-linearity of the medium".

المستخلص

علي الرغم من البحوث الكثيرة المنشورة في مفاهيم معادلات ماكسويل فان معادلة انتشار الموجة الكهرومغناطيسية وفكرة الموجة المنفردة سمحت باشتقاق نموذج معروف في علم البصريات باسم معادلة شرودنجرغير الخطية(NLSE) والتي تأخذ في الاعتبار التشت والتأثيرات اللا خطية. أن معظم الأنظمة في هذا الكون مؤهلة لتكون غيرخطية، لذا فإن الهدف المباشرمن هذا البحث هودراسة بعض الظواهر التي تحد ث في الألياف البصرية ثناء انتشار" الموجات الكهرومغناطيسية" ذا ت النبضا ت القصيرة جدًا، وهي غير الخطية.

أحد العوامل التي أدت إلى استخدام الطريقة العدية هو أن الحل التحليلي لمثل هذه المعادلات يصعب وأحيانا يكون مستحيل. ونتيجة لذلك فإن الأداة الأكثر ملاءمة لحل هذا النوع من المسائل، تسمي "طريقة فورير- الخطوة المنفصلة" (SSFM).

تحقيقا لهذه الغاية، تؤدي تكوين موجة منفردة ضوئية والتي تحتفظ بالشكل اثناء الانتشار، ميكانيكية الضغط تعتمد أساساعلي أن الموجات المنفردة ب عالية الت تيب. تزيد هذه المحاكاة العددية من الفهم لوصف تطور النبض في الالياف الضوئية مع الكشف عن ميزة الترابط بين الظاهرتين " التشتت واللاخطية للوسط".

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List of Symbols /Abbreviations

c	Speed of light in a vacuum.
D	Dispersion parameter.
n	Index of refraction in any medium
n_{NL}	Nonlinear refractive index.
v_ϕ	Phase speed.
v_g	Group speed.
z	propagation distance.
t	Time.
α	Linear attenuation coefficient.
β, k	Propagation constant (wave vector).
β_i	Derivative of order i of the propagation constant β with respect to ω.
γ	Coefficient of non-linearity.
λ	Wavelength in a medium.
τ	Time delayed.
ω	Pulse of the wave.
$\hat{D}$	Linearity operator of the SSF method.
$\hat{N}$	Nonlinearity operator in the SSF method.
GVD	Group- Velocity Dispersion.
NLSE	Nonlinear Schrödinger Equation (Nonlinear Schrödinger Equation).
SMF	Single Mode Fiber.
SPM	Self-phase modulation.
SSF	Split- Step Fourier (iterative Fourier).

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Chapter one

Introduction

1.1 Introduction

A system is said to be nonlinear if its output is not linearly proportional to input; on the basis of this definition, one can say that most of the systems in this universe qualify to be nonlinear. The science which deals with nonlinear systems is known as nonlinear science. In the past few decades, nonlinear science has emerged as a tool to study all those complex natural phenomena which cannot be studied completely by linear science. It is not a new subject or branch of science, although it delivers a whole set of fundamentally new ideas and surprising results. Nonlinear science qualifies to be a revolution due to its wide scope and coverage because it finds applications in almost all branches of science such as plasma physics, hydrodynamics, mechanics, biology, chemistry etc. Hence, due to feasibility of nonlinear science on system of every scale, it is possible to study same nonlinear phenomena in very distinct way, with the corresponding experimental tools.

The study of nonlinear system means to study the nonlinearity present in it.

Nonlinearity plays an important role in dynamics of various physical phenomena [1, 2], such as in electronic circuits, laser physics, nonlinear mechanical vibrations, population dynamics, astrophysics, plasma physics, chemical reactions, nonlinear wave motions, heartbeat, nonlinear diffusion, time-delay processes etc. Nonlinearity in any system make the system more complex and it became very difficult to study. A small disturbance induced in nonlinear system even by little variation in initial conditions can results into big difference in behavior in time evolution of the system. Hence a nonlinear system exhibits a sensitive dependence on initial conditions. However, linear systems are generally gradual, smooth, and regular, common example of linear system are slowly flowing streams, engines working at low power, slowly reacting chemicals, etc. Any system with large input generally shows nonlinear behavior. For example, the behavior of a spring is linear for small displacement, but if the initial displacement is large the spring shows nonlinear behavior.

In similar way, for small initial displacement simple pendulum behaves as linear system however as the initial displacement become large enough, its motion become nonlinear.

The nonlinear system which is to be studied is described by a nonlinear evolution equation (NLEE). These NLEE's are having complex structures due to linear and nonlinear effects.

The non-linearity is linked to the thresholds of excitation by an electric field, to multi-stability, to hysteresis, to phenomena which are modified qualitatively as the excitations occur, for example the propagation of a wave moving in a medium is determined by the properties of the medium. Nonlinearity leads to the distortion of the shape of large amplitude waves, for example, in turbulence. However, there is another source of distortion: the dispersion of a wave. The influence of these effects was a limit to the transmission of Alexandre Bell's photophone. Faced with the same problem, Tyndall John demonstrates that light can be guided, this experiment is currently used in optical fibers based on the "principle of total reflection". The invention of the laser in 1958 [Mah et al89] relaunched the transmission of information in waveguides.

The wave propagation in real mediums, for example a rolling wave in an optical fiber is distorted by dispersive and non-linear effects. The promising quick fix for these drawbacks is the "Soliton" concept, first discovered by Scotsman John Scott Russell in 1885.

More than 100 years ago, the mathematical equations describing the solitary waves were solved, among these there are the NLSE non-linear Schrodinger equations, difficult to solve analytically, in this case the resolution requires a numerical approach.

1.2 Statement of the problem:

In recent decades, soliton theory has become a very active area of research due to its importance in many branches of physics such as nonlinear optics. In general, the concept of soliton is always linked to nonlinear partial differential equations, whose analytical solution is difficult, for this reason it is necessary to use numerical methods. Which, led to study a model the propagation in a nonlinear dispersive medium.

This modeling leads to a nonlinear partial differential equation known optically as nonlinear Schrödinger, which requires numerical solution.

For the simulation used a method which is exclusively used to solve this kind of problems. An application will be made for the study of propagation in a single mono mode optical fiber. To this end two cases were investigated use of an input pulse in fundamental soliton form and chip pulse form.

1.3 Aims:

The objectives of this research are:

a) Obtaining mathematical modeling of study, the propagation in a nonlinear dispersive medium by the nonlinear Schrödinger Equation (NLSE).

b) Presenting numerical approach for mathematical modeling to simulate the NLSE to analyze the properties of optical solitons.

c) An applying the modest to study of propagation in a single mono mode optical fiber

d) Investigating the two cases: use of an input pulse in fundamental soliton form then in the form of chip pulse.

1.4 Methodology :

Many physical phenomena can be modelled by partial differential equations, but – apart from some very specific cases – it is generally not possible to write down the solution to these problems in closed form. In order to understand the behavior of the solution, it is thus often necessary to construct an approximation via a numerical solution.

this research concerns on a numerical simulation to describe the evolution of a pulse in an optical fiber while revealing the advantage of the coexistence of the two phenomena "dispersion and non-linearity of the medium by using Split Step Fourier Method" SSFM.

1.5 Questions:

1. What is the soliton optics?
2. How to solve nonlinear partial differential equations in solitons optics and how to apply them?
3. How these solutions analyzed?
4. How these solutions are plotted in graphs?

1.6 Thesis lay out:

chapter one introduce the study. After a few reminders on non-linearity,

we present in the chapter two the Maxwell equations to find a propagation equation. Then the process of propagation of an impulse in the dispersive medium, and not linear will be studied and at the end of the chapter we presented a small recall on the optical solitons, and how they are formed. As well as the derivation of the NLSE non-linear Schrödinger equation system.

In Chapter three, after a little reminder on optical fibers, devote this part to the methods of numerical resolution of the non-linear Schrodinger equation NLSE. For which used the so-called "Fourier fractional" numerical method (Split-Step Fourier in English), based on the fast Fourier transform algorithm. Therefore, the optical fiber will be cut into thin slices. Chapter four is devoted to simulations of the propagation of the soliton in an optical fiber (single mode). We will complete our work with an interpretation of the results. Then a conclusion in Chapter five.

Chapter two

Literature Review

2.1. Introduction

Non-linear effects are those that do not occur directly proportional to the action. This is the case with most real-world effects, and the reason for the difficulty in reproducing information faithfully by analog techniques.

This chapter is devoted to reminders of a few concepts related to non-linear optics, namely the notion of Kerr effect, dispersion, soliton, and the equation governing its propagation in a non-linear medium, emphasizing the importance of the dispersion compromise. Non-linearity of the medium.

2.2. The origin of non-linear optics

Nonlinear optics is the discipline of physics in which the density of the electrical polarization of the medium is studied as a nonlinear function of the electromagnetic field of light. Being a vast field of research activity on the propagation of electromagnetic waves, the non-linear interaction between light and matter tracks to a wide spectrum of phenomena, such as optical frequency conversion, optical solitons, phase conjugation and Raman scattering. In addition, many of the analytical tools used in non-linear studies of optics are general in nature, such as perturbation techniques and symmetry considerations, and may just as easily be applied to other disciplines in nonlinear dynamics. [1]

2.3 Electromagnetic properties of the medium

2.3.1. Wave

A wave is the propagation of a disturbance producing in its passage a variation of the physical properties of the medium, it is important to note that the wave results in a transport of energy and not of matter [2].

1.3.2. Electromagnetic wave:

As the name suggests, an electromagnetic wave is an electrical and magnetic wave. It is broken down into an electric field and a magnetic field. It takes place without transport of the material and these fields are called disturbances. These two disturbances oscillate at the same time but in two perpendicular planes (Figure 2.1).

An electromagnetic wave and a medium interact through three parameters [3]: Conductivity $\boldsymbol{\sigma}$, and Electrical permittivity $\boldsymbol{\varepsilon}$ and magnetic permeability $\boldsymbol{\mu}$. These parameters appear clearly in the Maxwell equations.

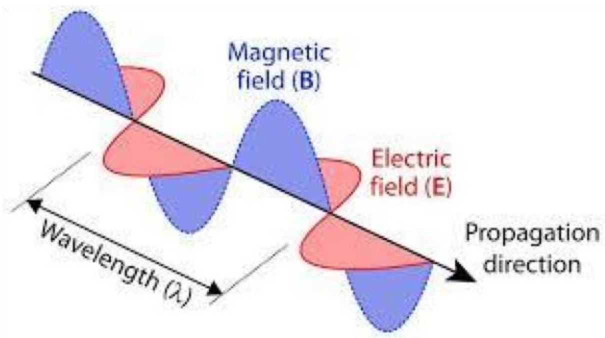

Figure 2.1. Electromagnetic wave

2.4. Nonlinear propagation equation:

2.4.1. Maxwell's equations:

If the material is insulating (non-conductive), linear, homogeneous, isotropic, and non-magnetic.

So, the dielectric constant (ε) is independent of orientation or location, so ε is treated as a scalar quantity. Under such conditions Maxwell gathered all the ideas on electromagnetic waves (their description and their interactions) in these four equations [3].

$$\vec{\nabla}.\vec{D} = 0 \tag{2.1}$$
$$\vec{\nabla}.\vec{B} = 0 \tag{2.2}$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.3}$$
$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{2.4}$$

○ $\vec{E}$is the electric field, (Volt / m). ○ $D^{\to\to}$ electrical displacement (or induction) (Coulomb / m²). ○ $B^{\to}$ the magnetic field (or induction), (Webber / m²).

○ $H^{\to}$ magnetic excitation (or field). (Ampere / m).

In a dielectric medium, the response of the medium to the excitation $E^{\to}$ and $^{\to}$ is given by:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \tag{2.5}$$
$$\vec{B} = \mu_0 \vec{H} + \vec{M} \tag{2.6}$$

Where μ_0 is the permeability of the vacuum and $P^{\to}$ is the electric polarization,$\vec{M}$ is the Magnetic polarization.

2.4-2. Electric field polarization

The polarization created by a light wave passing through a material is written in the form:

$$\vec{P} = \overrightarrow{p^1} + \overrightarrow{p^{(1)}} + \overrightarrow{p^{(2)}} + \overrightarrow{p^{(3)}} + \cdots \ldots \overrightarrow{p^{(i)}}$$

$\overrightarrow{p^{(i)}}$ is the order polarization i in powers of the electric field. More precisely, we can show that for i waves of frequencies $\omega_1, \cdots, \omega_i$ we note the amplitudes $\vec{E}\ (\omega_i)$, the polarization is written in the form:

$$\overrightarrow{p^{(i)}}(\omega_1 + \cdots + \omega_i) = \varepsilon_0 \chi^{(i)}(\omega_1 + \cdots + \omega_i)\vec{E}(\omega_1) + \cdots + \vec{E}(\omega_i)$$

Where ε_0 is the electrical permittivity of the vacuum, and $\chi^{(i)}(\omega_1, \cdots, \omega_i)$ is the electrical susceptibility tensor of order i which depends on the material used. This last expression shows that the wave creates a frequency different from the waves initially present.

An interpretation of the non-linearities appearing in the polarization comes from the microscopic aspect of the material. Each atom of a dielectric material is surrounded by an electronic cloud which can deform under the action of $\vec{E}$, which creates an electric dipole.

This dipole, for a small deformation, is proportional $\vec{E}$, but if the deformation is too large, this is no longer the case. The sum of all the dipoles is then the polarization introduced above, hence its non-linearity. Similar reasoning can be used in the case of metals and plasmas: the free electrons undergo, from the excitatory field, a Lorentz force depending on the speed of the electrons, and therefore on the polarization. Thus, these media can also exhibit non-linear effects. [18]

2.4.3. Equation of propagation of an electromagnetic wave:

In order to study the effects of the nonlinearities of a medium on the propagation of an electromagnetic wave, we first develop a simple wave equation suitable for a large class of important materials (dielectrics). To do this, we start by doing the rotational of the equation (2.3), we get [5]

$$\vec{\nabla} X (\vec{\nabla} \times \vec{E}) = \vec{\nabla} X \left(-\frac{\partial \vec{B}}{\partial t} \right) \qquad (2.7)$$

Knowing that

$$\vec{\nabla} X (\vec{\nabla} \times \vec{A}) = \vec{\nabla}. (\vec{\nabla}. \vec{A}) - \nabla^2 \vec{A} \qquad (2.8)$$

So, we have:

$$\vec{\nabla}X(\vec{\nabla} \times \vec{E}) = \vec{\nabla}.(\vec{\nabla}.\vec{E}) - \nabla^2\vec{E} \qquad (2.9)$$

In non-conductive materials the product

$$\vec{\nabla}.\vec{E} = 0 \qquad (2.10)$$

So, equation (2.7) becomes:

$$-\nabla^2\vec{E} = \vec{\nabla}X\left(\frac{\partial\vec{B}}{\partial t}\right) = \mu_0\frac{\partial}{\partial t}\vec{\nabla}X\vec{H} \qquad (2.11)$$

$$-\mu_0\frac{\partial}{\partial t}\left(\frac{\partial\vec{D}}{\partial t}\right) = \mu_0\varepsilon_0\left(\frac{\partial^2\vec{E}}{\partial t^2} + \frac{\partial^2\vec{P}}{\partial t^2}\right) \qquad (2.12)$$

The equation (2.11) is then written

$$\nabla^2\vec{E} - \mu_0\varepsilon_0\left(\frac{\partial^2\vec{E}}{\partial t^2}\right) = \mu_0\varepsilon_0\left(\frac{\partial^2\vec{P}}{\partial t^2}\right) = \mu_0\varepsilon_0\left(\frac{\partial^2\overrightarrow{P_L}}{\partial t^2} + \frac{\partial^2\overrightarrow{P_{NL}^{(2)}}}{\partial t^2} + \frac{\partial^2\overrightarrow{P_{NL}^{(3)}}}{\partial t^2}\right) \qquad (2.13)$$

With:

$$\frac{\partial^2\vec{P}_{NL}^{(2)}}{\partial t^2} + \frac{\partial^2\vec{P}_{NL}^{(3)}}{\partial t^2} \Leftrightarrow \frac{\partial^2\vec{P}^{NL}}{\partial t^2} \qquad (2.14)$$

2.4. The speed of propagation of electromagnetic waves:

A wave is a disturbance which moves in a medium. It is possible to associate two wave speeds with it, namely the phase speed and the group speed which sometimes are not equal.

2. **5.1 . Phase speed and group speed:**

The phase speed: let be a wave $U = U_0 \cos(\omega t - kz + \phi_0)$, U is the quantity which propagates, then the phase speed represents the speed of displacement of the wave plane, therefore the speed of propagation of the phase given by: $\phi = \omega t - kz$

Either: $v_\varphi = (dz/dt)_\phi$=c Where $v_\varphi = \omega k$
Where: ω is the pulse of the wave and k is the number of waves, ***c*** the speed of light in a vacuum.

Group speed: Group speed is generally presented as the speed at which energy or information is transported by a wave.

$$v_g = \frac{\partial \omega}{\partial k} \qquad (2.15)$$

2.6. Refractive index:

Refraction index often noted n, it is defined as the ratio of the speed of light in a vacuum to the speed of light in this medium [.3]

$$n = \frac{c}{v} \qquad (2.16)$$

So is a dimensionless quantity which characterizes the medium, it depends on the measurement wavelength but also depends on the environment (pressure and temperature).

In a material environment, the speed of light cannot exceed that in a vacuum, so a refractive index is always greater than or equal to 1 [4].

Table 1: Refractive index of some substances.

Matter	refractive index	Matter	refractive index
Air	1	Ruby	1.78
water	1.33	Diamond	2.46
Benzene	1.501	Sapphire	1.77
Quartz	1.55	Glass	1.5
Polystyrene	1.2	Pure Alcohol	1.32
Acetone	1.3	Glycerin	1.47

The refractive index plays an important role in extinction:

• deviation of the direction of the wave front (lens effect)
• dissipation (absorption)

2.7. Dispersion of a physical medium:

The speed of propagation of a wave in a medium can depend on its wavelength, hence a differential propagation phenomenon leading to the dispersion relationship.

Optics is a special case of this phenomenon for which the speed in a transparent medium is generally different from that in a vacuum.

As a rule, a pulse with several components contains different frequencies whose speed is not the same, which results in the distortion of the pulse.

The propagation of a pulse in a dispersive medium is a function of the order of dispersion, of the latter which is linked to the propagation constant β (ω), It is a Taylor development determined at the central frequency of the signal ω_0 [1].

2.7.1. The dispersion parameters:

Mathematically, the dispersion appears in the Taylor series development of the propagation constant around the central ω_0 pulse of the pulse [4,13].

$$K(\omega) = \beta(\omega) = n(\omega)\frac{\omega}{c}$$
$$= \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}\,(\omega - \omega_0)^2\beta_2 + \frac{1}{6}\,(\omega - \omega_0)^3\beta_3$$
$$+ \frac{1}{24}\,(\omega - \omega_0)^4(\omega - \omega_0)\beta_4 \qquad (2.16)$$

With$\beta_0 = \frac{n(\omega_0)}{\omega_0 c}$ is the propagation constant where n ($\omega_0 c$) is the refractive index at ω_0.

β_1 is the inverse of the group speed of the wave.

2.7.1.1. First order dispersion:

In the theory of first-order dispersion by the propagation constant

β (ω) is equal to:

$$\beta(\omega) = n(\omega)\frac{\omega}{c} = \beta_0 + (\omega - \omega_0)\beta_1 \qquad (2.17)$$

With: $\beta_0 = \frac{\omega_0}{v_\varphi \omega_0}$ And $\beta_1 = \left[\frac{d\beta}{d\omega}\right]_{\omega_0} = \frac{1}{v_g(\omega_0)}$

Consider an excitement in the form:

$$(z, t) = (z, t) - j(\omega 0 t - \beta(\omega 0) z)$$

where $A\,(z, t)$ is the complex amplitude which can be determined from the Fourier transform:

$$A(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} A_0(\acute{t})\ d\acute{t} \int_{-\infty}^{+\infty} \exp\{-j(t-\acute{t}) - j\beta_1 z(\omega - \omega_0)\} \qquad (2.19)$$

$A_0(t')$ is the complex amplitude at $z = 0$ verifying the relation:

$$(z = 0, t) = U_0 = A_0 e - (\omega_0 t) \qquad (2.20)$$

Which can show from equation (2.19) that the complex amplitude $A\,(z, t)$ follows the following evolution equation:

$$\frac{\partial A}{\partial z} + \frac{1}{v_g}\frac{\partial A}{\partial t} \qquad (2.21)$$

The solution of this last equation is:

$$A(z,t) = A\left(t - \frac{z}{v_g}\right) \qquad (2.22)$$

Therefore, the complex amplitude or the wave packet $A\,(z, t)$ moves in the space of the first order dispersion medium with a constant speed equal v_g around the point $\beta\,(\omega)$ as a single whole. without change of form. [1]

2.7.1.2. Second order dispersion:

The unchanged shape of the wave packet in first-order dispersion theory is not exactly exact, but it is approximated. Now consider dispersal as a real consequence. In this case, the second term must be introduced into the propagation constant. [14], [15]

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}\,(\omega - \omega_0)^2\beta_2 \qquad (2.23)$$

With β_2 being the speed of dispersion equal to:

$$\beta_2 = \left[\frac{d^2\beta}{d\omega^2}\right]_{\omega_0} = \frac{d}{d\omega}\left[\frac{1}{v_g(\omega_0)}\right]_{\omega_0} \qquad (2.24)$$

Let us suppose that the linear medium considered is subjected to the excitation of the electric field:

$$(z,t) = E_0(z,t)e^{-j(\omega 0t - \beta(\omega 0)z)} \qquad (2.25)$$

After some calculations, we find that the complex amplitude $E\,(z, t)$ satisfies the following differential equation:

$$j\frac{\partial E}{\partial z} + \frac{1}{2}\beta_2\frac{\partial^2 E}{\partial t^2} \qquad (2.26)$$

It should be noted that the equation (2.26) which describes the propagation of a pulse in a medium characterized by a second order dispersion, resembles the differential equation which governs the propagation of heat. So, the presence of the term dispersion of the second order $\frac{1}{2}\beta_2\frac{\partial^2 E}{\partial t^2}$ in equation

(2.26) acts as a type of complex term generalized diffusion for the envelope of the pulse $E\,(z, t)$ in the time domain.

Note that the dispersion of group speed is responsible for the appearance of several negative effects such as the enlargement effect which reduces the performance of transmission by optical fibers.
If β_2 is zero, the development of $\beta\,(\omega)$ must be pushed beyond the second order and a term $\beta_3 = \left[\frac{d^3\beta}{d\omega^3}\right]_{\omega 0}$ hence the dispersion of the medium is third order [1].

With regard to the influence of the dispersion effect, it can be said that a pulse propagating in a physical medium is thus deformed by the dispersion effect because its different spectral components do not undergo the same phase shift. This leads to an enlargement which leads to recovery of successive pulses leading to a detection error on reception.

Therefore, natural chromatic dispersion is considered to be a major problem which limits the performance of optical communications systems [1].

2.7.1.3. Compromise: dispersion-non-linearity:

To overcome the problem of dispersion, which is not only an experimental fact but also a consequence of general physical principles, theorists have proposed a solution capable of solving the problem: this new concept is "solitons". These are nonlinear excitations localized in space-time, and which propagate in the physical systems while retaining their original form almost indefinitely with a rigorous compensation between two antagonistic and inevitable characteristics: nonlinearity and dispersion.

2.8. Propagation of a wave in a dispersive medium:

A physical signal, of finite energy, decomposes as a sum of harmonic signals [2], we speak then of "packet of waves". So, we will be able to write:

$$A(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \tilde{A}\ (\omega) e^{j(kx-\omega(k)t)}\, dk \qquad (2.27)$$

The figure below represents, at t = 0, a "packet" of the harmonic waves of neighboring pulses as a function of x.

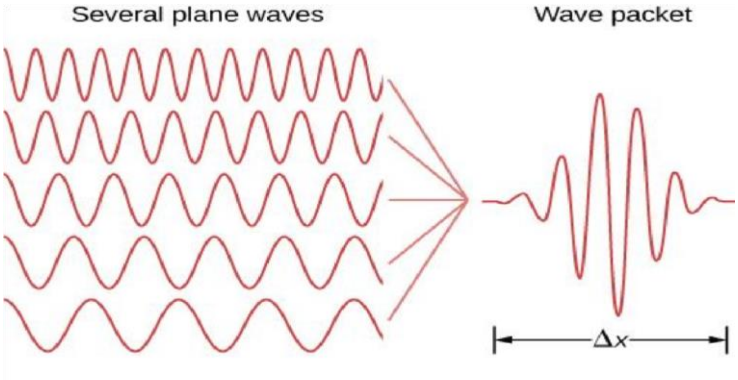

Figure 2.2. Envelope of a wave packet

To propagate a wave in a dispersive medium, we start by decomposing it into a sum of harmonic waves at time t = 0, then we propagate each harmonic component with its own phase speed, finally we reconstruct the wave by summing its components harmonics at the desired time (t).

In general, there is a distortion of the "wave packet" because the harmonic components do not sum up in the same way over time: this is often manifested by a spatial spread of the "wave packet". [4]

The figure below illustrates the propagation of a packet of waves:

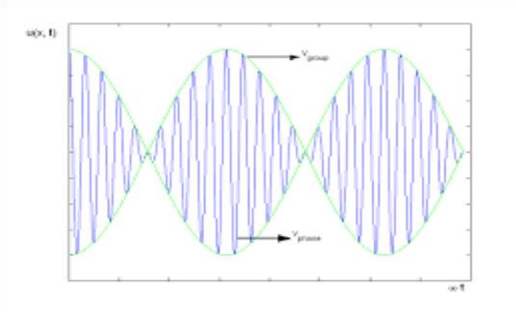

Figure (2.3): Propagation of the wave packet.

Indeed, when a wave propagates in a dispersive medium, the various frequency components of the wave propagate at different speeds, creating a temporal spread of the wave on arrival. This is called group speed dispersion (Group Velocity dispersion (GVD)) or chromatic dispersion [5].

2.8.1. Chromatic dispersion:

Chromatic dispersion is expressed in $ps/\ (nm\ \cdot km)$; it characterizes the spread of the signal linked to its spectral width (two different wavelengths do not propagate at exactly the same speed). This dispersion depends on the wavelength considered and results from the sum of two effects: the dispersion specific to the material, and the dispersion of the guide.

2.8.1.1. Dispersion due to the material:

The dispersion phenomenon results from a sensitivity of the medium to the frequency of the wave at the microscopic level.

2.8.1.2. Dispersion due to guidance:

This case of dispersion results from the wave nature of the wave and the desire to confine the wave in a limited volume so as to impose on the wave a direction of propagation.

2.9. Non-linearity and solitons

When a material medium is placed in the presence of an electric field $\vec{E}$, it is likely to modify this field by creating a polarization $\vec{P}$. This response of the material to the excitation can depend on the field $\vec{E}$in different ways. The nonlinear optic groups the set of optical phenomena having a nonlinear response with respect to this electric field, that is to say a response not proportional to E.

In the presence of an electromagnetic wave from the optical domain (wavelength on the order of 1000 nm), in other words light, many materials are transparent, and some of them are non-linear, c is why nonlinear optics is possible.

The main differences with linear optics are the possibilities of modifying the frequency of the wave or of making two waves interact between them via the material.

These very specific properties can only appear with strong light waves. This is why non-linear optics experiments could not be carried out until the 1960s thanks to the appearance of laser technology.

2.9.1. Nonlinear optical susceptibility:

The optical responses, including nonlinear ones are described as [7].

$$P(t) = \varepsilon\{\chi^{(1)}E(t) + \chi^{(2)}E^{(2)}(t) + \chi^{(3)}E^{(3)}(t) + \cdots .\}$$

$$= {}^{(1)} + {}^{(2)} + {}^{(3)} + \cdots \qquad (2.28)$$

Where we have expressed the polarization p (t) as a series of powers in the field strength E (t). The quantities χ (1), χ (2), χ (3) are called susceptibilities; χ (1) is a linear susceptibility and χ (2), χ (3) are called nonlinear second and third order.

For a typical solid-state system, $\chi^{(1)}$ is of the order of unity while $\chi^{(2)}$ is of the order of $1 / E_{at}$, and $\chi^{(3)}$ is of the order of $1 / E_{at}^2$. $E_{at} = e/(4\pi\varepsilon_0 a_0^2)$ is the atomic characteristic of the electric field.

$a_0 = 4\pi\varepsilon_0\hbar^2/me^2$ is the Bohr radius of the hydrogen atom. Explicitly [7].

$$\chi^{(2)} \simeq 1.94 \times 10^{-12} m / v$$

$$\chi^{(3)} \simeq 3.78 \times 10^{-24} m^2 / v^2$$

The formal expression for third order polarization is as follows:

$(\omega_0; \omega_n; \omega_m)$

$$= \varepsilon_0 D \sum_{JKL} {}^{(3)} (\omega_0 + \omega_n + \omega_m, \omega_0, \omega_n, \omega_m)\, X\, (\omega_0)(\omega_n) E_l(\omega_m) \qquad (2.29)$$

Where i, j, k, l refer to the Cartesian components of the fields and the degeneration factor D represents the number of distinct permutations of the frequencies ω_0, ω_n, ω_m.

$\chi^{(j)}$ j(= 1, 2,): Is the susceptibility of the j-th order. The linear susceptibility $\chi^{(1)}$ contributes to the linear refractive index $\bar{n}_0$ (real and imaginary parts; the imaginary part being responsible for attenuation). The second order susceptibility $\chi^{(2)}$ is responsible for the second harmonic generation. For SiO_2 the second order nonlinear effect is negligible since SiO_2 has inversion symmetry. This is why optical fibers do not exhibit second order nonlinear effects.

The third order susceptibility $\chi^{(3)}$ is responsible for the lower order nonlinear effects in optical fibers. Generally, it manifests itself by a modification of the refractive index with optical power or by a diffusion phenomenon. It is linked to the optical Kerr effect, four-wave mixing, third harmonic generation, stimulated Raman scattering, etc [9].

Assuming a linear polarization of the propagating light and neglecting the tensor character of $\chi_{ijkl}^{(3)}$, we find the following relation for nonlinear polarization:

$$(\omega) = 3\varepsilon_0{}^{(1)}(\omega = \omega + \omega - \omega)|E(\omega)|^2E(\omega) \quad (2.30)$$

The total polarization, which consists of linear and non-linear parts, is written as follows:

$$P(\omega) = 3\varepsilon_0\chi^{(1)}E(\omega) + 3\varepsilon_0\chi^{(3)}|E(\omega)|^2E(\omega) = \varepsilon_0\chi_{eff}E(\omega) \quad (2.31)$$

The actual susceptibility depends on the field as follows

$$\chi_{eff} = {}^{(1)} + 3^{(3)}|E(\omega)|^2 \quad (2.32)$$

And it is related to the refractive index like:

$$\bar{n} = 1 + {}^{(3)} \equiv \bar{n}_0 + \bar{n}_2 I \quad (2.33)$$

Here, I indicate *I* the mean intensity over time of the optical field. We start by discussing the main features of nonlinear effects

2.9.2 Kerr effect:

It was discovered by J. Kerr in 1875. He discovered that a transparent liquid becomes doubly refractory (birefringent) when placed in a strong electric field.

2.9.2.1. The optical Kerr effect:

The optical Kerr effect corresponds to a birefringence induced by an electric field varying at optical frequencies, proportional to the square of this field. It was observed for the first time, for molecules with directions of greater polarizability, by the French physicists Guy Mayer and François Gires in 1963. A sufficient light intensity was obtained thanks to a triggered laser. [7]

In general, the Kerr effect describes situations where the refractive index depends on the electric field as follows:

$$\bar{n}(n, |E|^2) = \bar{n}_0(\omega) + \bar{n}_2(\omega)|E|^2 \qquad (2.33)$$

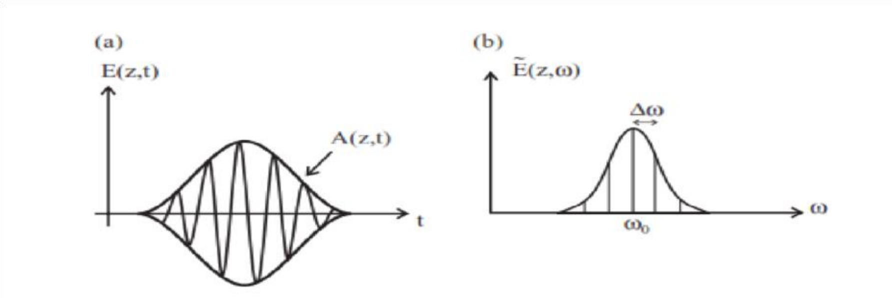

Figure (2.4). Illustration of the propagation-modulated pulse (a) and its spectrum (b) Here, $\bar{n}_2$ is known as the Kerr coefficient and it is related to susceptibility.

For a wave linearly polarized in the direction x for silica, its value is approximately $1.3 \times 10^{-22}\ m^2 / V^2$.

The Kerr effect comes from the non-harmonic movement of electrons bound in molecules. Consequently, it is a rapid effect, the response time being of the order of $10^{-15} s$.

2.9.3. Stimulated Raman scattering:

Diffusion phenomena are responsible for the Raman and Brillouin effects. During these diffusions, the energy of the optical field is transferred to local phonons: in Raman diffusing optical phonons are generated while in Brillouin the acoustic phonons are scattered. [8]

Because of the non-linearity and the dispersion, the solitons are existing, so what is a soliton.

2.10. The solitons

A solitary wave is a wave that propagates by ignoring the classical laws of energy dispersion. As a rule, this wave is strong enough to excite a non-linear effect which will compensate for the normal energy dispersion effect.by The solution of nonlinear partial differential equation which represent a solitary wave, which has permanent form.

Energy, through the nonlinear phenomenon, creates a potential well in its propagation medium. This well traps energy and prevents it from dispersing.

- It is localized within a region
- It does not disperse
- It does not obey the superposition principle [3].

Long after Scott-Russel's observation in 1850 of these spectacular phenomena by a wave in a canal, we realized that these energy packets could be subjected to forces which give them material properties. Hence the name of the word soliton stems from the geek (on), meaning particle. [10] The name was coined by Zabusky and Norman Kruskal (ZK) due to the particle-like behavior of the pulses they discovered.

2.10.1. The classes of solitons:

A rough description of a classical soliton is that of a solitary wave which shows great stability in collision with other solitary waves. There are several ways to classify solitons [11]. There are topological and non-topological solitons. Regardless of the topological nature of the solitons. All solitons can be divided into two groups considering their profile: permanent and time dependent. The third way to classify solitons agrees with the nonlinear equations which describe their evolution.

Here we discuss some classification of solitons [12].

2.10.1.1. Classification of solitons in bases of shape

a) bell soliton

The soliton solution of KdV equation have a bell shape and a low frequency soliton. This soliton referred to as non-topological solitons, the soliton solution of (NLS) equation have a bell-shaped hyperbolic secant envelope modulated a harmonic (Cosine) wave. This solution does not depend on the amplitude and high frequency soliton. b) Kink soliton

The solutions of (SC) equation are called kink or anti-kink solitons, and velocity does not depend on the wave amplitude. This soliton referred to as topological solitons.

The magnetic spins rotate from say spin down in one domain to spin up in the adjacent domain. The transition region between down and up is called Bloch wall.

c) breather soliton

Discrete breathers (DB), also known as intrinsic localized modes, or nonlinear localized excitations, are an important new phenomenon in physics, with potential applications of sufficient significance to rival or surpass the Soliton of integrable of partial differential equations.

2.10.1.2. Bright temporal envelope solitons:

Light pulses of a certain shape and energy that can propagate unchanged over large distances.

2.10.1.3. Dark temporal envelope solitons:

Pulses of "darkness" in a continuous wave, where the pulses have a certain shape and have propagation properties similar to bright solitons.

2.10.1.4. Spatial solitons:

Beams or pulses with continuous wave, with a transverse extent of the beam passing through the refractive index.

Changes due to Kerr optics can compensate for the direction of the beam. Optically, the induced change in refractive index works as an efficient light guide. [12.13]

2.10.1.5. Optical soliton:

The soliton arises from a balance between two compensating effects. In the case of an optical soliton, these effects are essentially phase self-modulation and abnormal dispersion. Imagine an electromagnetic pulse propagating. The phase auto-modulation shifts towards the lowest frequencies (therefore the longest wavelengths) the edge of the pulse, and conversely shifts towards the short wavelengths the lag of the pulse. The abnormal dispersion shifts the high frequencies towards the front of the pulse, the low frequencies falling behind (red propagates here less quickly than blue, unlike in the case of normal dispersion). Thus, between the phase self-modulation which acts on the spectrum of the pulse tends to make the front redder and the train more blue, and the abnormal dispersion which acts on the time profile of the pulse tends to make the front more blue and drags more red, the impulse finds a form that balances the two effects. Theory shows that it is a hyperbolic secant form [11].

This is the class of solitons on which we will focus in this work.

2.10.2 Formation of optical solitons

A light pulse is a bundle of electromagnetic waves of finite spectrum. Each of its spectral components propagates with a different group velocity, and as a result, the energy of the pulse extends over time along its propagation.

When the chromatic dispersion is negative (D <0), speak of the normal dispersion regime. In this case, long wavelengths (red frequencies) propagate faster than short wavelengths (blue frequencies). On the contrary, in socalled abnormal dispersion regime, the chromatic dispersion is positive (D> 0). Long wavelengths propagate more slowly than shorter wavelengths. In both cases, the pulse undergoes a temporal enlargement of its envelope.

The zero of the chromatic dispersion is around 1312 nm. For wavelengths less than this value, the dispersion is positive (normal regime). It is negative (abnormal regime) for longer wavelengths [11,14].

In the absence of nonlinear effects, the distortion of the optical pulse is mainly caused by chromatic dispersion and can be eliminated by the technique of dispersion compensation.

In reality, the response of the optical medium is not linear, because the refractive index depends on the intensity of the electric field (Kerr effect). This dependence induces a non-linear phase variation. This is called the auto phase modulation effect. This nonlinear effect introduces a frequency chirp.

In the abnormal dispersion regime, the direction of the frequency slip produced by the phase self-modulation effect is the opposite to that produced by the dispersion. This indicates that the frequency slip induced by the phase auto-modulation can compensate for that induced by the chromatic dispersion. This process leads to the formation of optical solitons which retain shape during propagation.

2.10.2.1 The soliton effects:

The soliton is an initially symmetrical light wave propagating without deformation of its shape in a dispersive and non-linear medium. In optics, the soliton is used to describe an impulse (temporal soliton) or a beam (spatial soliton). Mathematically, the soliton can be represented by the following equation [17,18]:

$$(z = 0, \tau) = N.\operatorname{sech}(\tau) \qquad (2.34)$$

N is the order of the soliton which is defined by [1,2]:

$$N = \sqrt{\frac{L_D}{L_{NL}}} = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}} \qquad (2.35)$$

Where P_0, L_D, L_{NL} are respectively the peak power of the pulse, the dispersion length, and the non-linear length. To determine the order of the soliton, we always take the nearest integer value.

In the case where $L_{NL} = L_D$, that is, the linear effect of the group speed dispersion is compensated by the non-linear effect, we will have a fundamental (or order one) soliton. Then, for $N = 1$, the fundamental soliton retains its shape during the propagation. See Figure 2.5

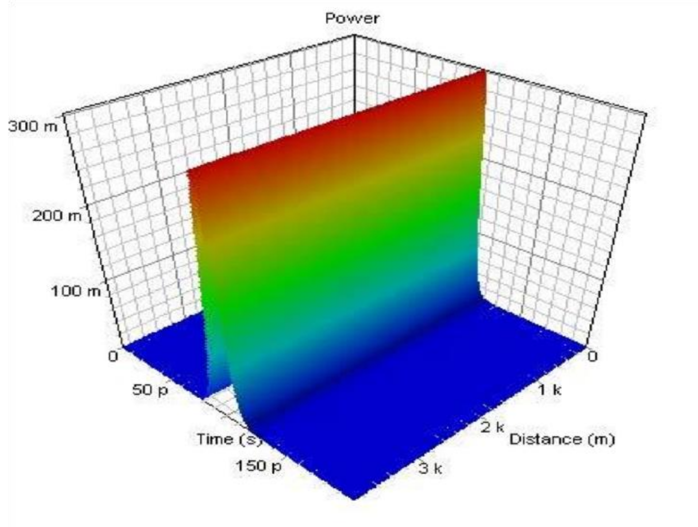

Figure 2.5. Fundamental soliton N = 1

Consequently, the peak power necessary for the existence of a fundamental soliton is:

$$P_0 = \sqrt{\frac{|\beta_2|}{\gamma T_0^2}} \tag{2.36}$$

several fundamental solitons propagating in a coupled manner and at the same group speed.

During its propagation inside an optical fiber, the fundamental solitons making up the higher-order soliton cause periodic interactions. Figure 2.6 shows the temporal evolution of the order two and order three soliton as a function of the propagation length. The evolution of these solitons can present several peaks where the impulse can regain its initial form periodically.

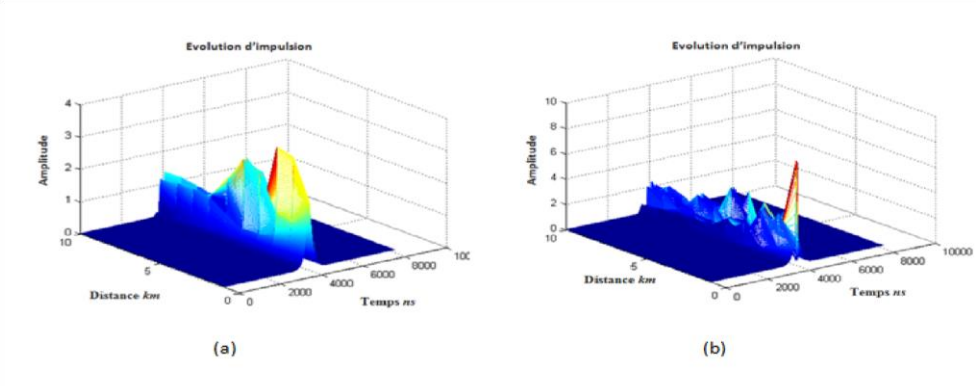

Figure 2.6 Temporal evolution of two solitons (N = 2, N = 3) as a function of the propagation length.

2.11. Equations with Soliton as solutions

In this part, we present some equations which admit the soliton as solutions. The Korteweg and Vries equation, the nonlinear Schrödinger equation, and the sine-Gordon equation.

2.11.1. Normalized non-linear Schrodinger equation for temporal solitons:

The starting point for the analysis of temporal solitons is the time-dependent wave equation for the spatial envelopes of electromagnetic fields in an optical Kerr medium, here for reasons of simplicity. For linearly polarized light in isotropic media, the propagation equation is given by [1]:

$$\left(j\frac{\partial}{\partial z}+j\frac{1}{v_g}\frac{\partial}{\partial t}-\frac{\beta}{2}\frac{\partial^2}{\partial t^2}\right)A_\omega(z,t)=-\frac{\omega n_2}{c}|A_\omega(z,t)|^2A_\omega(z,t) \qquad (2.37)$$

Where, as before, $v_g = (dk/d\omega)$ is the speed of the linear group, and where we have introduced the notation $\beta = \left.\frac{d^2k}{d\omega^2}\right|_{\omega\sigma}$

For the intensity-dependent refractive index $n = n_0 + n_2\ |E\omega|^2$. Since we are considering here the propagation of waves in isotropic media, with linearly polarized light (for which there is no crosstalk of polarization state). The wave equation is conveniently taken in scalar form:

$$\left(j\frac{\partial}{\partial z}+j\frac{1}{v_g}\frac{\partial}{\partial t}-\frac{\beta}{2}\frac{\partial^2}{\partial t^2}\right)A_\omega(z,t)=-\frac{\omega n_2}{c}|A_\omega(z,t)|^2A_\omega(z,t) \qquad (2.37)$$

The equation (2.37) consists of three interacting terms. The first two terms contain first order derivatives of the envelope, these terms can be considered as the homogeneous part of a wave equation for the envelope, giving solutions of progressive wave which depend on the two other terms, which rather act as source terms.

The third term contains a derivative of order 2 of the envelope, this term is also linearly dependent on the dispersion β of the medium, i.e. the variation of the group speed of the medium compared to the frequency angular ω of light. This term is generally responsible for spreading a short pulse when it crosses a dispersive medium.

Finally, the fourth term is a non-linear source term, according to the sign of n_2, will concentrate components of higher frequency either at the leading edge or at the trailing edge of the pulse as soon as it is displayed.

Chapter three

Numerical Methods

3.1. Introduction

In general, there are no analytical solutions to the complete Maxwell wave equation for a nonlinear optical system. Even numerical solutions to the wave equation are extremely difficult to implement due to the dimensionality of the problem. The vector form of the wave equation is a second order partial differential equation with four dimensions (three spatial and one temporal). Thus, approximations based on propagation conditions and experimental results are necessary to solve an approximate scalar form of the wave equation, i.e. the nonlinear Schrödinger equation. However, the approximations listed in the previous chapter limit the generality and validity of the solutions. For example, the condition of extreme non-linearity, as in the case of the generation of super continuum, is one where the approximation of the envelope, which varies slowly, can be violated.

The purpose of this chapter is to introduce a powerful method of numerical resolution of the NLSE, known as the Split-step Fourier method (SSFM). The chapter begins with a reminder on fiber optics followed by a reminder of NLSE numerical resolution methods list the advantages of SSFM compared to finite difference methods.

3.2. Optical fiber and light guiding:

3.2.1. Physical principle of Light guiding

The major physical principle that inspired fiber optic technology is what is known as "Total Internal Reflection". This follows from the law of refraction that a wave crossing through a boundary between two media of different density, deviated.

However, if the wave ever tries to pass from a medium of relatively high density to a less dense medium, there is a minimum angle between the direction of the wave and the normal of the border for which the wave will not be deflected but reflected. It is therefore possible for a light wave to propagate indefinitely in a glass cylinder [20].

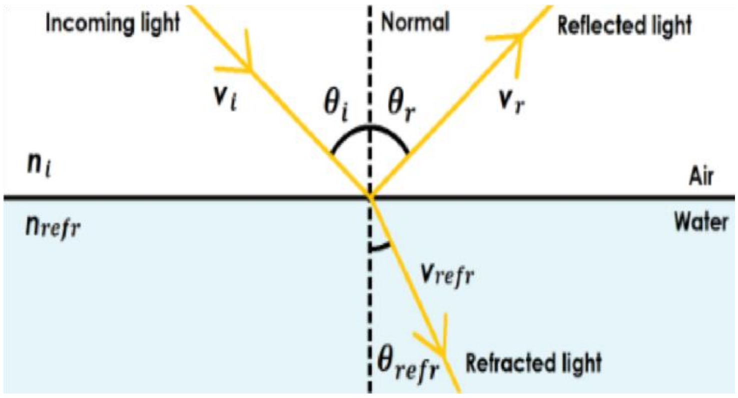

Figure 3.1 reflection / refraction.

3.2.2. Type of optical fibers:

A typical bare fiber consists of a core, a cladding, and a polymer jacket (buffer coating), the fulfill the conditions for TIR in the fiber, the angle of incidence of light launched into the fiber must be less than a certain angle, which is defined as the acceptance angle, θ_{acc}. which can be calculated by Snell's law.

3.2.2.1. Multi-mode fiber:

Only certain angles lead to modes. Obviously, the speed of a mode depends on the angle. The term "multimode" means that several modes can be guided. A typical number for a step index fiber is 1000 modes (one mode corresponds to a beam).

❖ Step index:

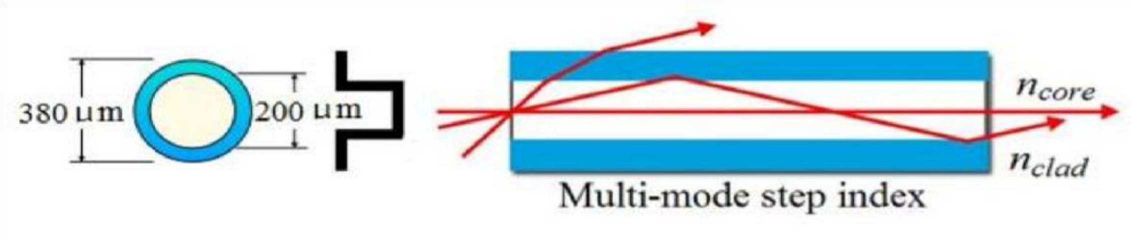

Figure (3.2) step index fiber

It is the simplest type of fiber. Indeed, it disperses the signal. In this fiber, the core is homogeneous and of index n1. It is surrounded by an optical cladding with an index n_2 less than n_1.

As for the optical cladding, it plays an active role in the propagation and should not be confused with the protective coating deposited on the fiber. The ray is guided by the total reflection at the level of the core-cladding interface, otherwise it is refracted in the cladding. [20].

❖ Graded index:

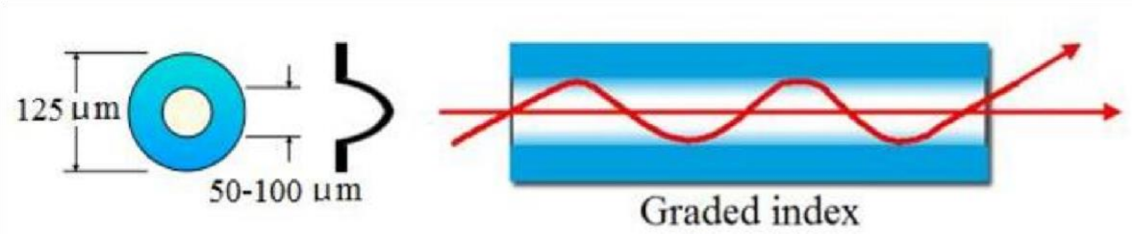

Figure (3.3) Graded index fiber

Their core, unlike step index fibers, is not homogeneous. Their core is in fact made up of several layers of glasses whose refractive index is different with each layer and the refractive index decreases from the axis to the cladding. Guidance is this time due to the effect of the index gradient. The rays follow a sinusoidal trajectory.

The cladding does not intervene directly but eliminates the too inclined rays. The advantage with this type of fiber is to minimize the dispersion of the propagation time between the rays [20].

3.2.2.2. Single mode fiber:

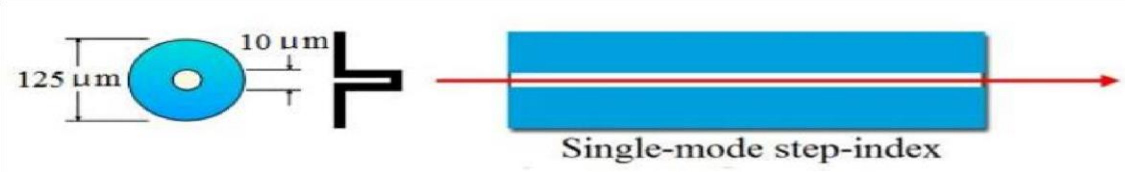

Figure (3.4) Single-mode fiber

The aim sought in this fiber is that the path that the beam must travel is as direct as possible. For this we strongly reduce the diameter of the heart which is in most cases less than 10µm. The modal dispersion is almost zero. Since we do not break the light beam the bandwidth is therefore increased, approximately 100 GHz * km or 1000 Mbits / s. Conventional single-mode fiber is stepping index. Its diameter allows the propagation of only one mode, the fundamental as only one mode propagates there is no difference in speed unlike multimode fibers. Because of these valuable advantages, it has gained considerable importance in long-distance transmissions [20].

3.2.3. Assessment on the various optical fibers

The following table gives a brief summary of the advantages and disadvantages of each structure: [20]

Table (3)

Structures	Benefits	Disadvantages
Step index multi-mode	Low price Ease of implementation	Significant signal loss and distortion
Graded index multimode	Reasonable bandwidth - Good transmission quality	Difficult to implement
Single mode	Very high bandwidth - No distortion	Very high price

As mentioned at the beginning of this chapter, an important application of solitons is the transmission of information in fiber optic systems. Soliton pulses are stable when propagated over long distances. Fiber losses are an important limiting factor, so it becomes necessary to periodically compensate for fiber losses.

3.3. Derivation of the nonlinear Schrödinger equation

The solitons in optical fibers are described by the equation known as the non-linear Schrödinger equation (NSLE), which we will derive it. In the derivation, we use the concept of Fourier spectrum for the propagation of pulses, see Figure 2.4.

A medium in which solitons propagate there is the non-linearity of Kerr effect. In such a medium, the refractive index depends on the intensity of the electric field $I\ (t)$ and can be written in the following form:

$$\bar{n}(t) = \bar{n}_0 + \bar{n}_2 I(t) \tag{3.1}$$

With [4]:

$$(t) = 2\bar{n}_0 \varepsilon_0 c|(z,t)|^2 \tag{3.2}$$

where $A(z, t)$ is the slowly varying envelope connected to the optical pulse described by the optical pulse $E(z, t)$ as:

$$(z, t) = (z, t)\ ^{(\omega 0 t - \beta 0 z)} \tag{3.3}$$

The Fourier transform of the optical field is [5,6]

$$E(z,t) = \int_{-\infty}^{+\infty} \tilde{E}(z,\omega)\ e^{j(\omega_0 t - \beta_0 z)}\ d\omega \tag{3.4}$$

Where $\tilde{E}\ (z, \omega)$ is the Fourier spectrum of the pulse, β the propagation constant and ω_0 the frequency at which the pulse spectrum is centered (also called carrier frequency), (See Figure. 2.4) For quasi-monochromatic pulses with $\Delta\omega \equiv \omega - \omega_0 \ll \omega_0$, it is useful to extend the propagation constant β (ω) in a Taylor series:

$$\beta(\omega) = \beta_0 - \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2\ (\omega - \omega_0)^2 + \Delta\ \beta_{NL} \tag{3.5}$$

Where we neglected higher order derivatives. Here $\Delta\beta_{NL} = \bar{n}_2 k_0 I$ is the nonlinear contribution to the propagation constant.

Replace the extension (3.5) with the equivalent (3.4):

$$E(z,t) = e^{-j\beta_0 z} \int_{-\infty}^{+\infty} \tilde{E}(z,\omega)\ e^{j\left(\omega t - \beta_1 z \Delta\omega^2 - z\Delta\omega_{NL}\right)}\ d(\Delta\omega)$$

$$E(z,t) = e^{j(\omega_0 t - \beta_0 z)} \int_{-\infty}^{+\infty} \tilde{E}(z,\omega_0 + \Delta\omega)\ e^{j\left(t\Delta\omega - \beta_1 z\Delta\omega - \frac{1}{2}\beta_1 z \Delta\omega^2 - z\Delta\omega_{NL}\right)}\ d(\Delta\omega)$$

$$= e^{j(\omega_0 t - \beta_0 z)} A(z,t) \tag{3.6}$$

Where we have introduced:

$$A(z,t) = \int_{-\infty}^{+\infty} \tilde{E}(z,\omega_0 + \Delta\omega)\ e^{j\left(t\Delta\omega - \beta_1 z\Delta\omega - \frac{1}{2}\beta_1 z \Delta\omega^2 - z\Delta\omega_{NL}\right)}\ d(\Delta\omega)$$

$$A(z,t) = \int_{-\infty}^{+\infty} \tilde{E}(z,\omega_0 + \Delta\omega)\ e^{jg(z,t)}\ d(\Delta\omega) \tag{3.7}$$

The next step consists in obtaining a differential equation describing the evolution of the amplitude $A\,(z,\,t)$ of equation (3.7) which is in integral form. To do this, we must take partial derivatives of the equation (3.7). We obtain

$$\frac{\partial A(z,t)}{\partial t} = \int_{-\infty}^{+\infty} \tilde{E}(z, \omega_0 + \Delta\omega)\; j\Delta\omega e^{jg(z,t)}\; d(\Delta\omega) \qquad (3.8)$$

$$\frac{\partial^2 A(z,t)}{\partial t^2} = \int_{-\infty}^{+\infty} \tilde{E}(z, \omega_0 + \Delta\omega)\; j\Delta\omega^2 e^{jg(z,t)}\; d(\Delta\omega) \qquad (3.9)$$

$$\frac{\partial A(z,t)}{\partial t} = \int_{-\infty}^{+\infty} \tilde{E}(z, \omega_0 + \Delta\omega)\; j\left(-\beta_1\Delta\omega - \frac{1}{2}\beta_2\,\Delta\omega^2 - \Delta\;\beta_{NL}\right) e^{jg(z,t)}\; d(\Delta\omega) \;(3.10)$$

When evaluating time derivatives, we have assumed in the above that $I\,(t)$ does not depend on time. The addition of the above derivative combination produces.

$$\frac{\partial A(z,t)}{\partial t} + \beta_1 \frac{\partial A(z,t)}{\partial t} - j\frac{1}{2}\beta_2 \frac{\partial^2 A(z,t)}{\partial t^2}$$
$$= \int_{-\infty}^{+\infty} \tilde{E}(z, \omega_0 + \Delta\omega)\; j\left(-\beta_1\Delta\omega - \frac{1}{2}\beta_2\,\Delta\omega^2 - \Delta\;\beta_{NL}\right) - j\beta_1\Delta\omega$$
$$- \frac{1}{2}\beta_2 (j\,\Delta\omega)^2 e^{jg(z,t)}\; d(\Delta\omega) \qquad (3.11)$$

The term in parentheses is. $[\ldots] = -i\Delta\beta_{NL} = -i\bar{n}_2 k_0 I$ the equation (3.11) therefore gives

$$\frac{\partial A(z,t)}{\partial t} + \beta_1 \frac{\partial A(z,t)}{\partial t} - j\frac{1}{2}\beta_2 \frac{\partial^2 A(z,t)}{\partial t^2}$$
$$= \int_{-\infty}^{+\infty} \tilde{E}(z, \omega_0 + \Delta\omega)\; (-j\bar{n}_2 k_0 I) e^{jg(z,t)}\; d(\Delta\omega)$$
$$= -j\bar{n}_2 k_0 I \int_{-\infty}^{+\infty} \tilde{E}(z, \omega)\; e^{jg(z,t)}\; d(\Delta\omega)$$
$$\equiv -j\bar{n}_2 k_0 I \equiv A(z,t) \qquad (3.12)$$

During use (3.7). The final equation describing the solitons is therefore [24], [26].

$$\frac{\partial A(z,t)}{\partial t} + \beta_1 \frac{\partial A(z,t)}{\partial t} - j\frac{1}{2}\beta_2 \frac{\partial^2 A(z,t)}{\partial t^2} = j\gamma|A(z,t)|^2 A(z,t) - \frac{\alpha}{2} A(z,t) \qquad (3.13)$$

Where defined the non-linear coefficient γ (after [3]) as follows:

$$\gamma = \frac{2\pi\bar{n}_2}{\lambda A_{eff}} \tag{3.14}$$

Here, A_{eff} is the effective central zone.

The interest here lies in the evolution of the impulse during propagation and not in the moment of arrival of the impulse. We can therefore simplify the above equation by transforming it into a coordinate system which moves with the group v_g. In this moving frame, the new time T and the new coordinate Z are the following [24]

$$Z = z$$

$$T = t - \beta_1 z$$

To get the transformed equation, we need to evaluate the derivatives against new variables as follows:

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial T}\frac{\partial T}{\partial t} + \frac{\partial A}{\partial Z}\frac{\partial Z}{\partial t} = \frac{\partial A}{\partial T} \tag{3.15}$$

Since $\frac{\partial T}{\partial t} = 1$ and $\frac{\partial Z}{\partial t} = 0$ from the above we find: $\frac{\partial^2 A}{\partial t^2} = \frac{\partial^2 A}{\partial T^2}$

Using the above results, we have:

$$\frac{\partial A}{\partial z} = \frac{\partial A}{\partial T}\frac{\partial T}{\partial z} + \frac{\partial A}{\partial Z}\frac{\partial Z}{\partial z} = -\beta_1\frac{\partial A}{\partial T} + \frac{\partial A}{\partial Z} \tag{3.16}$$

The last result is used in equation (3.13) to replace $\frac{\partial A}{\partial t}$. The transformed equation is

$$\frac{\partial A}{\partial Z} + j\beta_2\frac{\partial^2 A}{\partial T^2} - j\gamma|A|^2 A + \frac{1}{2}\alpha A = 0 \tag{3.17}$$

Where, in the final step, we replaced Z with z. It is a nonlinear Schrödinger equation (NLSE).

To deepen the analysis of the NLSE, we will introduce two characteristic lengths describing the dispersion (LD) and non-linearity (LNL). These are defined as follows:

$$L_D = \frac{T_0^2}{|\beta_2|} = \frac{T_0^2 2\pi c}{|D|\lambda^2} \qquad (3.18)$$

And

$$L_{NL} = \frac{1}{\gamma P_0} \qquad (3.19)$$

Where P_0 is the peak power of the envelope $A\,(z, T)$ which varies slowly, T_0 is a time characteristic value of the initial pulse which is often defined as being half of the maximum width of the pulse (l3 dB pulse). These two lengths characterize the distance at which an impulse must propagate to show the respective effect.

Physically, L_D is the propagation length at which a Gaussian pulse widens by a factor of $\sqrt{2}$ due to the dispersion of group velocities (GVD).

GVD dominates the propagation of pulses in fibers whose length L is $L \ll L_{NL}$ and $L \geq L_D$. In such a situation, the non-linearity of the NLSE can be ignored and the equation can be solved analytically. Non-linear effects dominate in fibers where $L \ll L_D$ and $L \geq L_{NL}$. Within this limit, the term dispersal can be ignored.

$$U = \frac{1}{\sqrt{P_0}} \quad and \quad \tau = \frac{T}{T_0} \qquad (3.20)$$

The width parameter $T0$ is related to the maximum intensity of the input pulse in half-maximum total width (FWHM). More precisely

$$T_s = 2T_0\, ln\left(1 + \sqrt{2}\right) \approx 1.763T_0 \qquad (3.21)$$

After a simple algebra, Eq. (3.17) takes the form:

$$\frac{\partial U}{\partial z} - j\frac{sign(\beta_2)}{2L_D}\frac{\partial^2 U}{\partial \tau^2} + j\frac{1}{L_{NL}}|U|^2 U + \frac{1}{2}\alpha U = 0 \qquad (3.22)$$

Another normalized form of the Schrödinger equation exists in the literature.

We obtain it in the lossless case, that is to say with α = 0. To calculate it, normalize the z coordinate as follows:

$$\xi = \frac{z}{L_D} \qquad (3.23)$$

After a few algebraic steps, we get:

$$\frac{\partial U}{\partial \xi} - j\frac{sign(\beta_2)}{2}\frac{\partial^2 U}{\partial \tau^2} + jN^2|U|^2 U = 0 \qquad (3.24)$$

Where N is known as the soliton order and is defined as follows:

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \qquad (3.25)$$

The last popular form of NLSE is found by introducing u as:

$$u = NU = \left(\frac{\gamma P_0 T_0^2}{|\beta_2|}\right)^{1/2} A \qquad (3.26)$$

Equation (3.22) then takes the following form:

$$\frac{\partial u}{\partial \xi} - j\frac{sign(\beta_2)}{2} - j|u|^2 u = 0 \qquad (3.27)$$

3.4. Numerical solution methods of the nonlinear Schrödinger equation

Optical pulses in media with properties of non-linearity of saturation are modeled by the nonlinear Schrödinger (NLS) Equation [Kato, 1989] with the following nonlinearity [Zemlyanaya and Alexeeva, 2011].

$$j\frac{\partial \psi}{\partial t} + \frac{1}{2}\frac{\partial^2 \psi}{\partial t} + \frac{|\psi|^2 \psi}{1+s|\psi|^2} = 0 \qquad (3.27)$$

It is a partial differential equation (PDE) because it describes a relation of ψ with regard to change in time and space. The solution to this equation is a so-called soliton of the form.

$$\psi(x,t) = \frac{2\sqrt{2}e^{\sqrt{2x}}}{1+\left(\frac{3}{2}-2S\right)e^{2\sqrt{2x}}}e^{jt+jv} \tag{3.29}$$

3.4.1. Fourier method with Split step

It is assumed in the cited method that the two effects of dispersion and non-linearity act separately. The principle of the method consists is considered to be the two effects on the same calculation step dz (z: direction of propagation) but in an alternating way by applying the nonlinear operator in the middle of the discretization step. The calculation is iterative and is done over the entire length of the fiber taking for each new calculation step as distribution initial of the field that obtained at the end of the previous step. This method is based on the Fourier transform [21].

3. 4.2. Finite difference method:

The finite difference method works by matching the derivatives of the expression with finite differences. In our PDE, we have $\frac{\partial\psi}{\partial t}$ and $\frac{\partial^2\psi}{\partial x^2}$ which must be approximated by finite differences. How approximated determines what type of finite difference system is used, which has various implications for accuracy, stability, and implementation.

1. The prospective difference is an explicit diagram which means that the solution at each point at the latest at the time level can be expressed by the solutions of the previous time levels. While this simplifies implementation, the regime suffers from stability issues.

2. The inverse difference is an implicit scheme which means that a system of equations must be solved in order to calculate the solution at the next time

level which makes the implementation non-trivial. However, this method has the superior property that it does not suffer from stability problems.

3. The central difference method has the advantage over the front difference and the rear difference with regard to accuracy since the error is O (τ^2) with respect to O (τ) of the other methods. This means that the total error of PDE 1 is O (τ^2+h^2) where τ is the time step and h the space. This method has the same drawback as the prospective difference method: Questions of stability [22].

3.4.3. The Inverse Scattering Method

The inverse diffusion method was the first method to solve the NLSE in the specific case of the propagation of solitons by Zakharov and Shabat [14]. The method uses the initial field E (z = 0, t) to obtain the initial diffusion data, then the propagation along z is found by solving the linear diffusion problem. The final field E (L, t) is reconstructed from the advanced diffusion data. Typically, this method is used for the propagation of solitons [22].

However, for the propagation of the soliton, the inverse diffusion method reduces numerically to a problem of eigenvalue and a system of linear equations. The complexity of this method may force the SSFM to be desired. However, the inverse scattering method does not suffer from element separation errors. the effects of dispersion and non-linearity of the fibers.

The form of the NLSE for the solitons to be solved by the inverse diffusion method is as follows:

$$j\frac{\partial U}{\partial \xi} = sign(\beta_2)\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} - \frac{L_D}{L_{NL}}|U|^2U = 0 \qquad (3.30)$$

When

$$U = \frac{A(z,t)}{\sqrt{P_0}}, \qquad \xi = \frac{z}{L_D} \ , and\ \tau = \frac{1}{T_0} \tag{3.31}$$

The above equation can be written to eliminate the soliton number $\frac{L_D}{L_{NL}}$

defining $u(\xi,\tau) = \frac{L_D}{L_{NL}}\ U(\xi,\tau)$. . The equation Eq (3.30) can be integrated and can be expressed in the form of two linear equations as follows:

$$(\xi)(\xi,\tau) \ = \xi\ (\xi,\tau) \tag{3.32}$$

$$\frac{\partial u(\xi,\tau)}{\partial t} = M(\xi)\ u(\xi,\tau) \tag{3.33}$$

Where L (ξ) and M (ξ) are differential operators in τ. Equation (3.32) is a problem of eigenvalue with eigenvalue ζ and Eq. (3.33) determines the evolution ξ of the function u (ξ, τ).

The term L (ξ) corresponds to the dispersion operator D̂ , since L (ξ) evolves so that the spectrum remains constant. L (ξ) and M (ξ) are known as a Lax pair of the integrable system given by [13].

$$L(\xi) = \begin{bmatrix} j\frac{\partial}{\partial\tau} & u(\xi,\tau) \\ -u^*(\xi,\tau) & -j\frac{\partial}{\partial\tau} \end{bmatrix} \tag{3.34}$$

And

$$M(\xi) = \begin{bmatrix} j\frac{\partial^2}{\partial\tau^2} + |u(\xi,\tau)|^2 & u(\xi,\tau)\frac{\partial^2}{\partial\tau^2} \\ -u^*(\xi,\tau)\frac{\partial^2}{\partial\tau^2} & -j\frac{\partial^2}{\partial\tau^2} - \frac{j}{2}|u(\xi,\tau)|^2 \end{bmatrix} \tag{3.35}$$

The field u (ξ = 0, τ) provides the initial diffusion information Σ (ξ = 0) from the eigenvalue solution of Eq (3.32). The evolution of the diffusion data Σ (ξ) is determined from Eq. (3.33). Finally, an inverse problem is solved to find the solution of propagation u (ξ, τ) from the diffusion data. In general, it is a question of solving a set of linear integral equations, which are reduced to a set of algebraic equations for the propagation of the soliton. The reverse diffusion method is summarized in the figure below: [3]

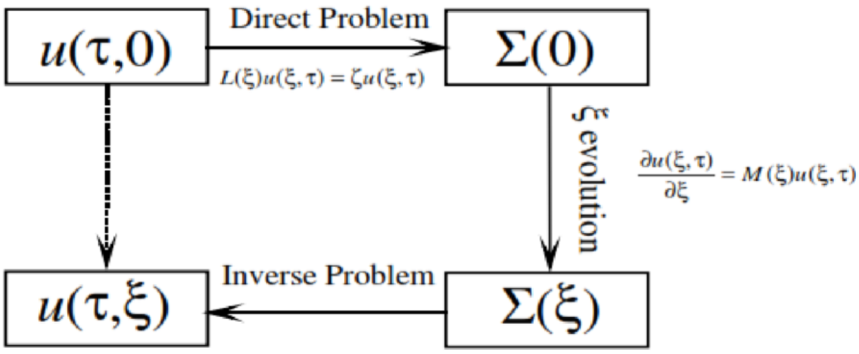

Figure 3.5 Block diagram of the reverse scattering method.

In the inverse scattering method, the scattering potential Σ serves as a conduit for solving the direct propagation of the field from u (t, 0) to u (t, ξ). First, the initial condition u (t, 0) is used with Eq (3.32) to determine the diffusion potential Σ (0). Then Eq. (3.41) is used to determine the propagation of the diffusion potential. Finally, the solution u (t, ξ) is found by solving the opposite problem involving the diffusion potential Σ (ξ).

3. 4.5. Fourier method with Split Step

3.4.5.1 Use of the Fourier transform method with fractional steps (The Split Step Fourier Method):

SSFM is the technique chosen to solve the NLSE because of its ease of implementation and its speed compared to other methods, in particular the finite time-domain difference methods. The finite difference method explicitly solves the Maxwell wave equation in the time domain under the assumption by axial approximation.

SSFM belongs to the category of pseudospectra methods, which are generally an order of magnitude faster compared to finite difference methods. The main difference between time domain techniques and SSFM is that it processes all electromagnetic components without eliminating the carrier frequency. As indicated in the previous chapter, the carrier frequency is removed from the derivation of the NLSE. Thus, finite difference methods can account for forward and backward propagation waves, while the derived NLSE for SSFM cannot. Since the carrier frequency is not decreased in the form of an electric field, finite difference methods allow the propagation of pulses to be described with precision almost one cycle. While the finish may be more precise than the SSFM method, it is only at the cost of a longer computation time.

In practice, the method chosen to solve the NLSE depends on the problem to be solved. For pulse propagation for telecommunication applications (~ 100 ps pulses through 80 km of fiber with dispersion and SPM) the SSFM works extremely well and produces results which are in excellent agreement with the experiments.

3.4.5.2 Presentation of the Method:

We will discuss the numerical solution of the nonlinear Schrödinger equation (NSE) which describes the propagation of optical solitons using the so-called Split Step Fourier Method (SSFM).

The propagation medium (for example, cylindrical optical fiber) is divided into small segments of length h each. In addition, each individual segment of length h is subdivided in two of equal length. The linear operator operates on each sub-segment of the frequency domain, while the nonlinear operator operates only locally at the central point see figure 3.2.

Operation of the linear operator $\hat{L}$, Eq. (3.34) on the first sub-segment is as follows:

$$e^{\frac{h\hat{L}}{2}}A(z,t) = F^{-1}\left\{e^{\frac{h\hat{L}}{2}}F\{A(z,t)\}\right\} \qquad (3.36)$$

That is to say, you have to transform the original Fourier amplitude from the time domain to the frequency domain, apply the linear operator $\hat{L}$, then apply the inverse Fourier transform to bring the amplitude back to the time domain.

The operation of the nonlinear operator defined by equation (3.35) is as follows:

$$A_{i+\frac{1}{2},L}(z,t) = A_{i+\frac{1}{2},R}(z,t)e^{h\widehat{N}} \quad (3.37)$$

Where $A_{i+1/2,L}$ is the value of the field amplitude at an infinitesimal point to the left of $i + 1$.

Finally, the operation of the linear operator on the second sub-segment of the length $h / 2$ is done exactly as follows in the same way as on the first segment.

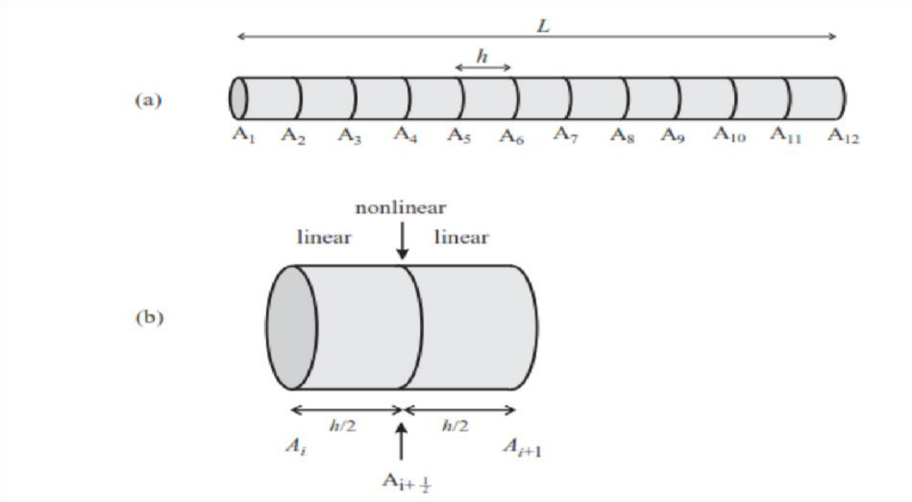

Figure. 3.6 Illustration of the Fourier method with separate steps. (a) Division of the optical fiber into N regions (here N = 11) of equal lengths. b) Illustration of the operation of linear operations and nonlinear on arbitrary segments.

SSFM is a numerical technique used to solve nonlinear partial differential equations like NLSE. The method is based on the calculation of the solution in small steps and on the separate considering of the linear and nonlinear steps. The linear step (dispersion) can be done in the frequency or time domain, while the nonlinear step takes place in the time domain, this method widely used to study the nonlinear propagation of pulses in optical fibers [27].

A nonlinear Schrödinger equation, Eq (3.17) contains dispersive and nonlinear terms. To introduce SSFM, write the NLSE equation in the following form:

$$\frac{\partial A(z,t)}{\partial z} = (\hat{L} + \hat{N})A(z,T) \tag{3.38}$$

$$\hat{L}A = -\frac{\alpha}{2} - \frac{j}{2}\beta_2 \frac{\partial^2 A}{\partial T^2} \tag{3.39}$$

Contains losses and dispersion in the linear mean and the nonlinear term.

$$\hat{N}A = J\gamma|A|^2 A \tag{3.40}$$

Non-linear effects in the environment are considered. The basis of the SSFM is to divide a propagation from z to z + h (h is a small step) in two operations (assuming that they act independently): during the first step, non-linear effects are included and at during the second step, we take into account the linear effects.

The formal solution of equation (3.36) on a small step h is therefore

$$(z + h, t) = e^{h(\hat{L}+\hat{N})}A(z, t) \tag{3.41}$$

In the first order approximation, the above formula can be written as follows

$$A\,(z + h, t) = e^{h\hat{L}}e^{h\hat{N}}(z, t) + O(h^2) \tag{3.42}$$

The basis of this approximation is established by the Baker-Hausdorf lemma [13], namely given $e^{\hat{A}}e^{\hat{B}} = e^{\hat{A}+\hat{B}}e^{\frac{1}{2}[\hat{A}+\hat{B}]}$ that operators A and B commute with [A, B].

The basis of the method is suggested by equation (3.42). He tells us that A

$(z + h, t)$ can be determined by applying the two operators independently. The propagation from z to z + h is divided into two operations: first the nonlinear step and then the linear step assuming that they act independently. If h is small enough, Eq. (3.42) gives good results.

The value of the step h can be determined by assuming that the maximum phase shift $\varphi_{max} = \gamma|AB|^2 h$, where AP is the peak value of A (z, t) due to the nonlinear operator is less than the predefined value . Iannone et al [21] reported that $\varphi_{max} \leq 0.05$ rad.

For a practical implementation of the SSFM, we need to establish practical expressions for the dispersive and nonlinear terms. In what follows, we will therefore analyze the effect of the two terms independently neglecting losses.

Let us analyze the effect of the single dispersive term. For this, we temporarily deactivate the non-linear term. After the Fourier transform, the linear equation "becomes

$$\frac{\partial A(z,\omega)}{\partial z} = -\frac{j}{2}\omega^2\beta_2\tilde{A}(z,\omega) \quad (3.43)$$

Who has the solution

$$\acute{A}(z,\omega) = \acute{A}(0,\omega)e^{-\omega 2\beta 2/}2 \quad (3.44)$$

The action of the nonlinear term alone is described by the following equation

$$\frac{\partial A(z,t)}{\partial z} = -j\gamma|A(z,t)|^2 A(z,t) \quad (3.45)$$

The "natural" solution lies in the time domain.It produces

$$(z,t) = (0,t)^{|A|2A} \quad (3.46)$$

In summary, the method on each segment of length h consists of three steps:

$$step1\begin{cases} \tilde{A}_{i-}(z,\omega) = F\{\tilde{A}_{i-}(z,\omega)\} \\ \tilde{A}_{i-}(z,\omega) = \tilde{A}_{i}(z,\omega)exp\left(-j\frac{1}{2}\omega^2\beta_2 h\right) \\ \tilde{A}_{i-}(z,\omega) = F^{-1}\{\tilde{A}_{i-}(z,\omega)\} \end{cases}$$

$$step2 \quad \tilde{A}_{i+}(z,t) = \tilde{A}_{i-}(z,t)exp(j\gamma|A|^2Ah)$$

$$step3\begin{cases} \tilde{A}_{i+1}(z,\omega) = F\{\tilde{A}_{i+1}(z,\omega)\} \\ \tilde{A}_{i+1}(z,\omega) = \tilde{A}_{i+1}(z,\omega)exp\left(-j\frac{1}{2}\omega^2\beta_2 h\right) \\ \tilde{A}_{i+1}(z,\omega) = F^{-1}\{\tilde{A}_{i+}(z,\omega)\} \end{cases}$$

Where F indicates the Fourier transform (TF) and F^{-1} the inverse Fourier transform.

Chapter Four

Simulation results

4.1. Introduction

To understand the phenomenon that occurs during the propagation of solitons, we will dissect the evolution of the impulse through the fiber. Its evolution will consider the effects of dispersions, non-linear effects, then combine these two effects in order to see them consequences on the impulse. The modeling is done by the Step Split Fourier method to solve the Schrödinger propagation equation in the case of a fundamental Soliton pulse then a Chirped pulse.

This study uses silica glass fiber as a research reference and the propagation medium because this type of fiber is widely used in several optical communication systems.

4.2. Digital simulation of the propagation of a pulse in a single-mode optical fiber

4.2.1 Propagation of an impulse in Soliton Fundamental form:

suppose an impulse given by: $A\,(z = 0, \tau)N\ \ sech\,(\tau)$ represented by the Figure (4.1). Its propagation in an optical fiber is modeled by the NLSE (4.1). to understand the phenomenon that occurs during this propagation, we will dissect the evolution of the impulse through the fiber. Its evolution will consider the effects of dispersion, nonlinear effects, then associate these two effects in order to see them consequences on the pulse (ultrashort wave).

$$\frac{\partial A(z,t)}{\partial z} = -\frac{\alpha}{2}A(z,t) + \frac{j}{2}\beta_2(z)\frac{\partial^2 A(z,t)}{\partial t^2} - j\gamma|A(z,t)|^2 A(z,t) \qquad (4.1)$$

Where $D = -j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} - \frac{\alpha}{2}$ and $NL = j\gamma|A|^2$, D is the dispersion operator and NL is the nonlinearity operator. Equation (4.1) takes the following form:

$$\frac{\partial A}{\partial z} = (D + NL)A \tag{4.2}$$

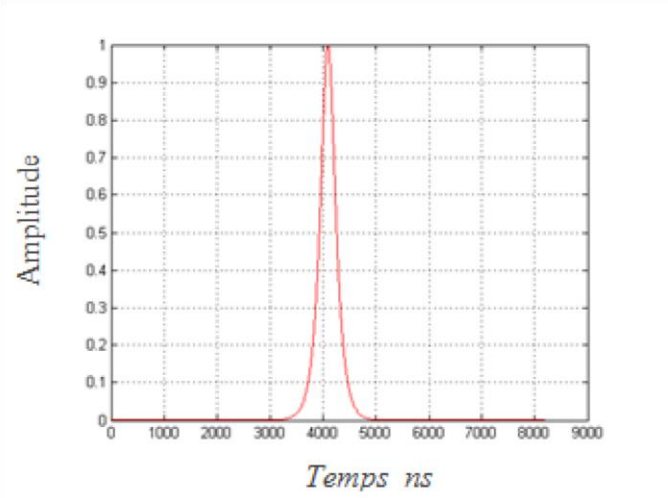

Figure 4.1: the input pulse.

4. 2.1.1 Numerical simulation of the propagation of a pulse in a linear medium.

When the distance of propagation satisfies the following parameters:

$$\begin{cases} z \sim L \\ z \ll L_{NL} \end{cases}$$

in this case the nonlinear part is null and consequently the dispersive effects play a preponderant role. This however applies if the fiber and pulse parameters meet the following conditions:

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \ll 1 \tag{4.2}$$

The equation (4.1) becomes:

$$\frac{\partial A_L}{\partial z} = -j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\alpha A}{2} \tag{4.3}$$

The term $\frac{\partial^2 A}{\partial t^2}$ is replaced by the Fourier transformation of the envelope A in the equation (4.3), which gives:

$$\frac{\partial A_L}{\partial z} = \left(j\frac{\beta_2}{2}\omega^2 - \frac{\alpha A}{2}\right)\tilde{A}(\omega) \tag{4.4}$$

Or:

$$\frac{\partial A_L}{\tilde{A}(\omega)} = \left(j\frac{\beta_2}{2}\omega^2 - \frac{\alpha A}{2}\right)\partial z \tag{4.5}$$

Let us integrate member to member, the left term from A_0 to A_1 and the right term from 0 to h.

$$\int_{A_0}^{A_1}\frac{\partial A_L}{\tilde{A}(\omega)} = \int_0^h\left(j\frac{\beta_2}{2}\omega^2 - \frac{\alpha A}{2}\right)\partial z \tag{4.6}$$

Equation (4.6) gives:

$$\ln\left|\frac{\tilde{A}_1}{\tilde{A}_0}\right| = \left(j\frac{\beta_2}{2}\omega^2 h - \frac{\alpha}{2}h\right) \tag{4.7}$$

Equation (4.7) has the solution:

$$\tilde{A}_1 = \tilde{A}_0 e^{\left(j\frac{\beta_2}{2}\omega^2 h - \frac{\alpha}{2}h\right)} \tag{4.8}$$

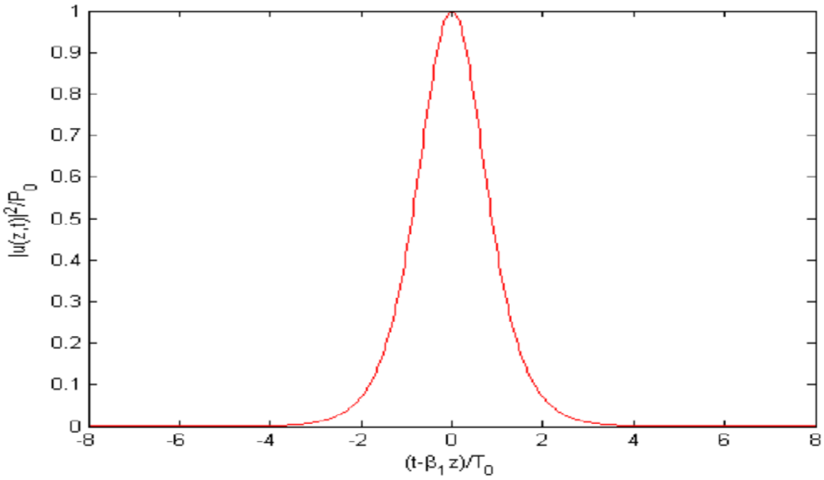

Figure 4.2: Propagation of a pulse in a medium of zero dispersion.

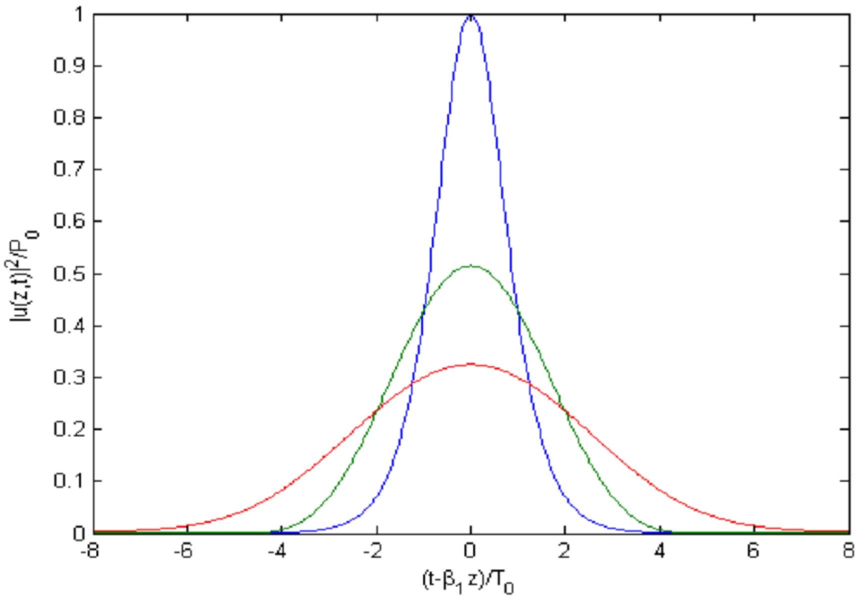

Figure 4.3: Propagation of a pulse in a dispersive medium (normal and abnormal dispersion).

The two previous figures indicate that in the two dispersion regimes (abnormal and normal), the different frequency components of the pulse propagate at different speeds. These different speeds create a temporal spread of the wave at the exit of the fiber. This is called group velocity dispersion (GVD).

On the contrary, with zero dispersion (the medium is non-dispersive), the pulse retains its initial shape since all the waves propagate at the same speed (Figure 4.2).

4. 2.2. Numerical simulation of the propagation of a pulse in a non-linear medium

4.2.2.1- Propagation of an impulse in fundamental soliton form:

In this case where the nonlinear part ($NL= i\gamma|A|^2$) in equation (4.1) is dominant (zero linear part D), i.e. the propagation satisfies the following condition: $\begin{cases} z \sim L_D \\ z \ll L_{NL} \end{cases}$ the objective is to show the influence of non-linear effects, in particular the Kerr effect which induces the phenomenon of selfphase modulation (SPM, Self-Phase Modulation).

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \gg 1 \tag{4.8}$$

The equation (4.1) becomes:

$$\frac{\partial A_{NL}}{\partial z} = -j\gamma |A|^2 A \tag{4.9}$$

Equation (4.9) becomes:

$$\frac{1}{A}\frac{\partial A_{NL}}{\partial z} = -j\gamma |A|^2 \tag{4.10}$$

Let us integrate the left term from $A0$ to $A1$ and the right term from 0 to h

$$\int_{A_0}^{A_1} \frac{\partial A_{NL}}{A} = \int_0^h j\gamma |A|^2 \partial z \tag{4.11}$$

The equation (4.11) becomes:

$$\ln \left|\frac{A_1}{A_0}\right| = j\gamma |A|^2 h \tag{4.12}$$

Equation (4.9) has the solution:

$$A_1 = A_0 e^{j\gamma |A| 2h} \tag{4.13}$$

Using the SSFM Method, with the choice of the following parameters, plus a variation of the propagation distance and the pulse represented by figure (4.1) as input pulse:

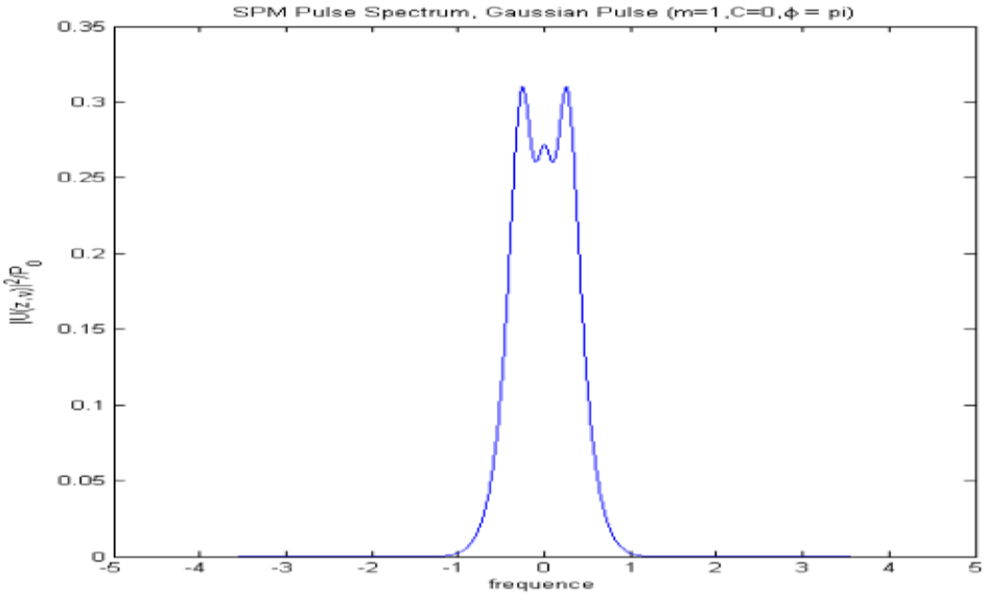

Figure 4.5: Propagation of a pulse in a non-linear dispersive medium

$$z = 1 \text{ to } \Phi_{max} = \pi \tag{4.14}$$

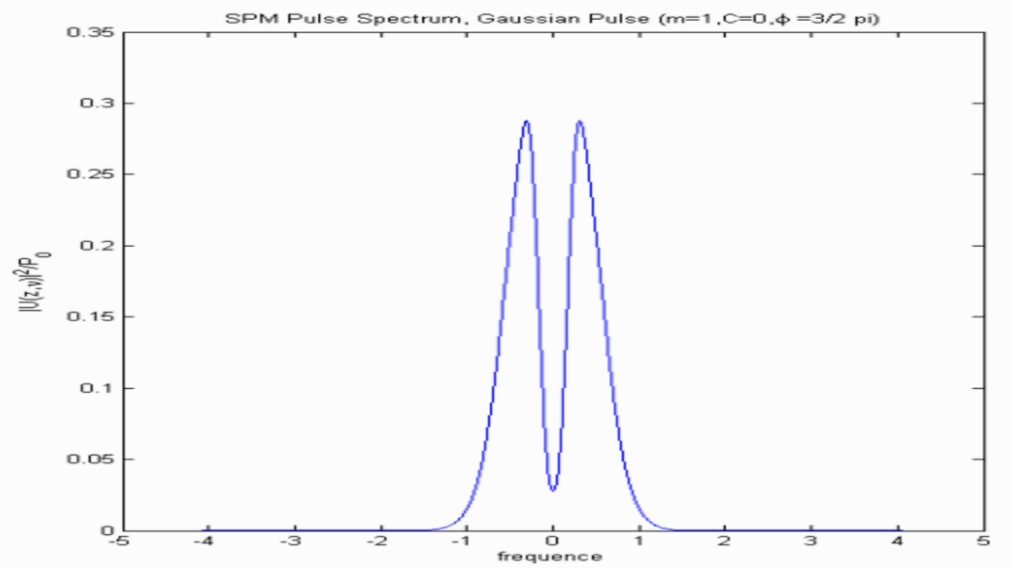

Figure 4.6: Propagation of a pulse in a non-linear dispersive medium at

$$z = 2 \ \Phi_{max} = 3\boldsymbol{\pi} / 2 \tag{4.15}$$

When a pulse is propagated in a nonlinear medium, the Kerr effect induces the phenomenon of auto phase modulation (SPM, Self-Phase Modulation) because of the propagation of an intense beam in the optical fiber, the nonlinear coefficient γ is positive producing a gradual increase in the refractive index. This passage of the wave changes the refractive index which in turn changes the phase of the pulse. But this produces a spectral widening of the pulse, unlike dispersion, a widening of the spectrum of pulses. The phase shift and this widening vary as a function of the propagation distance as shown in the two Figures (4.5) and (4.6).

4.2.2.1 Propagation of an impulse in Chirped Soliton Form:

One of the factors that limit the performance of the transmission capacity, the speed at which data can be transmitted, is the dispersion and nonlinear effects that occur during the process of light propagation in an optical fiber.

transport is limited by a widening of the pulses due to chromatic dispersion (GVD). In addition, there is the influence of the effects of parameters such as attenuation (α), dispersion (β_2), and non-linearity (γ).

We are going to consider all these parameters, always considering the propagation in the optical fiber. Our simulation work is based on the SSFM method for solving the NLSE (4.1). But this time using a chirped pulse as the input pulse, so what does the word chirped mean.

Definition: It is said that a light pulse is “chirped” (from the English chirped) when it undergoes a dispersion of its group velocity (GVD), as it can be induced by the nonlinear effect of high order at namely auto phase modulation (SPM), this causes a frequency variation over time.

The term comes from the English chirp, "chirping", by allusion to the song of certain birds which vary the frequency of their song. The figure below shows the electric field as a function of the time of an “chirped” optical pulse. Note that the frequency of the oscillation increases over time.

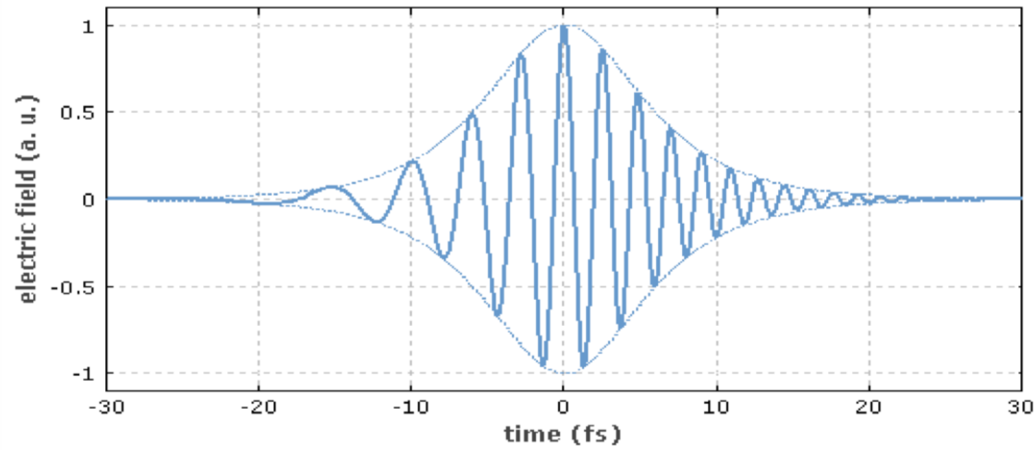

Figure (4.7): Represents a light pulse is chirped

To better study the influence of the value of the parameter of the chirp on the effect of the widening of the pulse, we propose to use a new approach based on the calculation of an indicator called enlargement ratio of l PBR pulse and its mean value given by the following expressions [28]:

$$PBR = \frac{\text{FWHM of propagation}}{\text{FWHM of first pulse}} \quad (4.15)$$

$$\overline{PBR} = \sum_{i=1}^{n} \frac{PBR_i}{n} \quad (4.16)$$

The following table (4.1) gathers the values used for the simulation.

Table (4.1): Value of the parameters used for the simulation.

Variable	Value (Unit)
$\boldsymbol{P_0}$ (the input power)	0.0000064 W
T_0 (width of the initial pulse)	125 e-12
Ld (the dispersion length corresponding to the order of the Soliton)	(N ^{2})/(g*$_{P0)}$
$\boldsymbol{\gamma}$ (Nonlinearity of the Fiber)	(0.003; 0.03; 0.3) in W/m
Alpha (Fiber Losses in dB / km)	3; 2; 1; 0
N (order of Soliton)	3; 2; 1; 0.5
C (chirp parameter)	3; -2; 0; 2; 3
$\boldsymbol{\beta_2}$ (2^{nd} order dispersion)	(T_0) ^ 2 / Ld (s2 / m)
h_1 (Step size)	1000

The parameters in table (4.1) are the GVD (Group Velocity Dispersion) speed parameters (β_2), and the non-linearity parameter $\boldsymbol{\gamma}$. The other approach to revealing the effect due to the parameters is a chirp (when the constants C are positive or negative) from -3 to 3.

In addition, the Fourier method with Fractional steps (SSFM) was applied to the system (4.1).

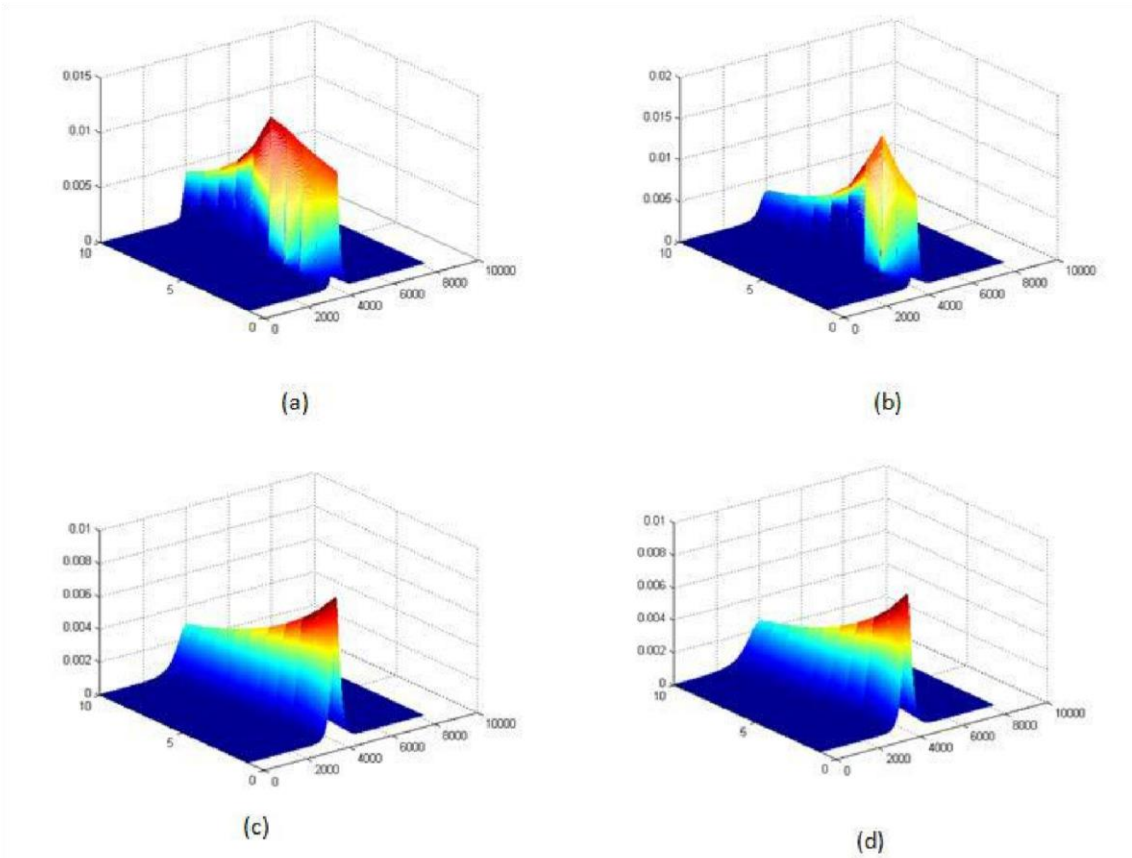

Figure 4.7: (a) Soliton pulse with chirping parameter C = 2. (b) Soliton pulse with chirping parameter C = 3 (c) with a chirping parameter C = -2.

(d) with C = -3.

From the simulation results obtained with regard to the second order dispersion and the nonlinear effects, it has been observed from Table 4.2 that the best PBR means have been obtained when the value of the parameter of the chirp is negative $C = -2$ and that the value of the parameter of the nonlinear γ remains constant at 0.003W / m.

When the mean value PBR for C = -2 is the lowest mean value of all the results of the simulation.

We can therefore say that the use of input pulses with a negative chirp value (C) with a constant non-linearity value is the best solution to reduce the pulse widening effects (reduce dispersion by GVD) and the non-linear oscillations.

Table 4.2: PBR value for a constant non-linearity parameter

Negative chirp parameter (C = -2) $\gamma = 0.003$		Negative chirp parameter (C = -3) $\gamma = 0.003$		Positive chirp parameter (C = 2) $\gamma = 0.003$		positive chirp parameter (C = 3) $\gamma = 0.003$	
β2	PBR	β2	PBR	β2	PBR	β2	PBR
-10	1.4081	-10	2.2820	-10	3.3205	-10	3.5737
-15	1.4260	-15	2.3581	-15	2.8494	-15	2.1856
-20	1.4224	-20	2.4136	-20	2.4689	-20	2.7107
-25	1.4092	-25	2.3235	-25	2.4589	-25	4.1683

For the following simulation, when the simulation system has modifications where the nonlinearity constants have become inconsistent variables and the $\beta 2$ was a constant, the simulation results are listed in the following table

TABLE 4. 3: PBR value with constant $\boldsymbol{\beta_2}$

Negative chirp parameter (C = -2) $\gamma = 15$		Negative chirp parameter (C = -3) $\gamma = 20$		Positive chirp parameter (C = 2) $\gamma = 15$		positive chirp parameter (C = 3) $\gamma = 20$	
β2	PBR	β2	PBR	β2	PBR	β2	PBR
0.3	1.4051	0.3	2.2915	0.3	3.2110	0.3	4.1661
0.03	1.4125	0.03	2.2911	0.03	3.2171	0.03	4.1254
0.003	1.4260	0.003	2.4136	0.003	2.8494	0.003	2.7107
0.002	1.4451	0.002	2.1458	0.002	2.0481	0.002	

According to the results of the simulation which are illustrated in figure (4.7) to (c), the table (4.2) and the table (4.3). can see that with a certain variation in the parameter of the chirped (C), β2, and γ. However, the opposite occurs when the chirped process has been applied for the value C = 2 (where C > 0), the instantaneous frequency increases linearly from its front edge to the back edge. The opposite occurs when C = -2 (where C <0) which shows that the instantaneous frequency decreases linearly from the trailing edge to the leading edge.

It can be seen that the widening of the pulse depends on the sign of the GVD parameter (β_2) and the Chirp parameter (C).

Table (4.2) shows that the best value of the Chirp parameter C = -2. This proves that its positive value and its negative value can affect the state of the pulse during its propagation in the fiber. The advantage of using a value of the negative chirp parameter makes it possible to lower the effect of widening of the pulse and the nonlinear oscillations. This can be explained by the fact that the "delay" of a longer wavelength can inhibit the effect of GVD, i.e. when the value of GVD decreases due to the existence of a delay, the widening effects of the pulse are minimized.

Chapter Five
conclusion

5.1 Introduction:

The world of telecommunications continues to be revolutionized thanks to the advent of a jewel the size of a hair (optical fiber) which is used to transport information by light from one end to the other. During this transfer, there are effects leading to its distortion, in particular, chromatic dispersion and the Kerr effect. Under certain conditions, these effects compensate for each other and the signal propagates without distortion: it is the solitary impulse which is called soliton solution of the NLSE equation and known for a long time, which resists disturbances and which can in principle spread without spreading over long distances.

In order to understand the main effects that impede the propagation of a pulse (signal), we are interested in this thesis to numerically solve the non-linear Schrödinger equation by the method of `` Fourier with fractional step ''. Method introduced because of its simplicity of use, easy to develop and has good performance in computing time; also provides natural access to the spectral evolution of the field and is fast up to two orders of magnitude compared to finite difference methods.

5.2 Conclusion:

From the results of the simulation, can conclude that the order of variation of the dispersion parameter takes effect where the chirped method was performed in the input pulse of the propagation process. Furthermore, it has been found that the negative chirp has the best value among all the results in terms of PBR ratio. To this end, the dispersion parameters, the nonlinearity, and the input power value must be determined due to the shape, quantity, speed, and amplitude of the input pulse which are respectively affected by these factors.

We can conclude that in general in the abnormal dispersion regime, the direction of frequency slip (chirping) produced by the self-phase modulation effect (SPM) and due to the nonlinearity effect, is l opposite to that produced by dispersion (GVD).

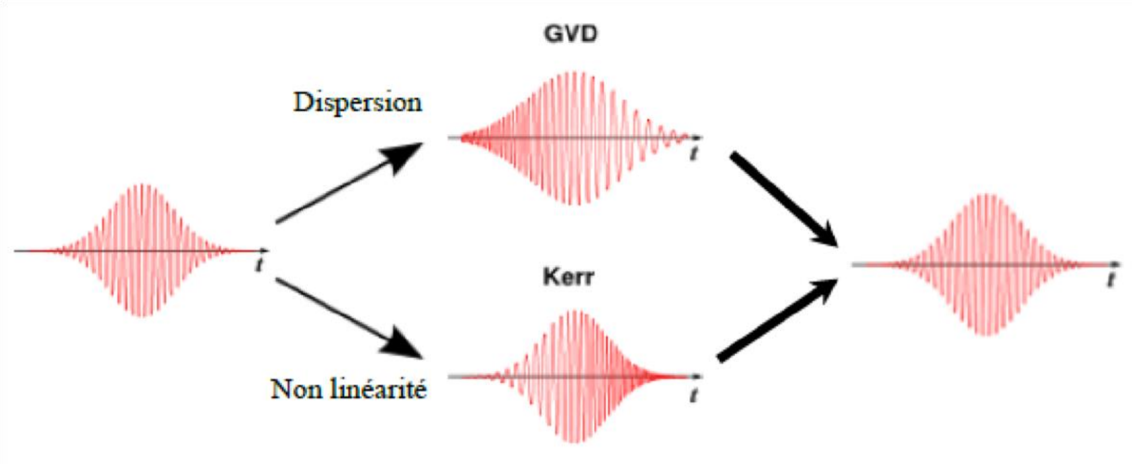

Figure 5.1 compensation process between self-phase modulation (Kerr effect) and GVD chromatic dispersion.

This indicates that the frequency slip induced by the phase auto-modulation can compensate for that induced by the chromatic dispersion (See Figure 4.8). This process leads to the formation of optical solitons which retain shape during propagation, the compression mechanism fundamentally due to high order solitons.

After solution of the NLSE by the SSFM method, we studied the effects which limit the power of the signal during its propagation in the optical fiber. This allowed us to conclude that:

- When the dispersion effects dominate, the pulse undergoes a broadening of its spectrum leading to a loss of information. These effects can be resolved by sending the signal at the zero-dispersion wavelength.
- When non-linear effects dominate, SPM introduce a phase shift which limits the maximum power of the signal to be transmitted over the fiber. As well as SPM increases the enlargement rate for a normal dispersion regime (β_2> 0) and reduces this rate for an abnormal dispersion regime ((β_2 <0)).

- When the pulse intensities are large enough to involve non-linear effects, these can be used to compensate for dispersion.

- Following the coupling of the two effects in the fiber, two regime cases are taken into account: normal dispersion regime, abnormal dispersion regime, the latter is very important because the auto phase modulation can compensate for the abnormal dispersion because it induces a phase term of sign opposite to that introduced by the dispersion of the fiber.

- The results of our simulations using the method of "Fourier with Split-step" for the study of the propagation of an impulse in the form of "Soliton Chirp" allows to conclude that the negative chirp has the best value among all the results in terms of the average of the PBR ratio. To this end, the dispersion parameters, the non-linearity, and the input power value must be determined due to the shape, quantity, speed, and amplitude of the input pulse which are respectively affected by these factors.

5.3 Future Work

Finally, the work undertaken within the framework of this thesis has opened a direction of research which in our opinion deserves to be deepened, in particular:

- Solving the nonlinear Schrödinger's equation considering other nonlinear effects, third order dispersion, the Brillouin or Raman effect.

- Study of the propagation of solitons in periodic structures, namely, optical fibers with photonic crystals

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