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Application of Continuous-Time Markov Chain Model for Repairing Machines (Case study: Asalaya Sugar Factory during the Season of 2019)

تطبيق نموذج سلسلة ماركوف ذات الزمن المستمر لإصلاح الماكينات
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قال تعالى:

وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

صدق الله العظيم

سورة الإسراء–الآية(85)

Dedication

To spirit of my father, Allah's Mercy upon he.

To my mother , my family

Acknowledgement

I really express my deep thanks gratitude to **Allah** for completing this research, also my full thanks to my supervisors ***Dr.Khalid RahamtallaKhedir***,Dr.***Mohammedelameen Essia Qurashi***, for their supervision. Special thanks to my, also my great thanks to my friend ***Eng***.***Khalid EltahirAbdall-Basit***(Asalaya Sugar Factory) and all his colleges for help in data collection. And my college and special friend***Mr.Osama Abdelaziem Mohammed*** for his support.

Abstract

This study aims at dealing with ContinuoustimeMarkov chain Model application on fault time of two machines (Mill troup &Boiler) an important machines in Asalaya Sugar Factory on season (January/2019–December/2019), which affiliated to Sudanese Sugar Company. The study conduces that failure time of machines follows Exponential distribution estimation fault distribution. Failure time represent transition matrix in the ContinuoustimeMarkov chain. The probability of machine in operating state is greater than probability of machine in a fail state. The high probability of the machine in operating state and the mean time of a machine stay an estimated by 4 hours in state (1) (operating state) meanwhile the machine stay in state (0) (fail state) estimated by one hour which indicates the efficiency of the maintenance unit, it is clear that, the probability of available time to repair machines when it fault approximately (0.80), this indicates that the machines has high availability.

Through estimating the failure rate and repair rate andthe transition probabilitiesfrom operational state to another , it was found thatfailure rate of the machine(Mill troup) is greater than the failure rate of the machine (Boiler),the probability of both machines in working condition is high ,probability of machines (Mill troup) expose more failure, which requiresmore effort in maintenance than machine (Boiler).The probability of the overall failure rate of the machines is negligible probability for the machines,which isa good indicator as it is unlikely that both machines will fail at the same time. That means maintenance work should take place immediately for the machine that suffers a malfunction.

ملخص الدراسة:

تهدف هذه الدراسة إلى تطبيق نموذج سلسلة ماركوف ذات الزمن المستمر على زمن الفشل (العطل) للماكنتين (Mill troup &Boiler) وهي من أهم الماكينات في مصنع سكر عسلاية التابعة لشركة السكر السودانية للموسم (يناير / 2019 – ديسمبر / 2019). توصلت الدراسة: إلى أن وقت زمن (العطل) الماكينات يتبع التوزيع الأسي. ويمثل زمن الفشل (العطل) مصفوفة إنتقالية في سلسلة ماركوف ذات الزمن المستمر. إحتمالية وجود الماكينة في حالة التشغيل أكبر من احتمال وجود الماكينة في حالة عطل، ويبقى الاحتمال الكبير للماكينة في حالة التشغيل. ويقدر وجود الماكينة في الحالة (1) (حالة التشغيل) بـ 4 ساعات .و بقاء الماكينة في الحالة (0) (حالة عطل) مقدرة بساعة واحدة ،مما يدل على كفاءة وحدة الصيانة في المصنع، ومن الواضح أن احتمال الزمن المتاح لإصلاح الماكينات عند حدوث عطل تقريبا يقدر (80%)، وهذا يشير إلى أن للماكينات إتاحية عالية.

من خلال دالة الموثوقية لتقدير معدل الفشل(العطل) ومعدل الإصلاح واحتمالات وتكوين مصفوقة الانتقال من حالة تشغيلية إلى أخرى، وجد أن معدل فشل(عطل) الماكينة (Mill troup) أكبر من معدل فشل الماكينة (Boiler)، وإحتمالية وجود الماكينتين في حالة عمل (حالة تشغلية)عالية، واحتمال تعرض الماكينة (Mill troup) لعطل كبير، الأمر الذي يتطلب مجهودًا أكبر في الصيانة للماكينة (Mill troup)، وإحتمال معدل الفشل (العطل) الكلي للماكينات هو احتمال ضئيل ، وهو مؤشر جيد لأن من غير المحتمل أن تتعطل كلا الماكينتين في نفس الزمن. وهذا يعني تتم إجراء الصيانة على الفور للماكينةالتي تعاني من العطل.

أهم ما أوصت به الدراسة: تحسين الكفاءة التشغيلية للماكينات عن طريق الصيانة الكلية أوالجزئية. اعتماداًعلى النتائج التي تقدمها نموذج سلسلة ماركوف ذات الزمن المستمر فهي تقدم مقياس الأكثر دقة للحالة التشغيلية للماكينات،الاهتمام العالي بالتسجيل الحقيقي للاعطال. حيث أن جميع الأساليب الكمية والرياضية التي تستخدم في مجال الصيانة ، تعتمد بصفة خاصة على مدى دقة تسجيل البيانات.

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1.1: Introduction:

The concept of maintenance refers to a group of activities aimed at increasing the actual use of machines. The productivity of a machine is defined by how it is operated and maintained throughout its life cycle, proper preparation and installation as well as regular maintenance, inspection and replacement of spare parts contribute to increasing working time and improving performance. the machines of the industrial plant are subject to many faults with the passage of time and a failure is defined as the loss of the machine's ability to perform an operation or set of operations that are necessary for the machine to provide a specific service (Lotfi, 2011,p5). The faults are also known as repairing, preventing and avoiding damage resulting from use (xioaun &Lifeng, p.294).Successful maintenance operations require the manufacturing facility to create plans and effective measures that support implementation processes, one of which is the use of mathematical methods that can provide an indication characterized by a high degree of accuracy about the operational condition of the machines. One of these methods is the use of a Markov chain to measure the total failure rate of production line machines and the transition probabilities form state to another. The research aims to apply one of the models of stochastic operations, which is the Markov chains model which can be used to calculate the failure rates and repair rates of machines and the transition probabilities the machines from one operational state to another. In order to ascertain the operating condition of the machines.

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1.2: The Research Problem:

The machines operate via two ways while functioning broken down. when machine breakdown it can be fixed and gets again as new well-functioning as it was before .the adaption and the use of markers continuous –time chain model in dealing with machine that happen to have default help in the predication regarding the machines age and default. The adaption and application of this model lead to a clear improvement in service system and in producing a machine data base for decision makers.

1.3 :The Importance of the Research:

The importance of this present study lies in the fact that Continuous –time Markov chain Model is one stochastic models that contribute in the study of default rates through limited durations. These models are used in working out solutions to the problem of the increasing via indorsing clear decisive policies to determine the running cost of the used and changeable derives.

1.4: Research Objectives:

This research aims to achieve the following objectives:

- Application of Continuous –Time Markov Chain in repairable machine in Asalaya Sugar Factory.
- Construct Markov chain transitions matrix of machines.
- Estimate availability of the machines in Asalaya Sugar Factory .
- Identifying the time durations of machine default .
- Calculating Over all failure rate for machines is negligible probability.
- Predict the remain age of running machines.

1.5: Research Hypotheses:

1.The failure time of machines follows Exponential distribution.

2.The observed transition matrix is embeddable in the continuous-time Markov chain

3.The machines have high availability

4. Markov chain can be applied in Asalaya Sugar Company to calculate probabilities transmission of machines from one operational state to another.

5. Over all failure rate for machines is negligible probability.

1.6 :Research Methodology:

In this research I will use the analytical method, which studies the continuous –time markov chain and its use in the formation continuous – time markov chain model in the machines Asalaya Sugar Factory for the purpose of forecasting. Also I will use the descriptive approach to present the applied study data with some descriptive measures and graphs to identify the general characteristics of the study data.

1.7 :Research limits:

Spatial limits: Asalaya Sugar Factory.

Temporal limits: faults time of machines for season (2019).

1.8: The Research Data:

The research is based on data for number of failures and time repair during 12 consecutive months for the year (2019) for machines depended on mechanical faults and the use of the data to Construct continuous –time Markov chain model .The system under study consists of two machines

(Mill troup) related to mill Suger cane and machine (Boiler) related to boil cane juice. The system performs the required function if both machines are in operating condition or that one of them is valid for work because one of them has a fault that does not affect the function of the second machine, it is possible to continue operating until the repair of the faulty machine is completed.

1.9 : Previous Studies:

1. Study of Boualem Rabta, Bart van den Boom and Vasco Molini entitled (Continuous-time Markov Chain Approach for Modeling of Poverty Dynamics: Application to Mozambique) 2016 .This paper explores the use of continuous-time Markov chain theory to describe poverty dynamics. It is shown how poverty measures can be derived beyond the commonly reported headcounts and transition probabilities. The added measures include the stationary situation, the mean sojourn time in a given poverty state and an index for mobility. Probit regression is employed to identify the most influential factors on the transition probabilities. Moreover, sensitivity analysis shows that the results are robust against perturbations of the transition matrix. We illustrate the approach with pseudo-panel data constructed from a repeated cross-section survey in Mozambique, using a pairwise matching method to connect households in the 2003 sample to similar households in 2009. Results reflect high and persistent poverty levels with considerable movements into and out of poverty. An estimated 57 percent of the poor in the first wave remained poor in the second wave and 43 percent moved out. Likewise, 64 percent remained non-poor and 36 percent moved in. The corresponding stationary poverty head counties 45 percent with respective mean sojourn time of 6.9 years in poverty and 8.4 years out of poverty. Conditioning the Markov chain on covariates identified by probit regressions indicates that poverty dynamics are responsive to household characteristic and livelihoods

2. Study of Hassan Abdul Hadi Hassan entitled (Calculating The Overall Failure Rate And The Transition Operational Status Probabilities For machines Using Markov Chains) 2016. The concept of maintenance refers to several activities that aim to increase the efficient usage of the equipment and industrial machines, in order to achieve high productivity levels. And enhance the quality of products. The maintenance aims to ensure that the machines and production equipment are kept in optimal operating condition. So it's a very important to stand on the actual operational status for that machines. There are many quantitative models can help the industrial facility to achieve this goal.The research aims to apply one of operations research models, which is called Markov chains to measure the overall machines failure rate. And the probability transmission from one operational condition to another. The Ibn Majid Grneral Company had been chosen as a research field, to apply the Markov chains model, on the rolling and the dish-end machine which were selected from the tanks workshop. The research found that the overall failure rate is (0.15018). And the probability that the two machines are in operating (0.62445). And the probability that the rolling machine is fault and the dish-end machine is operating (0.18590). And the probability that the rolling machine is operating and the dish-end machine is fault (0.18964) is fault (0.18964).

3. Study of Tamás Jónás , Noémi Kalló , Zsuzsanna Eszter Tóth entitled (Application of Markov Chains for Modeling and Managing Industrial Electronic Repair Processes) 2014. This paper presents a research of Markov chain based modeling possibilities of electronic repair processes provided by electronics manufacturing service (EMS) companies. These stochastic processes are considered as business-like, industrialized activities that are typically complex with a high number of process states and many possible paths from the start state to the absorbing end states. Two models based on absorbing and acyclic absorbing Markov chains are introduced in order to model these processes. The presented method provides a quick tool for determining the most important operational and statistical parameters of the process and mapping the paths that contribute the most to the total load of the process. These results support several managerial applications concerning e.g. process improvement, quality control and resource allocation. The proposed model is illustrated with an industrial application.

4. Study of Mohammad Saber Fallahnezhad ,Alie Ranjbar, Faeze Zahmatkesh Sredorahi entitled (**A** Markov Model for Production and Maintenance Decision) 2020. In this paper, we consider a production machine which may fail and it inecessary to repair the machine after each failure and there are two statusesfor each repair; in one case, we should replace the machine because of catastrophic failure and in the other case, only small repairs are neededTimes.

1.10 Compare between the research and Previous Studies :

We find most of the studies that dealt with of application of Continuous-Time Markov Chain poverty, while they were not interested in building Continuous-Time Markov Chain models to forecast faults of machines or repairable machines.

1.11 :Structure of Research:

This research contains five chapters; the first chapter includes an introduction, a problem, the importance, objectives, hypothesis, limitations, data and the methodology of the research.

Chapter two: contains Maintenance and Availability, which includes an introduction, Types of machine maintenance, Condition-based maintenance , Four ways to improve machine maintenance , important of machine maintenance, Purpose of Maintenance , Failure Modes Effects Analysis and Availability.

chapter four: contains the application of the research, in which data of the study described and the model continuous –time markov chain.

Chapter three: contains Continuous-Time Markov Chain Model, Introduction, Markov processes, Markov Property, Reliability Function, Markov Modules in Complex Systems, Markov Analysis in Fault Tree Analysis, Laplace Transforms, The Exponential Distribution, The Generator Matrix, Steady-state probabilities.

Chapter five: contains the results that have been findings and the proposed recommendations.

Chapter two

Maintenance and Availability

2.1: Introduction

2.2: Types of machine maintenance

2.3: Condition-based maintenance

2.4: Four ways to improve machine maintenance

2.5: important of machine maintenance

2.6: Purpose of Maintenance

2.7: Failure Modes Effects Analysis

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2.1 Introduction:

Machine maintenance is the work that keeps mechanical assets running with minimal downtime. Machine maintenance can include regularly scheduled service, routine checks, and both scheduled and emergency repairs. It also includes replacement or realignment of parts that are worn, damaged, or misaligned. Machine maintenance can be done either in advance of failure or after failure occurs. Machine maintenance is critical at any plant or facility that uses mechanical assets. It helps organizations meet production schedules, minimize costly downtime, and lower the risk of workplace accidents and injuries.

2.2 Types of machine maintenance

There are nine types of machine maintenance. Each one has its pros and cons (except reactive maintenance, which is all cons), and can be mixed and matched with assets to create a balanced maintenance program.

2.2.1 Reactive maintenance

Reactive maintenance refers to repairs done when a machine has already reached failure. Since it's unexpected, unplanned, and usually leads to rushed, emergency repairs, It's often called "fighting fires."

2.2.2 Run to fail maintenance

Run to fail maintenance is very similar to reactive maintenance. It involves letting a piece of equipment run until it breaks down. However, run to fail is a deliberate choice, whereas reactive maintenance is not. A plan is in place to ensure parts and labour are available to get the asset up and running, or replaced, as soon as possible.

2.2.3 Routine maintenance

Routine maintenance consists of basic maintenance tasks, such as checking, testing, lubricating, and replacing worn or damaged parts on a planned and ongoing basis

2.2.4 Corrective maintenance

Corrective maintenance is any work that gets assets back into proper working order, although it's most commonly associated with smaller, non-invasive tasks that fix a problem before a complete failure occurs. For example, realigning a part during a routine inspection.

2.2.5 Preventive maintenance

Preventive maintenance refers to any regularly scheduled machine maintenance intended to identify problems and repair them before failure occurs. Preventive maintenance can be split up into two predominant types: Time-based preventive maintenance and usage-based preventive maintenance. Time-based preventive maintenance are tasks scheduled at a certain time interval, such as the last day of every month or every 10 days. Usage-based preventive maintenance is when work is scheduled based on the operation of equipment, such as after 500 miles or 15 production cycles.

2.3: Condition-based maintenance

Condition-based maintenance depends on monitoring the actual condition of assets in order to perform maintenance when there is evidence of decreased performance or upcoming failure. This evidence can be obtained through inspection, performance data, or scheduled tests, and it can be gathered either on a regular basis or continuously, through the use of internal sensors

2.4: Four ways to improve machine maintenance

There are four main areas to focus on when aiming to improve machine maintenance at your facility: Planning, precision, protection, and measurement.

1. Planning

Having a machine maintenance plan in place will ensure that parts, equipment, and labour are available when they're needed, and that there is a strategy in place to use those resources effectively. Many maintenance plans will include both planned and scheduled maintenance, which will identify problems before failure occurs, and planned unscheduled maintenance, which ensures that failures are repaired, and assets are returned to working order as quickly as possible.

2. Precision

Establishing a precision maintenance strategy will ensure that maintenance tasks are performed consistently, accurately, and according to industry best practices.

In order for precision maintenance to be effective, you must ensure you have these four elements:

- Workers must have the training and skills to perform maintenance tasks quickly and accurately
- Tools and equipment that are needed to perform maintenance tasks must be available
- All maintenance materials, such as lubricants and spare parts, must be high quality and free from contamination

- Maintenance plans and workflows must be accessible and easy to follow

3. Protection

A key part of improving maintenance involves keeping workers safe. Workers must have adequate personal protective equipment (PPE) and be trained in how to use it correctly. There are a few important tasks to keep on top of when you're looking to improve safety**:**

- Frequent checks to ensure guards or barriers are in use and are not damaged
- Inspections of electrical equipment, power cords and switches to identify exposed wires
- Regular workplace safety training for every employee

4. Measurement

The final piece of the machine maintenance puzzle is measuring asset performance. Without data, condition-based, predictive, and prescriptive maintenance plans will not work. Accurate data about how your machinery performs lets you choose the right maintenance strategy, which will lead to better, more reliable performance.

2.5: Important of Machine Maintenance

The importance of an effective maintenance program cannot be overlooked because it plays such an important role in the effectiveness of Lean manufacturing. As in personal health care insurance, maintenance may be considered the heath care of our manufacturing machines and equipment. It is required to effectively reduce waste and run an efficient, continuous manufacturing operation, business, or service operation. The cost of regular maintenance is very small when it is compared to the cost of a major breakdown at which time there is no production.

2.6: Purpose of Maintenance

Purpose of Maintenance The main purpose of regular maintenance is to ensure that all equipment required for production is operating at 100% efficiency at all times. Through short daily inspections, cleaning, lubricating, and making minor adjustments, minor problems can be detected and corrected before they become a major problem that can shut down a production line. A good maintenance program requires company-wide participation and support by everyone ranging.

2.6.1: Breakdowns

A machine's breakdown true cost is sometimes difficult to measure. A recent survey showed that the cost for a machine breakdown is more than just the maintenance labor and materials to make the repair. A recent survey showed the actual cost for a breakdown between four to fifteen times the maintenance costs. When the breakdown causes production to stop, the costs are very high because no parts are being produced.For years, maintenance has been treated as a dirty, boring and often overlooked job. It is very important to get the best productivity from a company's equipment but it is not recognized as a part of the operation that produces revenue. The simple question is often, "Why do we need to maintain things regularly?" The answer is, "To keep things as reliable as possible.

2.6.2: General Maintenance

The challenge for reliability is dealing with data from the past. Failure is modeled, analyzed and, to some extent, predicted. Unfortunately, the prediction does not take into account users or working environment-related restrictions, and often the results are not those useful. Machine conditions are monitored at the machine level, one machine at a time. It is a "Fail and Fix approach." Troubleshooting is the primary purpose.

2.6.3: Autonomous Maintenance

The purpose of autonomous maintenance is to develop operators to be able to take care of small maintenance jobs on the equipment they use so skilled maintenance people can concentrate on value-added activity and technical repairs.

2.6.4: The Maintenance World of Tomorrow

With modern computing and information technologies, more products and machines are equipped with sensors on critical parts of machines to warn of potential failures long before they may fail so they can be corrected before they stop production.

2.6.5: Intelligent Maintenance

Systems Intelligent maintenance systems (IMS) Predict and Forecast equipment performance so "near-zero breakdown" status is possible. Near-zero downtime focuses on machine performance techniques to minimize failures. Data comes from two sources: sensors (mounted on the machines) and the entire enterprise system (including quality data, past history and trending). By looking at data from these sources (current and historical), it can predict future performance.

2.7: Failure Modes Effects Analysis

Failure Modes and Effects Analysis (FMEA) is methodology for analyzing potential reliability problems early in the development cycle where it is easier to take actions to overcome these issues, thereby enhancing reliability through design. FMEA is used to identify potential failure modes, determine their effect on the operation of the product, and identify actions to mitigate the failures. A crucial step is anticipating what might go wrong with a product. Failure Modes and Effects Analysis (FMEA) is applied to each system, subsystem, and component identified in the boundary definition. For every function identified, there can be multiple failure modes. The FMEA addresses each system function, all possible failures, and the dominant failure modes associated with each failure. The FMEA then examines the consequences of failure to determine what effect failure has on the mission or operation, on the system, and on the machine. Even though there are multiple failure modes, often the effects of failure are the same or very similar in nature. From a system function perspective, the outcome of any component failure may result in the system function being degraded. Similar systems and machines will often have the same failure modes, but the system use will determine the failure consequences. Other term that is used in this area is FMECA Failure Mode Critically Analysis. The most important contribution of FMECA with respect to FMEA that is focusing mainly on Criticality of the identified failures therefore sometimes, called single point failure mode .However in many texts and sources, the terms FMEA and FMECA are used to explain the same methodology and usually both include the criticality analysis. FMECA is an essential tool when Reliability Centered Maintenance (RCM) approach is adopted. It is used to identify what are the most critical components, their failure modes and to rank them according with the consequences they might have on the system The FMECA procedure is divided into the following steps:

•Identifying all potential failure modes and their causes.

•Evaluation of the effects on the system of each failure mode.

•Identifying failure detection methods.

•Identifying corrective measures for failure modes.

•Assessing the frequency and severity of important failures for criticality analysis.

2.7.1: Failure Effects

The effects of the failure for each failure mode are to be listed as follows:

•The Local Effect: is to describe the initial change in the equipment item or component operation when the failure mode occurs; failure detection methods, if any, are to be identified and availability of standby system/equipment to provide the same function.

•The Functional Failure is to describe the effect of the failure mode on the system or functional group; such as potential physical damage to the system/equipment item; or potential secondary damage to either other equipment items in the system or unrelated equipment items in the vicinity.

•The End Effect is to describe the overall effect on the vessel addressing propulsion, directional control, environment, fire and/or explosion. For offshore drilling units and offshore oil and gas production facilities, the End Effects would address drilling,

2.8: Failure Rate

A fault is defined as the loss of the machine's ability to perform an operation or set of operations that are necessary for the machine to provide a specific service (Lotfi, 2011,p.5)

faulty behavior can be described by a number of mathematical functions and quantitative methods, which differ in complexity according to the nature of the machines system (Dhillo, 2002, p.191), The commonly used mathematical function to calculate fault rate in an exponential distribution is:

$$\lambda = \frac{f(t)}{R(t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.1)}$$

Where $f(t)$ probability density function (pdf) of fault and R(t) Reliability function

2.8.1: Mean time to failure (MTTF)

One of basic measures of reliability is mean time to failure (MTTF) .This statistical value is defined as the average time expected until the first failure of machine. MTTF can calculated by the failure rate inverse, $(1/\lambda)$. Assuming failure rate

2.8.2: Mean time between failures (MTBF)

The basic measure of reliability is mean time between failures (MTBF) for repairable equipment. MTBF can be expressed as the time passed before a component, assembly, or system breakdowns, under the condition of a constant failure rate. On the other hand, MTBF of repairable systems is the predicted value of time between two successive failures. It is a commonly used variable in reliability and maintainability analyses. MTBF can be calculated as the inverse of the failure rate, λ, for constant failure rate systems. For example, for a component with a failure rateof 2 failures per million hours, the MTBF would be the inverse of that failure rate λ, or:

$$\text{MTBF} = \frac{1}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.2)$$

2.8.3: Mean time to repair (MTTR)

Mean time to repair (MTTR) can described as the total time that spent to perform all corrective or preventative maintenance repairs divided by the total of repair numbers. It is the anticipated time period from a failure (or shut down) to the repair or maintenance fulfillment. This is a term that typically only used in repairable systems.

2.9: Failure types

Failures generally be grouped into three basic types, though there may be more than one cause for a particular case. The three types included: early failures, random failures and wear-out

2.9.1: Failure Analysis and Prevention Failures.

In the early life stage, failures as infant mortality often due to defects that escape the manufacturing process. In general, when the defective parts fail leaving a group of defect free products, the number of failures caused by manufacture problems decrease. Consequently the early stage failure rate decreases with age. During the useful life, failures may related to freak accidents and mishandling that subject the product to unexpected stress conditions. Suppose the failure rate over the useful life is generally very low and constant. As the equipment reaches to the wear-out stage, the degradation of equipment is related to repetitious or constant stress conditions. The failure rate during the wear-out stage increases dramatically as more and more occurs failure in equipment that caused by wear-out failures.

2.9.2: Early life period

To ensure the integrity of design, we used many methods. Some of the design techniques include: burn-in (to stress devices under constant operating conditions); power cycling (to stress devices under the surges of turn-on and turn-off); temperature cycling (to mechanically and electrically stress devices over the temperature extremes); vibration; testing at the thermal destruct limits; highly accelerated stress and life testing; etc. Despite usage of all these design tools and manufacturing tools such as six sigma and quality improvement techniques, there will still be some early failures because we will not able to control processes at the molecular level. There is always the risk that, although the most up to date techniques are used in design and manufacture, early breakdowns will happen. In order to remove these risks — especially in newer product consumes some of the early useful life of a module via stress screening.The start of operating life in initial peak represents the highest risk of failure.

2.10: MTBF vs. useful life

Sometimes MTBF is Mistakenly used instead of component's useful life. Consider, the useful life of a battery is 10 hours and the measure of MTBF is 100,000 hours. This means that in a set of 100,000 batteries, there will be about one battery failure every 1 hour during their useful lives. Sometimes these numbers are so much high, it is related to the basis calculations of failure rate in usefulness period of component, and we suppose that the component will remain in this stage for a long period of time. In the above example, wear-out period decreases the component life, and the usefulness period becomes much smaller than its MTBF so there is not necessarily direct correlation between these two. Consider another example, there are 15,000 18-year-old humans in the sample. Our investigation is related to 1 year. During this period, the death rate became 15/15,000 = 0.1%/year. The inverse of the failure rate or MTBF is 1/0.001 = 1000. This example represents that high MTBF values is different from the life expectancy. As people become older, more deaths occur, so the best way to calculate MTBF would be monitor the sample to reach their end of life. Then, the average of these life spans are computed. Then we approach to the order of 75–80 which would be very realistic.

2.11: Wear-Out Period

As fatigue or wear-out occurs in components, failure rates increasing high. Power wear-out supplies is usually due to the electrical components breakdown that are subject to physical wear and electrical and thermal stress. Furthermore, the MTBFs or FIT rates calculated in the useful life period no longer apply in this area of the graph. A product with a MTBF of 10 years

can still exhibit wear-out in 2 years. The wear-out time of components cannot predict by parts count method. Electronics in general, and Vicar power supplies in particular, are designed so that the useful life extends past the design life. This way wear-out should never occur during the useful life of a module.

2.12: Failure sources

There are two major categories for system outages: 1. Unplanned outages (failure) and 2.Planned outages (maintenance) that both conducted to downtime. In terms of cost, unplanned and planned outages are compared but use the redundant components maybe mitigate it. The planned outage usually has a sustainable impact on the system availability, if their schematization be appropriate. They are mostly happen due to maintenance. Some causes included periodic backup, changes in configuration, software upgrades and patches can caused by planned downtime. According to prior research studies 44% of downtime in service providers is unscheduled. This downtime period can spent lots of money. Another categorization can be:

• Internal outage

• External outage

Specification and design flaws, manufacturing defects and wear-out categorized as internal factors. The radiation, electromagnetic interference, operator error and natural disasters can considered as external factors. However, a well-designed system or the components are highly reliable, the failures are unavoidable, but their impact mitigation on the system is possible.

2.13: Failure Rate Data

The most common ways that failure rate data can be obtained as following:

• Historical data about the device or system under consideration.

Many organizations register the failure information of the equipment or systems that they produce, in which calculation of failure rates can be used for those devices or systems. For equipment or systems that produce recently, the historical data of similar equipment or systems can serve as a useful estimate.

• Government and commercial failure rate data.

The available handbooks of failure rate data for various equipment can be obtained from government and commercial sources. MIL-HDBK-217F, reliability prediction of electrical equipment, is a military standard that provides failure rate data for many military electronic components. Several failure rate data sources are available commercially that focus on commercial components, including some non-electronic components.

• Testing

The most accurate source of data is to test samples of the actual devices or systems in order to generate failure data. This is often prohibitively expensive or impractical, so that the previous data sources are often used instead.

2.14 Reliability Model

We recall the definition of the most common stochastic models which are used to of repairable system the model is reliability models, homogenous Poisson process and non-homogenous process and families of life time distribution.

2.14.1: Measures of Reliability

The probability of fault is continued for a time can be defined as follows

$$P(t \le t) = F(t) \ldots, t \ge 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.3)$$

Where : t is random variable indicates the time of fault i.e. $F(t)$ denoted to probability machine in time t and is called follow non reliability.

If we define reliability the possibility of machine work successfully through time t , can be written follow reliability as follows:

$$R(t) = 1 - F(t) = P(t > t) \ldots\ldots\ldots\ldots\ldots\ldots(2.4)$$

If we denoted to follow density random variable for fault with $f(t)$ can be expression for follow reliability as follows:

$$R(t)=1-F(t)=1-\int_{\theta}^{t} f(t)dt=\int_{t}^{\theta} f(t)dt \qquad \ldots\ldots\ldots\ldots(2.5)$$

If the time of fault described with exponentional density the follow density it:

$$f(t)=\frac{1}{\theta}e^{-\frac{t}{\theta}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots, t\geq 0, \theta>0 \ldots\ldots\ldots\ldots(2.6)$$

So it can be rewritten the follow reliability form:

$$R(t)=\int_{t}^{\infty}\frac{1}{\theta}e^{-\frac{t}{\theta}}dt=e^{-\frac{t}{\theta}} \qquad \ldots\ldots\ldots\ldots, t\geq 0 \ldots\ldots\ldots\ldots\ldots\ldots(2.7)$$

2.14.2: Fault Rate:

The possibility of fault machine in specific time period t_1, t_2 can be expressed with follow non reliability.

$$\int_{t_1}^{t_2} f(t)dt=\int_{t_1}^{\infty} f(t)dt-\int_{t_2}^{\infty} f(t)dt=R(t_1)-R(t_2)$$

The called the at which get faults in specific time periodt_1, t_2, with rate faults throught period .denoted with t_1 for the not get fault at the beginning of the period and equation of the faults as :

$$\frac{R(t)-R(t_2)}{(t_2-t_1)R(t_1)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.8)$$

And not the faults rate for time .if we denoted for the period t_2 with: $(t+\Delta t)$,be equation (6.3).

$$\frac{R(t)-R(t+\Delta t)}{\Delta t.R(t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.9)$$

And means with rate it number of faults in each unit time.

2.14.3: Hazard Rate

Define as limits of rate of faults for a period of near-zero equation can be written in the form:

$$h(t)=\lim_{\Delta t\to 0}\frac{R(t)-R(t+\Delta t)}{\Delta t.R(t)}=\frac{1}{R(t)}\left[-\frac{dR(t)}{dt}\right]$$

$$h(t)=\frac{f(t)}{R(t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.10)$$

To find out possibility of fault machine it have age t in time period $[t,t+\Delta t]$ written as:

$$f_{pos}=h(t).dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.11)$$

The hazard rate refer to change in rate fault through age of machine. To find out hazard rate for the sample machines N (machine consisting of n element). We will assume $N_s(t)$ is random variable denotes to number of machines working successfully at time t thus, the $N_s(t)$ is binomial distribution.

$$P\left[N_s(t)=n\right]=\frac{N}{N.(N-n)}=\left[R(t)\right]^n\left[1-R(t)\right]^{N-n}$$

$$n=0,1,\ldots,N$$

The expected value for $N_s(t)$:

$$E\left[N_s(t)\right]=N.\left[R(t)\right]=N(t)$$

$$R(t)=\frac{E(N_s(t))}{N}=\frac{\bar{N}(t)}{N} \quad \text{..........................(2.12)}$$

And reliability in time t, it is arithmetic mean for rate success in t thus:

$$F(t)=1-R(t)=1-\frac{\bar{N}(t)}{N}=\frac{N-\bar{N}(t)}{N} \quad \text{..................(2.13)}$$

And rate density fall equal

$$F(t)=\frac{dF(t)}{dt}=-\frac{1}{N}.\frac{d\bar{N}(t)}{dt}$$

2.15: Availability:

Availability is an important metric used to assess the performance of repairable systems, accounting for both the reliability and maintainability properties of a component or system.

2.15.1: Availability Classifications

The classification of availability is somewhat flexible and is largely based on the types of downtimes used in the computation and on the relationship with time (i.e., the span of time to which the availability refers). As a result, there are a number of different classifications of availability, including:

- Instantaneous (or Point) Availability
- Average Uptime Availability (or Mean Availability)
- Steady State Availability
- Inherent Availability
- Achieved Availability
- Operational Availability

2.15.1.1: Instantaneous or Point Availability, A(t)

Instantaneous (or point) availability is the probability that a system (or component) will be operational (up and running) at a specific time, t. This classification is typically used in the military, as it is sometimes necessary to estimate the availability of a system at a specific time of interest (e.g., when a certain mission is to happen). The point availability is very similar to the reliability function in that it gives a probability that a system will function at the give time, t. Unlike reliability, however, the instantaneous availability measure incorporates maintainability information. At a given time, t, the system will be operational if one of the following conditions is met .(Elsayed, E.,1996).

The system functioned properly since the last repair at time u, $0 < u < t$. The probability of this condition is:

$$\int_0^t R(t-u)m(u)du \qquad \text{………………….……….(2.14)}$$

with *m(u)* being the renewal density function of the system. Consequently, the point availability is the summation of the above two probabilities, or:

$$A(t) = R(t) + \int_0^t R(t-u)m(u)du \qquad \text{…………..……….(2.15)}$$

2.15.1.2: Average Uptime Availability (or Mean Availability)

The mean availability is the proportion of time during a mission nor time period that the system is available for use. It represents the mean value of the instantaneous availability function over the period (0, T] and is given by:

$$\overline{A}(t) = \frac{1}{t}\int_0^t A(u)du \qquad \text{……………...………(2.16)}$$

2.15.1.2: Steady State Availability

The steady state availability of the system is the limit of the availability function as time tends to infinity. Steady state availability is also called the long-run or asymptotic availability. A common equation for the steady state availability found in literature is:

$$A(\infty) = \lim_{t\to\infty} A(t) \qquad \text{…………...………(2.17)}$$

However, it must be noted that the steady state also applies to mean availability. The next figure illustrates the steady state availability graphically:

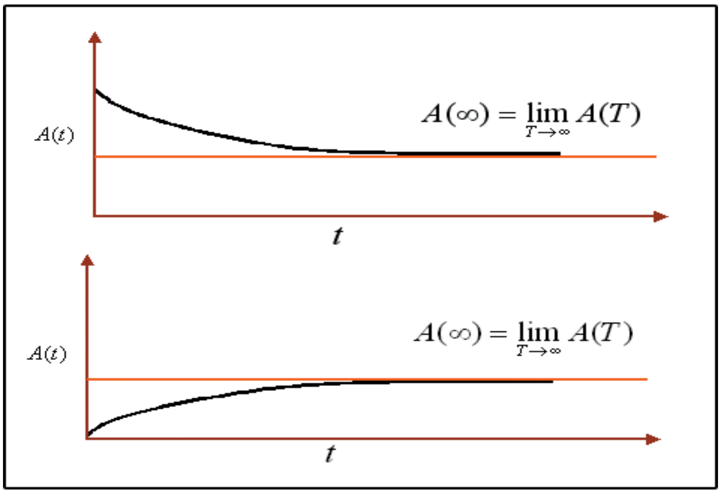

Figure (2.1) Illustration of point availability approaching steady state.

For practical considerations, the availability function will start approaching the steady state availability value after a time period of approximately four times the average time-to-failure. This varies depending on the maintainability issues and complexity of the system. In other words, you can think of the steady state availability as a stabilizing point where the system's availability is roughly a constant value.

2.15.1.3: Inherent Availability

Inherent availability is the steady state availability when considering only the corrective maintenance (CM) downtime of the system. This classification is what is sometimes referred to as the availability as seen by maintenance personnel. This classification excludes preventive maintenance downtime, logistic delays, supply delays and administrative delays. Since these other causes of delay can be minimized or eliminated, an availability value that considers only the corrective downtime is the inherent or intrinsic property of the system. Many times, this is the type of availability that companies use to report the availability of their products (e.g., computer servers) because they see downtime other than actual repair time as out of their control and too unpredictable.

The corrective downtime reflects the efficiency and speed of the maintenance personnel, as well as their expertise and training level. It also reflects characteristics that should be of importance to the engineers who design the system, such as the complexity of necessary repairs, ergonomics factors and whether ease of repair (maintainability) was adequately considered in the design .For a single component, the inherent availability can be computed by:

$$A_I = \frac{MTBF}{\mathrm{MTBF} + \mathrm{MTTR}} \quad \text{……..............……….(2.18)}$$

Where:

MTBF = Uptime / Number of System Failures

MTTR = CM Downtime / Number of System Failures

2.15.1.4:Achieved Availability

Achieved availability is very similar to inherent availability with the exception that preventive maintenance (PM) downtimes are also included. Specifically, it is the steady state availability when considering corrective and preventive downtime of the system. The achieved availability is sometimes referred to as the availability seen by the maintenance department (includes both corrective and preventive maintenance but does not include logistic delays, supply delays or administrative delays).Achieved availability can be computed by looking at the mean time between maintenance actions, MTBM, and the mean maintenance downtime, :

$$A_A = \frac{MTBM}{\mathrm{MTBM} + \overline{\mathrm{M}}} \quad \text{……..............………(2.19)}$$

2.15.1.5:Operational Availability

Operational availability is a measure of the "real" average availability over a period of time and includes all experienced sources of downtime, such as administrative downtime, logistic downtime, etc. The operational availability is the availability that the customer actually experiences. It is essentially the posterior availability based on actual events that happened to the system. The previously discussed availability classifications are a priori estimates based on models of the system failure and downtime distributions. In many cases, operational availability cannot be controlled by the manufacturer due to variation in location, resources and other factors that are the sole province of the end user of the product.

Operational availability is the ratio of the system uptime to total time. Mathematically, it is given by:

$$A_0 = \frac{UPtime}{\text{Operating Cycle}} \quad \text{……..............……….(2.20)}$$

where the operating cycle is the overall time period of operation being investigated and uptime is the total time the system was functioning during the operating cycle. (Note: The operational availability is a function of time, t, or operating cycle.)

$$A_o = \frac{Uptime}{Operating\ Cycle}$$

Chapter three

Continuous-Time Markov Chain Model

3.1 Introduction:

3.2 Markov processes

3.3 Markov Property

3.4 Chapman-Kolmogorov Equations

3.5 Steady-state probabilities

3.6: System Performance Characteristics

3.7 System Availability

3.8 Mean Duration of a System Failure

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3.10 Reliability Function

3.11 Markov Modules in Complex Systems

3.12 Markov Analysis in Fault Tree Analysis

3.13 Laplace Transforms:

3.14 Semi-Markov processes:

3.15 Continuous-Markov processes

3.16 The Exponential Distribution

3.17 The Generator Matrix

3.18 Steady-state probabilities

3.1 Introduction:

The models in the first six chapters are all based on the assumption that the components and the systems can be in one out of two possible states: functioning state or a failed state. We have also seen that the models are rather static and not well suited for analysis of repairable systems.In this chapter we will intro- duce a special type of stochastic processes, called Markov' chains, to model systems with several states and the transitions between the states. A Markov chain is a stochas- tic process $(x(t), t \geq 0)$ that possesses the Markov property. (We will define the Markov property clearly later.) The random variable X (t) denotes the stute of the process at time t. The collection of all possible states is called the state space, and we will denote it by X. The state space X is either finite or countable infinite. In most of our applications the state space will be finite and the states will correspond to real states of a system (see Example 8.1). Unless stated otherwise, we take X to be {O, 1,2, . . . , r), such that X contains r + 1 different states. The time may be discrete, taking values in (0, 1, 2, . . .}, or continuous. When the time is discrete, we have a discrete-time Markov chain; and when the time is continuous, we have a continuous- time Markov chain. A continuous-time Markov chain is also called a Markovprocess. When the time is discrete, we denote the time by n and the discrete-time Markov chain by (X,l,n = 0, I, 2, ...)

Table 3.1

State	Component 1	Component 2
3	Functioning	Functioning
2	Functioning	Failed
1	Failed	Functioning
0	Failed	Failed

The presentation of the theoretical basis of the Markov chains is rather brief and limited. The reader should consult a textbook on stochastic processes for more details. An excellent introduction to Markov chains may be found in, for example, Ross (1996). A very good description of continuous-time Markov chains and their application in reliability engineering is given by (Cocozza-Thivent (1997). The main focus in this book is on continuous-time Markov chains and how these chains can be used to model the reliability and availability of a system. In the follow- ing, a continuous-time Markov chain will be called a Markov process. In this chapter, we start by defining the Markov property and Markov processes. A set of linear, first order differential equations, called the Kolmogorov equations, are established to de- termine the probability distribution $p(t)=[p_o(t), p_I(t),\dots, p_r(t)]$ of the Markov process at time t, where Pi (t) is the probability that the process (the system) is in state i at time t. We then show that P(t), under specific conditions, will approach a limit P when $t+\infty$. This limit is called the steady-state distribution of the process (the system). Several system performance measures - like state visit frequency, system availability, and mean time to first system failure - are introduced. The steady-state distribution and the system performance measures are then determined for some sim- ple systems like series and parallel systems, systems with dependent components, and various types of standby systems. Some approaches to analysis of complex sys- tems are discussed. The time-dependent solution of the Kolmogorov equations is briefly discussed. The chapter ends by a brief discussion of semi-Markov processes, a generalization of the Markov processes.

3.2 Markov Processes:

With 100% capacity, 80% capacity, and so on. In other applications it is important to distinguish the various failure modes of an item, and we may define the failure modes as states. For a complex system, the number of states may hence be overwhelming, and we may need to simplify the system model, and separately consider modules of the system.

3.3 Markov Property

Consider a stochastic process ($x(t)\ t \geq 0$) with continuous time and state space X = (0, 1,2, . . . , r). Assume that the state of the process at time s is X (s) = i. The conditional probability that the process will be in state j at time t + s is Pr(X (t + s) = j I X(s) = i, X (u) = ($x(u), 0 \leq u < s$)

Where (x (u), 0 ≤ u < s} denotes the “history” of the process up to, but not including, time s. The process is said to have the Markov property if:

Pr (X (t + s) = j 1 X(t) = i, X(u) = x(u), 0 ≤ u < s)

= Pr(X(t + s) = j I X(s) = i)…………………………….(3.1)

F or all possible x (u), 0 ≤ u < s

In other words, when the present state of the process is known, the future development of the process is independent of anything that has happened in the past. A stochastic process satisfying the Markov property (8.1) is called a Markov pro- cess (or a continuous-time Markov chain). We will further assume that the Markov process for all i, j in X fulfills.

Pr (X (t + s) = j 1 X(s) = i) = Pr(X (t) = j I X(0) = i) for all s, t $\geq$ 0

which says that the probability of a transition from state i to state j does not depend on the global time and only depends on the time interval available for the transi- tion. A process with this property is known as a process with stationary transition probabilities, or as a time-homogeneous process. From now on we will only consider Markov processes (i.e., processes fulfilling the Markov property) that have stationary transition probabilities. A consequence of this assumption is that a Markov process cannot be used to model a system where the transition probabilities are influenced by long-term trends and/or seasonal variations. To use a Markov process, we have to assume that the environmental and operational conditions for the system are relatively stable as a function of time.

Consider a Markov process (X (t), t 0) with state space X = (0, 1,2, . . . , r} and stationary transition probabilities. The transition probabilities of the Markov process P_{ij} (t) = Pr(X (t) = j I X (0) = i) for all i, j $\in$ X

may be arranged as a matrix

$$p(t) = \begin{pmatrix} p_{00}(t) & p_{01}(t) & . & . & . & p_{0r}(t) \\ p_{10}(t) & p_{11}(t) & . & . & . & p_{1r}(t) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ p_{r0}(t) & p_{r1}(t) & . & . & . & p_{rr}(t) \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots(3.2)$$

Since all entries in P(t) are probabilities, we have that

$0 \leq pj,(t) \leq 1$for all f 10, i, j$\in$ x

When a process is in state i at time 0, it must either be in state i at time t or have made a transition to a different state. We must therefore have $\sum_{j=0}^{r} pij(t) = 1$ For all i $\in$ X

The sum of each row in the matrix P is therefore equal to 1. Note that the entries in row i represent the transitions out of state i (for j $\neq$ i), and that the entries in column j represent the transition into state j (for i $\neq$ j).

Let 0 = $S_o \leq S_1 \leq S_2 \leq$. . . be the times at which transitions occur, and let $T_i = s_{i+1} - s_i$ the it intercurrence time, or sojourn time, for i = 1,2, ... A possible "path" of a Markov process is illustrated in Fig. 8.1. The path is sometimes called the trajectory of the process. We define Sj such that transition i takes place immediately before Si, in which case the trajectory of the process is continuous from the right. The Markov process in Fig. 8.1 starts out at time t = 0 in state 6, and stays in this state a time T_I. At time

$S_1 = T_I$ the process has a transition to state 0 where it stays a time T_2. At time $S_2 = T_I + T_2$ the process has a transition to state 4, and so on.

Consider a Markov process that enters state i at time 0, such that X(0) = i. Let $\widehat{T}_i$ be the sojourn time in state i. [Note that Ti denotes the i th interoccurrence time,

We may now (Ross, 1996, p. 232) construct a Markov process as a stochastic process having the properties that each time it enters a state i:

1. The amount of time z. the process spends in state i before making a transition into a different state is exponentially distributed with rate, say α_i

. 2. When the process leaves state i, it will next enter state j with some probability P_{ij}, where

$$\sum_{\substack{j=0 \\ j \neq i}}^{r} p_{ij} = 1 .$$

The mean sojourn time in state i is therefore

$$E(\hat{t}_i) = \frac{1}{\alpha_i}$$

If $\alpha_i = \infty$, state i is called an instantaneous state, since the mean sojourn time in such a state is zero. When the Markov process enters such a state, the state is instantaneously left. In this book, we will assume that the Markov process has no instantaneous states, and that 0 $\leq \alpha_i < \infty$ for all i. If $\alpha_i = 0$, then state i is called absorbing since once entered it is never left. In Sections 8.2 and 8.3 we will assume that there are no absorbing states. Absorbing states are further discussed in Section 8.5.

We may therefore consider a Markov process as a stochastic process that moves from state to state in accordance with a discrete-time Markov chain. The amount of time it spends in each state, before going to the next state, is exponentially distributed.

Let T_{ij} be the time the process spends in state i before entering into state (j $\neq$ i). The time T_{ij} is exponentially distributed with rate aij.

Consider a short time interval at. Since T_{ij} and $\tilde{T}_i$ are exponentially distributed, we have that

$$p_{ij}(\Delta t) = p_r(\tilde{T}_i > \Delta t) = \ell^{-\alpha_i \Delta t} \approx 1 - \alpha_i \Delta t$$

$$p_{ij}(\Delta t) = p_r(\tilde{T}_i \leq \Delta t) = 1 - \ell^{-\alpha_i \Delta t} \approx \alpha_{ij} \Delta t$$

when Δt is "small". We therefore have that

$$\lim_{\Delta t \to 0} \frac{1 - p_{ij}(\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{p_r(\widetilde{T}_i < \Delta t)}{\Delta t} = \alpha_i$$

$$\lim_{\Delta t \to 0} \frac{p_{ij}(\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{pr(\widetilde{T}_i < \Delta t)}{\Delta t} = \alpha_{ij} \text{ for } i \neq j$$

For a formal proof, see Ross (1996, p. 239),

Since we, from (8.4) and (8.5), can deduce α_i and P_{ij} when we know a_{ij} for all i, j in X, we may equally well define a Markov process by specifying (i) the state space X and (ii) the transition rates a_{ij} for all i $\neq$ j in X. The second definition is often more natural and will be our main approach in the following. We may arrange the transition rates a_{ij} as a matrix

$$A = \begin{pmatrix} a_{00} & a_{01} & . & . & . & a_{0r} \\ a_{10} & a_{11} & . & . & . & a_{1r} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{r0} & a_{r1} & . & . & . & a_{rr} \end{pmatrix} \qquad \text{.....................(3.3)}$$

Where we have introduced the following notation for the diagonal elements

$$\alpha ii = -\alpha_i = -\sum_{\substack{j=0 \\ j \neq i}}^{r} \alpha_{ij}$$

We will call A the transition rate matrix of the Markov process. Some authors refer to the matrix A as the infinitesimal generator of the process. Observe that the entries of row i are the transition rates out of state i (for j

$\neq$ i). We will call them departure rates from state i. According to (8.5) $-\alpha_i = \alpha_i$ is the sum of the departure rates from state i, and hence the total departure rate from state i. The entries of column i are transition rates into state i (for j $\neq$ i). Notice that the sum of the entries in row i is equal to 0, for all i $\in$ X

Procedure to Establish the Transition Rate Matrix to establish the transition rate matrix A, we have to:

1. List and describe all relevant system states. Non-relevant states should be removed, and identical states should be merged (e.g, see Example 8.3). Each of the remaining states must be given a unique identification. In this book we use the integers from 0 up to r. We let r denote the best functioning state of the system and 0 denote the worst state. The state space of the system is thus X = (0, 1 . . . r]. Any other sequence of numbers, or letters may, however, also be used.

2. Specify the transition rates aij for all i $\neq$ j and i, j $\in$ X. Each transition will usually involve a failure or a repair. The transition rates will therefore be failure rates and repair rates, and combinations of these.

3. Arrange the transition rates aij for i $\neq$ j as a matrix, similar to the matrix (8.8). (Leave the diagonal entries aij open.)

4. Fill in the diagonal elements a_{ii} such that the sum of all entries in each row is equal to zero, or by using (8.9)

A Markov process may be represented graphically by a state transition diagram that records the a_{ij} of the possible transitions of the Markov process. The state transition diagram is also known as a Markov diagram. In the state transition diagram, circles are used to represent states, and directed arcs are used to represent transitions between the states. An example of a state transition diagram is given in Fig. 8.2.

Transition rate to state 2 is $a_{32} = \lambda_2$, and the transition rate to state 1 is $a_{31} = \lambda_1$. The sojourn time in state 3 is therefore $\bar{T}_3$ = min (T_{31}, T_{32}], where T_{ij} is the time to the first transition from state i to state j. T_3 has an exponential distribution with rate a31 + $a_{32} = \lambda_1 + \lambda_2$, and the mean sojourn time in state 3 is l/($\lambda_1 + \lambda_2$).

When the system is in state 2, the next transition may either be to state 3 (with rate $a_{23} = p_2$) or to state 0 (with rate $a_{20} = \lambda_1$). The probability that the transition is to state 3 is $\mu_2/(\mu_2 + \lambda_1)$, and the probability that it goes to state 0 is $\lambda_1/(\mu_2 + \lambda_1)$. The memoryless property of the exponential distribution ensures that component 1 is as good as new when the system enters state 2. In this example we assume that component 1 has the same failure rate).I in state 3, where both components are functioning, as it has in state 2, where only component 1 is functioning. The failure rate a_{20} of component 1 in state 2 may, however, easily be changed to a failure rate $\lambda^{\wedge}{}_1$ that is different from (e.g., higher than) λ_1.

When the system is in state 0, both components are in a failed state and two independent repair crews are working to bring the components back to a functioning state. The repair times T_{o1} and T_{o2} are independent and exponentially distributed with repair rates p_1 and p_2, respectively. The sojourn time T_o in state 0, min {T_{ol}, T_{o2} is exponentially distributed with rat ($\mu_1 + \mu_2$), and the mean downtime (MDT) of the system is therefore 1/ ($\mu_1 + \mu_2$). When the system enters state 0, one of the components will already have failed and be under repair when the other component fails.

The memoryless property of the exponential distribution ensures, however, that the time to complete the repair is independent of how long the component has been under repair

The transition rate matrix of the system is thus:

$$A = \begin{pmatrix} -(\mu_1+\mu_2) & \mu_2 & \mu_1 & 0 \\ \lambda_2 & -(\lambda_2+\mu_1) & 0 & \mu_1 \\ \lambda_1 & 0 & -(\lambda_1+\mu_2) & \mu_2 \\ 0 & \lambda_1 & \lambda_2 & -(\lambda_1+\lambda_2) \end{pmatrix} \quad \text{......................(3.4)}$$

3.4 Chapman-Kolmogorov Equations

By using the Markov property and the law of total probability, we realize that

$$p_{ij}(t+s) = \sum_{k=0}^{r} p_{ik}(t)\,p_{kj}(s) \text{ for all } i, j \in x, t, s > 0 \quad \text{……………………….(3.5)}$$

known as the Chapman-Kolmogorov equations. The equations may, by using (8.2), be written in matrix terms as

$$p(t+s) = p(t).p(s) \quad \text{……………………….(3.6)}$$

Notice that P (0) = II is the identity matrix. Notice also that if t is an integer, it follows that P (t) = $[P(1)]^{t}$. It can be shown that this also holds when t is not an integer.

Kolmogorov Differential Equations

We will try to establish a set of differential equations that may be used to find Pij (t), and therefore start by considering the Chapman-Kolmogorov equations

$$p_{ij}(t+\Delta t) = \sum_{k=0}^{r} p_{ik}(\Delta t)\,p_{kj}(t) \quad \text{………………………….(3.7)}$$

Note that we here split the interval ($0, t+\Delta t$) in two parts. First, we consider a transition from state i to state k in the small interval ($0, \Delta t$), and thereafter a transition from state k to state j in the rest of the interval. We now consider

$$p_{ij}(t+\Delta t) - p_{ij}(t) = \sum_{\substack{k=0 \\ k\neq i}}^{r} p_{ik}(\Delta t) p_{kj}(t) - [1 - p_{ii}(\Delta t)] p_{ij}(t)$$

…………………..(3.8)

By dividing by Δt and then taking the limit as $\Delta t \to 0$, we obtain

$$\lim_{\Delta t \to 0} \frac{p_{ij}(t+\Delta t) - p_{ij}(t)}{\Delta t} = \lim_{\Delta t \to 0} \sum_{\substack{k=0 \\ k\neq i}}^{r} \frac{p_{ik}(\Delta t)}{\Delta t} p_{kj}(t) - \alpha_i p_{ij}(t)$$

……………(3.9)

Since the summing index is finite, we may interchange the limit and summation on the right-hand side of (8.12) and obtain, using (8.6) and (8.7),

$$p_{ij}(t) = \sum_{\substack{k=0 \\ k\neq i}}^{r} a_{ik} p_{kj}(t) - \alpha_i p_{ij}(t) = \sum_{\substack{k=0 \\ k\neq i}}^{r} a_{ik} p_{kj}(t)$$

………………(3.10)

Where $a_{ii} = \alpha_i$, and the following notation for the time derivative is introduced:

$$p_{ij}(t) = \frac{d}{d_t} p_{ij}(t)$$

The differential equations (8.13) are known as the Kolmogorov backward equations. They are called backward equations because we start with a transition back by the start of the interval.

The Kolmogorov backward equations may also be written in matrix format as

$$p(t) = A.p(t)$$

We may also start with the following equation:

$$p_{ij}(t+\Delta t) = \sum_{k=0}^{r} p_{ik}(t) p_{kj}(\Delta t)$$
……………………(3.11)

Here we split the time interval ($0, t+\Delta t$) into two parts. We consider a transition from i to k in the interval (0, t), and then a transition from k to j in the small interval ($t, t+\Delta t$). We consider

$$p_{ij}(t+\Delta t) - p_{ij}(t) = \sum_{\substack{k=0\\k\neq j}}^{r} p_{ik}(t) p_{kj}(\Delta t) - [1 - p_{jj}(\Delta t)] p_{ij}(t)$$
………(3.12)

Named after the Russian mathematician Andrey N. Kolmogorov (1903-1987).

By dividing by Δt and then taking the limit as $\Delta t \to 0$ we obtain

$$\lim_{\Delta t \to 0} \frac{p_{ij}(t+\Delta t) - p_{ij}(t)}{\Delta t} = \lim_{\Delta t \to 0} \left[\sum_{\substack{k=0\\k\neq j}}^{r} p_{ik}(t) \frac{p_{kj}(\Delta t)}{\Delta t} - \frac{1 - p_{jj}(\Delta t)}{\Delta t} p_{ij}(t) \right]$$

Since the summation index is finite, we may interchange limit with summation and obtain

$$p_{ij}(t) = \left[\sum_{\substack{k=0\\k\neq j}}^{r} a_{kj} p_{ik}(t) - \alpha_j p_{ij}(t) = \sum_{k=0}^{r} a_{kj} p_{ik}(t) \right]$$
………………(3.12)

Where, as before, $a_{jj} = -\alpha_j$. The differential equations (8.15) are known as the Kolmogorov forward equations. The interchange of the limit and the sum above does not hold in all cases but is always valid when the state space is finite. The Kolmogorov forward equations may be written in matrix terms as

$$\tilde{p}(t) = p(t).A$$

For the Markov processes we are studying in this book the backward and the forward equations have the same unique solution P (t), where $\sum_{k=0}^{r} p_{ij}(t) = 1$ for all i in X. In the following, we will mainly use the forward equations.

3.4.1 State Equations:

Let us assume that we know that the Markov process has state i at time 0, that is, X (0) = i. This can be expressed as

$$p_i(0) = p_r(x(0) = i) = 1$$

$$p_k(0) = p_r(x(0) = k) = 0$$

Since we know the state at time 0, we may simplify the notation by writing P_{ij} (t) as P_j (t). The vector P(t) = [P_o(t). P_1 (t). . . P_r (t)] then denotes the distribution of the Markov process at time t, when we know that the process started in state i at time 0. As in (8.3) we know that $\sum_{j=1}^{r} p_i(t) = 1$.

The distribution P (t) may be found from the Kolmogorov forward equations (8.15)

$$\tilde{p}_j(t)\sum_{k=0}^{r} a_{kj} p_k(t)$$

Where, as before, $a_{jj} = \alpha_j$. In matrix terms, this may be written

$$[p_0(t),..., p_r(t)].\begin{pmatrix} p_{00} & p_{01} & . & . & . & p_{0r} \\ p_{10} & p_{11} & . & . & . & p_{1r} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ p_{r0} & p_{r1} & . & . & . & p_{rr} \end{pmatrix} = [p_0^*(t),...., p_r^*(t)] \quad \text{...............(3.13)}$$

or in a more compact form as

$$p(t).A = \tilde{p}(t)$$

Equations (8.19) are called the state equations for the Markov process.

Remark: Some authors prefer to present the state equations as the transpose of that is A^T. P (t) $^T = \tilde{p}(t)^T$. In this case the vectors will be column vectors, and equations (8.18) can be written in a more compact form as

$$\begin{pmatrix} p_{00} & p_{01} & . & . & . & p_{0r} \\ p_{10} & p_{11} & . & . & . & p_{1r} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ p_{r0} & p_{r1} & . & . & . & p_{rr} \end{pmatrix} . \begin{bmatrix} p_0(t) \\ p_1(t) \\ . \\ . \\ . \\ p_r(t) \end{bmatrix} = \begin{pmatrix} p^*{}_0(t) \\ p^*{}_1(t) \\ . \\ . \\ . \\ p^*{}_r(t) \end{pmatrix} \quad \text{...............(3.14)}$$

In this format the indexes do not follow standard matrix notation. The entries in column i represent the departure rates from state i, and the sum of all the entries in a column will be 0. The reader may choose in which format he wants to present the state equations. Both formats will give the same result. In this book, we will, however, present the state equations in the format of (8.18) and (8.19). Since the sum of the entries in each row in A is equal to 0, the determinant of A is 0 and the matrix is singular.

Consequently, equations (8.19) do not have a unique solution. However, by using that

$$\sum_{j=0}^{r} p_j(t) = 1 \qquad \text{..............(3.15)}$$

and the known initial state [Pi (0) = 1], we are often able to compute the probabilities P_j (t) for j = 0, 1,2, . . . , r . [Conditions for existence and uniqueness of the solutions are discussed, for example, by Cox and Miller (1965).The mean sojourn time in state 1 is the mean time to failure, MTTF $= \frac{1}{\lambda}$, and the mean sojourn time in state 0 is the mean downtime, MDT = $\frac{1}{\mu}$. The mean downtime is sometimes called the mean time to repair (MTTR). The state transition diagram for the single component is illustrated in Fig. 8.5. The state equations are.

$$p_0(t), p_1(t), \begin{pmatrix} -\mu \\ \lambda \end{pmatrix} = [\tilde{p}_0(t), \tilde{p}_1(t)] \qquad \text{..............(3.16)}$$

The component is assumed to be functioning at time t = 0

$$p_1(0) = 1, \quad p_0(0) = 0$$

Since the two equations we get from (8.20) are linearly dependent, we use only one of them, for example

$$-\mu p_0(t) + \lambda p_1(t) = \tilde{p}(t)$$

and combine this equation with $P_o(t) + P_I(t) = 1$. The solution is

$$p_1(t) = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda}\ell^{-(\lambda+\mu)t} \quad \text{……………(3.19)}$$

$$p_0(t) = \frac{\lambda}{\mu+\lambda} - \frac{\lambda}{\mu+\lambda}\ell^{-(\lambda+\mu)t} \quad \text{……………(3.17)}$$

For a detailed solution of the differential equation, see Ross (1996, p. 243). The availability of the component PI (t) denotes the probability that the component is functioning at time t, that is, the limiting availability $p_1 = \lim_{1 \to \infty}$ $p_1(t)$ is from

$$p_1 = \lim_{t \to \infty} p_1(t) = \frac{\mu}{\lambda+\mu} \quad \text{……………(3.18)}$$

The limiting availability may therefore be written as the well-known formula

$$p_1 = \frac{MTTF}{MTTF + MDT} \quad \text{……………(3.19)}$$

When there is no repair ($\mu = 0$), the availability is $p_1(t) = \ell^{-\lambda t}$ which coincides with the survivor function of the component. The availability $p_1(t)$.

In many applications only the long-run (steady-state) probabilities are of interest, that is, the values of Pj (t) when $t \to \infty$. In Example 8.5 the state probabilities P_j (t) (j = 0, 1) approached a steady-state P_j when $t \to \infty$. The same steady-state value would have been found irrespective of whether the system started in the operating state or in the failed state.

Convergence toward steady-state probabilities is assumed of the Markov processes we are studying in this chapter. The process is said to be irreducible if every state is reachable from every other state (see Ross 1996). For an irreducible Markov process, it can be shown that the limits

$$\lim_{t\to\infty} p_j(t) = p_j \quad \text{For} \quad j = 0,1,2....,r$$

Always exist and are independent of the initial state of the process (at time t = 0). For a proof, see Ross (1996, p. 25 1). Hence a process that has been running for a long time has lost its dependency of its initial state X (0). The process will converge to a process where the probability of being in state j is

$$p_j = p_j(\infty) = \lim_{t\to\infty} p_j(t) \quad \text{For} \quad j = 0,1,2....,r$$

3.5 Steady-state probabilities:

The steady-state probabilities P = [P_o. P_1. . . P_r] must therefore satisfy the matrix equation:

$$[p_0, p_1, ..., p_r] \begin{pmatrix} a_{00} & a_{01} & . & . & . & a_{0r} \\ a_{10} & a_{11} & . & . & . & a_{1r} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{r0} & a_{r1} & . & . & . & a_{rr} \end{pmatrix} = [0,0,....,0] \qquad ....(3.20)$$

Which may be abbreviated to

A.p=0

where as before

$$\sum_{j=0}^{r} p_j = 1$$

To calculate the steady-state probabilities, P_O, P_1. . . P_r, of such a process, we use r of the r + 1 linear algebraic equation from the matrix equation (8.25) and in addition the fact that the sum of the state probabilities is always equal to I. The initial state of the process has no influence on the steady-state probabilities. Note that P, also may be interpreted as the average, long-run proportion of time the system spends in state j

Matrix is

$$A = \begin{pmatrix} -(\mu_1+\mu_2) & \mu_2 & \mu_1 & 0 \\ \lambda_2 & -(\lambda_2+\mu_1) & 0 & \mu_1 \\ \lambda_1 & 0 & -(\lambda_1+\mu_2) & \mu_2 \\ 0 & \lambda_1 & \lambda_2 & -(\lambda_1+\lambda_2) \end{pmatrix} \quad \ldots\ldots(3.21)$$

We can use (8.26) to find the steady-state probabilities Pj for j = 0, 1, 2, 3, and we get the following equations:

$$\begin{aligned} &-(\mu+\mu_2)p_0+\lambda_2 p_1+\lambda_2 p_1+\lambda_1 p_2=0 \\ &\mu_2 p_0-(\lambda_2+\mu_1)p_1+\lambda_1 p_3=0 \\ &\mu_1[p_2-(\lambda_1+\mu_2)p_2+\lambda_2 p_3=0 \\ &p_0+p_1+p_2+p_3=1 \end{aligned}$$

Note that we use three of the steady-state equations from () and in addition the fact that $P_o + P_1 + P_2 + P_3 = 1$. Note also that we may choose any three of the four steady-state equations, and get the same solution.

The solution i

$$p_0 = \frac{\lambda_1\lambda_2}{(\lambda_1+\mu_2)(\lambda_2+\mu_2)}$$

$$p_1 = \frac{\lambda_1\mu_2}{(\lambda_1+\mu_2)(\lambda_2+\mu_2)}$$

$$p_2 = \frac{\mu_1 \lambda_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}$$

$$p_3 = \frac{\mu_1 \lambda_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}$$

Now for i=1, 2 let

$$q_i = \frac{\lambda_i}{(\lambda_i + \mu_i)} = \frac{MDT_i}{MTT_i + MDT_i}$$

$$q_i = \frac{\mu_i}{\lambda_i + \mu_i} = \frac{MTTF_i}{MTTF_i + MDT_i}$$

where $MDT_i = 1/\mu_1$ is the mean downtime required to repair component i, and $MTTF_i = 1/\lambda_1$ is the mean time to failure of component i (i = 1,2). Thus q_i denotes the average, or limiting, unavailability of component i, while p_i denotes the average (limiting) availability of component i (i = 1, 2). The steady-state probabilities may thus be written as

$$p_0 = q_1 q_2$$

$$p_1 = q_1 p_2$$

$$p_2 = p_1 q_2$$

$$p_3 = p_1 p_2$$

In this simple example, the components fail and are repaired independently of each other. We may therefore use direct reasoning to obtain the results in (8.28)

P_o = Pr (component 1 is failed). Pr (component 2 is failed) = $q_1 q_2$

P_1 = Pr (component 1 is failed). Pr (component 2 is functioning) = $q_1 p_2$

P_2 = Pr (component 1 is functioning). Pr (component 2 is failed) = p_1q_2

P_3= Pr (component 1 is functioning). Pr (component 2 is functioning) = p_1p_2

Note: In this simple example, where all failures and repairs are independent events, we do not need to use Markov methods to find the steady-state probabilities. The steady-state probabilities may easily be found by using standard probability rules for independent events. Please notice that this only applies for systems with independent failures and repairs.

3.6: System Performance Characteristics:

Several system performance measures that may be used in the steady-state situation are introduced in this section.

Visit Frequency the Kolmogorov forward equation () was

$$p^{*}_{ij}(t) = \sum_{\substack{k=0 \\ k\neq j}}^{r} a_{kj} p_{ik} - \alpha_j p_{ij}(t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.22)$$

When we let $t_i \rightarrow \infty$, then $p_{ij}(t) \rightarrow p_j$, and $p^{*}_{ij}(t) \rightarrow 0$ since the summation index in (8.15) is finite, we may interchange the limit and the sum and get, as $t \rightarrow \infty$,

$$0 = \sum_{\substack{k=0 \\ k\neq j}}^{r} a_{kj} p_k - \alpha_j p_j$$

$$p_j \alpha_j = \sum_{\substack{k=0 \\ k\neq j}}^{r} p_k a_{kj} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.23)$$

The (unconditional) probability of a departure from state j in the time interval $(t, t+\Delta t)$ is

$$\sum_{\substack{k=0\\k\neq j}}^{r} p_r((x(t+\Delta t)=k \cap (x(t)=j))$$

$$\sum_{\substack{k=0\\k\neq j}}^{r} p_r((x(t+\Delta t)=k / x(t)=j).p_r(x(t)=j)=\sum_{\substack{k=0\\k\neq j}}^{r} p_{jk}(\Delta t).p_j(t)$$

When $t \to \infty$, this probability tends to $\sum_{\substack{k=0\\k\neq j}}^{r} p_{jk}(\Delta t).p_j$, and the steady-state frequency of departures from state j is, with the same argument as we used to derive equation (),

$$v_j^{dop} = \lim_{\Delta t \to 0} \frac{\sum_{\substack{k=0\\k\neq j}}^{r} p_{jk}(\Delta t).p_j}{\Delta t} = p_j \alpha_j \quad \text{………………(3.24)}$$

The left-hand side of (8.29) is hence the steady-state frequency of departures from state j. The frequency of departures from state j is seen to be the proportion of time P_j spent in state j times the transition rate a, out of state j. Similarly, the frequency of transitions from state k into state j is P_k a_{kj}. The total frequency of arrivals into state j is therefore

$$v_j^{arr} = \sum_{\substack{k=0\\k\neq j}}^{r} p_k \alpha_{jk}$$

Equation (8.29) says that the frequency of departures from state j is equal to the frequency of arrivals into state j, for j = 0, 1, . . . , r, and is therefore sometimes referred to as the balance equations. In the steady-state situation, we define the visit frequency to state j as

$$v_j = p_j \alpha_j \sum_{\substack{k=0\\k\neq j}}^{r} p_k \alpha_{jk} \quad \text{…………………………(3.25)}$$

and the mean time between visits to state j is l/v_j.

Mean Duration of a Visit When the process arrives at state j, the system will stay in this state 5 times? j until the process departs from that stase, j = 0, 1, . . . , r. We have called $\widehat{T}_j$ the sojourn time in state j and shown that $\widehat{T}_j$ is exponentially distributed with rate α_j. The mean sojourn time, or mean duration of a visit, is hence

$$\theta_j = E(\tilde{T}_j) = \frac{1}{\alpha_j} \quad \text{for } j = 0,1,2,\ldots,r \ldots\ldots\ldots\ldots\ldots (3.26)$$

The mean proportion of time, Pi, the system is spending in state j is thus equal to the visit frequency to state j multiplied by the mean duration of a visit in state j for j=0,l, ..., r

3.7 System Availability :

Let X = (0, 1, . . . , r} be the set of all possible states of a system. Some of these states represent system functioning according to some specified criteria. Let B denote the subset of states in which the system is functioning, and let F = X - B denote the states in which the system is failed. The average or long-term availability of the system is the mean proportion of time when the system is functioning; that is, its state is a member of B. The average system availability A, is thus defined as

$$A_s = \sum_{j \in B} p_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.27)$$

In the following we will omit the term average and call As the system availability. The system unavailability (1 - As) is then

$$1 - A_s = \sum_{j \in F} p_j$$

The unavailability (1 - As) of the system is the mean proportion of time when the system is in a failed state.

Frequency of System Failures The frequency WF of system failures is the steady-state frequency of transitions from a functioning state (in B) to a failed state (in F):

$$\omega_F = \sum_{j \in B} \sum_{k \in F} p_j . \alpha_{jk}$$

3.8 Mean Duration of a System Failure

The mean duration θ_F of a system failure is defined as the mean time from when the system enters into a failed state (F) until it is repaired estored and brought back into a functioning state (B).

Analogous with (8.32) it is obvious that the system unavailability (1 - As) is equal to the frequency of system failures multiplied by the mean duration of a system failure. Hence

$$1 - A_s = \omega_F . \theta_F$$

3.9 Mean Time between System Failures

The mean time between system failures, MTBFs, is the mean time between consecutive transitions from a functioning state (B) into a failed state (F). The MTBFs may be computed from the frequency of system failures by

$$MTBF_s = \frac{1}{\omega_F} \quad \text{.........(3.28)}$$

3.10 Reliability Function

As discussed on page 321, the set of states X of a system may be grouped in a set B of functioning states and a set F = X - B of failed states. In the present section we will assume that the failed states are absorbing states. Consider a system that is in a specified functioning state at time t = 0. The survivor function R (t) determines the probability that a system does not leave the set B of functioning states during the time interval (0, t]. The survivor function is thus

$$R(t) = \sum_{j \in B} p_j(t) \qquad \text{……………………..…(3.29)}$$

The Laplace transform of the survivor function is

$$R^*(s) = \sum_{j \in B} p_j(s)$$

Mean Time to System Failure

The mean time to system failure, MTTFs, may according to Section 2.6 be determined by

$$MTTF_s = \int_0^\infty R(t)dt \qquad \text{…………………………..…(3.30)}$$

The Laplace transform of R (t) is given by

$$R^*(s) = \int_0^\infty R(t)\ell^{-st}dt \qquad \text{………………………(3.31)}$$

The MTTFs of the system may thus be determined from (8.76) by inserting s = 0. Thus

$$R^*(0) = \int_0^\infty R(t)dt = MTTF_s \qquad \text{………………..…(3.32)}$$

3.11 Markov Modules in Complex Systems

In Chapters 3 and 4 we discussed how to model complex systems by reliability block diagrams and by fault trees. We found that these approaches were suitable for rather static systems but were not able to account for dynamic features like complex maintenance and complex switching systems. Most systems will, however, have some modules that are rather static and other modules that have dynamic features. A possible approach is then to isolate the dynamic effects in as small modules as possible and treat these modules by Markov analysis. As far as possible, these modules should be defined in such a way that they are independent of each other. Thereafter, we may introduce these modules as supercomponents into a reliability block diagram (or the fault tree), as illustrated in the reliability block diagram in Fig. 8.20, and do the system calculations according to the approach we presented in Chapter 4. Dependencies between the various items in the reliability block diagram that we are not able to model explicitly, may be analyzed by the methods described in Chapter 6

The supercomponents 3 and 5 in Fig. 8.20 will normally comprise several compo- nents. When we establish a Markov model for the supercomponents, we will usually define several states for each of them and find the steady-state probability for each state.

To find the system reliability by the methods we presented in Chapter 4, we have to define two states for each item in the reliability block diagram. The states, resulting from the Markov analysis, must therefore be merged into a functioning state (1) and a failed state (0). By this merging we will loose a lot of information that might be useful.

3.7.1 Independent Modules

In some cases we may be able to split a complex system into manageable modules that may be regarded as independent. If we are able to establish Markov models for each module and calculate the steady-state probabilities, we may use standard probability rules to find the system steady-state probabilities. This approach is analogous to what we did in Example 8.6 for a simple parallel system. When we have a large number of independent modules with several states for each module, the total number of possible system states may be overwhelming. An alternative approach is Kronecker[4] sums and products (see Appendix C). We will illustrate this approach by a simple example. Which we recognize as the transition rate matrix for the parallel system in Example 8.6. The Kronecker sum of the transition rate matrices for the two independent modules 0 is therefore equal to the transition rate matrix for the whole system

4Named after the German mathematician Leopold Kronecker (1 823-1891)

It has been shown that the result in Example 8.10 also is valid in the general case. If a system comprises n independent modules with transition rate matrices A_1, A_2. A_n, then the transition rate matrix A of the system may be written as

$$A = A_1 \oplus A_2 ... \oplus A_n = \bigoplus_{j=1}^{n} A_i$$

For details, see Amoia and Santomauro (1977).

If we are able to split the system into manageable and independent modules and establish transition rate matrices for the various modules, the Kronecker sum may then be used to establish the total system transition rate matrix. Several of the most popular mathematical programs have specific subroutines that may be used to find the system transition rate matrix.

The Kronecker product is very efficient when it comes to solving linear equations. Let hi denote the transition rate matrix of module i, and let P (;) = [Pi'), . . . , Pri] denote the steady-state probabilities of module i. We know from (8.26) that PCi). $A = 0$. Let us now assume that we have a system with two independent modules with transition rate matrices A_1 and

A_2 and steady-state probabilities P(l) and P(2), respectively. We know from (8.96) that the transition rate matrix for the system is given by $A = A_1 \oplus A_2$. The system steady-state probabilities P must fulfill

$$p.A = p.(A_1 \oplus A_2) = 0$$

The question is then: Will it be possible to find P from P (1) and P (2)? Before we answer this question, we look at an example

1. Split the complex system into a set of n manageable and coherent modules. The various modules must be independent. Components within a module may, however, be dependent. Dependent components must belong to the same mod- ule.

2. Find the transition rate matrix Ai and the corresponding steady-state probability- ties P(i) for each module i = 1,2, . . . , n. (It might be wise to build a "library" of standard modules)

3. If of interest, the transition rate matrix for the system may be determined by $A_1 \oplus A_2 \oplus ... \oplus A_n$.

4. Determine the steady-state probabilities for the system by $p = p^{(1)} \oplus p^{(2)} \oplus ... \oplus p^{(n)}$

In practice, it might be a problem to keep track of the indexes in P, that is, to realize which system state corresponds to a specific index. It is therefore important to be very systematic when defining the indexes for each module. The Kronecker product approach has been applied to protective relays in trans- former stations by Svendsen (2002). Application of the Kronecker product to depen- dent modules was discussed by Lesanovskgd (1988).

3.12 Markov Analysis in Fault Tree Analysis

We will now illustrate how results from Markov analysis can be used in fault tree analysis. Assume that a fault tree has been established with respect .to a TOP event (a system failure or accident) in a specific system. The fault tree has n basic events (components) and k minimal cut sets K_1, K_2, . . . , K_k. The probability of the fault tree TOP event may be approximated by the upper bound approximation (4.50)

$$Q_0(t) \approx 1 - \prod_{j=1}^{k}(1 - \tilde{Q}_j(t)) \quad \text{..................(3.30)}$$

Let us assume that the TOP event is a system failure, such that $Q_0(t)$ is the system unavailability. The average (limiting) system unavailability is thus approximately

$$Q_0 \approx 1 - \prod_{j=1}^{k}(1 - \tilde{Q}_j) \quad \text{.........................(3.31)}$$

Where $\tilde{Q}_j$ denotes the average unavailability of the minimal cut parallel structure corresponding to the minimal cut set K_j, j = 1.2, . . . , k. In the rest of this section we will assume that component i has constant failure rate A;, mean downtime to repair MDT_i, and constant repair rate $\mu_i = 1/MDTi$ for i = 1,2, . . . , n. Furthermore, we assume that $\lambda_i = \mu_i$ for all i = 1,2, . . . , n.

The average unavailability q_i of component i is $\mu_i/(\mu_i + \lambda_i)$, which may be approximated by λ_I . MDT_i, such that

$$\tilde{Q}_j = \prod_{i \in k_j} \frac{\mu_i}{\mu_i + \lambda_i} \approx \prod_{i \in k_j} \lambda_i . MDT_i \quad \text{........................(3.33)}$$

The TOP event probability (system unavailability) is thus approximately

$$Q_0 \approx 1-\prod_{j=1}^{k}(1-\prod_{i\in k_j}\lambda_i.MDT_i)$$

$$Q_0 \approx \sum_{j=1}^{k}\prod_{i\in k_j}\lambda_i.MDT_i$$

Cut Set Information Consider a specific minimal cut parallel structure Kj, for j = 1,2, . . . , k. As before we assume that the components fail and are repaired independent of each other. When all the components of the cut set K_j are in a failed state, we have a cut set failure. The mean duration of a failure of cut set K_j is from ()

$$MDT_i = \frac{1}{\sum_{i\in k_j}\mu_i} \qquad \text{...............(3.34)}$$

The expected frequency of cut set failures ω_j is from (8.48)

$$\omega_j \approx \left(\prod_{i\in k_j}\frac{\lambda_i}{\mu_i}\right).\left(\sum_{i\in k_j}\mu_i\right)$$

and, the mean time between failures (MTBF) of cut set K_j is

$$MTBF_j = \frac{1}{\omega_k} \qquad \text{..................................(3.35)}$$

Note that $MTBF_j$ also includes the mean downtime of the cut parallel structure. The downtime is, however, usually negligible compared to the uptime. System information the system may be considered as a series structure of its k minimal cut parallel structures. If the cut parallel structures

3.13 Laplace Transforms:

An alternative approach is to use Laplace transforms. An introduction to Laplace transforms is given in Appendix B. Again, assume that we know P (0), the distribution of the Markov process at time 0. The state equations (8.19) for the Markov process at time t are seen to be a set of linear, first order differential equations. The easiest and most widely used method to solve such equations is by Laplace transforms. The Laplace transform of the state probability P_j (t) is denoted by $p_j^*(s)$, and the Laplace transform of the time derivative of P_j (t) is, according to Appendix B,

$$\xi\left[p_j^*(t)\right] = sp_j^*(s) - p_j(0) \quad \text{For } j=0,1,2,\ldots,r \ldots\ldots\ldots\ldots\ldots (3.35)$$

The Laplace transform of the state equations () is thus in matrix terms

$$p^*(s).A = sp^*(s) - p(0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.36)$$

By introducing the Laplace transforms, we have reduced the differential equations to a set of linear equations. The Laplace transforms P_T (s) may now be computed from (8.108). Afterwards the state probabilities P_j (t) may be determined from the inverse Laplace transforms

3.14 Semi-Markov Processes:

In Section 8.2 we defined a Markov process as a stochastic process having the prop- erties that each time it enters a state i:

1. The amount of time the process spends in state i before making a transition into a different state is exponentially distributed with rate, say α_i

2. When the process leaves state i, it will next enter state j with some probability p_{ij} ,where $\sum_{\substack{j=0 \\ j\neq i}}^{r} p_{ij} = 1$

An obvious extension to this definition is to allow the time the process spends in state i (the sojourn time in state i) to have a general "life" distribution, and also to let this distribution be dependent on the state to which the process will go. Ross (1996, p. 213) therefore defines a semi-Marko process as a stochastic process { $x(t), t \geq 0$ } with state space X = (0, 1,2, . . . , r) such that whenever the process enters state i

1. The next state it will enter is state j with probability P_{ij}, for i, j in X.

2. Given that the next state to be entered is state j, the time until the transition from i to j (OCCUTS) has distribution F_{ij} .

The skeleton of the semi-Markov process is defined in the same way as for the Markov process (see Section 8.2), and will be a discrete-time Markov chain. The semi-Markov process is said to be irreducible if the skeleton is irreducible. The distribution of the sojourn time $\widehat{T}_i$ in state i is

$$F_i(t) = \sum_{\substack{j=0 \\ j\neq i}}^{r} p_{ij} F_{ij}(t) \qquad \text{......................(3.37)}$$

The mean sojourn time in state i is

$$\mu_i = \mathrm{E}(\widehat{T}_i) = \int_0^\infty t dF_i(t) \qquad \text{........... ....(3.38)}$$

We notice that if $F_{ij}(t) = 1 - \ell^{\alpha_i t}$, the semi-Markov process is an ordinary Markov process. Let ci denote the time between successive transitions into state i, and let $\mu_{ii} = \mathrm{E}(T_{ii})$ the visits to state i will now be a renewal process, and we may use the theory of renewal processes described in Chapter 7.

If we let Ni (t) denote the number of times in [0, r] that the process is in state i, the family of vectors is called a Markov renewal process.

$$[N_0(t), N_1(t), ..., N_r(t)] \text{ For } t \geq 0$$

If the semi-Markov process is irreducible and if 7'ii has a nonlattice distribution with finite mean, then

$$\lim_{t \to \infty} pr(x(t) = i \mid x(0) = j) = p_i$$

exists and is independent of the initial state. Furthermore

$$p_i = \frac{\mu_i}{\mu_{ii}} \qquad \text{……………………………….……(3.39)}$$

For proof, see Ross (1996, p. 214). Pi is the proportion of transitions into state i and is also equal to the long-run proportion of time the process is in state i .

When the skeleton (the embedded process) is irreducible and positive recurrent, we may find the stationary distribution of the skeleton $\pi = [\pi_0, \pi_1, ..., \pi_r]$ as the unique solution of

$$\pi_j = \sum_{i=0}^{r} \pi_i p_{ij} \qquad \text{………………..…(3.40)}$$

where $\sum_i \pi_j = 1$ and $\pi_j = \lim_{n \to \infty} pr(x_n = j)$ [since we assume that the Markov process is aperiodic]. Since the nj is the proportion of transitions that are into state j, and p, is the mean time spent in state j per transition, it seems intuitive that the limiting probabilities should be proportional to $\pi_j \mu_j$. In fact

$$p_j = \frac{\pi_j \mu_j}{\sum_i p_{ij} \mu_j} \quad \text{……………….……(3.41)}$$

For a proof, see Ross (1996, p. 215). Semi-Markov processes are not discussed any further in this book. Details about semi-Markov processes may be found in Ross (1996), Cocozza-Thivent (1997), and Limnios and Oprisan (2001)

3.15: Continuous-Time Markov Chain

This section begins our study of Markov processes in continuous time and with discrete state spaces. Recall that a Markov process with a discrete state space is called a Markov chain, so we are studying continuous-time Markov chains. It will be helpful if you review the section on general Markov processes, at least briefly, to become familiar with the basic notation and concepts. Also, discrete-time chains plays a fundamental role, so you will need review this topic also. We will study continuous-time Markov chains from different points of view. Our point of view in this section, involving holding times and the embedded discrete-time chain, is the most intuitive from a probabilistic point of view, and so is the best place to start. In the next section, we study the transition probability matrices in continuous time. This point of view is somewhat less intuitive, but is closest to how other types of Markov processes are treated. Finally, in the third introductory section we study the Markov chain from the view point of potential matrices. This is the least intuitive approach, but analytically one of the best. Naturally, the interconnections between the various approaches are particularly important. Continuous-time. Markov Chain model use to represent and calculate the transition probabilities of the machine from being in operating to being down at specific points in time. In addition to calculating the long run (stationary) probability for machine to be state operating / state down, the mean sojourn time in state operating / state down.

A continuous-time Markov chain X(t) is defined by two components: a jump chain, and a set of holding time parameters λi. The jump chain consists of a countable set of states S⊂{0,1,2,… } along with transition probabilities pij. We assume $p_{ii=0}$, for all non-absorbing states i∈S. We assume

1. if $X(t)=i$, the time until the state changes has $Exponential(\lambda i)$ distribution;
2. if $X(t)=i$, the next state will be j with probability pij.

The process satisfies the Markov property. That is, for all 0≤ $t1< t2< \cdots < tn < tn+1$, we have

$$P\left(X\left(t_{n+1}\right)= j / X\left(t_{n}\right)=i, X\left(t_{n-1}\right)=i_{n-1,\ldots,} X\left(t\right)=i\right)= P\left(X\left(t_{n+1}\right)= j / X\left(t_{n}\right)=i\right) \qquad .(3.42)$$

let's define the transition probability $Pij(t)$ as

$$P(t)=\left(X\left(t+s\right)= j / X\left(s\right)=i\right)= P\left(X\left(t\right)= j / X\left(0\right)=i\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.43)$$

For $s,t \in [0,\infty]$

We can then define the *transition matrix*, $P(t)$. Assuming the states are 1, 2, ⋯, r, then the state transition matrix for any t≥0 is given by

$$P(t)=\begin{pmatrix} P_{00}(t) & P_{01}(t) \\ P_{10}(t) & P_{11}(t) \end{pmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.44)$$

Where P(t) are probabilities $0 \leq P_{ij}(t) \leq 1$, we have $P_{ij} = (X(t) = j / X(0) = i)$ for all $t \geq 0$ $i, j \in x$ is the probability for a machine to move from state i to state j at time . The sum of each row in the matrix P is therefore equal to 1:

$$\sum_{j=0}^{1} P_{ij}(t) = 1 \qquad \text{...(3.45)}$$

3.16: The Exponential Distribution :

A random variable t has the *exponential distribution* with rate parameter $\lambda \in (0, \infty)$ if t has a continuous distribution on $(0, \infty)$ with probability density function f given by:

$$f(t) = \lambda e^{-\lambda t} \quad \text{for } t \in (0, \infty) \quad \text{.. ....} \quad (3.45)$$

Equivalently, the right distribution function *F* is given by:

$$F(t) = P(T > t) = e^{-\lambda t} \quad \text{for } t \in (0, \infty) \quad \text{..........} \quad (3.46)$$

The mean of the distribution is $1/\lambda$ and the variance is $1/\lambda^2$ .The exponential distribution has an amazing number of characterizations. One important is the memoryless property which states that a random variable t with values in $(0, \infty)$ has an exponential distribution if and only if the conditional distribution of $t - s$ given $t > S$ is the same as the distribution of τ itself, for every $S \in (0, \infty)$. It's easy to see that the memoryless property is equivalent to the law of exponents for right distribution function F, namely

$F=(s+t)=F(s)F(t)$ for $s,t\in(0,\infty)$. Since F is right continuous, the only solutions are exponential functions.

2.17 The Generator Matrix

The generator matrix, usually shown by Q, gives us an alternative way of analyzing continuous-time Markov chains. Consider a continuous-time Markov chain $X(t)$ .Assume $X(0)=i$. The chain will jump to the next state at time T_1, where T_1 ~*Exponential* (λ). In particular, for a very small t>0, we can write:

$$P(T_1<\delta)=1-e^{-\lambda t} \quad \approx 1-(1-\lambda t)=\lambda t \qquad \ldots\ldots \qquad (3.47)$$

Thus, in a short interval of length δ, the probability of leaving state i is approximately For this reason, λ is often called the transition rate out of state I, we compute positive constants μ and λ. The generator matrix for the continuous Markov chain:

$$Q=\begin{bmatrix} -\mu & \mu \\ \lambda & -\lambda \end{bmatrix} \qquad \ldots.. \qquad (3.48)$$

We use generator matrix (6) to estimate the transition probability (Ross 1996, p.243) we get.

$$P_{00}(t)=\frac{\lambda}{\mu+\lambda}+\frac{\mu}{\mu+\lambda}e^{-(\mu+\lambda)t} \qquad (3.49)$$

$$P_{01}(t)=\frac{\mu}{\mu+\lambda}-\frac{\mu}{\mu+\lambda}e^{-(\mu+\lambda)t} \qquad (3.50)$$

$$P_{10}(t)=\frac{\lambda}{\mu+\lambda}-\frac{\lambda}{\mu+\lambda}e^{-(\mu+\lambda)t} \qquad (3.51)$$

$$P_{10}(t)=\frac{\mu}{\mu+\lambda}+\frac{\lambda}{\mu+\lambda}e^{-(\mu+\lambda)t} \qquad (3.52)$$

3.18 Steady-State Probabilities

The steady-state probabilities machine stays in the long run in state 0 (non working) and state 1(working) equal to:

$$V_0 = \frac{\lambda}{\mu + \lambda} \qquad (3.53)$$

$$V_1 = \frac{\mu}{\mu + \lambda} \qquad (3.55)$$

The stationary vector:

$$V = \left[\frac{\lambda}{\mu + \lambda} \quad , \frac{\mu}{\mu + \lambda} \right] \qquad (3.56)$$

The mean sojourn time in state 1 is the mean time to failure, MTTF = $\frac{1}{\lambda}$,

and the mean sojourn time in state 0 is the mean downtime, MDT = $\frac{1}{\mu}$.

Chapter four

Application

4.1:Introduction

Transition probabilities of machines from state0 to 1 are estimated from failure time of machines described data by using some descriptive measures. Test the distribution of data used Exponential distribution; we compute the generator matrix of the underlying continuous-time Markov chain after checking an conditions. It was applied to each machine separately, then the two machines.The system under study consists of two machines (Mill troup) related to mill Suger cane and machine (Boiler) related to boil cane juice. The system performs the required function if both machines are in operating condition or that one of them is valid for work because one of them has a fault that does not affect the function of the second machine, it is possible to continue operating until the repair of the faulty machine is completed. The probabilities of the system will be one of the following four states:

State (0): Both machines working.

State (1): Machine (Mill troup) working - machine (Boiler) non-working.
.

State (2): Machine (Boiler) working - machines (Mill troup) non-working.

State (3): Both machines non – working.

4.2: Description of Failure Times:

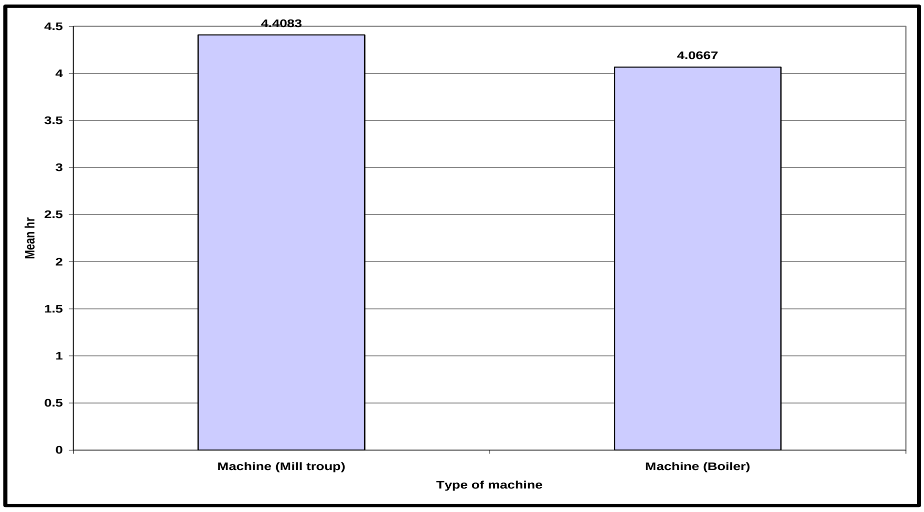
.

Table (4.1)

Ratesoffailure times for Both machine

Machine	Total failure no	Total failure time	Mean (hr)	Std. (hr)	95% C.I. for Mean	
					Lower Bound	Upper Bound
Machine (Mill troup)	61	52.900	4.4083	2.0725	3.0915	5.7252
Machine (Boiler)	51	48.800	4.0667	2.5296	2.4594	5.6937
Both machines	112	101.7	4.2376	2.2683	3.2779	5.1953

Source: The researcher from applied study, SPSS Package, 2021



Source: The researcher from applied study, Excel Package, 2021

Figure (4.1): Shows meanoffailure times for bothmachines

From above table and figuer, it has shown that according to the mean values for the each machines ,There is a convergence between the mean failure of the two machines, as the mean failure of machine (Mill troup) is (4.4083) hours, while the mean failure of machine (Boiler) is (4.0667) during the year.

4.3: Results of machine (Mill troup):

4.3.1: Test of failure time distribution:

Here we test the following hypothesis**:**

H_o: The failure time follows Exponential distribution

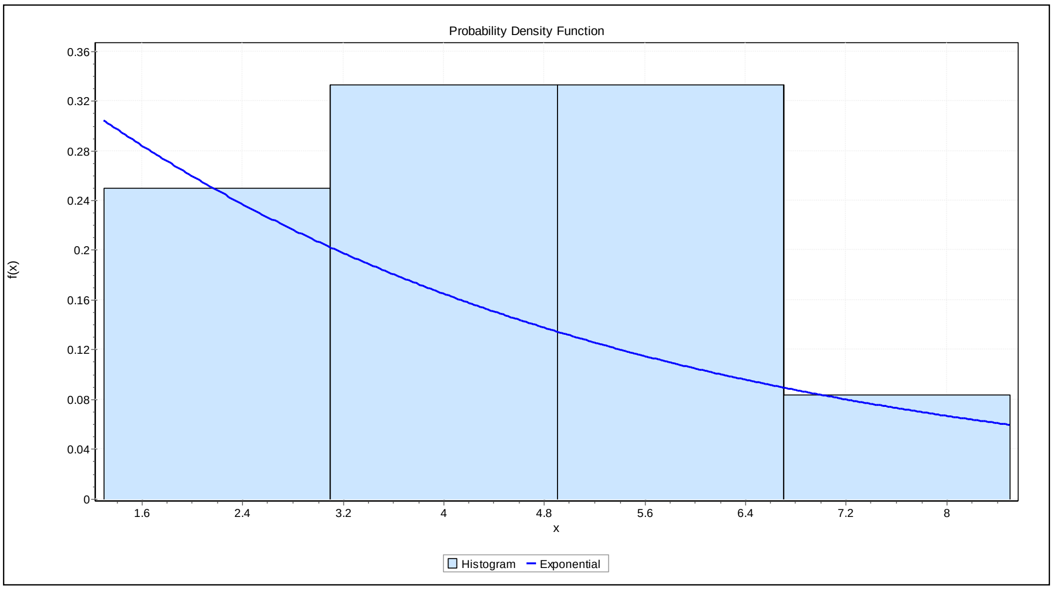
H_1: The failure data follows Exponential distribution

Table (4.2)

Kolmogorov-Smirnov test for failure time distributionof machine (Mill troup)

Sample Size Statistic P-Value Rank λ	12 0.2956 0.2003 54 0.2268				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.2958	0.3382	0.3754	0.4192	0.4491
Reject?	No	No	No	No	No

Source: The researcher from applied study, Easyfit Package, 2021



Source: The researcher from applied study, Easy fit Package, 2021

Figure (4.2): Shows meanoffailure times for machine Mill troup

From above table, it shows the p-value of Kolmogorov-Smirnov test of machine **(**Mill troup) is greater than significant level (0.05) that mean the failures time follows Exponential distribution with rate (□□□□0.2268), and repair rate μ =0.8721.

4.3.2: Estimate Continuous-Time Markov Chain for Machine (Mill troup)

Table (4.3)

Estimate continuous-time Markov chain for machine (Mill troup)

	Failure time 1/2019–12/2019 Machine (Mill troup)
Generator matrix	$Q = \begin{bmatrix} -0.8721 & 0.8721 \\ 0.2268 & -0.2268 \end{bmatrix}$
Transition matrix $p = \begin{bmatrix} 0.4708 & 0.5292 \\ 0.1376 & 0.8624 \end{bmatrix}$	
Stationary vecto $V^T = \begin{bmatrix} 0.2064 & 0.7936 \end{bmatrix}$	
Mean sojourn time in state 1 (hour) hours	4.4091 ≈ 4
Mean sojourn time in state 0 (hour) hour	1.1467 ≈ 1
Availability $\approx 0.79\%$	0.7935

Source: The researcher from applied study, Mathcad2000 Package, 2021

The results in Table 2 show , the transition probability of machine in state 0 is (0.4708) means 47% of operating time a machine in a failed state, transition probability from state 0 to state1 is (0.5292) means 53% operating time a machine under repaired, transition probability form state1 to state 0 is (0.1376) that means 14% the operating time a machinefails state , transition probability of machine in state 1(0.8624) which indicates a machine is operating state. The stationary probabilities of the continuous-time Markov chain indicate that in the long run around 21% the available operating time machine in state 0 (fail) and 79% of time in state 1 (operating), On average, a machine stays an estimated 4 hours in state

1(operating),while the machine stays in state 0 one hour. The availability percent of the machine 80%.

4.4 : Results of machine (Boiler):

4.4.1: Test of failure time distribution:

Here we test the following hypothesis**:**

H_o: The failure time follows Exponential distribution

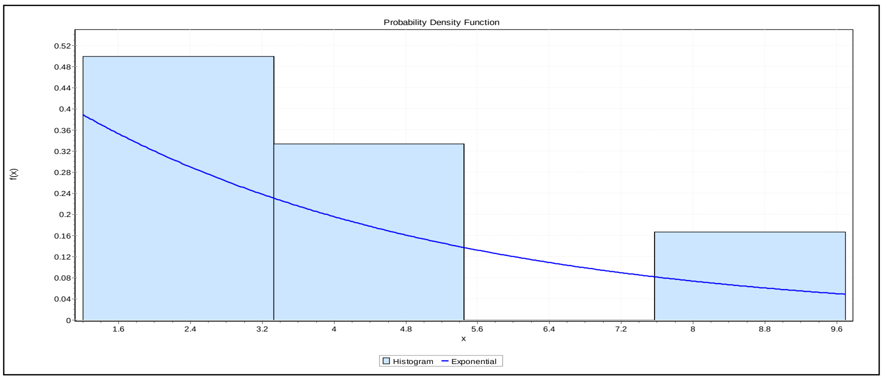
H_1: The failure data follows Exponential distribution

Table(4.4)

Kolmogorov-Smirnov test for failure time of machine (Boiler)

Sample Size Statistic P-Value Rank λ	12 0.27432 0.27351 49 0.2459				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.2958	0.3382	0.3754	0.4192	0.4491
Reject?	No	No	No	No	No

Source: The researcher from applied study, Easyfit Package, 2021



Source: The researcher from applied study, Easyfit Package, 2021

Figure (4.3): Shows meanof failure times for machine Boiler

From above table, it shows the p-value of Kolmogorov-Smirnov test of machine **(**Boiler) is greater than significant level (0.05) that mean the failures time follows Exponential distribution with rate (□□□□0.2459), and repair rate $\mu =$ 0.968.

4.4.2 Estimate continuous-time Markov chain for machine (Boiler) .

Table (4.5)

Estimate continuous-time Markov chain for machine (Boiler) :

Failure time 1/2019–12/2019Machine (Boiler)	
Q-Matrices	$Q = \begin{bmatrix} -0.9687 & 0.9687 \\ 0.2459 & -0.2459 \end{bmatrix}$
Transition matrix $p = \begin{bmatrix} 0.4392 & 0.5608 \\ 0.1424 & 0.8576 \end{bmatrix}$	
Stationary vector $V = [0.2025 \quad 0.7975]$	
Mean sojourn time in state 1 (hour) hours	4.0667 ≈ 4
Mean sojourn time in state 0 (hour) hour	1.0323 ≈ 1
Availability $\approx 0.80\%$	0.7975

Source: The researcher from applied study, Mathcad2000 Package, 2021

The results in Table (3) show , the transition probability of machine in state (0) is (0.4392) means 43% of time a machine in a failed state, transition probability from state 0 to state1 is (0.5608) means 56% of time a machine under repaired, transition probability form state(1) to state (0) is (0.1424) that means 14% of time a machine isfails, transition probability of machine in state (1) (0.7975) which indicates 80% of time a machine in operating state. The stationary probabilities of the continuous-time Markov chain indicate that in the long run around 20% the available operating time machine in state (0) (fail) and 80% of time in state (1) (operating), On average, a machine stays an estimated 4 hours in state 1(operating),while the machine stays in state (0) one hour. The availability percent of the machine 80%.

4.5 Results of both machines

4.5.1: Test of failure time distribution:

Here we test the following hypothesis**:**

H_o: The failure time follows Exponential distribution

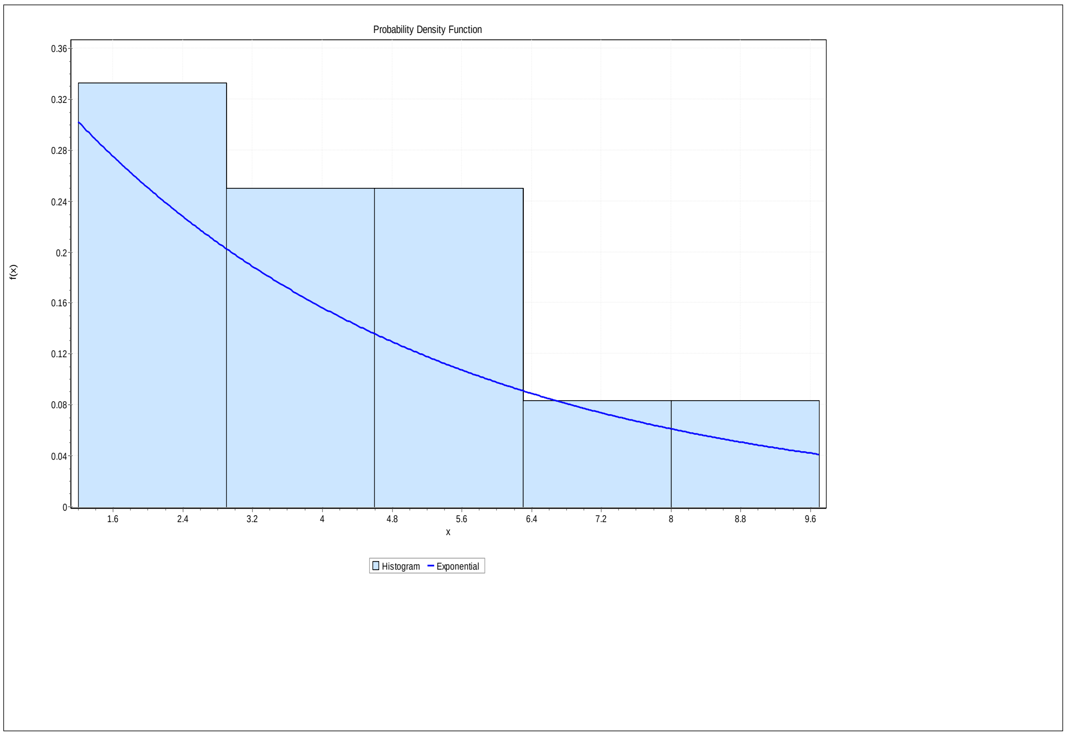
H_1: The failure data follows Exponential distribution

Table (4.6)

Kolmogorov-Smirnov test for failure time both machines

Sample Size Statistic P-Value Rank λ	24 0.26275 0.05962 54 0.2360				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.2958	0.3382	0.3754	0.4192	0.4491
Reject?	No	No	No	No	No

Source: The researcher from applied study, Easyfit Package, 2021



Source: The researcher from applied study, Easyfit Package, 2021

Figure (4.4): **Shows meanof failure times for both machines**

From above table, it shows the p-value of Kolmogorov-Smirnov test of machine **(**Boiler) is greater than significant level (0.05) that mean the failures time follows Exponential distribution with rate (□□□□0.2360), and repair rate $\mu =$ 0.9417.

4.5.2 Estimate continuous-time Markov chain for machine (both) .

Table (4.7)

Estimate continuous-time Markov chain for both machine :

Failure time 1/2019–12/2019	**Bo th machines**
Q-Matrices	$Q=\begin{bmatrix}-0.9147 & 0.9417\\ 0.2360 & -0.2360\end{bmatrix}$
Transition matrix $p=\begin{bmatrix}0.4467 & 0.5533\\ 0.1387 & 0.8613\end{bmatrix}$	
Stationary vector $V^T=[0.2025 \quad 0.7975]$	
Mean sojourn time in state 1 (hour) hours	$4.2373 \approx 4$
Mean sojourn time in state 0 (hour) hour	$1.0619 \approx 1$
Availability $\approx 0.80\%$	0.7996

Source: The researcher from applied study, Mathcad2000 Package, 2021

The results in Table 5 show , the transition probability of bothmachines in state (0) is (0.4467) means 45% of time a both machines area failed state, transition probability from state (0) to state (1) is (0.5533) means 55% of time a bothmachines under repaired, transition probability form state (1) to state (0) is (0.1387) that means 14% of time a bothmachines arefails, transition probability of bothmachines in state (1) (0.7975) which indicates 80% of time a machine in operating state. The stationary probabilities of the continuous-time Markov chain indicate that in the long run around 20% the available operating time both machine in state (0) (fail) and 80% of time in state (1) (operating), On average, a both machine stay an estimated 4 hours in state 1 (operating),while the both machine stay in state (0) one hour. The availability percent of the both machine 80%.

4.6 Estimating failure rate and repairs.

For the purpose of applying the Markov chain model in maintenance, data for number of failures and time repair during 12 consecutive months for the year (2019) for machines depended on mechanical faults. The tabular method was used to calculate the density function and the Reliability function (Adolfo,2007,p.51) .The failure rate and repair were calculated for each machine as follows

4.6.1 Failure rate and repairs rateof machine (Mill troup):

Table(4.8)

Calculate failure rate and repair rate for machine (Mill troup)

Month	Failure no	Repair time	PDF $\hat{f}(t)$	Reliability function $\hat{R}(t)$	Failure rate $\hat{\lambda} = \frac{f(t)}{R(t-1)}$
1	3	2.1	0.05	0.95	0.05
2	6	4.8	0.10	0.85	0.11
3	9	5.7	0.15	0.70	0.18
4	6	6.3	0.10	0.61	0.14
5	9	8.5	0.21	0.39	0.34
6	6	3.9	0.10	0.30	0.26

7	2	3.5	0.03	0.26	0.10
8	2	2.3	0.03	0.23	0.12
9	5	5.5	0.08	0.15	0.35
10	1	5.8	0.02	0.13	0.13
11	2	1.3	0.03	0.11	0.23
12	6	3.2	0.10	0.01	1
Total	**61**	**52.9**			**3.01**

Source: The researcher from applied study, Mathcad2000 Package, 2021

From above table :

1.Failure rate for machine (Mill troup):

$$\lambda_1 = \frac{\sum_{i=1}^{12} \hat{\lambda}_i}{12} = \frac{3.01}{12} = 0.2508$$

2.Repairrate for machine (Mill troup):

$$\mu_1 = \frac{\text{Repair time}}{\text{Failure no}} = \frac{52.9}{61} = 0.8721$$

3.The mean time to failure:

$$\text{MTTF} = \frac{1}{\lambda_1} = \frac{1}{0.2508} = 3.9872 \approx 4 \text{ month}$$

3.The mean downtime:

$$\text{MDF} = \frac{1}{\mu_1} = \frac{1}{0.8721} = 1.1467 \approx 1 \text{ hour}$$

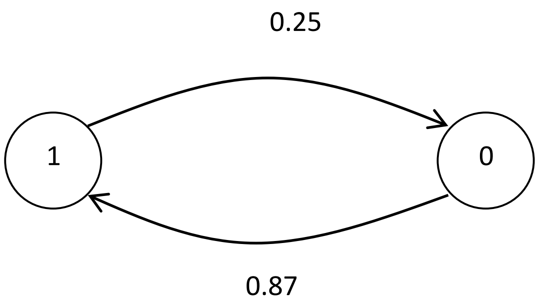


Figure (4.5) : Markov Chain Model diagram for a machine (Mill troup).

From table. no (1) and figure. no (2), the number of failures of machine(Mill troup) is (61) failure with failure rate (0.25) and repairrate (0.87). The mean time to failure is (4) month and mean downtime (1) hour.

4.6.2 Failure rate and repairs rateof machine (Boiler):

Table(4.9)

Calculate failure rate and repair rate for machine (Boiler)

Month	Failure no	Repair time	PDF $\hat{f}(t)$	Reliability function $\hat{R}(t)$	Failure rate $\hat{\lambda} = \frac{f(t)}{R(t-1)}$
1	2	2.8	0.04	0.96	0.04
2	3	4.2	0.06	0.90	0.06
3	6	9.7	0.12	0.78	0.13
4	5	4.8	0.10	0.68	0.13
5	4	5.1	0.08	0.61	0.12
6	5	3.9	0.10	0.51	0.16
7	3	1.8	0.06	0.45	0.12
8	2	1.2	0.04	0.41	0.09
9	3	2.8	0.06	0.35	0.15
10	4	1.8	0.08	0.27	0.23
11	8	7.8	0.16	0.12	0.59
12	6	2.9	0.12	0.00	1
Total	**51**	**48.8**			**2.82**

Source: The researcher from applied study, Mathcad2000 Package, 2021

From above table :

1.Failure rate for machine (Boiler):

$$\hat{\lambda}_2 = \frac{\sum_{i=1}^{12} \hat{\lambda}_i}{12} = \frac{2.82}{12} = 0.2350$$

2.Repairrate for machine (Boiler):

$$\hat{\mu}_2 = \frac{\text{Repair time}}{\text{Failure no}} = \frac{48.8}{51} = 0.9687$$

3.The mean time to failure:

$$\text{MTTF} = \frac{1}{\lambda_2} = \frac{1}{0.2350} = 4.2553 \approx 4 \text{ month}$$

4.The mean downtime:

$$\text{MDF} = \frac{1}{\mu_2} = \frac{1}{0.9687} = 1.0323 \approx 1 \text{ hour}$$

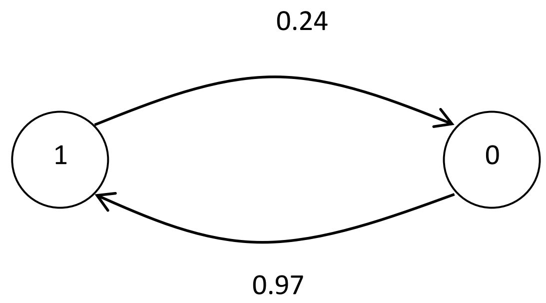


Figure (4.6): Markov Chain Model diagram for a machine (Boiler).

From table. no (1) and figure. no (3), the Number of failures of machine(Boiler) is (51) failure with failure rate (0.24) and repairrate

(0.96).The mean time to failure is (4) month and mean downtime (1) hour. The four states of system according to the failure rate and repair rate in following figure:

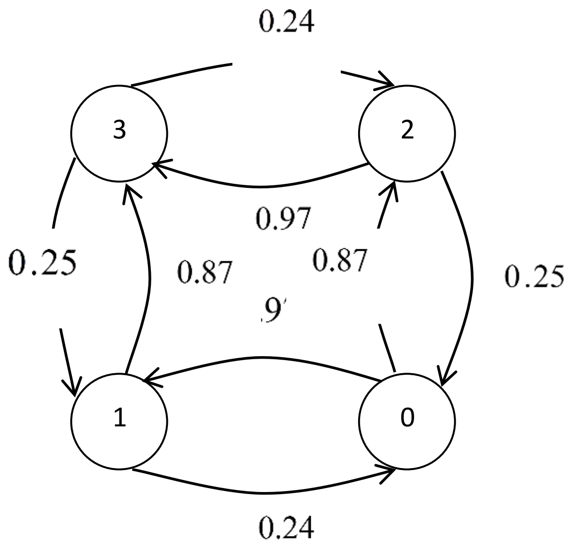


Figure (4.7): Markov Chain Model diagram for two machines.

From above figure ,It is clear that The failure rate of failures of machine (Mill troup) is greater than the failure rate of the machine(Boiler).That explains length of the repair time of machine (Mill troup).

4.7: Estimate Markov chain for both machine:

1.**State (0):** Both machines are working.

$$P_0 = \frac{1}{1+\frac{\lambda_1}{\lambda_2+\mu_1}+\frac{\lambda_2}{\lambda_1+\mu_2}} = \frac{1}{1+\frac{0.2508}{0.2350+0.8721}+\frac{0.2350}{0.2508+0.9687}} = 0.7046$$

2**. State (1):** Machine (Mill troup) down - machine (Boiler) up. .

$$P_1 = \frac{\lambda_1}{\lambda_2+\mu_1}P_0 = \frac{0.2508}{0.2350+0.8721}0.7046 = 0.1596$$

3. State (2): Machine (Boiler) up- machines (Mill troup**)** down**.**

$$P_2 = \frac{\lambda_2}{\lambda_1 + \mu_2} P_0 = \frac{0.2350}{0.2508 + 0.9687} 0.7046 = 0.1358$$

The initial condition equation $P_0 + P_1 + P_2 = 1$

as:

$$0.7046 + 0.1596 + 0.1358 = 1$$

4.8: Overall failure rate of machines:

$$\lambda sys = \lambda_2 P_1 + \lambda_1 P_2 = (0.2350)(0.1596) + (0.2508)(0.1358) = (0.0375) + (0.0340)$$

$= 0.0715 \approx 0$

Through above results:

- Probability of both machines are working is (0.7046).that means %70of the available operating time of the both machine are in working condition.
- Probability of Machine (Mill troup) non- working - machine (Boiler) workingis (0.1596). that means %16of the available operating time machine (Mill troup) non- working and machine (Boiler) working.
- Probability of Machine (Boiler) working - machines (Mill troup**)** non- workingis (0.1358). that means %14of the available operating time machine (Boiler) working and machine (Mill troup) non-working.
- The probability of the overall failure rate of the machines (0.0715) is negligible probability for the machines which is a good indicator

as it is unlikely that both machines will fail at the same time. That means a maintenance work is taking place immediately for the machine that suffers a malfunction

Table(4.10) :

Compare between machine (Mill troup) and machine (Boiler) in rate failure and repair

Rate	**Type of machine**	
	Mill troup	**Boiler**
Failure rate (λ)	0.2508	0.2350
Repairrate (μ)	0.8721	0.9687
The mean time to failure (MTTF)	3.9872 ≈ 4 month	1.2553 ≈ 4 month
The mean downtime (MDF)	1.1467 ≈ 1 hours	1.0323 ≈ 1 hours

Source: The researcher from applied study, Mathcad2000 Package, 2021

From the above table : The failure probability for both machines is close, but the failure probability of machines (Mill troup) (0.2508**)** is greater than the failure probability of machine (Boiler) (0.2350). The mean time to failure and The mean downtime of two machines are equal.

4.9: The steady-state probability of Machines:

1**. State (0):**

$P_0 = \frac{\lambda_1\lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{(0.2508)(0.2350)}{(0.2508+0.8721)(0.2350+0.9687)} = \frac{0.0589}{1.3516} = 0.0436$ year

Mean hours in state (0) per year: 0.0436*8760 =381.934 ≈ 382 hour/year

In the long run the machines will stay in state (0) approximately 382 hours per year

3. State (1):

$P_1 = \frac{\lambda_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{(0.2508)(0.9687)}{(0.2508+0.8721)(0.2350+0.9687)} = \frac{0.2429}{1.3516} = 0.1797$

Mean hours in state (1) per year: 0.1797*8760 =1574.172 ≈ 1574 hour/year

In the long run the machines will stay in state (1) approximately 1574 hours per year.

4**. State (2):**

$P_2 = \frac{\lambda_2\mu_1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{(0.2350)(0.8721)}{(0.2508+0.8721)(0.2350+0.9687)} = \frac{0.2049}{1.3516} =$ 0.1516

Mean hours in state (2) per year: 0.1516*8760 =1328.016 ≈ 1328 hour/year

In the long run the machines will stay in state (2) approximately 1328 hours per year.

4**. State (3):**

$$P_2 = \frac{\mu_1 \mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} = \frac{(0.2350)(0.8721)}{(0.2508 + 0.8721)(0.2350 + 0.9687)} = \frac{0.8448}{1.3516} = 0.6250$$

Mean hours in state (3) per year: 0.6250*8760 =5475.000 ≈ 5475 hour/year in the long run the machines will stay in state (3) approximately 5475 hours per year.

Chapter five

Results& Recommendations

Results: 5.1

5.2: Recommendations

5.1 Results

1. The failure time of machines follows Exponential distribution.
2. The diagonal elements of the transition matrix P are bigger than the off-diagonal ones which indicates that the observed transition matrix is embeddable in the continuous-time Markov chain.
3. The transition probability from state (1) to state (0) andand transition probability from state (0) to state (1) for machine (Mill troup) ,machine (Boiler) and both machines was very close.
4. The mean sojourn time in state the a both machine stay an estimated 4 hours in state 1 (operating), while the both machine stay in state (0) one hour.
5. The probability of available time to repair machines when it fault approximately(0.80).
6. Markov chains machines conducted for the failure time of two machines in Asalaya Sugar Company.
7. The probability of both machines in working condition is high.
8. failure rate and repair, probability of machines (Mill troup) has more failure which requiresmore effort in maintenance than machine (Boiler**).**
9. The mean amount of time the machine operates to failure is 4 hours for both machines and the mean time to return a non-working machine to its working condition is 1 hour, which indicates the efficiency of the factory maintenance unit.

10. The probability of the overall failure rate of the machines (0.0715) is negligible probability for the machines which is a good indicator as it is unlikely that both machines will fail at the same time. That means a maintenance work is taking place immediately for the machine that suffers a malfunction.

5.2 Recommendations:

1. Improving the operational efficiency of the machines through total or partial maintenance. Depending on the results provided by the Continuous Time Markov Chain Model, it provides a more accurate measure of the operating state of the machines..
2. Improving the operational efficiency of the machines through total maintenance based on the results obtained by the CTMC model, it provides a more accurate measure of the condition of machines.
3. High interest in true registration of faults. As all quantitative and mathematical methods made they are used in the field of maintenance, depending in particular on the accuracy of recording that data.
4. Work and prepare for the faults that occurred to machines. As well as creating a database that includes the names of the machines for their repair times.
5. Some data on failure rates may include data on some faults that are not due to operating performance. This affects the accuracy of the failure rate calculations and the repair rates, so it is necessary to mark these faults in the database.

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Appendix

Month	Machine			
	Mill troup		Boiler	
	Repair no	Failure time	Repair no	Failure time
1	3	2.1	2	2.8
2	6	4.8	3	4.2
3	9	5.7	6	9.7
4	6	6.3	5	4.8
5	9	8.5	4	5.1
6	6	3.9	5	3.9
7	2	3.5	3	1.8
8	2	2.3	2	1.2
9	5	5.5	3	2.8
10	1	5.8	4	1.8
11	2	1.3	8	7.8
12	6	3.2	6	2.9

Source: Sudanese Sugar Company,2019