Comparison Between Gross Errors Detection Methods in Surveying Measurements

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ABSTRACT- The least squares estimation method is commonly used to process measurements. In practice, redundant measurements are carried out to ensure quality control and to check for errors that could affect the results. Therefore, an insurance of the quality of these measurements is an important issue. Measurement errors of collected data have different levels of influence due to their number, measured accuracy and redundancy. The aim of this paper is to examine the detection of gross error capabilities in vertical control networks using three methods; Global Test, Data Snooping and Tau Test to compare the effectiveness of these three methods. With the least squares' method, if there are gross errors in the observations, the sizes of the corresponding residuals may not always be larger than for other residuals that do not have gross errors. This makes it difficult to find (detect) it. Therefore, it is not certain that serious errors should be detected by just examining the magnitudes of the residuals alone. These methods are used in conjunction with developed programs to calculate critical values for the distributions (in real time) rather than look for these in statistical tables. The main conclusion reached is that the tau (τ) statistic is the most sensitive to the presence gross error detection; therefore, it is the one recommended to be used in gross error detection.

Key words*: gross error, statistical test, data snooping, redundancy, quality control.*

المستخلص- تُستخدم طريقة تقدير أقل التربيعات بشكل شائع لمعالجة القياسات. في الممارسة العملية ، يتم إجراء قياسات زائدة عن الحاجة لضمان مراقبة الجودة وللتحقق من الأخطاء التي قد تؤثر على النتائج. وبالتالي ، يصبح تأمين جودة هذه الأرصاد مهمة. أخطاء القياس للبيانات المراد جمعها لها مستويات مختلفة من التـأثير كنتاج لعددها ، دقة قياسها والأرصاد الزائدة. الهدف من هذه الورقة هو فحص إمكانيات الكشف عن الخطأ الجسيم في شبكات التحكم الرأسية باستخدام ثلاثة طرق ؛ اختبار عالمي ، تطفل على البيانات واختبار تاو لمقارنة فعالية هذه الطرق الثلاث. باستخدام طريقة أقل التربيعات ، إذا كانت هناك أخطاء جسيمة في الأرصاد ، فقد لا تكون أحجام الأخطاء المتبقية دائمًا أكبر من القيم المتبقية الأخرى التي لا تحتوي على أخطاء جسيمة. هذا يجعل من الصعب العثور عليها (اكتشافها). لذلك ، ليس من المؤكد أنه يجب اكتشاف الأخطاء الجسيمة بمجرد فحص مقادير الأخطاء المتبقية وحدها. هذه الطرق تم استخدامها وربطها ببرنامج الحاسوب الذي تم تطويره لإيجاد القيم الحرجة ، في الزمن الفعلي ، للتوزيعات المستخدمة بدلاً من استنباطها من جداول الإحصاء. من النتائج المستخلصة وجد أن المعلمة تاو هي الأكثر حساسية تجاه الأخطاء الجسيمة وبالتالي يفضل استخدامها في كشف الأخطاء الجسيمة.

1. INTRODUCTION

Surveying is the art of making appropriate measurements in horizontal or vertical planes. The basic measurements in engineering surveying are horizontal distance and vertical distance (height) as well as horizontal, vertical and zenith angles, as shown in Figure 1. Various techniques are used to measure these quantities and different tools and methods have been developed for this. Surveying is the process of making observations and

measurements using various electronic, optical and mechanical devices, some of which are very complex. However, if only the best equipment and methods were used, it is impossible to take notes completely free of small differences caused by errors. These errors are sourced from instruments (*Systematic Errors*), environmental (*Random Errors*) and human operator (*Gross Errors*).

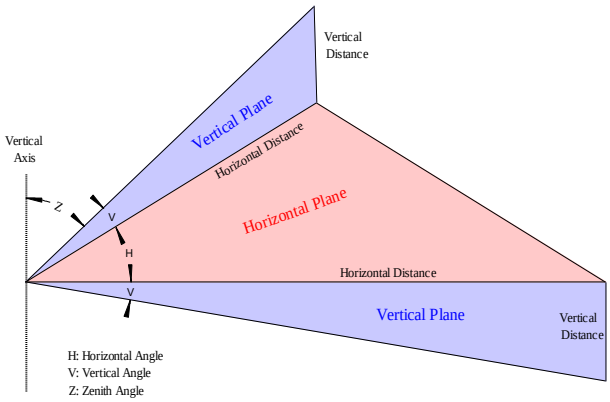

Figure 1: Surveying measurements

Systematic errors behave according to a certain system or physical law of nature which may or may not be known.

If the law of occurrence is known, systematic errors can be calculated and eliminated - they always appear with the same sign and magnitude and are therefore often referred to as constant error.

The following systematic errors corrections are applied to taped distances, height differences (Δh), and angles in order to improve their precision: slope, standardization, tension, temperature, sag, combined curvature and refraction, and atmospheric refraction. As well as US standard accuracy required for measuring length, height differences and angles, as shown in Table 1. In the table the symbols are defined as: Δh*:* the height difference, *L*: measured length, l_B: length of baseline, l_F: length of field tape along baseline, T_F: tension applied to the tape (*N*), T_S*:* standard tension (*N*), *A:* cross-sectional area of the tape (mm^2), *E:* modulus of elasticity for the tape material (N mm^{-2}), *W:* the weight of the tape per meter length ($N\ m^{-2}$), *a:* the coefficient of expansion of the tape material, t_f: mean field temperature (°*C*), t_s*:* temperature of standardization (*20°C*), *r*: refraction (*l/7*), c: curvature in meters, *D*: sighting distance in kilometers, *S*: distance between the stations. *P*: barometric pressure (*m bar*), *T*: atmospheric temperature in Kelvin (*273.15 + t °C*), *t:* atmospheric temperature in °*C*, *k*: length of leveling line in kilometers, *n*: number of angles.

TABLE 1: SYSTEMATIC ERRORS CORRECTIONS AND US STANDARD ACURACY OF THE MEASUREMENTS

Measurements (Observations)	Systematic error	Correction	Orders of accuracy	Max closures
Length	Slope < 10%	$-\Delta h^2/2L$	First Order (*1:M*)	1:100 000
	Standardization	$\pm L(l_B - l_F)/l_B$	Second Or. Class I	1:50 000
	Tension	$\pm L(T_F - T_S)/AE$	Second Or. Class II	1:20 000
	Temperature	$\pm \alpha L(t_f - t_s)$	Third Or. Class I	1:10 000
	Sag (Catenary)	$-W^2L/24T_F^2$	Third Or. Class II	1:5000
Δh	Curvature of the earth (***c***)	$-0.0785D^2$	First Or. Class I	$\pm 4\sqrt{k}\ mm$
			First Or. Class II	$\pm 5\sqrt{k}\ mm$
	Combined curvature and refraction (***c*** + ***r***)	$-0.0673\ D^2$	Second Or. Class I	$\pm 6\sqrt{k}\ mm$
			Second Or. Class II	$\pm 8\sqrt{k}\ mm$
	If $D = 0.120\ km$, then $c + r = -0.001\ m$, and neglected if $D < 0.120\ km$		Third Order	$\pm 12\sqrt{k}\ mm$
			Water way (*Small ponds*)	$\pm 20\sqrt{k}\ mm$
Angles	Atmospheric Refraction ($d\delta$)	$-\frac{8PS}{T^2}\frac{\partial t}{\partial y}\ sec$	First Order	$\pm 1.7\sqrt{n}\ sec$
			Second Or. Class I	$\pm 3\sqrt{n}\ sec$
			Second Or. Class II	$\pm 4\sqrt{n}\ sec$
	applied as corrections to the raw data before use in network adjustment		Third Or. Class I	$\pm 10\sqrt{n}\ sec$
			Third Or. Class II	$\pm 12\sqrt{n}\ sec$
			Water way (*Small ponds*)	$\pm 60\sqrt{n}\ sec$

TABLE 2: ERRORS OF CLOSURE (E_C)

Measurements (Observations)	Calculation of (E_C)	Accuracy required to correct the (E_C)
Length	From difference between two independent measurements: $E_C = AB - BA$	Relative accuracy ($1: L/E_C$) compare with US standard accuracy ($1:M$) i.e., second order class I, $M = 50000$
Δh	From height differences: $E_C = \Delta h_{given} - \Delta h_{observe}$	E_C compare with US standard Accuracy ($\pm Constant\sqrt{k}\ mm$) i.e., second order class I $= \pm 6\sqrt{k}\ mm$
	From level traverses: $E_C = \sum BS - \sum FS = \sum Rise - \sum Fall = Last\ RL - First\ RL$	
*Angle*s	From interior angles: $E_C = (n-2)\times 180^o - \sum(obs.\ angles)$	E_C compare with US standard Accuracy ($\pm Constant\sqrt{n}\ sec$) i.e., second order class I $= \pm 3\sqrt{n}\ sec$
	From exterior angles: $E_C = (n+2)\times 180^o - \sum(obs.\ angles)$	

The corrections shown in Table 1 are applied to the measurements (raw data) before any modification is made to them. Then the closing errors are calculated for the measurements, and their quality is determined according to the conditions of accuracy required for them as shown in Table 2.

Random errors are occasional errors that result from limitations or defects in the tool used, either due to manufacturing defects or improper parts fit. It is also due to the inability of the tools used to determine the values. Random errors occur according to the laws of chance. Can reduce by making repeated measurements. The accidental error in the final result varies with the square root of the number of individual measurements.

Gross errors are the result of a malfunctioning of either the instrument or the surveyor [1-2]. Typical examples are the incorrect reading or incorrect recording of results and failure of the instrument due to weak power supply or extreme environmental conditions. At least theoretically gross errors can be avoided by due care or they can be detected by carefully designed observation schemes [3].

For high-precision applications, such as strain control, it is necessary to detect and locate serious errors prior to strain analysis. Whenever possible, gross errors should be tackled before Least Squares Estimation (LSE), by means of screening and independent checks [4-5-6].

2. RESEARCH OBJECTIVES

The main objective of this paper is to see the sensitive method to the gross errors' detection in the vertical control networks, and study the effect of the magnitudes of them on the three statistics methods and on the variance factor.

3. MATERIALS AND METHODS

3.1 Least squares Estimation (LSE)

The least squares method is one of the estimation techniques used in the survey. It has been widely used in most practical applications due to its simplicity and also because statistical information is widely available. In addition, it gives estimated values which are statistically equal to their true values (Unbiasedness). Also, it gives variances which are smaller than the variance resulting from any other estimation method. For these last two reasons, LSE is considered to be the most efficient method of estimation.

We need to have some method of analyzing the results of a least squares computation to determine whether or not any of the observations are outliers. These methods depend on the analysis of residuals after an estimation process has been carried out. If we assume that the observed quantities are normally distributed which are generally so, then the residuals of these observations are also normally distributed with zero mean because the least squares method tends to minimize the weighted sum of residuals.

$$v_1^2 + v_2^2 + \ldots + v_n^2 \rightarrow \quad minimum \qquad (1)$$

The relationship between the true values of the observed quantities ($\bar{l}$) and the true parameters ($\bar{x}$) is the basic mathematical model and is expressed as a general vector function

$$F(\bar{x},\bar{l}) = 0 \quad (2)$$

And the coefficient matrix (A) of the unknowns will simply obtained by partial differential of $F(\bar{x},\bar{l})$ with respect to the parameters ($x_1, x_2, \dots, x_m$) hence

$$A = \begin{bmatrix} \partial f_1/\partial \bar{x}_1 & \cdots & \partial f_1/\partial \bar{x}_m \\ \vdots & \ddots & \vdots \\ \partial f_n/\partial \bar{x}_1 & \cdots & \partial f_n/\partial \bar{x}_m \end{bmatrix} \quad (3)$$

The main equations for LSE using observation equations method are shown here without further derivation. More details are found extensively in surveying literature for example (Cross[7]). The fundamental Equations for LSE with n observations, m parameters and redundancy r are as follows:

$$\hat{x} = (A^T W A)^{-1} A^T W b \quad (4)$$

$$\hat{v} = A\hat{x} - b \quad (5)$$

$$\hat{\sigma}_0^2 = \hat{v}^T W \hat{v} / r \quad (6)$$

$$C_{\hat{v}} = W^{-1} - A(A^T W A)^{-1} A^T \quad (7)$$

$$C_{\hat{l}} = A(A^T W A)^{-1} A^T \quad (8)$$

$$r = n - m \quad (9)$$

where: $\hat{v}$: Vector of estimated residuals, $\hat{x}$: estimated parameters, $C_{\hat{v}}$: Covariance matrix of the residuals, W: weight matrix, and obtained from the following:

$$W = \begin{bmatrix} 1/S_{11}^2 & 0 & \dots & 0 \\ 0 & 1/S_{22}^2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 1/S_{nn}^2 \end{bmatrix} \quad (10)$$

The variances ($S_{11}^2, S_{22}^2, \dots, S_{nn}^2$) are measures of precisions of the observations ($\ell_1, \ell_2, \dots, \ell_n$), respectively. b: vector of the difference between observed values (l_i) and corresponding computed values using approximate value (x^0) for the parameters.

$$b = \begin{bmatrix} l_1 - f_1(x^0, l) \\ \vdots \\ l_n - f_n(x^0, l) \end{bmatrix} \quad (11)$$

3.2 Methods of Gross Error Detection

In recent years the detection of gross errors and the reliability of observations has been one of the main research directions in surveying.

A gross error in one observation usually affects residuals in other observations. If an observation fails a statistical test, it does not mean that there is a serious error in that observation. Therefore, a statistical test should be used to detect errors or significant errors. The methods used in this paper to identify gross errors include global tests, data snooping (ω), and tau tests.

3.2.1 Global Test

It is the first test to be applied to the post-variance factor $\hat{\sigma}_0^2$ after any estimation process, when there is prior knowledge about the accuracy of the observations, that is, when the pre-variance factor σ_0^2 is assumed to be known. Otherwise, the test has no meaning.

Under the null hypothesis H_0 the statistic $\hat{\sigma}_0^2/\sigma_0^2$ follows the $F_{r,\infty}$ - distribution. It is to be remembered that $F_{r,\infty} = \chi_r^2/r$. The decision for this comprehensive test (one-tailed or two-tailed) depends on the purpose of the test determined by the null hypothesis H_0.

The two- tailed test takes the form:

$$H_0: \hat{\sigma}_0^2 = \sigma_0^2$$
$$H_a: \hat{\sigma}_0^2 \neq \sigma_0^2$$

Where, σ_0^2 represents the variance factor and $\hat{\sigma}_0^2$ is its estimated value; this gives the following $100(1-\alpha)\%$ confidence interval for the variance factor σ_0^2:

$$P\left[\frac{r\hat{\sigma}_0^2}{\chi^2_{r,1-\alpha/2}} \le \sigma_0^2 \le \frac{r\hat{\sigma}_0^2}{\chi^2_{r,\alpha/2}}\right] = 1-\alpha \quad (12)$$

When the global test is used for the detection of gross errors it is normally expected that $\hat{\sigma}_0^2$ will be greater than σ_0^2. Therefore, a one-tailed test is recommended which takes the form:

$$H_0: \hat{\sigma}_0^2 \ge \sigma_0^2$$
$$H_a: \hat{\sigma}_0^2 < \sigma_0^2$$

and the one-tailed, right hand, test is recommended: i.e.

$$\frac{\hat{\sigma}_0^2}{\sigma_0^2} \sim \mathrm{F}_{1-\alpha;\mathrm{r},\infty} \quad (13)$$

Since $\hat{\sigma}_0^2/\sigma_0^2 = \hat{v}^T W\hat{v}/r\sigma_0^2$, equation (13) can be written as:

$$\hat{v}^T W\hat{v}/r\sigma_0^2 \sim F_{1-\alpha;r,\infty} \quad (14)$$

Since $\frac{\chi^2_{1-\alpha,r}}{r} = F_{1-\alpha,r,\infty}$, *then*

$$\hat{v}^T W\hat{v}/\sigma_0^2 \sim \chi^2_{1-\alpha;r} \quad . \quad (15)$$

If the data contains gross errors, the above quadratic model will increase and the test may or may not fail depending on the magnitude of the gross errors and how they are reflected in the residual values. If this test equation (14) fails, then H_0 is rejected. Unfortunately, there may be more than one reason for rejection [8-9-10]. for example:

i. Weights were estimated incorrectly.
ii. The mathematical model is incorrect.
iii. The observations contain gross errors.

The above-mentioned reasons are not known which one failed the test, and also the test does not provide any additional information. Therefore, the source must be studied, whatever the reason, and not ignored. If we restrict ourselves to the third possible reason for the rejection, namely, the gross errors in the observations, an alternative hypothesis H_a can be presented, see (Van Mierlo[9])

3.2.2 Data Snooping (ω - Test)

The theory of this technique is developed and introduced by (Baarda[1]) for use in geodetic control networks. Assuming residual values indicate a linear function of observations, so it can be used for evaluation.

The statistic $\hat{\sigma}_0^2/\sigma_0^2$ is used first to test the global model as described previously. If this statistic is below the threshold, then the global model is considered correct, that is, there are no major errors in the observations, in other words, no errors in the observations. The threshold value is obtained from the $F_{1-\alpha,r,\infty}$ distribution with the commonly applied significance level α, i.e. probability of 100(1-α)%.

At Baarda's suggestion, the global test (14) is used to detect gross errors and the "Data Snooping" test (16) is used to localize it. Decisions from both tests must be consistent, i.e., the same boundary values must be found whether the global or single, ω, test is taken [8-9-10].

The residual values and α should be standardized to obtain standard ω_i, and standardized residues α0 and used to detect each individual observation separately, as follows:

$$\omega_i = \left|\frac{\hat{v}_i}{\sigma_{\hat{v}_i}}\right| \sim \sqrt{F_{1-\alpha_0;1,\infty}} \quad . \quad (16a)$$

Which follows a standardized normal distribution (*N* (0,1)) i.e

$$\omega_i = \left|\frac{\hat{v}_i}{\sigma_{\hat{v}_i}}\right| \sim N_{1-\alpha_0/2} \quad (16b)$$

$$\alpha_0 = 1-(1-\alpha)^{1/n} \cong \alpha/n \quad (17)$$

where: $\sigma_{\hat{v}_i}$ is the posterior standard deviation given by the square root of the *ith* diagonal element of matrix $C_{\hat{v}}$ in (7). The test can be applied as follows:

i. The least squares estimation is used to estimate $\hat{v}$ and $C_{\hat{v}}$ from (5) and (7) respectively.
ii. The level of significance α is determined and standardized to α_0 using (17).
iii. The critical value ω_c is determined from the available program written for this purpose using the level of significance α_0.
iv. The statistic ω_i is computed for each observation using (16) a.
v. The computed value, ω_i , is compared with the critical value, ω_c
vi. Check if the maximum standardized residual does not reflect the presence of any gross error. i.e if $\omega_i \leq \omega_C$ Otherwise remove the observation containing a gross error and repeat until all data is screened.

Baarda's method [1], assumes that σ_0^2 is known a priori, and employs a multi-dimensional test. In the actual implementation of Baarda's method, both *Type* I and *Type* II errors should be taken into account.

3.2.3 Tau Test

The variance in unit weight σ_0^2 is assumed to be known as shown in the null hypothesis of previous

tests, which means that all variances are measured correctly. However, if σ_0^2 is not sufficiently known, or no one wish to rely on a priori estimates, then a posteriori estimate $\hat{\sigma}_0^2$ is available from LSE. In this case, global testing of variance is not performed and the data snooping method must be modified. The new test statistic, suggested by Pope, is the one to use, which takes the form given below [8-9-10].

$$\tau_i = \frac{\hat{v}_i}{\hat{\sigma}_0 \sigma_{\hat{v}_i}} = \frac{\omega_i}{\hat{\sigma}_0} \quad (18)$$

This statistic follows the so-called tau distribution. Since the residuals are used for the estimation of τ statistic through $\hat{\sigma}_0^2$, Pope's, or Tau, method assumes $\hat{\sigma}_0^2$ as unknown and applies its LSE to estimate it in computing the normalized residuals. The test statistic is one dimensional i.e.

$$\tau_i = \left| \frac{\hat{v}_i}{\hat{\sigma}_0 \sigma_{\hat{v}_i}} \right| \sim \tau_{1-\alpha;n,r} \quad (19)$$

where:

$$\alpha = n\alpha_0\% \quad (20)$$

It should be noted that this test is a one-tailed, left hand, test. That is H_0 is accepted if:

$$\tau_i \leq \tau_{1-\alpha;n,r} \quad (21)$$

Otherwise H_0 is rejected, and the corresponding observation is suspected of having a gross error, provided the mathematical model is correct and the weights are correctly determined. The test does not take into account the probability of *Type* II error.

The Tau test can be setup using least squares results as follows:

i. The least squares estimation is used to estimate $\hat{v}$ and $C_{\hat{v}}$ from (5) and (7) respectively.
ii. The level of significance α is determined and standardized to α_0 using (16).
iii. The critical value τ_c , is determined from the developed program using the level of significance $n\alpha_0\%$.
iv. The statistic τ_i is computed for each observation using (18).
v. The computed value, τ_i , is compared with the critical value, τ_c .
vi. Check whether H_0 is accepted or not; if not accepted, that indicates the presence of a gross error in that observation, otherwise it is not.
vii. Remove the observation having a gross error and repeat the test for the remaining observations until all data is screened.

4. RESULTS AND DISCUSSION

The following example illustrates the application of the three different gross error detection methods [8-9-10]. A vertical control network as shown in Figure 2, below with measurements as shown in Table 3, was used.

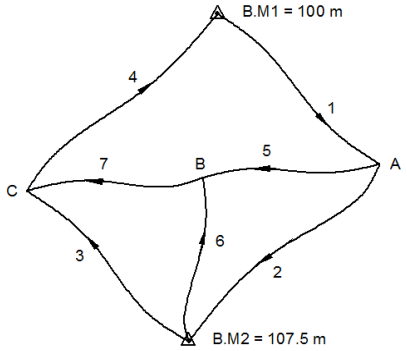

Figure 2: The vertical network

TABLE 3: VERTICAL CONTROL NETWORK OBSERVATIONS

Obs	From	To	Difference in elevations (meters)	Weights (W)
1	B.M1	A	5.100	3
2	A	B.M2	2.340	4
3	B.M2	C	-1.250	6
4	C	B.M1	-6.130	4
5	A	B	-6.800	6
6	B.M2	B	-3.000	6
7	B	C	1.700	6

From Table 3 and equations 3, 10, and 11, we can obtain:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix} \quad b = \begin{bmatrix} 105.10 \\ -105.16 \\ 106.25 \\ -106.13 \\ -0.68 \\ 104.50 \\ 1.70 \end{bmatrix}$$

The LSE solution resulted in:

$\hat{v}^T = [0.05, 0.01, -0.053, -0.067, 0.019, -0.011, 0.008]$

$\sigma_{\hat{v}}^T = [0.49, 0.396, 0.306, 0.42, 0.26, 0.299, 0.272]$

$\hat{\sigma}_0^2 = 0.011$

The following values are used in all tests carried out:

$\sigma_0^2 = 1, \alpha = 0.05,$ and $r = 7 - 3 = 4$

4.1 The Global Test

The hypotheses are set as follows:

$$H_0: \frac{\hat{\sigma}_0^2}{\sigma_0^2} \leq F_C \quad , \quad H_a: \frac{\hat{\sigma}_0^2}{\sigma_0^2} > F_C$$

The critical value of $F_{0.95,4,\infty}$, (F_C) as determined from the program is 2.364. With these values, $\frac{\hat{\sigma}_0^2}{\sigma_0^2} = \frac{0.011}{1} = 0.011$, Therefore, H_0 is accepted; hence no ω test is required.

4.2 The Tau Test

The critical value of τ determined using the developed program resulted in:

$$\tau_{1-\alpha;n,r} = \tau_{0.95;7,4} = 1.932$$

From equation (17), the test statistic τ_i computed for the seven observations resulted in the following:

$\tau^T = [0.96, 0.226, 1.612, 1.491, 0.675, 0.336, 0.274]$

These are compared with τ_c . Since none of the values exceeds τ_c, no gross error is present. The same figure with the same seven observations was used for the comparing tests. The least squares result of the four tests are as shown in Table 4.

TABLE 4: LSE OF THE RESIDUALS

$\hat{V}$	Size of gross error (m)			
	0.54	**0.55**	**1.90**	**2.20**
$\hat{v}_1$	-0.338	-0.346	-1.318	-1.534
$\hat{v}_2$	-0.142	-0.144	-0.522	-0.606
$\hat{v}_3$	-0.031	-0.031	0.023	0.035
$\hat{v}_4$	-0.089	-0.089	-0.143	-0.155
$\hat{v}_5$	-0.075	-0.077	-0.311	-0.363
$\hat{v}_6$	0.047	0.048	0.192	0.224
$\hat{v}_7$	-0.028	-0.029	-0.119	-0.139

Observation (1) is assumed to have gross errors of magnitudes 0.54, 0.55, 1.90, 2.20 m. Tests are carried out using the new observations having gross errors of magnitudes mentioned above. With significant level, α , of 0.05, r = 4, and $\sigma_0^2 = 1$. The results of calculated statistics are shown in Table 5.

TABLE 5: TEST RESULTS

Size of gross error (m)	$\hat{\sigma}_0^2$	Global test $H_0: F_{max} \leq F_C$ $F_C = 2.364$	ω – test $H_0: \omega_{max} \leq \omega_C$ $\omega_C = 2.683$	τ – test $H_0: \tau_{max} \leq \tau_C$ $\tau_C = 1.932$
0	0.011	$F_{max} = 0.011$ H_0: Accept (No gross error)	Not needed	$\tau_{max} = 0.173$ H_0: Accept (No gross error)
0.54	0.128	$F_{max} = 0.128$ H_0: Accept (No gross error)	Not needed	$\tau_{max} = 1.930$ H_0: Accept (No gross error)
0.55	0.133	$F_{max} = 0.133$ H_0: Accept (No gross error)	Not needed	$\tau_{max} = 1.935$ H_0: Reject (gross error present = 0.69σ)
1.90	1.817	$F_{max} = 1.817$ H_0: Accept (No gross error)	$\omega_{max} = 2.690$ H_0: Reject (gross error present = 2.5σ)	$\tau_{max} = 1.995$ H_0: Reject (gross error present)
2.20	2.459	$F_{max} = 2.459$ H_0: Reject (gross error present = 2.8σ)	$\omega_{max} = 3.130$ H_0: Reject (gross error present)	$\tau_{max} = 1.997$ H_0: Reject (gross error present)

5. DISCUSSION

It can be seen from Table (5) that in the presence of a gross error, the variance factor increases with an increase in the size of that gross error. It increased from 0.011 to 2.459 when the size of a gross error increased from 0 (no gross error) to 2.20 meters. The size of the calculated F distribution increases by the same amount ($\boldsymbol{\sigma_0^2 = 1}$**).** When the size of a gross error is increased, the ratio of the aposteriori variance factor to its apriori value (F_{max}) also increased. However, the F_{max} value increases more rapidly than the increase in the size of a gross error. This can be seen clearly from the Table. An increase of, approximately, four times the size of a gross error (0.55 to 2.2 m) resulted in an increase in the F_{max} value of 19 times the size of a gross error (0.128 to 2.459).

All three statistics, associated with the three methods also increase with an increase of the size of a gross error. However, the ratio with which the statistic associated with the three methods increase at different rates. The global method's test statistic (F_{max}) increases more rapidly than the other two methods. For the other two methods, the statistic associated with Pope's method (τ statistic) increase with a considerably slower rate than Baarda's method of data snooping test statistic (ω). An increase of 0.3 meters in the size of a gross error lead to an increase of 0.642, 0.440, and 0.002 for the three statistic's related to the three methods of gross error detection: F_{max}, ω_{max}, and τ_{max} respectively. This is an indication that points to the fact that the τ statistic is the most sensitive to gross errors compared to the other two statistics. Very small errors can be reflected in the τ statistic and can, therefore be detected.

Comparing the size of a gross error that any method can detect, with a probability of 0.95 (significance level of 0.05) is approximately 2.8σ, 2.5σ, and 0.7σ meters for global, Baarda's data snooping (ω), and Pope's method (τ) respectively. This result conforms to the foregoing result discussed in the previous paragraph. Namely, the τ statistic is the most sensitive of the three statistics and is, therefore, the one recommended to be used in gross error detection. It can detect gross errors as small as 0.7σ meters (in the test σ = 0.769 meters).

6. Conclusions

The variance factor increases in size with an increase in the size of a gross error. Its size reflects whether there is a gross error or not. The τ (tau) statistic is the most sensitive to gross errors compared to the other two statistics (F and ω). Errors as small as 0.7σ meters can be detect using the τ statistic. The ω statistic is better than the global test in gross error detection.

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