

# Chapter One

## Introduction

### 1.1 General Introduction

Cantilever Beam structure has been widely used in various mechanical , civil and aerospace industries for various application . In cantilever beams there is a possibility of various flaws, edge cracks are one of them . The existence of edge cracks in beams can affect the structural behavior of beam to a greater extent and reduces its safety. Therefore reliable model designed by the researchers to study the structural behavior of cantilever beam and tray to enhancement it is strength by using carbon fiber reinforced polymer (CFRP)as a strengthening technique .

Cantilever beam means a rigid beam or bar that is fixed to a support usually a vertical structure or wall and the beams other end is free . It is Y a vertical loads . The beams fixed end has a reaction force and moment created by the load acting at the free end . The intention of cantilever beam is to create a bending effect to certain limit . diving board at swimming pool is a perfect example for cantilever beam . Aircraft wing that carries wind force is another good example for cantilever beam It highlights that the unique design of cantilever structures pushes forward the physical possibilities of architecture and construction, challenges the attractive force and breaks stereotypes .

Unique engineering solutions with a cantilever span of over 20m are paid special attention to . There are technologies, design systems, materials and testing techniques that make it possible to create "flying" structures

Special attention is paid to the economic, safe and environmentally sound technology for the use of lightweight concrete with void formers reducing the amount of concrete and the mass of element without losing strength in cantilever span.

It should be noted that nowadays there is a great variety of cantilever architectural and structural elements of buildings that perform certain functional and aesthetic tasks . They can be classified as follows [18R] : type1- a roof that is an element of the building that has enclosure and structural functions (the roof can be walkable); type 2-part of a building's volume of the height of one or more floors (from small oriel windows to great volumes of a building); type 3-open sites that protrude from a facade's flat area (balconies, terraces) and have different functions and equipment (see Fig.1).

The use of innovative technologies, materials and design systems enables to increase the application of cantilever structures. It allows choosing the best solutions for them and overcoming different risks.

The unique buildings with introduced cantilever architectural and structural elements it can be concluded that their aesthetic and symbolic characteristics are subordinated to urban and functional planning tasks and are generally innovative solutions.

The design of cantilevered structures should satisfy the following General Requirements [15R] :

1. The span to effective depth ratio of cantilevered beams or slabs should Comply with clause( 9.2) of the Code of Practice for Structural Use of Concrete 2004 (mentionable in chapter 2 clause 2.1.1 ) .
2. Dead loads due to finishes, parapets and waterproofing materials, and imposed loads due to maintenance work and possible ponding resulting from malfunctioning of the drainage

system should be accurately assessed and allowed for in the design

3. Cantilevered structures , especially those projecting over streets, should be detailed in such a manner that they may be demolished or replaced without affecting the safety and integrity of the main structure of the building.

The construction of cantilevered structures should satisfy the following General Requirements [17R] :

1. All cantilevered structures should be cast monolithically with and at the same time as the directly supporting members . Construction joints should not be located along the external edge of the supporting members . In case this is unavoidable , any alternative construction method must be submitted for prior approval. Such method should ensure that the finished product would be able to attain a structural strength no less than that provided by monolithic construction , and that it would not allow the ingress of water through the joint.
2. Adequate bar spacers should be provided to maintain the position and alignment of the steel reinforcing bars.
3. Effective waterproofing should be provided.

In past the local flexibility approach first introduced by Okamura et al. is one of the most important one of these models [19R] . In this method , carbon fiber reinforced polymer (CFRP) is considered as the source of local flexibility in the beam .

To resolve this problem , some researchers have focused on developing especial finite element models based on the concept of local flexibility for studying practical engineering problems .

The main objective of this research is to study structural behavior of a cantilever beam that includes deflection , stress distribution with the varied cantilever beam having different long .

Deflection in structural engineering terms refers to the movement of a beam or node from its original position due to the forces and loads being applied to the member . Also known as displacement can occur from external applied load or from the weight of the structure itself and the force of gravity in which this applies.

Concrete gets cracked due to deflection since it is a brittle material If it is gets cracked the steel nearer to the bottom face may get atmospheric exposure and it may start rusting . it will make it to lose its strength . if we provide adequate concrete cover and limit deflection it may avoided .

At the research , we will see the effect of using carbon fiber reinforced polymer (CFRP) in the deflection of cantilever beam .

Most research on using FRP plate bonding for flexural strengthening was carried out in the last decade [9R] . There has been an explosive growth in the recent years , which resulted from the increasing global need for structural performance updating and retrofitting works . the strengthening and repair of RC structures has become increasingly important , especially in the last decade . Strengthening is usually needed to improve the performance of existing RC structures . A change in the capacity of a structure in service could be due to an increase or change in applied loads , for example , increase in traffic above bridges , addition of extra floors on an existing structure , or installation of new equipment . many deterioration , like cracks or large deflections . these are affected by different factors , such as earthquakes , vibrations , corrosion of reinforced bars and environmental changes .

Externally , Carbon Fiber Reinforced Polymer (CFRP) is one of the new materials used to strengthen or repair RC structures . It is particularly suitable for in-situ rehabilitation , and has become an increasingly applied and important technology because of CFRP advantages , high tensile strength , low weight , low installation cost and flexibility of storage ,

transportation and use . Many experimental and analytical studies have been carried out on strengthening or repairing RC beams using various types of FRP , including those related to design criteria and failure modes . The present study aims to investigate the behavior of cantilever beam and the effect of using Carbon Fiber Reinforced Polymer (CFRP) as strengthened technique for long cantilever beam .

***What is the purpose of a cantilever ?***

It is able to project and externalize rooms and maximize space serving not only aesthetic but also functional purposes . Appearing to defy gravity , these strikingly disassociated forms are thought-provoking in their ability to solve functional problems and even on to logical ones: how to create positive , inhabitable spaces literally out of nothing .



**Fig . (1.1) Shapes of Cantilever Buildings**

- (a) Lucerne Culture and Congress Centre (KKL), architect – Jean Nouvel, 2000  
 (b) BMW Welt, Munich, Germany, architects – Coop Himmelb(l)au, 2007, (c) 28 Social Housing, Paris, France, KOZ Architectes , 2010, (d) UNASUR General Secretary Headquarters in Quito, Ecuador, architect – Diego Guayasamin, 2014, (e) Prater Street 30-32, architects – PLANT Atelier Peter Kis, Budapest, Hungary, 2007, (f) Bosco Verticale Torre, architects – Stefano Boeri Architetti , 2014.

## **1.2 Problem Statement .**

What can do to use long cantilever beam without fear of the deformation or damage . Although it is beauteous and majestic it is carry , complexity in the procedure of the analyses and design .

## **1.3 Objectives of The Research .**

1. The main objective is to study structural behavior of a cantilever beam that includes deflection, stress distribution with the varied on the beam having different lengths and sectional area .
2. The efficiency of using Carbon Fiber Reinforced Polymer . (CFRP) as a strengthening technique for reinforced cantilever beams .
3. To using Finite Element Program ( ABAQUS ) to modeling and analysis structural element ( cantilever beam ) .

## **1.4 Methodology of The Research .**

This study is aimed to understanding the behavior of cantilever beam , and try to modeling RCC beam by using Finite Element Program (ABAQUS) and then compare experimental results (which taken from previous study) with the ABAQUS results , and investigate the efficiency of using carbon Fiber Reinforced Polymer (CFRP) as a strengthening technique for reinforced cantilever beams .

After study the behavior of cantilever beam (ch2) , searching in the innovative technologies, materials and design systems which are use as strengthening technique for reinforced cantilever beams to choosing the best solutions , in this study used Carbon Fiber Reinforced Polymer (CFRP) as strengthening technique for reinforced cantilever beams .

This research consist a Finite Element (FE) Model to study behavior of cantilever ( deflection and stress) , the modeling procedure done by using (FE) program (ABAQUS) . To use (ABAQUS) program , at first should compare experimental results which taken from previous study with the FE program (ABAQUS) results to sure the effectiveness and efficiency of the program (ABAQUS) in Reinforcement Concrete beam modeling , after checked the performance of Abaqus program , cantilever beam at length 3.5m was model with and without (CFRP) and comparison between it . But in the first the properties of beam was concluded from previous study (B3) which mentioned in chapter (4) and which was at length 2.2m , B3 modeled with (CFRP) to check the deflection and then modeled B4 at length 3.5m .

## **1.5 Thesis Outline :**

**Chapter 1** : Presents General Introduction , a Statement of the Problem, the Research Objectives , Methodology , and The outline for each chapter.

**Chapter 2** : Overviews what others have already done in this field of research ( Literature Review ) .

**Chapter 3** : Presents Research Methodology used in this Project (Modeling Strategy) .

**Chapter 4** : Uses a Finite Element Program to Modeling Cantilever Beam with Length 3.5m ( Modeling) .

**Chapter 5** : Presents the Results & Discussions .

**Chapter 6** : Conclusions and Recommendations



## Chapter Two

### Literature Review

#### 2.1 Introduction

Past research related to this research comes from different areas of studies. there is many of researches and studies was done to analyses and understand the behaviour of cantilever beam to enhance the strength of cantilever beam and become able to create long cantilever beam with better deflection and higher sensitivity .

A cantilever beam is a beam anchored at only one end . The beam carries the load to the support where it is forced against by a moment and shear stress.

The moments , shear and deflections for a cantilever beam are substantially greater than those for an equivalently loaded span that is supported at both its ends. Also the moments in a cantilever can never be redistributed to other parts of the structure – the beam must always be capable of resisting the full static moment . Because of these factors and the problems that often occur with increased deflections due to creep , the design and detailing of a cantilever beam should be done with care .

The provision of additional steel in the compressive zone of the beam can help to restrain the increased deflection caused by creep .

There have been many researchers that study the behavior of cantilever beam and at the first place it must be mentioned that Timoshenko (1963) is the first that gives solution for critical moment of behavior of

cantilever beam due to uniform bending moment . Timoshenko solution is based on the assumption that at the ends of the beam, torsional rotation is prevented but warping is allowed . Timoshenko's formula is adopted in AISC Specification for Structural Steel Building .

For non uniform bending moment , AISC [7R] developed modification factor for non uniform moment diagram  $C_b$  For cantilever beam the specification states that the value of  $C_b$  (modification factor) is taken 1. In other words , AISC use the same equation as for simply supported beam. Actually the boundary conditions of cantilever beam are not the same as simply supported beam .

Both ends of simply supported beam are prevented for torsional rotation and free to warp .

But for cantilever, at fixed end torsional rotation and warping are prevented but at free end torsional rotation and warping are free .

Guide to Stability Design Criteria (Ziemian, 2010) gives some equation to calculate elastic critical moment of cantilever beam for point load at free end and uniformly distributed load. Two different location of loading are considered, at shear center, at top flange .

-At the book entitled (loads, analysis, materials and design of structural elements.) [7R] it talked about the Shear force and bending moment of beams and mentioning that , A beam is a structural member subject to lateral loading in which the developed resistance to deformation is of a flexural character. The primary load effect that a beam is designed to resist is that of bending moments but, in addition, the effects of transverse or vertical shearing forces must be considered .

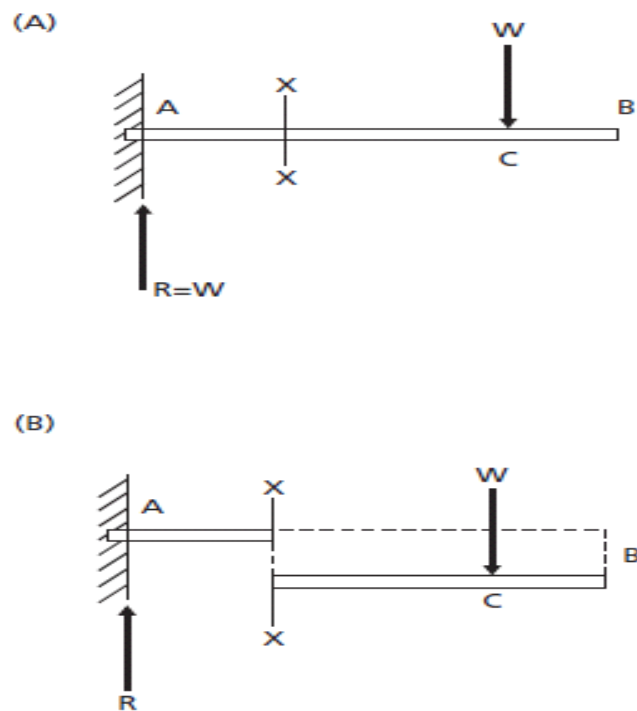
Shear force ( $V$ ) is the algebraic sum of all the transverse forces acting to the left or to the right of the chosen section.

Bending moment ( $M$ ) at any transverse cross-section of a straight beam is the algebraic sum of the moments, taken about an axis passing through

the centroid of the cross-section, of all the forces applied to the beam on either side of the chosen cross-section .

Consider the cantilever AB shown in (fig 2.1) . For equilibrium, the reaction force at A must be vertical and equal to the load W.

The cantilever must therefore transmit the effect of load W to the support at A by developing resistance (on vertical cross-section planes between the load and the support) to the load effect called shearing force. Failure to transmit the shearing force at any given section, e.g. section x-x, will cause the beam to fracture as in (B) .



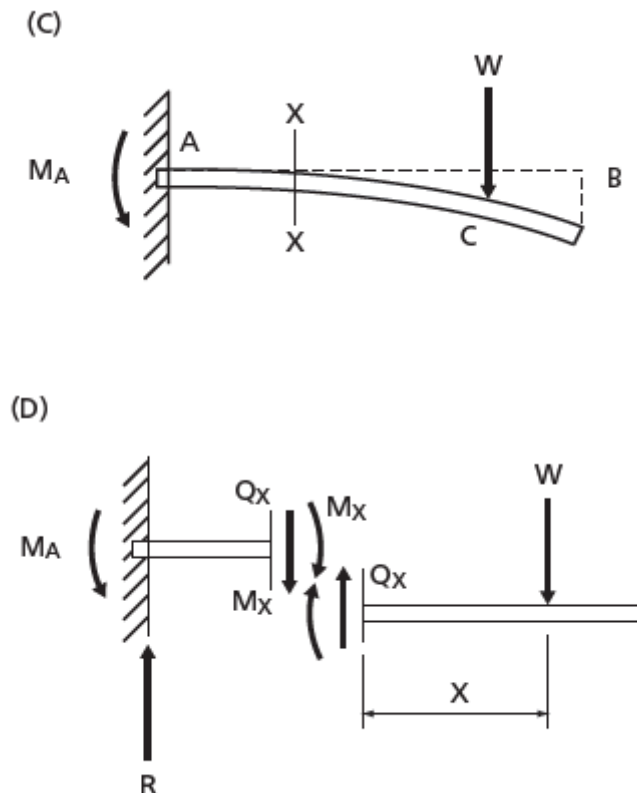
**Fig . (2.1 ) Cantilever AB**

To deform as in (C). To prevent rotation of the beam at the support A, there must be a reaction moment at A , shown as  $M_A$  , which is equal to the product of load W and the distance from W to point A .

The shearing force and the bending moment transmitted across the section x-x may be considered as the force and moment respectively that

are necessary to maintain equilibrium if a cut is made severing the beam at  $x-x$ . The free-body diagrams of the two portions of the beam are shown in (D) . Then the shearing force between A and C =  $Q_x = W$  and the bending moment between A and C =  $M_x = W \cdot AC$ .

Note: Both the shearing force and the bending moment will be zero between C and B .



**Fig . (2.2) Cantilever AB Bending Moment& Shearing Force**

***Definitions***

*Shear force (Q)* is the algebraic sum of all the transverse forces acting to the left or to the right of the chosen section . *Bending moment (M)* at any transverse cross section of a straight beam is the algebraic sum of the moments , taken about an axis passing through the centroid of the cross section , of all the forces applied to the beam on either side of the

chosen cross section. Shearing forces, which tend to make the part of the beam to the left move up and the right part move down, are considered positive. The bending moment is considered positive if the resultant moment is clockwise on the left and anticlockwise on the right. These tend to make the beam concave upwards and are called *sagging* bending moments. If the moment is anticlockwise on the left and clockwise on the right, the beam will tend to become convex upwards – an effect called hogging.

Force remains constant in between. When the load is uniformly distributed, however, the shear force will vary at a uniform rate. Thus it will be seen that uniform loads cause gradual and uniform change of shear, while concentrated loads bring a sudden change in the value of the shear force.

Shear failure at section of beams and cantilevers without shear reinforcement will normally occur, shear reinforcement is provided in the form of vertical stirrup member or a combination of stirrups and bent-up bars, the links should be placed in the top two-thirds of the effective depth. If ( $V$ ) is the shear force at a section, then the shear stress  $v$  is given by ( $v=V/bd$ ), the shear stress must never exceed the lesser of  $0.8(F_{cu})^{0.5}$  or  $5\text{N/mm}^2$ .

The tension steel must be fully anchored into the support, and at the front edge, it should be anchored by welding to a transverse bar of equal strength or by bending the bars back to form a loop.

### ***Bending moment variation***

Concentrated loads will cause a uniform change of the bending moment between the points of action of the loads. In the case of uniformly distributed loads, the rate of change of the bending moment will be parabolic. Maximum bending moment values will occur where the shear force is zero or where it changes sign.

## ***Deflection***

Deflection in structural engineering terms refers to the movement of a beam or node from its original position due to the forces and loads being applied to the member . Also known as displacement can occur from external applied load or from the weight of the structure itself and the force of gravity in which this applies. The deflection of the beam should not adversely affect its efficiency or appearance . deflection may be calculated and then compared with the serviceability requirements given in section (2.1.2) .

## **2.2 Design Considerations**

According to Manual for Design and Detailing of Reinforced Concrete to Code of Practice for Structural Use of Concrete 2004 [17R] , Design considerations for a cantilevered beam in General as follows :

- 1.** The span to overall depth of cantilever beams should not be Greater than 7 .
- 2 .**The minimum percentage of top tension longitudinal reinforcement based on the gross cross-sectional concrete area should be 0.25% for all reinforcement grades generally (PNAP173 App. A 6(c)) . However, if the

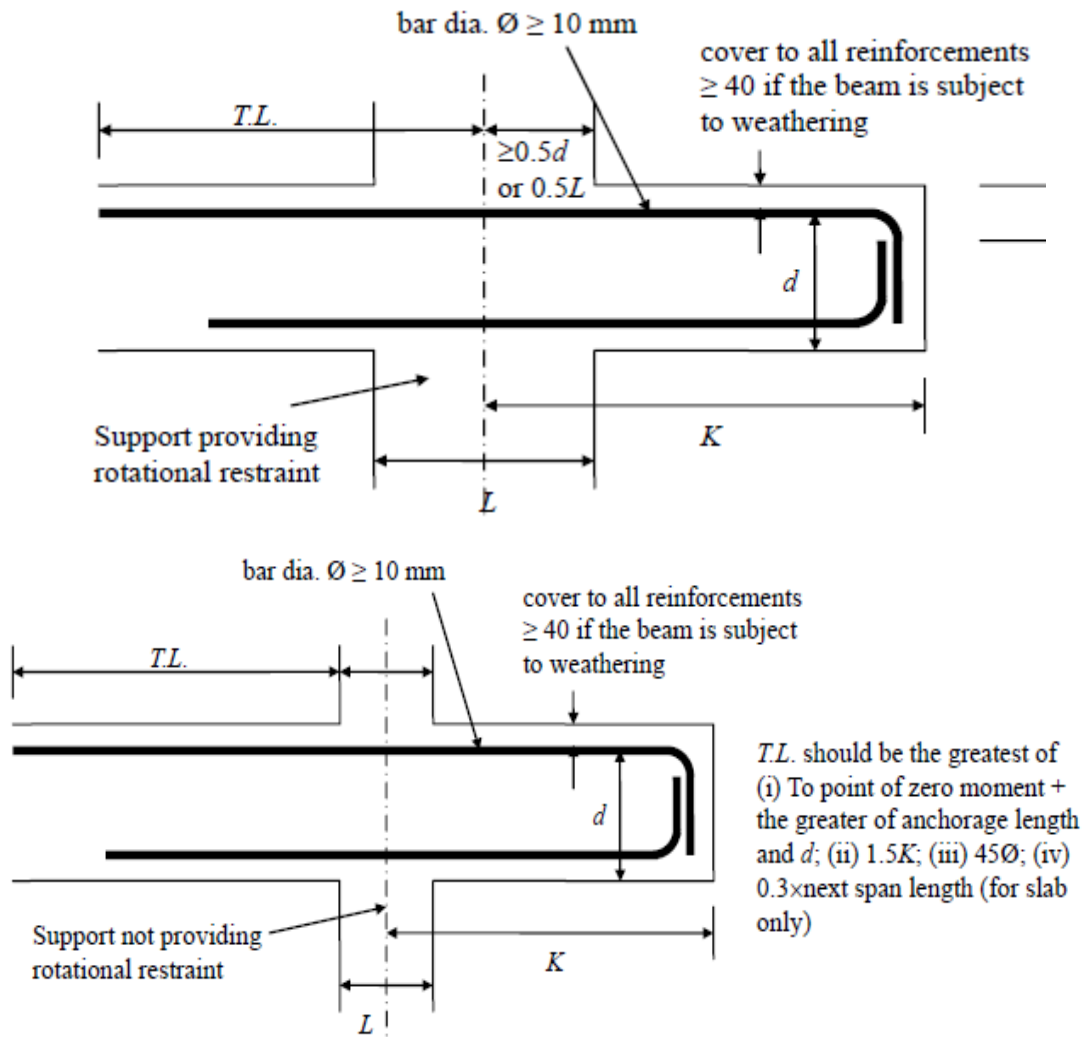
Cantilever structure is a flanged beam where the flange is in tension , the minimum steel percentage is 0.26% for T-section and 0.2% for L-section but based on the gross area of the rectangular portion of width of the web times the structural depth as per Table 9.1 of the Code. The more stringent requirement shall prevail .

- 3.** Diameter of the longitudinal reinforcement  $\geq 10$  mm as illustrated in Figure (2.3) (PNAP173 App . A 6(c)).
- 4.**The centre-to-centre spacing of the top tension longitudinal bars  $\leq 150$ mm as illustrated in Figure(2.3) (PNAP173 App. A 6(c))

5. For cantilevered structure exposed to weathering, cover to all reinforcement  $\geq 40$  mm (PNAP173 App. A 8(a))

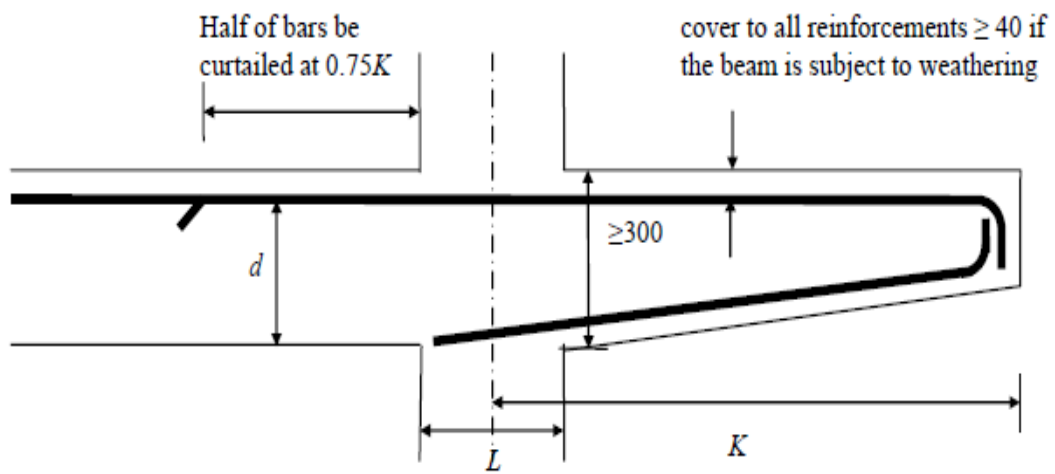
6. Anchorage of tension reinforcement shall be based on steel stress of  $y 0.87 f_y$  and (a) full anchorage length should be provided with location of commencement in accordance with Cl. 9.4.3 of the Code as illustrated in 125 Version 2.3 May 2008 Figures (2.3) and (2.4) and (b) minimum anchorage length of 45 times the longitudinal bar diameter in accordance with PNAP 173 App. A 6(d). The different commencement points of anchorage lengths as indicated by PNAP 173 Appendices B and C are not adopted in this Manual. However, requirements for the lengths of curtailment of tension reinforcement bars PNAP173 and Cl. 9.2.1.6 of the Code in relation to curtailment of tension reinforcements are amalgamated. They are shown in Figures (2.3) and (2.4) .

7. The overall depth at support should be at least 300 mm as shown in Figure (2.4) .



**Fig . (2.3) Anchorage and Maximum Longitudinal Bar Spacing in Cantilevers as Required by the Code and PNAP173**





**Fig (2.4) . Particular Requirements for Cantilever Beams as Required by the Code and PNAP 173**

### 2.2.1 Span / Effective Depth Ratio for a Rectangular or Flanged Beam .

The calculations has been shown to be a tedious operation , however for general use rules based on limiting the span-effective depth ratio of a member are adequate to ensure that the deflection are not excessive .

According to Code of practice for design and construction (BSI 8110-1) [5R] , The basic span/effective depth ratios for beams are given in table (2.1) .

These are based on limiting the total deflection to Span/250 and this should normally ensure that the part of the deflection occurring after construction of finishes and partitions will be limited to span/500 or 20mm , whichever is the lesser , for spans up to 10m .

**Table 2.1- Basic Span/Effective Depth Ratio for Rectangular or Flanged Beams .**

<b>Support Conditions</b>	<b>Rectangular Section</b>	<b>Flanged Beam With <math>b_w/b \leq 0.3</math></b>
<b>Cantilever</b>	<b>7</b>	<b>5.6</b>
<b>Simply Supported</b>	<b>20</b>	<b>16.0</b>
<b>Continuous</b>	<b>26</b>	<b>20.8</b>

### 2.2.2 Effective Length of Cantilever

According to Code of practice for design and construction (BSI 8110-1) [5R] , The effective length of a cantilever should be taken as its length to the face of the support plus half its effective depth except where it forms the end of a continuous beam where the length to the centre of the support should be used .

## **2.3 General Steps in FE Analysis**

### **1. Pre-Processing**

- Creation of Geometry
- Assigning Material Property
- Selection of Element Type
- Discretisation of model

### **2. Analysis**

- Applying Boundary Conditions
- Applying Load (Pressure/Moment)
- Submission for Solving.

### **3. Post-Processing**

- Selecting the Type of Field Variable Interested
- Visualization of Selected Variable.
- Generation of Graphs/Plots

## **2.4 Various FEA Software's for Structural Analysis**

### **1. Ansys**

- Ansys Workbench
- Ansys - LS Dyna

### **2. Abaqus**

- CAE
- Standard
- Explicit

### **3. MSc Products**

- Patran
- Nastran
- Dytran

## **2.5 The ABAQUS**

Abaqus is a suite of powerful engineering simulation programs [16R] , based on the finite element method, that can solve problems ranging from relatively simple linear analyses to the most challenging nonlinear simulations .

Abaqus contains an extensive library of elements that can model virtually any geometry. It has an equally extensive list of material models that can simulate the behavior of most typical engineering materials including metals, rubber, polymers, composites, reinforced concrete, crushable and resilient foams, and geotechnical materials such as soils and rock. Designed as a general-purpose simulation tool, Abaqus can be used to study more than just structural (stress/displacement) problems.

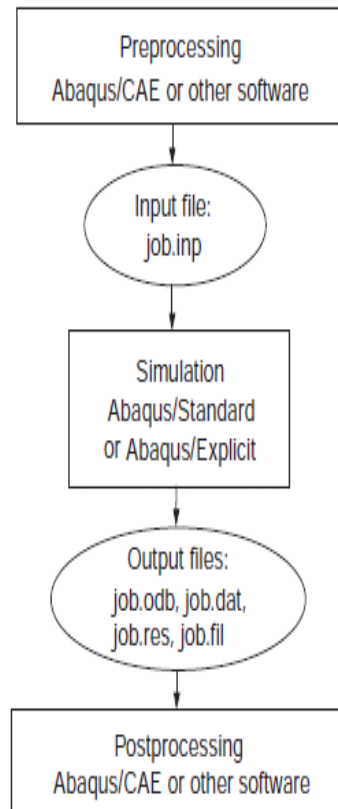
It can simulate problems in such diverse areas as heat transfer, mass diffusion, thermal management of analyses), electrical components (coupled thermal-electrical acoustics, soil mechanics (coupled pore fluid-stress analyses), and piezoelectric analysis. Abaqus offers a wide range of capabilities for simulation of linear and nonlinear applications.

Problems with multiple components are modeled by associating the geometry defining each component with the appropriate material models and specifying component interactions. In a nonlinear analysis Abaqus automatically chooses appropriate load increments and convergence tolerances and continually adjusts them during the analysis to ensure that an accurate solution is obtained efficiently.

## 2.6 ABAQUS Basics

A complete Abaqus analysis usually consists of three distinct stages: preprocessing, simulation, and postprocessing [16R] .

These three stages are linked together by files as shown below:



**Fig . (2.5 ) Basics Stages in Abaqus**

**Also Modeling is Addressed as Follows:**

- Elements type
- Material property
- Assigning sections
- Defining step
- Interaction between elements
- Specify boundary conditions and load
- Meshing
- Assigning job
- Evaluating the results

## 2.7 Definition of Model

Modeling is a digital representation of physical and functional characteristics of a facility .

As mentioned earlier [16R] , Abaqus/CAE is divided into functional units called modules. Each module contains only those tools that are relevant to a specific portion of the modeling task. For example, the Mesh module .

Contains only the tools needed to create finite element meshes, while the Job module contains only the tools used to create, edit, submit, and monitor analysis jobs. You select a module from the Module list in the context bar the modules in the menu corresponds to a logical sequence you may follow to create a model. In many circumstances you must follow this natural progression to complete a modeling task; for example, you must create parts before you create an assembly. Although the order of the modules follows a logical sequence, Abaqus/CAE allows you to select any module at any time, regardless of the state of your model.

However, certain obvious restrictions apply; for example, you cannot assign section properties, such as cross-sectional dimensions of an I-beam, to geometry that has not yet been created.

A completed model contains everything that Abaqus needs to start the analysis. Abaqus/CAE uses a model database to store your models.

When you start Abaqus/CAE, the Start Session dialog box allows you to Create a new , empty model database in memory. After you start Abaqus/CAE, you can save your model database to a disk by selecting File→Save from the main menu bar; to retrieve a model database from a disk, select File → Open .

## 2.8 Previous Research

1-In study by Bindu A Thomas (Design of Highly Sensitive MEMS cantilever beam Using COMSOL Multiphysics ) [6R] whose Consider The construction of cantilever beam is an art , after construction in order to study the behavior of that can done by applying various amount of loads on it , It helps in usage of the beam in MEMS applications in various fields , less loading on cantilever beam surface gives negligible deflection at its free end and poor sensitivity. In this study new micro cantilever designs are presented , which hold promises for better deflection and higher sensitivity and comparison as been done by constructing beam with different materials along with that the design simulations of MEMS based micro-cantilever made up of single crystal silicon using FEM (COMSOL Multiphysics) is carried out to study the stress , displacement and Eigen frequency measurements of the cantilever . The work is carried out by using COMSOL Multiphysics software. The result shows that cantilever beam can be constructed by various materials like silicon, silicon nitride and silicon dioxide. Out of three structures beam with silicon will give the very high Eigen frequency of 0.22GHz compare to other two with the same dimension. Further it's possible to improve the performance of the beam by adding different combinations of the materials.

2-In study in January 2016 by T. Eswara Rao of Analysis of Stress and Deflection of Cantilever Beam and its Validation Using ANSYS [1R] investigates the deflection and stress distribution in a long , slender cantilever beam of uniform rectangular cross section made of linear elastic material properties that are homogeneous and isotropic. The deflection of a cantilever beam is essentially a three dimensional problem

An elastic stretching in one direction is accompanied by a compression in perpendicular directions .

The beam is modeled under the action of three different loading conditions: vertical concentrated load applied at the free end, uniformly distributed load and uniformly varying load which runs over the whole span. The weight of the beam is assumed to be negligible. It is also assumed that the beam is inextensible and so the strains are also negligible. Considering these assumptions at first using the Bernoulli-Euler's bending moment curvature relationship , the approximate solutions of the cantilever beam were obtained from the general set of equations.

Then assuming a particular set of dimensions, the deflection and stress values of the beam are calculated analytically .

Finite element analysis of the beam was done considering various types of elements under different loading conditions in ANSYS 14.5. The various numerical results were generated at different nodal points by taking the origin of the Cartesian coordinate system at the fixed end of the beam . The nodal solutions were analyzed and compared . On comparing the computational and analytical solutions it was found that for stresses the 8 node brick element gives the most consistent results and the variation with the analytical results is minimum .

**3-**In November 2017 Ibrahim M. Abu-Alshaikh Published his research (Closed-Form Solution of Large Deflected Cantilever Beam against Follower Loading Using Complex Analysis ) [18R] .

This research is aimed to obtain a closed-form solution for solving the large deflection of a cantilever beam opposed to a concentrated point follower load at its free end. This closed-form solution when compared with other conventional numerical approaches is characterized by



simplicity, stability and straightforwardness in getting the beam deflection and slopes even for extremely large loading conditions. The closed-form solution is obtained by applying complex analysis along with elliptic-integral approach.

Very good results were obtained when the elastic of the beam compared with that of various numerical methods which are used in analyzing similar problem .

The result of this research shows that The closed-form solutions of LD of CB subjected to a concentrated non-conservative inclined tip force are derived and verified by applying elliptic integral along with complex analysis . The exact solution for the tip-angle  $\varphi(0)$  is mathematically extracted while it is determined numerically by other researchers . The closed-form solution for any arbitrary slope angle  $\varphi(s)$  is characterized by its stability due to the mathematical fact that it contains exponential and complex functions.

**4-**In October 2015 Indian Journal of Science and Technology Published research entitled (Investigating Behavior of Cantilever Beams of Normal and Lightweight Reinforced Concrete under Cyclic Load ) [16R] .

This study is focused on the experimental investigation of the behavior of the beams constructed of scoria lightweight aggregate concrete . Five samples of cantilever beams are developed; three of which are made of lightweight aggregate concrete, while two are made of normal weight concrete. In this paper, performance of the plastic hinge in the flexural beams made of lightweight aggregate concrete is investigated based on the stiffness reduction parameters , pinching phenomena, strength reduction , and ductility. Findings : The results showed that beam stiffness, independent from strength , is reduced with increase in the displacement cycles and in the high deformation cycles , pinching

phenomenon occurs randomly. Also, strength is not reduced with increase in amplitude of the cycles until reaching displacement ductility 4 and flexural behaviors of normal and reinforced lightweight aggregate concretes are the same value. In deformation cycles with the amplitude of 80mm, equivalent to ductility 4, no significance decrease in the strength is observed in force–deformation curve. With respect to ratio of the shear span to significant depth of the test specimens, the flexural behavior is dominant and the beams endure ductility of greater than 4. It is concluded that the reinforced lightweight aggregate concrete beams with maximum lightweight aggregate size of smaller than 5mm have ductility behavior during the bending , similar to the reinforced normal concrete beams. Application / Improvements : The results demonstrated that in flexural beams , type of concrete (lightweight or normal aggregate) does not have particular influence on the stiffness reduction .

5- In study was done by Edgardo Solano Carrillo entitled (The cantilevered beam : An analytical solution for general deflections of linear-elastic materials ) [19R] , says that The deflections of a cantilevered beam made of a linear-elastic material under the influence of an external vertical concentrated force at the free end are analysed in detail and it is found that a factor that is always ignored in the theory commonly developed in the literature concerned permits the solution of the elastic curve to be obtained analytically. he introduce this analytical solution and show that the widely known simple solution for small deflections is a limiting case, which holds when forces applied are much smaller than the force needed for breaking the beam. A simple relation for an upper bound to this latter force is obtained which may be of practical importance for engineering. Finally, the most simple results obtained theoretically are compared with experiments in the laboratory.

He Concluded that Based on the supposition that the beam we are considering is made of a linear-elastic material and its length is greater than its lateral dimensions (thin beam) , we have obtained the correct equation for the elastic curve when the weight of the beam is neglected . An analytical solution for the function describing the shape of the beam when it is bent has been found and simple relations such as the slopes at each point along the beam and a superior limit for the force needed for rupture are characteristic of this new development for the cantilevered beam. The usual terms appearing in the theory for small curvatures such as the vertical deflections of the free end of the beam are shown to be just a part of the general solution.

6-Reporting tests and investigations have been reviewed by Almakht et al. (1998) to develop a thorough understanding of the behaviour of beams strengthened by CFRP plates [9R]. CFRP plates were found to increase the flexural capacity within certain limits (Almakht et al. 1998). Externally bonded CFRP plates were found to perform well under the effect of the impact loading (Erki, Meier 1999). Adding an anchoring system at the end of the plates can improve the impact performance of the strengthened beam (Erki, Meier 1999). Repair of a real bridge with externally bonded FRP plates was found to decrease the flexural stresses in the steel reinforcements and the mid-span deflection (Stallings et al. 2000). Strengthening of concrete beams with externally bonded FRP plates was found to increase the ultimate capacity by 70% and reduce the size and the density of the cracks along the beam length (Fanning, Kelly 2001). A significant increase in the ultimate capacity was observed after adding the externally bonded CFRP sheets (Nguyen et al. 2001). Ultimate capacity of strengthened beams increased by up to 230%, and even for the preloaded beam before strengthening, the ultimate capacity significantly

increased, which indicates good performance for repair situations (Rahimi, Hutchinson 2001) .

Repairs of damaged RC beams with externally bonded CFRP sheets were carried out by Benjeddou et al. (2007). The study validates the effectiveness of the CFRP sheet as repairing technique for all the damage degrees. The peeling off failure mode was controlling the failure mechanism. The load capacity had increased by 87% for the strengthening beam when no pre-crack load was applied, and it was 44% for the highest damage degree. Choo et al. (2007) investigated the retrofitting of an actual bridge damaged under extreme loading using externally bonded CFRP sheets. The FE modeling was used to estimate the force emanated due to the extreme loads, and it also showed that repairing with CFRP sheets made a significant difference for the ultimate limit .

7- Undoubtedly , the Finite Element method represents one of the most significant achievements in the field of computational methods in the last century. Historically, it has its roots in the analysis of weight-critical framed aerospace structures . These framed structures were treated as an assemblage of one-dimensional members , for which the exact solutions to the differential equations for each member were well known . These solutions were cast in the form of a matrix relationship between the forces and displacements at the ends of the member. Hence, the method was initially termed matrix analysis of structures. Later, it was extended to include the analysis of continuum structures . Since continuum structures have complex geometries, they had to be subdivided into simple components or “elements” interconnected at nodes. It was at this stage in the development of the method that the term “finite element” appeared. However, unlike framed structures, closed form solutions to the

differential equations governing the behavior of continuum elements were not available. Energy principles such as the theorem of virtual work, the principle of minimum potential energy, which were well known, combined with a piece-wise polynomial interpolation of the unknown displacement, were used to establish the matrix relationship between the forces and the interpolated displacements at the nodes numerically. In the late 1960s, when the method was recognized as being equivalent to a minimization process, it was reformulated in the form of weighted residuals and variational calculus, and expanded to the simulation of nonstructural problems in fluids.

Thermo mechanics, and electromagnetic. More recently, the method is extended to cover multiphysics applications where, for example, it is possible to study the effects of temperature on electromagnetic properties that might affect the performance of electric motors[2R].

### **FINITE ELEMENT ANALYSIS AND THE USER**

Nowadays, [2R] in structural design, the analysis of all but simple structures is carried out using the Finite Element method. When graduate structural engineers enter the design office, they will encounter advanced commercial finite element software whose capabilities, and the theories behind its development, are far superior to the training they have received during their undergraduate studies. Indeed, current commercial finite element software is capable of simulating nonlinearity, whether material or geometrical, contact, structural interaction with fluids, metal forming, crash simulations, and so on. . . . Commercial software also come with advanced pre- and postprocessing abilities. Most of the time, these are the only components the user will interact with, and learning how to use them is often a matter of trial and error assisted by the documentation accompanying the software.

However, proficiency in using the pre- and postprocessors is by no means related to the accuracy of the results. The preprocessor is just a means of facilitating the data input, since the finite element method requires a large amount of data, while the postprocessor is another means for presenting the results in the form of contour maps. The user must realize that the core of the analysis is what happens in between the two processes . To achieve proficiency in finite element analysis , the user must understand what happens in this essential part , often referred to as the “black box.” This will only come after many years of high-level exposure to the fields that comprise FEA technology (differential equations, numerical analysis, and vector calculus) . A formal training in numerical procedures and matrix algebra as applied in the finite element method would be helpful to the user, particularly if he/she is one of the many design engineers applying finite element techniques in their work without a prior training in numerical procedures.

## **Chapter Three**

### **Modeling Strategy**

#### **3.1 Introduction**

Abaqus is a suite of powerful engineering simulation programs, based on the finite element method, as shown in (ch2) we illustrate briefly the basic stages of the Abaqus program, and then a 3D geometrically finite element analysis was carried out in the Abaqus workbench. For the finite element analysis, a structural beam of various lengths was used. Further boundary conditions were given to the model to fix the beam from one end and free it from another end in order to behave like a cantilever beam and to have only one direction of displacement.

In this chapter we will compare experimental results taken from a previous study with the FE program (ABAQUS) results to ensure the effectiveness and efficiency of the program (ABAQUS) in Reinforcement Concrete beam modeling. Then the numerical results from the FEA are compared with the experimental results.

### **3.2 Modeling Strategy Steps :**

#### **1- Comparison**

- a. Between previous experimental work and FE program (ABAQUS) result of cantilever beam .
- b. Between previous experimental work and FE program (ABAQUS) result of simple supported beam with and without externally CFRP .
- c. Between previous experimental work and FE program (ABAQUS) result of cantilever beam with and without externally CFRP  
(ch 4).

#### **2- Modeling of cantilever beam at length 3.5m with and without CFRP (ch 4) .**



### 3.3 Comparison :

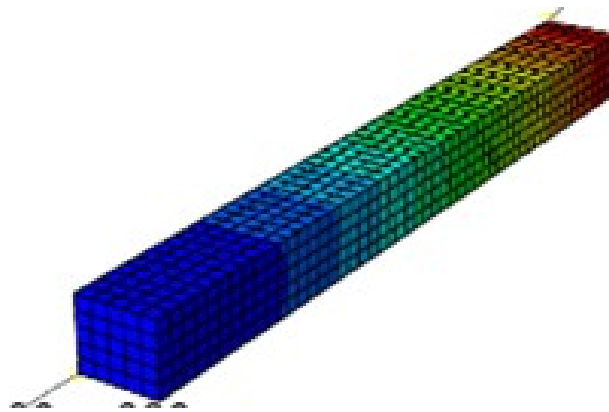
1- In study was done in January 2016 [1R] investigate the deflection and stress distribution in a long, slender cantilever beam of uniform rectangular cross section made of linear elastic material properties that are homogeneous and isotropic. The deflection of a cantilever beam is essentially a three dimensional problem. An elastic stretching in one direction is accompanied by a compression in perpendicular directions. The beam is modeled under the action of three different loading conditions: vertical concentrated load applied at the free end, uniformly distributed load and uniformly varying load which runs over the whole span. The weight of the beam is assumed to be negligible. It is also assumed that the beam is in extensible and so the strains are also negligible. Considering this assumptions at first using the Bernoulli-Euler's bending- moment curvature relationship , the approximate solutions of the cantilever beam was obtained from the general set of equations. Then assuming a particular set of dimensions, the deflection and stress values of the beam are calculated analytically. Finite element analysis of the beam was done considering various types of elements under different loading conditions in ANSYS 14.5. The various numerical results were generated at different nodal points by taking the origin of the Cartesian coordinate system at the fixed end of the beam. The nodal solutions were analyzed and compared. On comparing the computational and analytical solutions it was found that for stresses the 8 node brick element gives the most consistent results and the variation with the analytical results is minimum .

Now , will modeling cantilever beam (B1) and take it is properties from this precedent study as following :

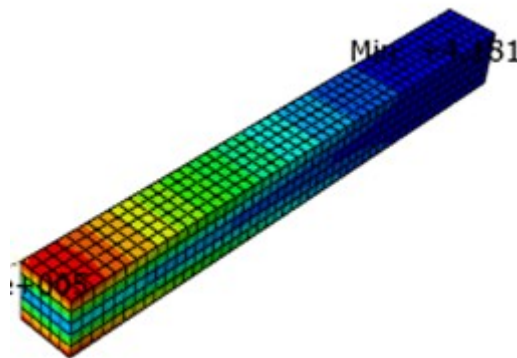
- $b=10m$
- $h=10m$

- $L=100\text{m}$
- $\nu=0.3$
- $E=2E+5$
- The modeling of the RCC beam was done using the 10-node brick elements to represent the concrete .

And later will compare the result with the result of the studied , a uniform distribution load was applied on the free end having a magnitude of 500N. Through this approach will get the deflection and stress distribution on the entire beam where the value was maximum .



**Fig . (3.1) Deflection of B1**



**Fig . (3.2 ) Stress Distribution of B1**

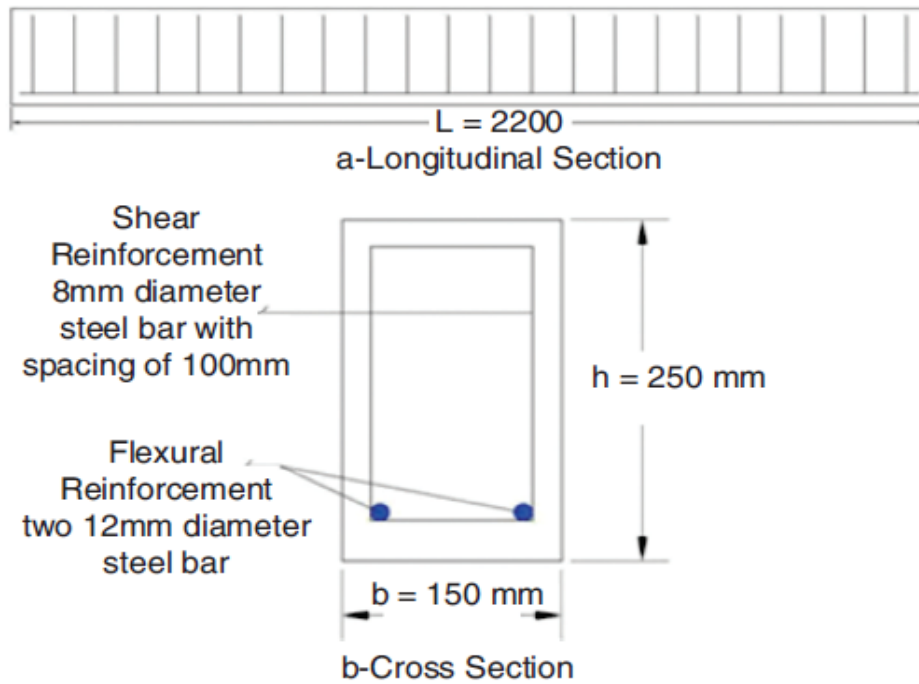
2- Since want To study The efficiency of using Carbon Fiber Reinforced Polymer (CFRP) as a strengthening technique for reinforced cantilever beams , at first will try to simulate The paper [9R] which presents the results of both analytical and experimental study on the repair effectiveness of Carbon Fiber Reinforced Polymer (CFRP) sheets for RC beams with different levels of pre-repair damage severity. It highlights the effect of fixing CFRP sheets to damaged beams on the load capacity, mid-span deflection, the steel strain and the CFRP strain and failure modes. The analytical study was based on a Finite Element (FE) model of the beam using brick and embedded bar elements for the concrete and steel reinforcement, respectively. The CFRP sheets and adhesive interface were modeled using shell elements with orthotropic material properties and incorporating the ultimate adhesive strain obtained experimentally to define the limit for debonding. In order to validate the analytical model, the FE results were compared with the results obtained from laboratory tests conducted on a control beam and three other beams subjected to different damage loads prior to repair with CFRP sheets.

Now modeling RC beam For pre-repair phase and for the post-repair phase , Beams were designed according to ACI 318 (2008) [3R] Code requirements .

properties of beam (B2) as following :

- $b = 150\text{mm}$
- $h = 250\text{mm}$
- $L = 2200\text{mm}$
- $\nu = 0.2$
- $E = 2E+5$
- Flexural reinforcement tow 12mm diameter steel bar

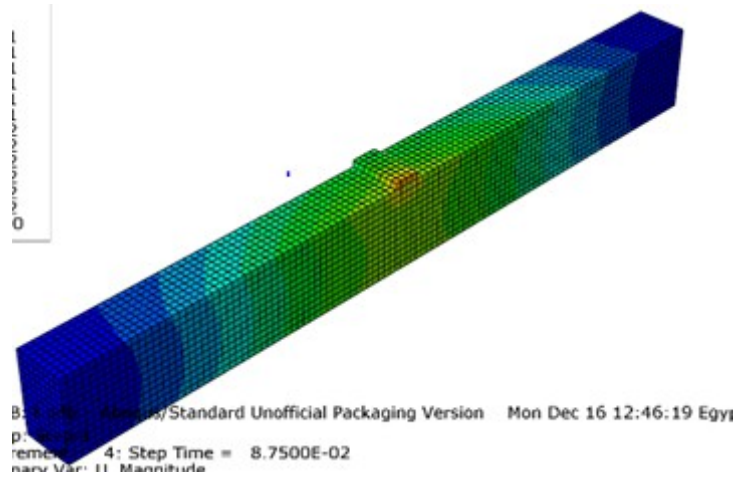
- Shear reinforcement 8mm diameter steel bar with spacing of 100mm



**Fig . (3.3 ) Details of B2**

- Repairing with CFRP sheet was designed according to ACI440. [3R] (200) code requirements with a 100mm width and 1.2mm thickness and the length was the clear span of the beam.
- The modeling of the RC beam was done using the 20-node brick elements to represent the concrete .

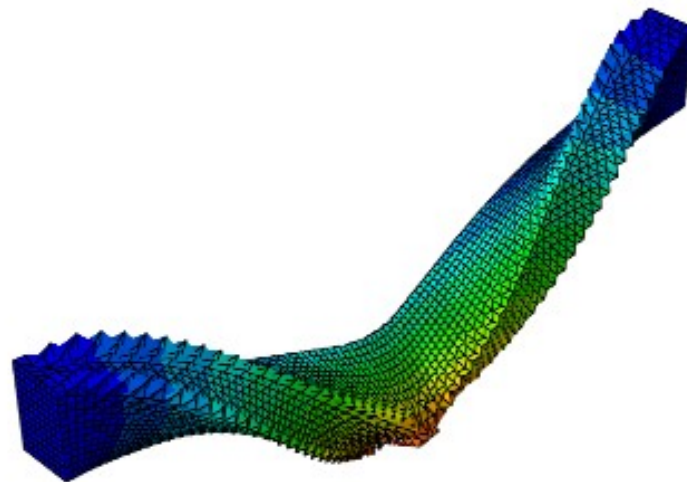
**a.** RC beam (B2a) without externally bonded CFRP sheet . The beam was loaded with point load applied at mid-span (10kN)



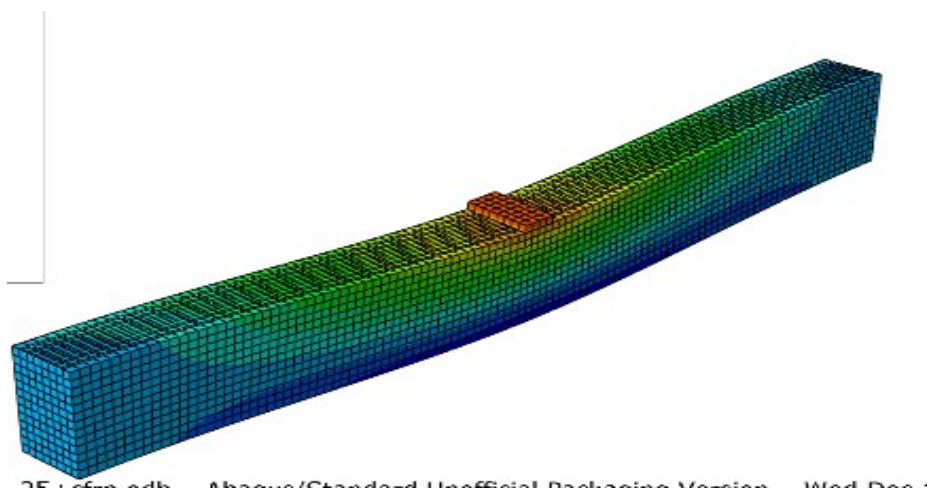
**Fig . (3.4 ) Deflection of B2a**

**b.** (B2b) Was exposure to failure load was the same at 71kN after repaired beam , it was first exposed to the design limit load at 25kN , then was strengthened with externally bonded CFRP sheets .

Now review the difference between the two cases and show the effect of using CFRP at the beam (B2b)

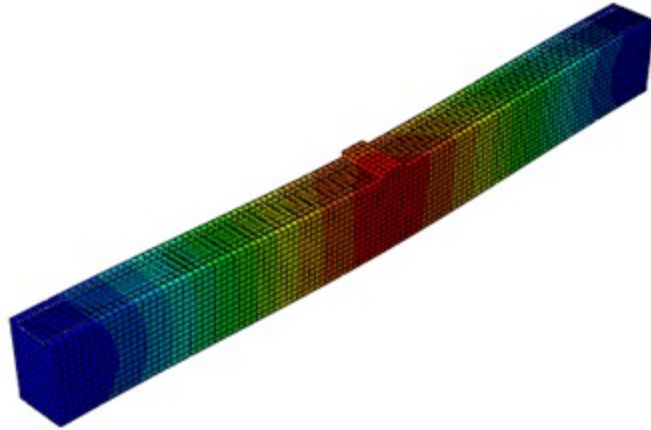


**Fig . (3.5) Deflection of B2b without CFRP**



**Fig . ( 3.6) Deflection of B2b with CFRP**

**c.** RC beam (B2c) with externally bonded CFRP sheets and load amount 100kN



**Fig . (3.7 ) Deflection of B2c with CFRP**

## Chapter Four

### Modeling

#### 4.1 Introduction

This chapter consider as complement to previous chapter , it is essentially constant of two stage , in the first will modeling the cantilever beam in length 2.2m which extracted from previous study to check deflection work , then will modeling cantilever beam at length 3.5m and show the effect of CFRP in the deflection of beam .

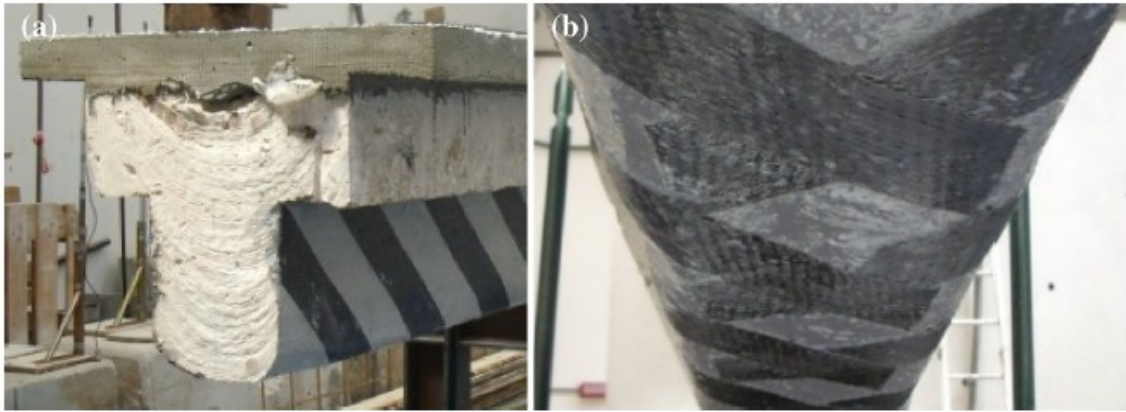
#### 4.2 Using of CFRP With Cantilever Beam :

Based on previous study [8R] , and which was used CFRP as the best solution to improve beam shear strength as the retrofitting intervention that choice was based on it fast and the cost is modest between solution .

In this study, two beams extracted from an existing building constructed in the 1930s in Rome and retrofitted by carbon fiber-reinforced polymer (C-FRP) U strips placed at beam ends, where also negative bending moments were present, and have been evaluated with experimental tests at the laboratory of the Department of Architecture of Roma Tre University. Beam steel and concrete characteristics were evaluated by means of different tests. The experimental results are discussed considering the final results in terms of maximum shear resistance in the presence of negative bending moments. Load deflections at different points along the beam, shear-C-FRP deformation along the reinforcement strips and the damage state for different load levels, are presented .



The beams were integrated with a new cantilever at one support to reproduce, in the lab, the negative moments and shear due to adequate vertical concentrated loads. The cantilever length are equal to 2.2 m .



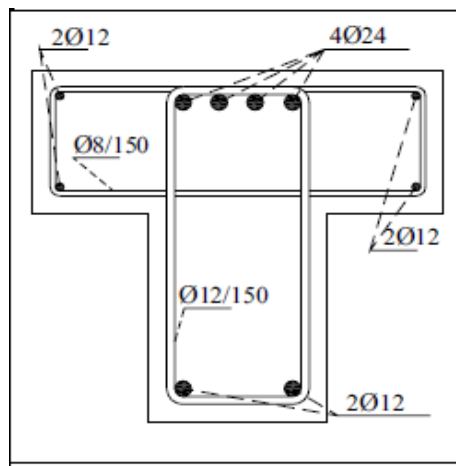
**Fig . (4.1 ) Retrofitting of the beam specimens: a C-FRP Strips for shear reinforcement and RC slab for flexural reinforcement; b intersection of the C-FRP strips at the beam intrados[8R] .**



**Fig . (4.2 ) Interleaving Between Cantilever and Existing Beam[8R] .**

The study use CFRP (U) sheet anchorage in the area of negative bending moments , whereas in the study mentioned in the second paragraph of item 6 in chapter 3 , which also used CFRP but as externally bonded plates along the beam length , In this research we will use CFRP as externally bonded 2sheet .

Dimensions of beam which we will modeling obtained from the study which use CFRP (U) sheet as strengthing technique to beam with cantilever eadg



**Fig . (4.3 ) Reinforcement Details of B3 & B4**

As it scenes in fig ( 4.3 ) , the section is Flanged beam , the flange was use as slab to sure interleaving between existing beam and new cantilever beam as illustrated in fig (4.2) , but in this study use rectangular beam and just conclude the dimensions from fig (4.3) to beam (B3) as follows :

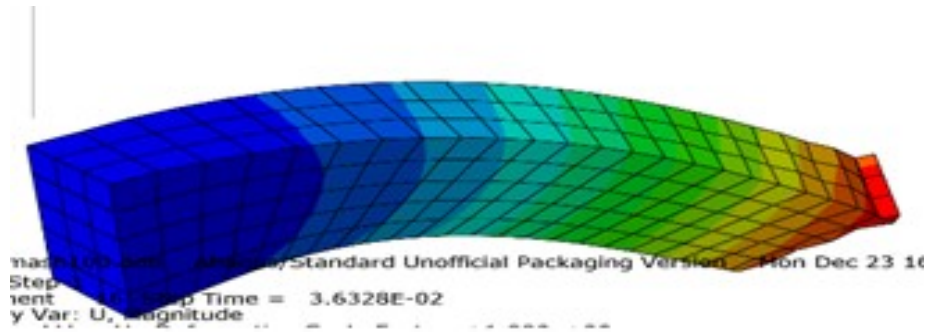
- L = 2200mm
- B = 250mm
- D = 500mm
- Load = 27kN

- Stirrups = 12mm @ 150mm
- $v=0.2$
- $E=2E+5$
- Longitudinal upper rebars: 4  $\phi$ 24 mm
- Longitudinal lower rebars: 2  $\phi$ 12 mm
- $F_y = 450$  Mpa
- $F_{cu} = 28$  Mpa
- Repairing with CFRP sheet was designed according to ACI440.2R (200) code requirements with a 100mm width and 1.2mm thickness and the length was the clear span of the beam.
- The modeling of the RCC beam was done using the 20-node brick elements to represent the concrete .

The length of the beam will began from 2.2m and then up to 3.5m , in the first will comparison the result of deflection in length 2.2m with the experimental approach (with and without CFRP ) , and then we will try to arrive to 3.5m length of cantilever beam .

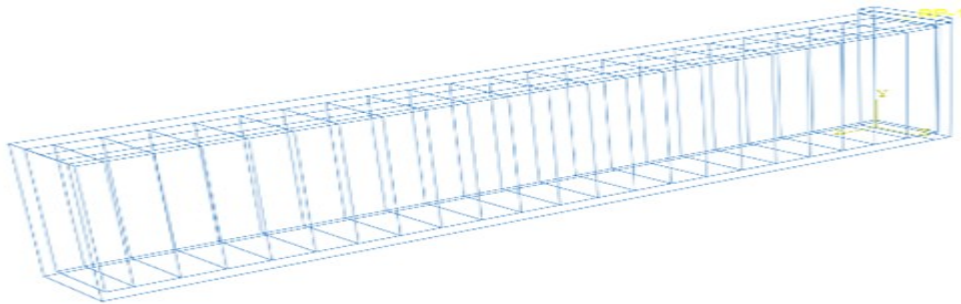
## 4.2.1 Cantilever Beam (B3) in Length 2.2m

### a . B3a without using CFRP

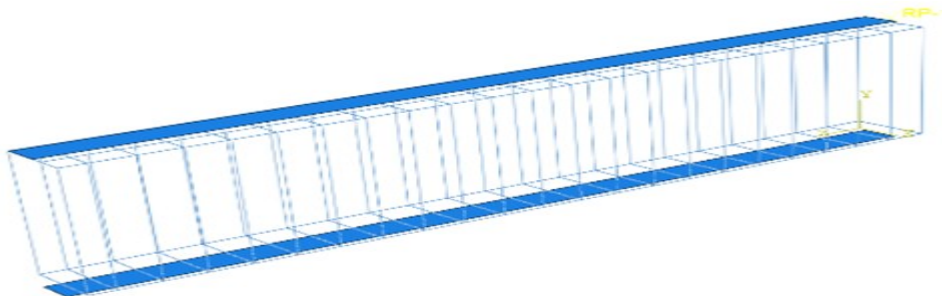


**Fig . (4.4 ) Deflection of B3a**

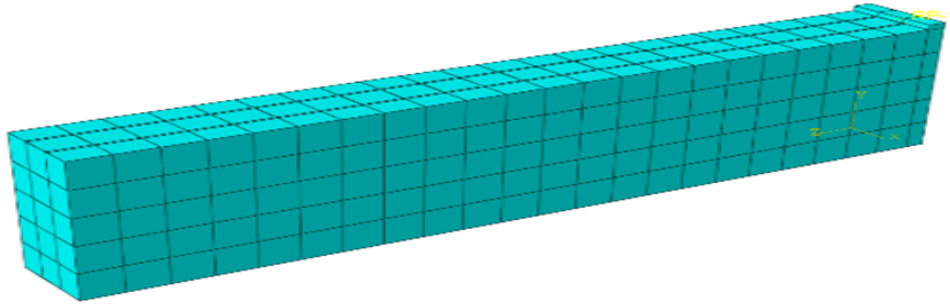
### b . B3b with using CFRP



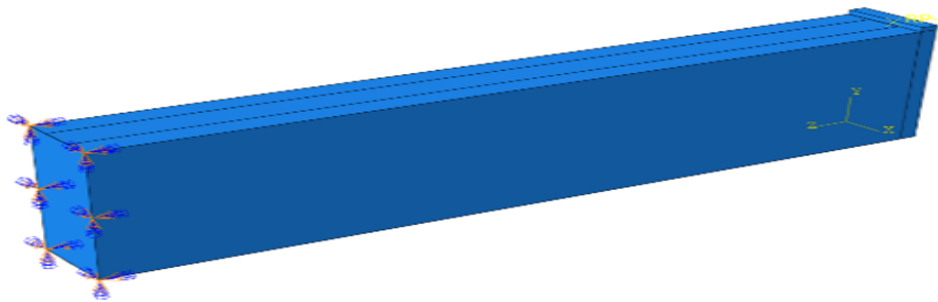
**b.1 Reinforced Bar Modeling**



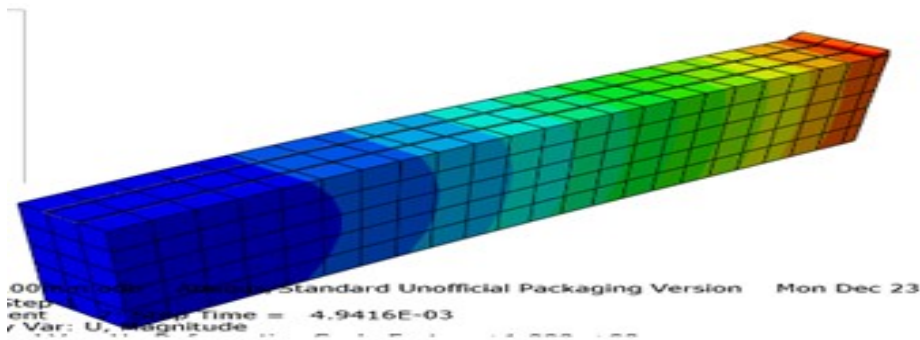
**b.2 CFRP Modeling**



### b.3 Mach Modeling



### b.4 Support Modeling

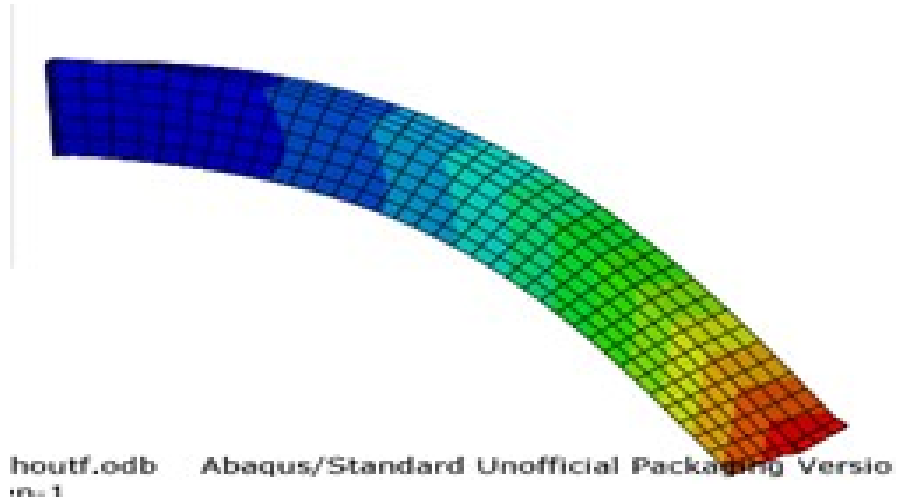


**Fig . (4.5 ) Deflection of B3b**

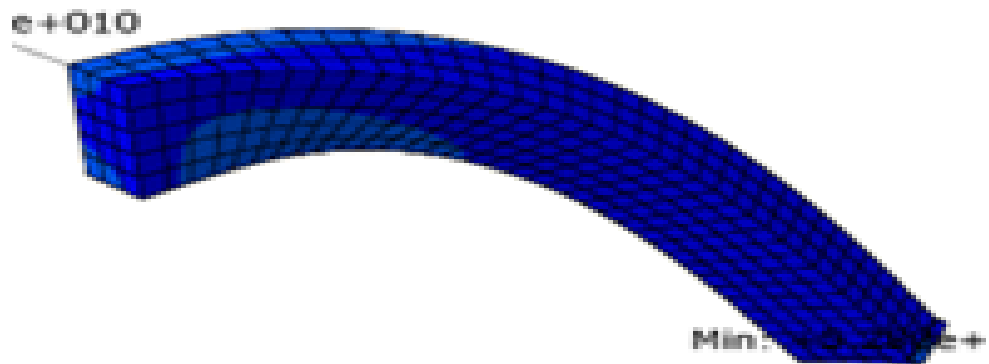
- As it scenes , there is a big difference between the two cases and a significant effect on the structural behavior of cantilever beam when use CFRP , This is also evident in the scheme obtained from the study from which the model was extracted .

## 4.2.2 Cantilever Beam ( B4) in Length 3.5m

### a . B4a without using CFRP

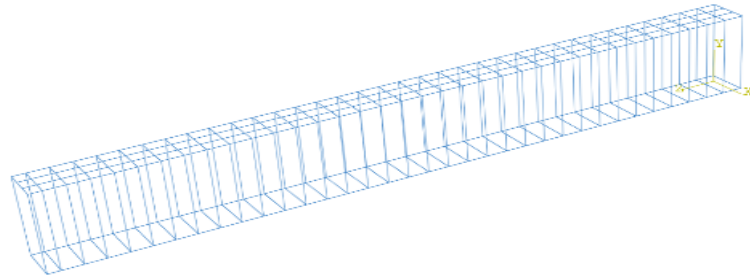


**Fig . (4.6) Deflection of B4a**

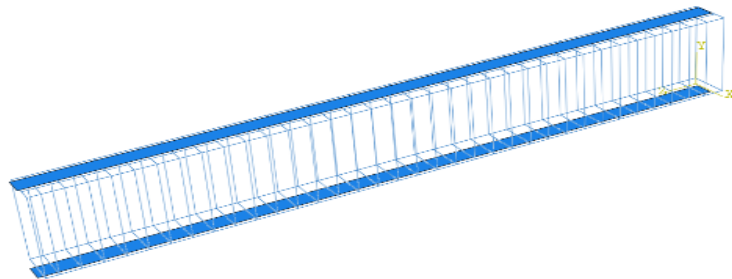


**Fig . (4.7) Stress Distributions of B4a**

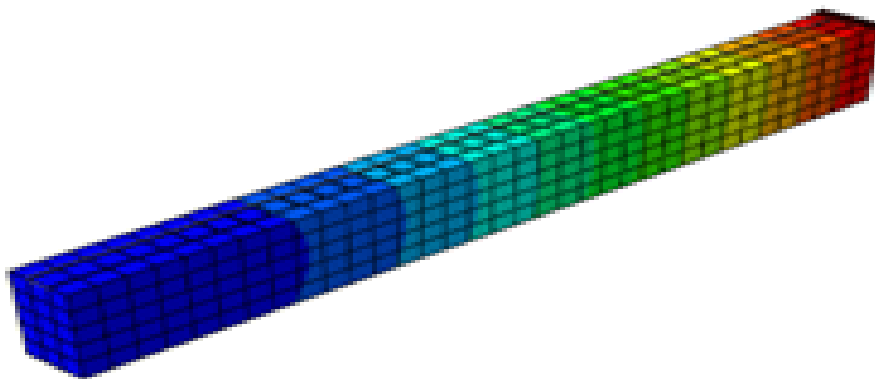
**b . B4b with using CFRP**



**b.1 Reinforced Bar Modeling**

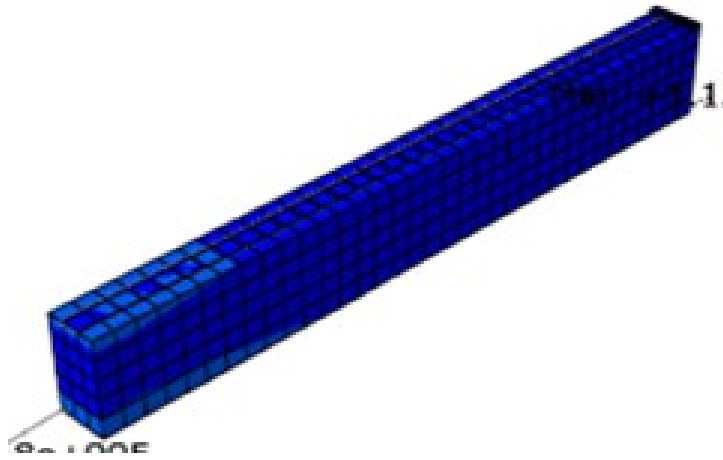


**b.2 CFRP Modeling**



**b.3 Cantilever Beam Modeling**

**Fig . (4.8) Deflection of B4b**



**Fig . (4.9) Stress Distribution of B4b**



## Chapter Five

### Results & Discussion

#### 5.1 Introduction

As mentioned earlier, The cantilever is rigidly fixed in one end the other end is hanged in the air .It does not need a column or wall to support this end . So , the space below the cantilever can be used efficiently .

Cantilevers require care and ingenuity in their design , erection , assessment , and rehabilitation by structural engineers .

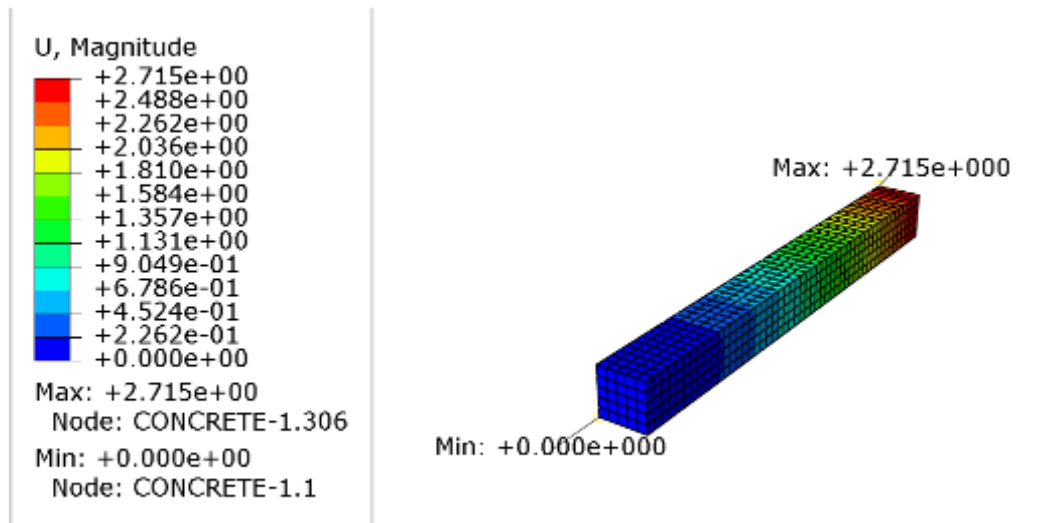
This study is aimed to understanding the behavior of cantilever beam , and try to modeling RCC beam by using finite element program (ABAQUS) and then compare experimental results (which taken from previous study) with the ABAQUS results. and investigate the efficiency of using carbon fiber reinforced polymer (CFRP) as a strengthening technique for reinforced cantilever beams .

As scenes , The results obtained showed good agreement between experimental results (from previous study) and the ABAQUS , and there is a significant effect on the structural behavior of cantilever beam whene use CFRP , This is also evident in the scheme obtained from the study from which the model was extracted .

This part will present the results of study and trying to discuss what was reached in the research .

## 5.2 Results

### 5.2.1 Results of B1

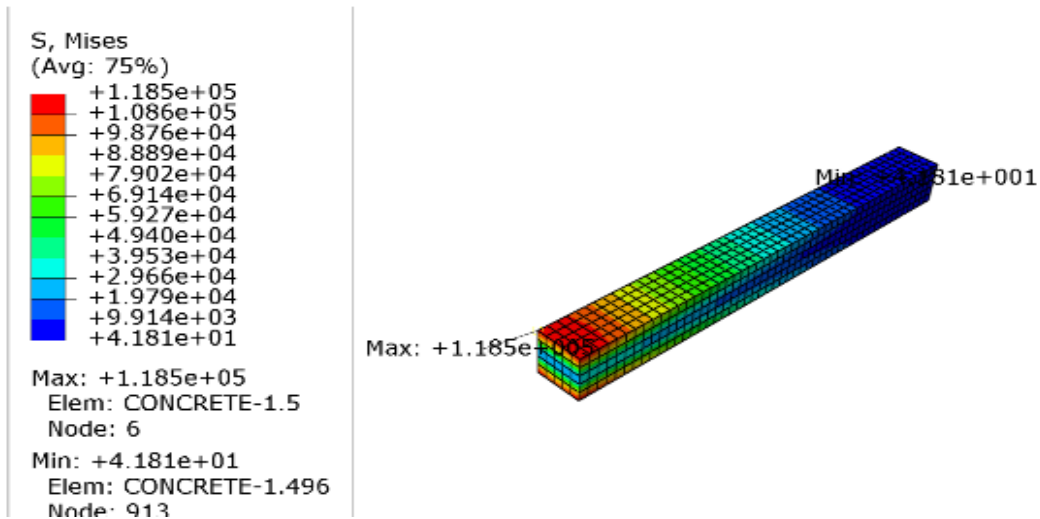


**Fig . (5.1) Deflection Values of B1**

Max Deflection Obtained = 27.15m

Max Deflection was Obtained in Ansys = 24.858m

Error = 2.292m



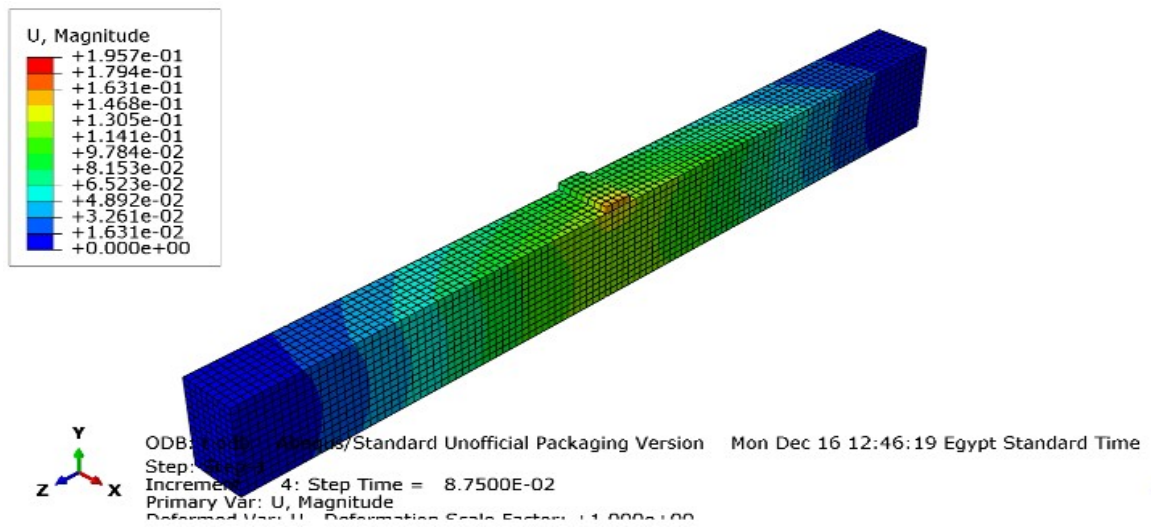
**Fig . (5.2 ) Stress Distribution of B1**

Mises Stress Obtained = 11800 N/mm<sup>2</sup>

Mises Stress was Obtained in Ansys = 11277N/mm<sup>2</sup>

Error = 523N/mm<sup>2</sup>

## 5.2.2 Results of B2

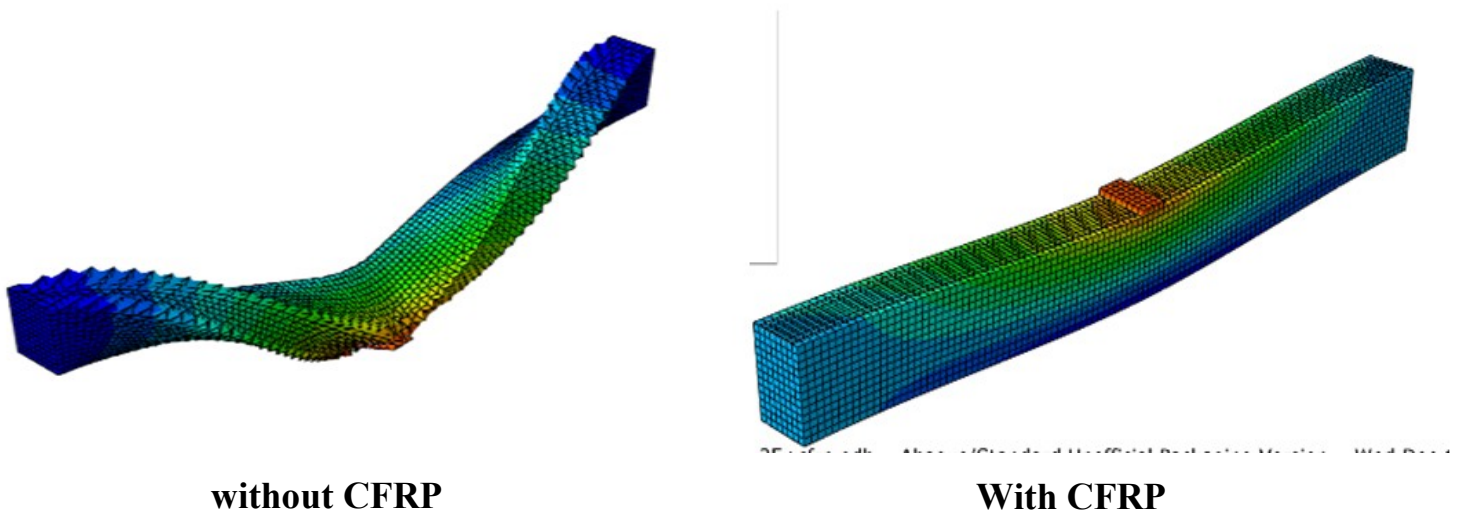


**Fig . (5.3 ) Deflection Values of B2a**

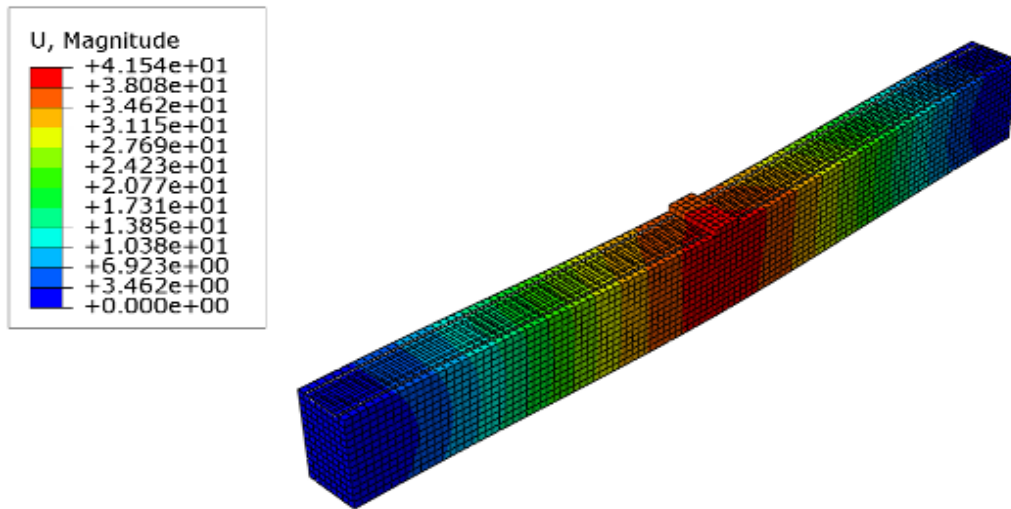
Max Deflection Obtained = 0.1957mm

The Deflection was Obtained in Study = 0.2mm

Error = 0.00043mm



**Fig . (5.4 ) Deflection of B2b**



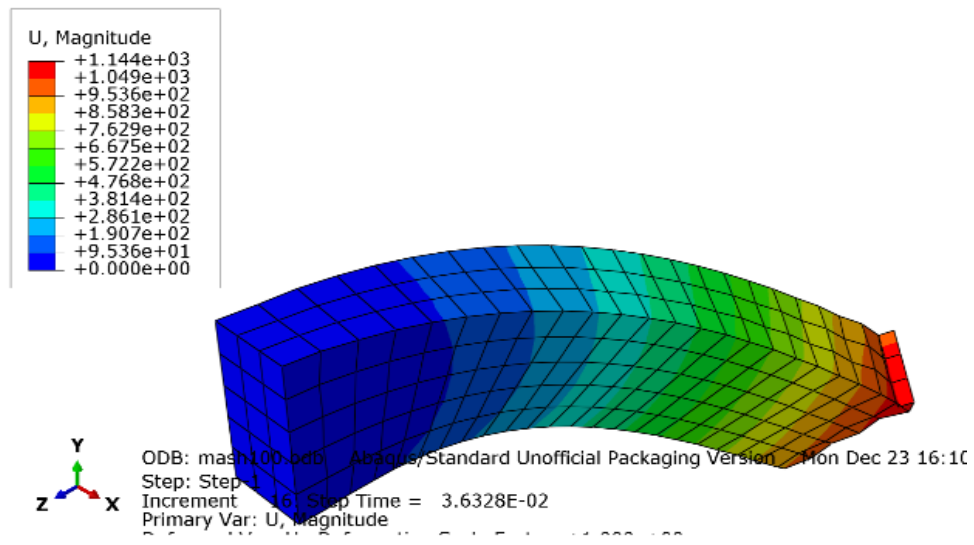
**Fig . (5.5 ) Deflection of B2c With CFRP**

Max Deflection Obtained = 41.54 mm

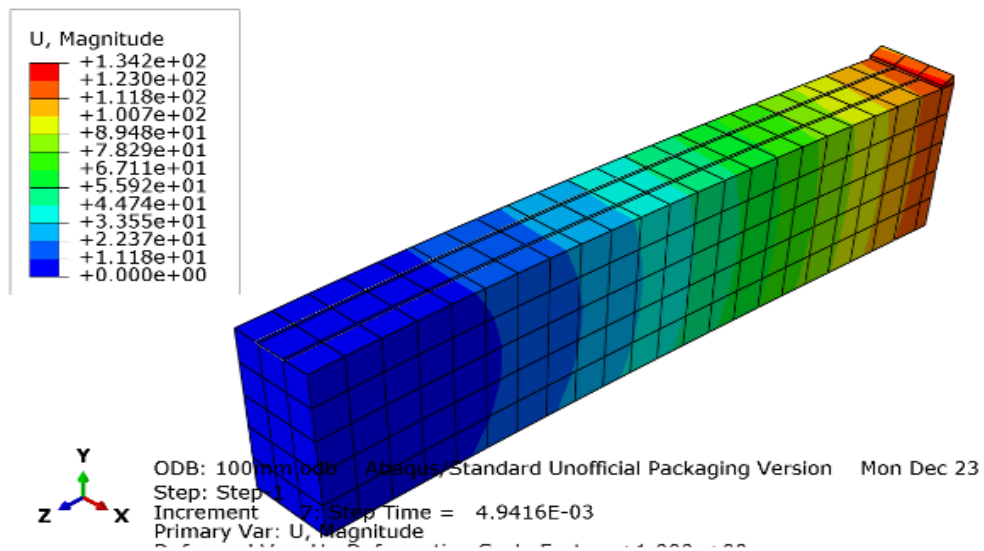
The Deflection was Obtained in Study = 37 mm

Error = 4.54 mm

### 5.2.3 Results of B3



a . B3a without using CFRP



b . B3b with using CFRP

Fig . (5.6) Deflection of B3

## 5.2.4 Results of B4

### a . B4a without using CFRP

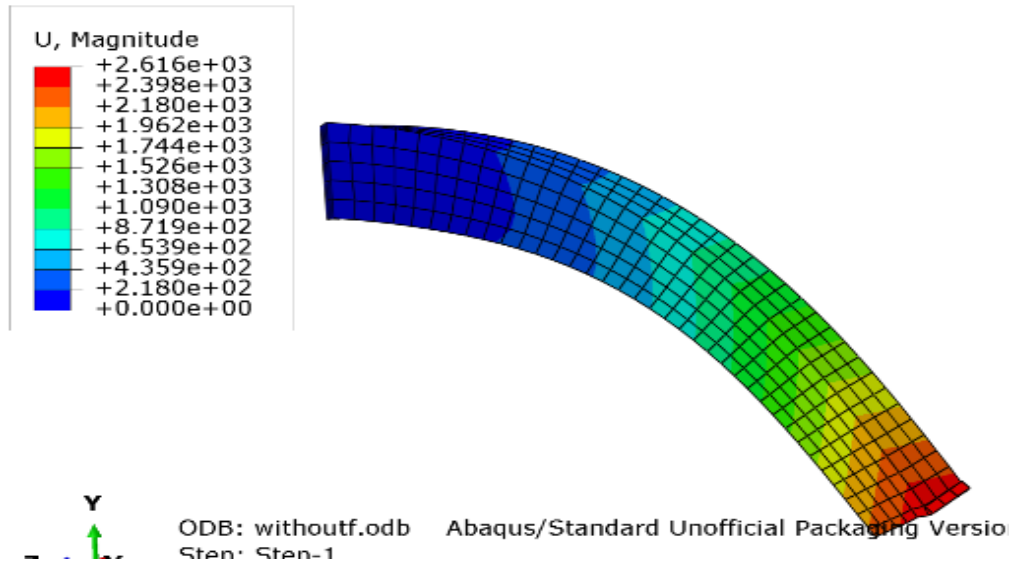


Fig . (5.7) Deflection of B4a

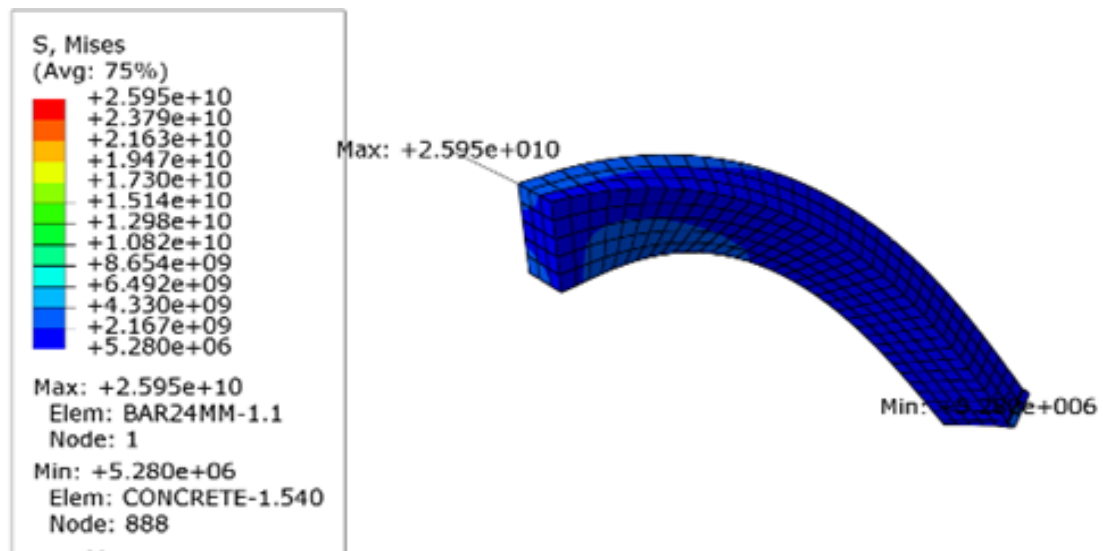
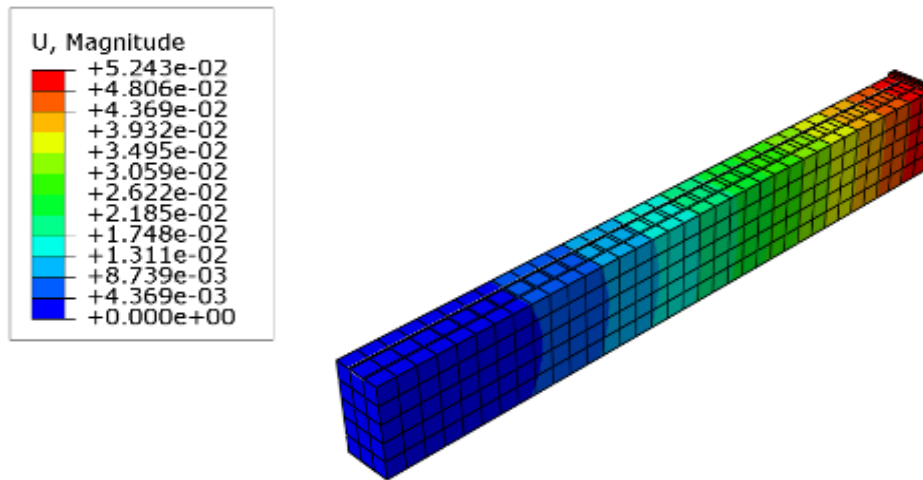
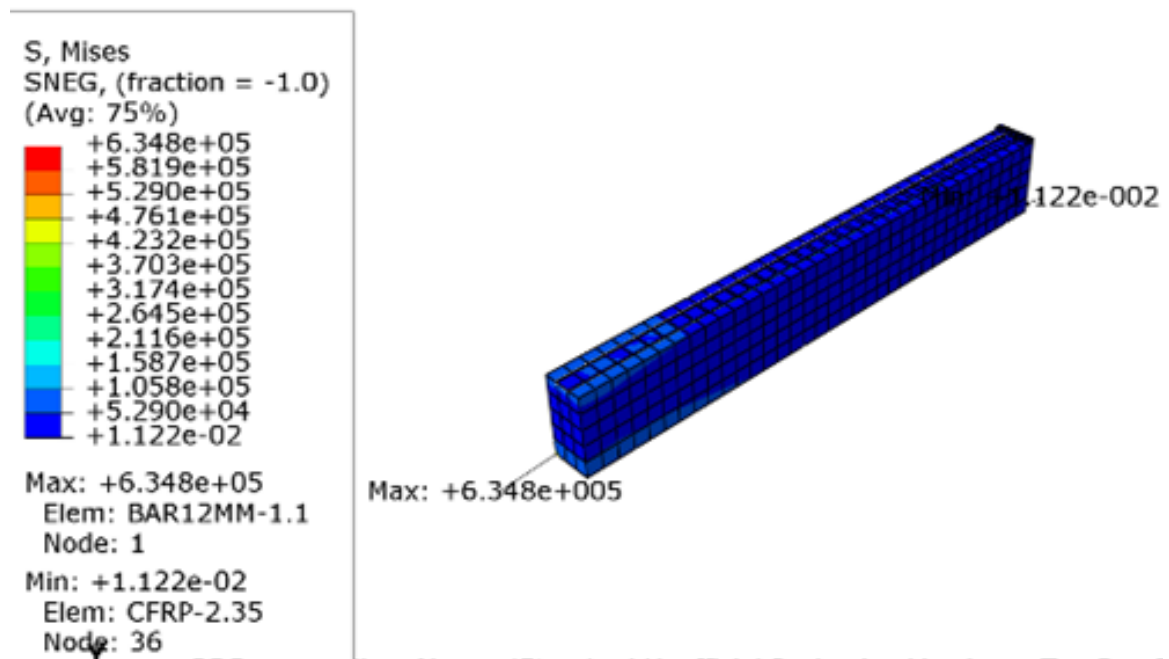


Fig . (5.8) Stress Distributions of B4a

**b . B4b with using CFRP**



**Fig . (5.9) Deflection of B4b**



**Fig . (5.10) Stress Distribution of B4b**

### 5.3 Discussion of Results

This section will present the results obtained from the figures , and discussion the analytical results of finite element program (ABAQUS )

#### 5.3.1. Discussion Results of B1

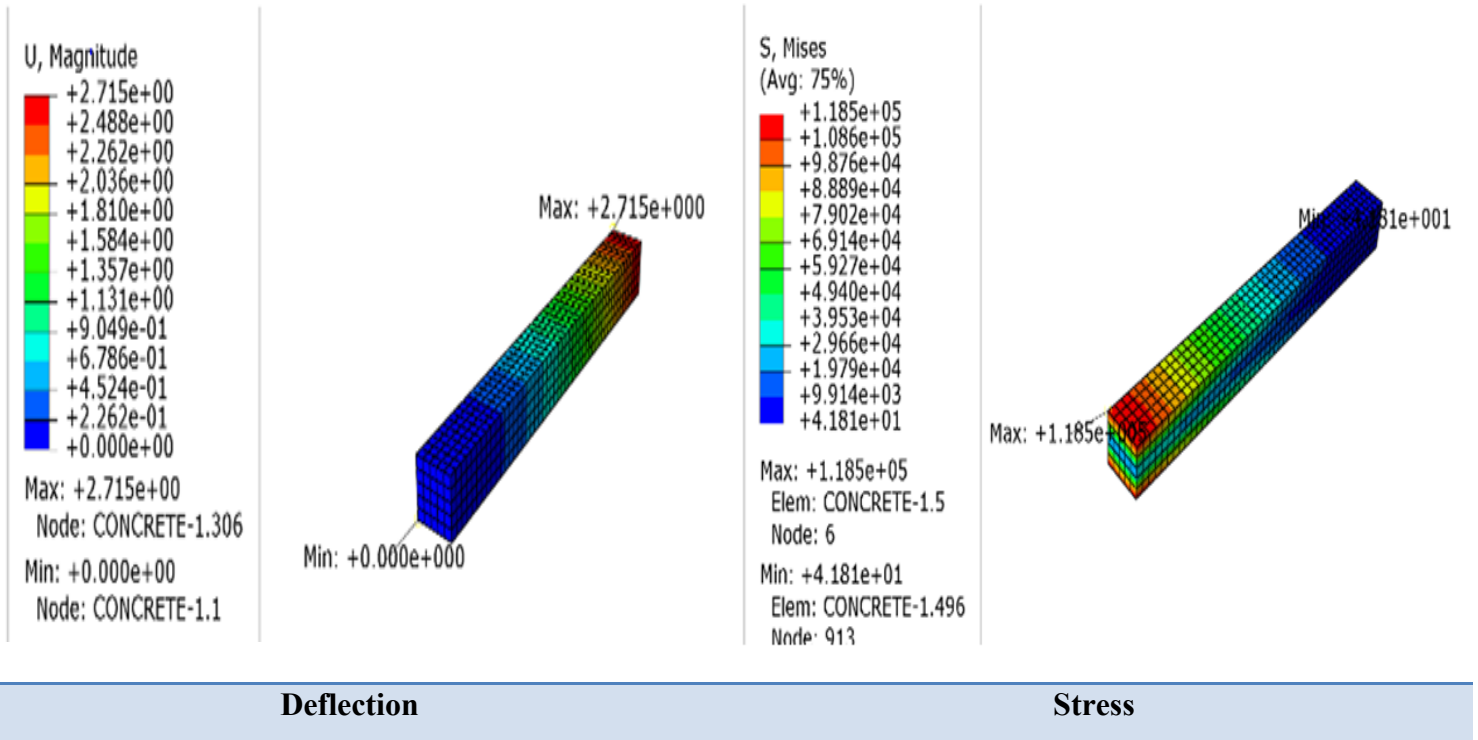


Fig . (5.11 ) Deflection and Stress of B1

Table 5.1 - Deflection and Stress of B1 at Load 500N :

Beam NO	Deflection (m)		Error (m)	variance (%)	Stress (N/mm <sup>2</sup> )		Error (N/mm <sup>2</sup> )	variance (%)
	FEA(Abaqus)	Exp (ansys)			FE(abaqus)	Exp(ansys)		
B1	27.15	24.858	2.292	0.92	11800	11277	523	0.96



Figures (5.11) , show modeling of reinforced concrete cantilever (RCC) beam in length 10m & load 500N ,the beam was used to sure the effectiveness and efficiency of the program (ABAQUS) in reinforced cantilever beam modeling , The results of deflection to experimental work ( previous study) was 24.858m wile FEA to Abaqus program show 27.15m , and the stress value to experimental work was 11277 N/mm<sup>2</sup> wile FEA to Abaqus program show 11800 N/mm<sup>2</sup> , That is mean there are acceptable agreement between the analytical (FEA)and the experimental results ( from previous study) as illustrated in the table (5.1) .

### 5.3.2. Discussion Results of B2

To study the efficiency of using Carbon Fiber Reinforced Polymer (CFRP) as a strengthening technique ( external reinforced ) for reinforced cantilever beams , in the first need to sure the effectiveness and efficiency of the program (ABAQUS) in modeling of reinforced concrete (RC) beam with CFRP , for this purpose was use beam (B2) to simulate the previous study which presents the results of experimental study for effectiveness of Carbon Fiber Reinforced Polymer (CFRP) sheets for RC beams , and comparison between ( ABAQUS) program result and experimental work which was use CFRP . The compression of (B2) was done in three phases , (B2a) , (B2b) and (B2c) , the purpose of this beams are illustrated as follows :

#### 1. B2a & B2c

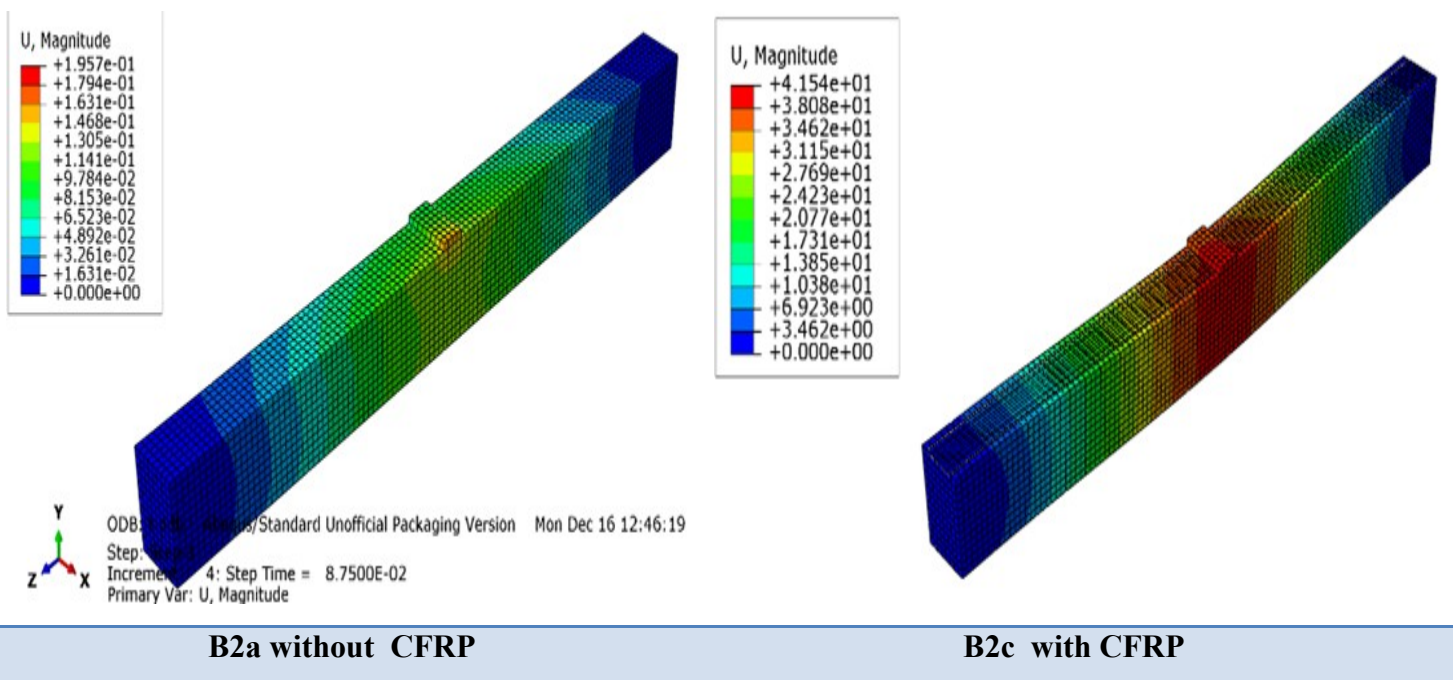


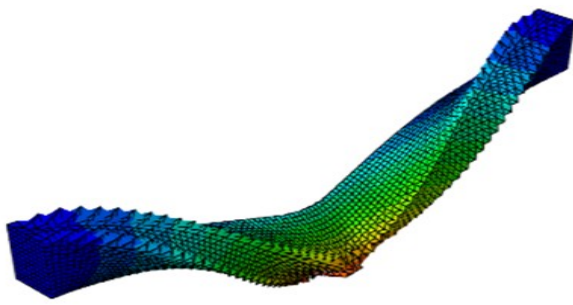
Fig . (5.12 ) Deflection of B2a & B2c

**Table 5.2 - Deflection of B2a at Load 10KN & B2c al Load 100KN:**

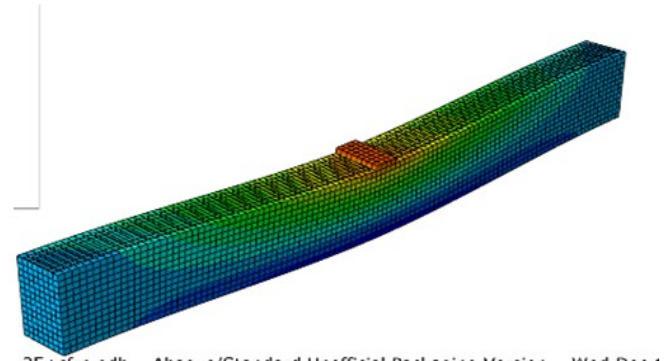
Beam No	Deflection (mm)		Error(mm)	Variance (%)
	FEA(Abaqus)	EXPR		
B2a	0.1957	0.2	0.00043	0.98
B2c	41.54	37	4.54	0.89

Figure (5.12) to B2 , show modeling of RC beam in length 2.2 m , (B2a) with concentrated load 10 kN , while (B2c) with concentrated load 100kN and externally reinforced CFRP. The beam (B2a) was used to sure the effectiveness and efficiency of the program (ABAQUS) in RC beam modeling , The results of experimental work (previous study) were 0.2mm while the result of FEA was 0.1957mm , as clear the result almost the same and there was no noticeable difference between the experimental and the analytical (FEA) results as illustrated in the table (5.2) . While (B2c) in the figure (5.12) was used to sure the effectiveness and efficiency of the program (ABAQUS) in RC beam modeling with CFRP , The results of experimental work (previous study) was 37mm while the result of FEA was 41.54mm ,as clear there are good agreement between the analytical ( FEA) and the experimental results as illustrated in the table (5.2) .

## 2. B2b



Without CFRP

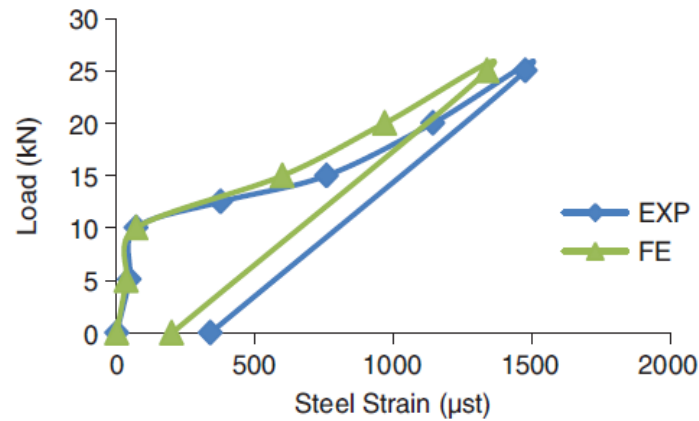


With CFRP

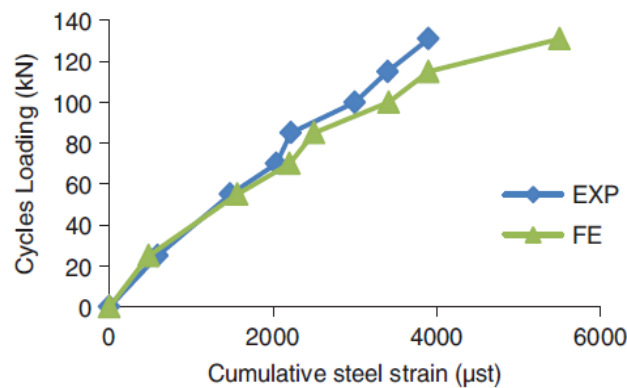
**Fig . (5.13) Deflection of B2b**

This figures (5.13) to (B2b) , show modeling of RC beam in length 2.2m with design limit load 25kN , the beam was used just to Comparison behavior of RC beam with and without using CFRP , The results show a big difference between the two cases as illustrated in the figure ( 5.13) and a significant effect on the structural behavior of Reinforcement beam when use CFRP as external reinforcement .

The CFRP sheet leading to an increase in the steel yield loading rate with the CFRP sharing the tensile stresses with the steel bars as illustrated in figure (5.14) . At failure, the analytical (FEA) results ( from previous study) [9R] show that steel reinforcement has reached the rupture strain at 5500  $\mu\text{st}$ , which leads to full failure of the beam after debonding of the CFRP at 131 kN . The experimental results show that the steel reached less than 4000  $\mu\text{st}$  as illustrated in figure (5.14) b , which means that steel was still in the hardening zone and could take more loading; this is the reason behind the ability of the beam to take loading after the CFRP debonding , and behind the importance of experimental work .



a. Load against the Steel Strain Curve for B2b without CFRP



b. Load against the cumulative Steel Strain for B2b with CFRP

**Fig . (5.14) Steel Strain of B2b (Experimental)**

- After checked the performance of Abaqus program , cantilever beam at length 3.5m was model . But in the first the properties of beam was concluded from previous study which mentioned in chapter (4) and which was with length of the beam (B3) 2.2m , B3 modeled with CFRP to check the deflection and then modeled B4 at length 3.5m as follow :

### 5.3.3. Discussion Results of B3

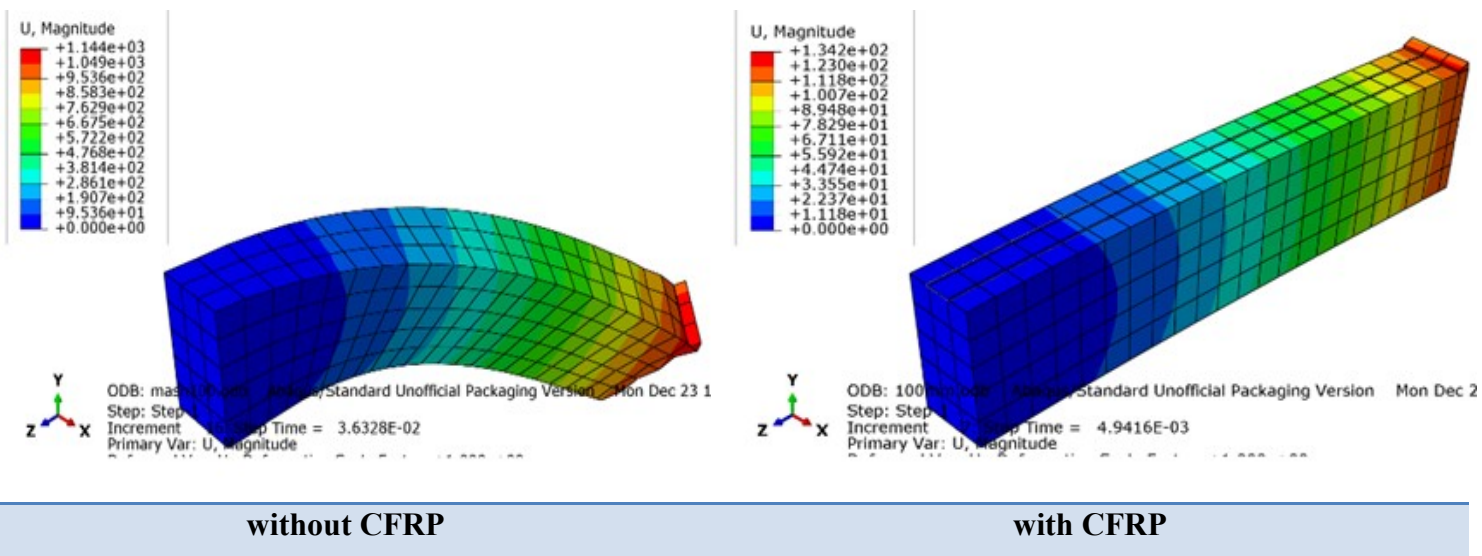


Fig . (5.15) Deflection of B3

Table 5.3 - Deflection of B3 at Load 27kN :

Deflection (mm)		Difference (mm)
B3a	B3b	
1144	134.2	1009.8

Figure (5.15) to ( B3) , show modeling of RCC beam in length 2.2m with concentrated load 27kN , (B3a) RCC without CFRP while (B3b) RCC with externally bonded of CFRP 2sheet , the beam was used to sure the effectiveness and efficiency of CFRP , and Comparison RCC beam with and without using CFRP , The results of deflection to B3a was 1144mm while to B3b with CFRP is 134.2mm , as clear from the result there are a big difference between the two cases and a significant effect on the structural deflection of RCC beam when use CFRP as externally bonded to enhance the deflection of long cantilever beam, and as clear in the figure (5.15) the big difference between two cases , that is also illustrated in the value of deflection on the table (5.3) .

### 5.3.4. Discussion Results of B4

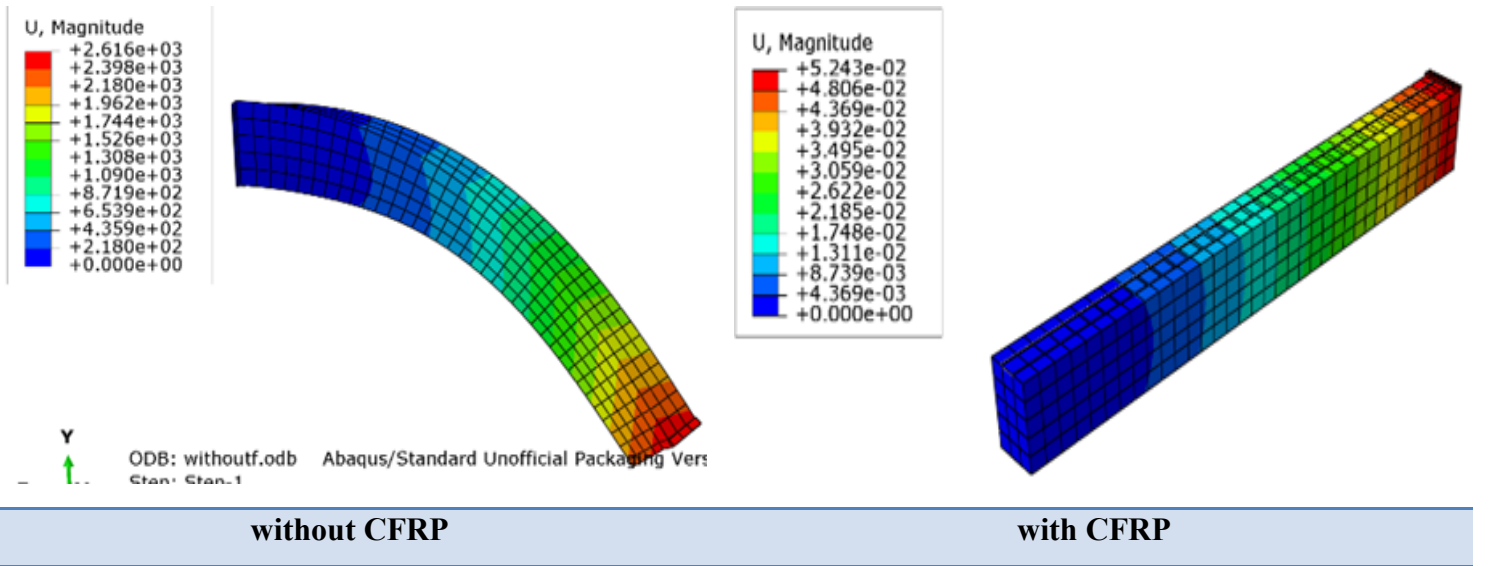


Fig . (5.16) Deflection of B4

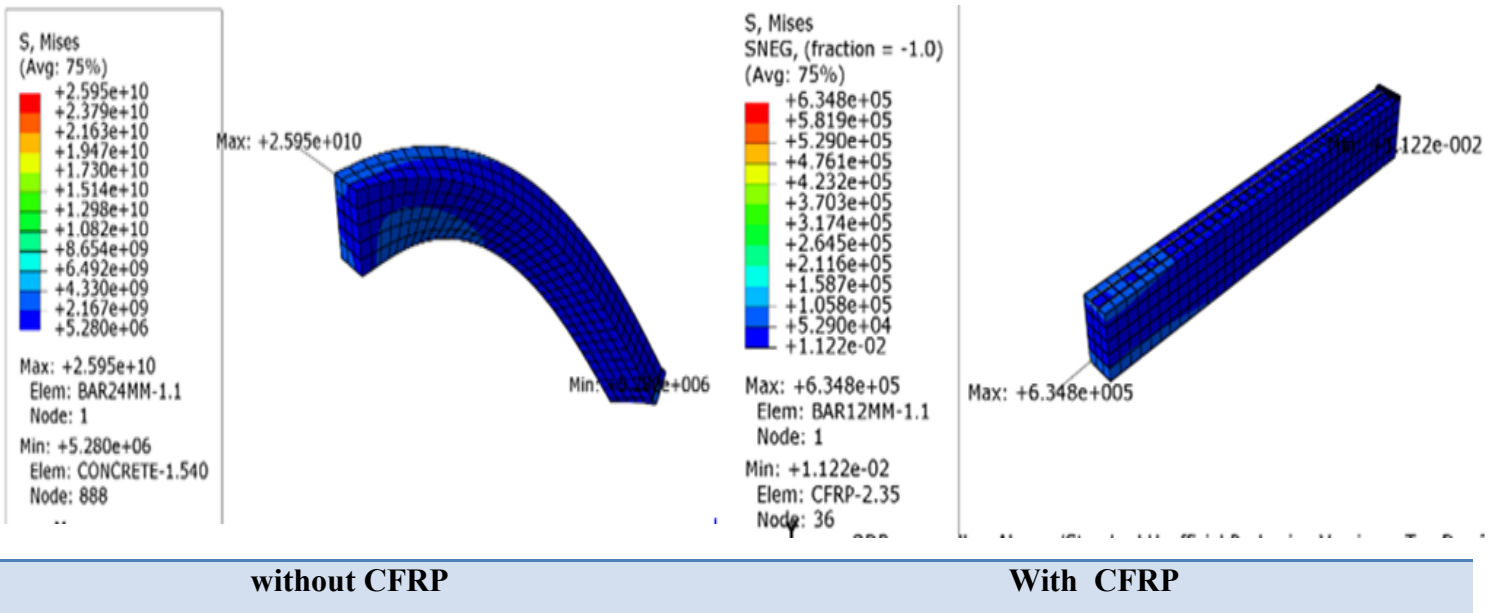


Fig . (5.17) Stress of B4

**Table 5.4 - Deflection and Stress of B4 at Load 27KN:**

Deflection (mm)		Difference (mm)	Stress (N/mm <sup>2</sup> )		Difference (N/mm <sup>2</sup> )
B4a	B4b	2615.94757	B4a	B4b	2.595e <sup>10</sup>
2616	0.05243		2.595e <sup>10</sup>	6.348e <sup>5</sup>	

Figure (5.16) and (5.17) show modeling of RCC beam (B4) in length 3.5m with concentrated load 27kN , (B4a) RCC beam without CFRP , while (B4b) RCC with externally bonded of CFRP. The beam was used to Comparison of behavior of RCC beam with and without using CFRP and try to reach to length 3.5m to cantilever beam without big deflection .

The results of deflection to B4a without CFRP was 2616mm while to B4b with CFRP 2sheet is 0.05243 mm , and the stress value to B4a was 2.595e<sup>10</sup> N/mm<sup>2</sup> , but B4b show 6.348e<sup>5</sup> N/mm<sup>2</sup> value of stress . The results show a big difference between the beams and a significant effect on the deflection behavior of cantilever beam when use CFRP 2sheet as externally bonded to enhance the behavior ( deflection & stress ) of long cantilever beam , and as clear in the figure (5.16) & (5.17) the big difference between the two cases , that is also illustrated in the values at the table (5.4) .

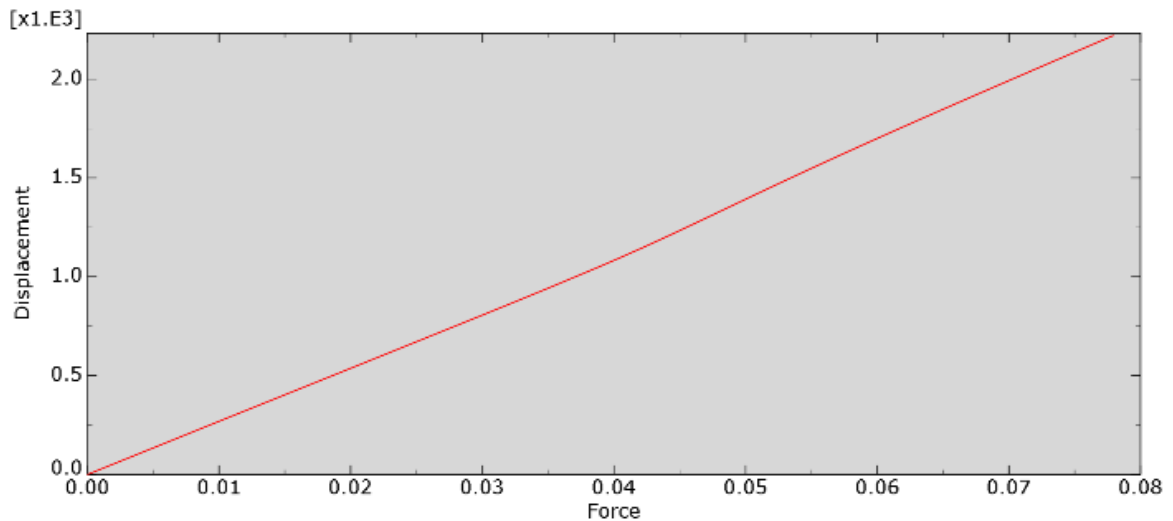
- The obtained results indicated that a significant increasing in stiffness and the ultimate load of the strengthened beams almost up to two times , which lead to significant decrease in deflection of beam as it is clear in the behavior ( deflection & stress ) to the beam modeling after adding CFRP as external reinforcement .



## 5.4 Influence of the CFRP on the RCC Beams

This section explain the features of Carbon Fiber Reinforced Polymer (CFRP) to understand the big effect to it in the ultimate load and deflection behavior to cantilever beam . As scenes in ch (3&4) , there is a significant effect on the structural behavior (deflection & stress distribution ) of cantilever beam when use CFRP , This is also evident in the scheme obtained from the study from which the model was extracted .

### a. B4a Force vs Displacement without using CFRP



### b. B4b Force vs Displacement with CFRP

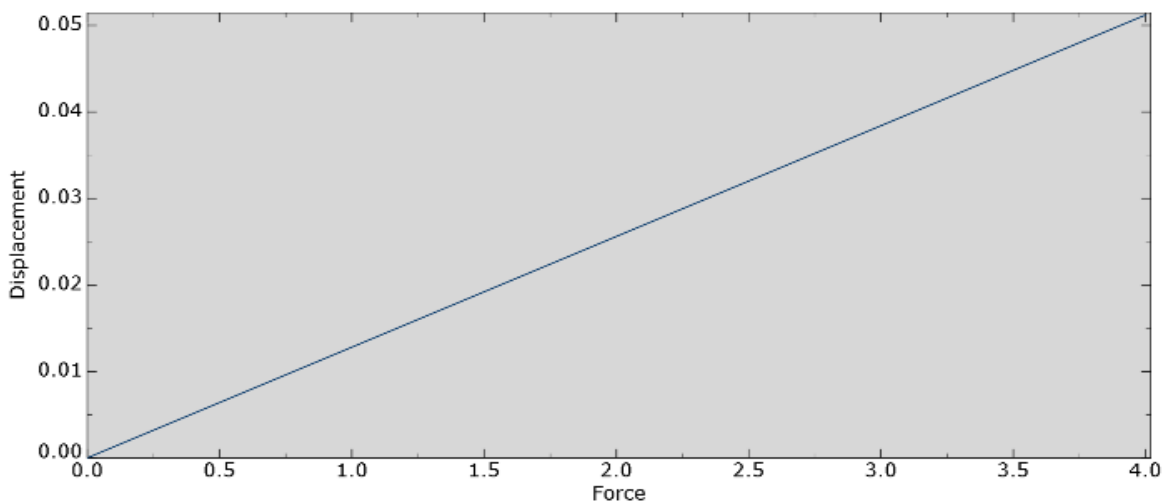


Fig . (5.18) Force vs Displacement of B4

Figure (5.18) shows the load against displacement curves for the two cases, with and without using CFRP, the diagram shows a big difference between the two cases. As illustrated in the diagram when force was 0.07N in the B4a (without CFRP) the deflection value is 2 mm, while in beam B4b (with CFRP) when force or load is 4N the deflection is almost 0.05mm, so that diagram shows us how the small value of load can make big displacement, but with CFRP external reinforced in the same beam, big value of load makes small magnitude of deflection, and with CFRP when we take the same magnitude of load as beam without CFRP (0.07N) the displacement value is almost 0.005mm that shows how CFRP can decrease the deflection more than 50%. Based on early studies of the last decade, Strengthening of RC beams with externally bonded CFRP plates was found to increase the ultimate capacity by 37–87%, Strengthening of the RC beam with one layer of the CFRP plate was found to increase the ultimate capacity by 200%, and strengthening with two layers increased it by 250% [9R]. The results show that fixing CFRP sheets to the tension face of the beam have increased the load capacity of the beams, Fixing the CFRP has reduced the deflection of the beam (as we see in the figures), and as mentioned, the CFRP has high tensile strength, and it is sharing the tensile stresses with the steel bars, that will retard the incidence of rupture strain by maintaining steel in the hardening zone and could take more loading, this is the reason behind the ability of the beam to take loading after the CFRP debonding.

Externally Carbon Fiber Reinforced Polymer (CFRP) is one of the new materials used to strengthen or repair RC structures. It is particularly suitable for in situ rehabilitation, and has become an increasingly applied and important technology.

## **5.5 To use Long Cantilever Beam without Fear of the Deformation or Damage .**

A cantilever is an important structural element that provide less support and efficient use of space in building design , and it is essential design elements that provide functional and aesthetic architectural benefits . As cantilever serving not only aesthetic but also functional purposes , their structural safety needs more attention and it is require care and ingenuity in their design .

And as it is known that , increase in length of cantilever beam lead to increase in complexities of design and construction procedure , so What can do to use long cantilever beam without fear of the deformation or damage!

The use of innovative technologies, materials and design systems enables to increase the application of cantilever structures. It allows choosing the best solutions for them and overcoming different risks.

There is a lots of techniques which used to enhance and strengthing cantilever , such as, using Reinforced concrete (RC) jacket with or without shear connector bars , two layers of Fiber Reinforced Polymer , Glass Fiber (GFRP) or Carbon Fiber (CFRP) , tow steel plates using expansion bolts [13R], and different materials [10R] such as, silicon , silicon oxide and silicon nitride .

When cantilever have great length there are new materials and technologies use to make this length is possible , like , a pre-tensioned and post-tensioned concrete structure of high strength concrete , structural steel high strength rebar and self-consolidating concrete [18R]

In this study used CFRP as strengthening technique for reinforced cantilever beams , and As scenes , there is a significant effect on the structural behavior of cantilever beam when use CFRP .

CFRP material are recently commonly used for strengthening of structural reinforced concrete [9R] . The low weight reduces both the duration and cost of construction . The composites can be applied as a thin plate or layer by layer. CFRP become an increasingly applied and important technology because of it is advantages , such as , availability in any length , corrosion-resistance , high tensile strength , low weight , low installation cost and flexibility of storage , transportation and use , moreover , the properties of the composite material with epoxy resin have advantages such as resistance to corrosion , high tensile strength , reasonable stiffness consequently .

## Chapter Six

### Conclusions and Recommendations

#### 6.1 Introduction

Cantilever beam is one of the important structural element which give the building aesthetic and unique feature , it is need more attention in the design and constructing . sure there is a limits but undoubtedly there is always another choice and palpably there is a solution to any structural problem , cantilever beam and cantilever structure in general , in addition to its show the creativity to the structural design which make construct of long cantilever structure is possible , it can addition spaces to the building literally out of nothing .

The present study aimed is to study structural behavior of a cantilever beam that includes deflection, stress distribution with the varied on the beam having different long sectional area , and investigate the effectiveness of externally reinforcement CFRP sheets to cantilever beam by using FEA program (ABAQUS) to model RCC beam . Also , the present study try to explain how the CFRP work to decrease the deflection and increase the load capacity .

In this part of research will review the conclusion and recommendations which concluded from study .

## 6.2 Conclusions

Following are the main conclusions that can be drawn from on the results of the present study :

1. To improve structural behavior of cantilever beam and in general cantilever structure, more careful studies must be conducted, and acquirement of knowledge about the modern techniques used ,to reach to buildings virtuosic in it is architectural and structural design .
2. Finite element program (ABAQUS) modeling is in good agreement with experimental approach results which taken from previous study. As illustrated in ( ch5) when modeling reinforcement cantilever beam B1 the variance between ABAQUS results and experimental work was 0.92 % , As clear from the ratio there are acceptable agreement between the values .
3. ABAQUS can be used to study more than just structural (Stress/Displacement) problems.
4. It should be noted that , When using Finite Element program (ABAQUS) , whenever the mesh small the value of the deflection will be reduce .
5. There is important and significant effect of CFRP on the structural behavior of cantilever beam it is illustrated in the varied models of beam and in the value of deflection in the tables (ch 5) .
6. Strengthening RCC beam with externally bonded CFRP sheets increases the carrying load and decreases the deflection and the steel strain . When modeled B4 at length 3.5m without CFRP the value of deflection was 2616mm , while with externally reinforcement CFRP the deflection was 0.05243mm , as clear the big effect to the CFRP , and big difference between the two value which lead to significant decrease in deflection of beam (ch5) .

7. The CFRP sheet sharing the tensile stresses with the steel bars , and according to it is position ( in the external part of beam ) , CFRP caring maximum amount of tensile stress that will retards incidence of rupture strain by maintain steel in the hardening zone and could take more loading , this is the reason behind the ability of the beam to take loading after the CFRP debonding .
8. B4 with it is properties and design system , and strengthening technique with CFRP which mentioned in ch(4) can reach to length 3.5m of cantilever beam with good performance .

### 6.3 Recommendations

1. When using Finite element program (ABAQUS) and to reach to higher possible accuracy it must to knowing and obtain beon all detail of materials properties and information witch need it to modeling procedure .
2. To achieve proficiency in Finite Element analysis Program (ABAQUS) , the user must understand what happens in the essential part , This will only come after long times of high-level exposure to the program .
3. Specific tests are required to validate of CFRP effect .
4. According to pervious study , and after debonding of the CFRP sheets the FE software stopped at specific limited , while for the experimental results can take more loading . So , It must to conducting experimental work to support and check on of the simulations works and Results.
5. Study stopped in the length 3.5m , but it is not the purpose of the research or the using of CFRP , after surely of the effect of CFRP in the behavior of concrete beam by experimental work should try to arrive to more than 3.5m length of cantilever beam .
6. When design cantilever beam like B4 recommended that keeping construction requirements which mentioned in (ch1) [17R] , and design considerations which mentioned in (ch2) [17R] , as such suggested that design upper steel with 6 rebars and lower steel with 4 rebars and don't decrees the diameter of steel .
7. There is a lots of techniques which used to enhance and strengthing cantilever beam [17R] , recommended that study that techniques and comparison it is effect with the effect of CFRP .



Lastly , a cantilever is one of the most magically tools in building design. In essence , it allows part of a construction to stand unsupported . This might sound like it is defying all known laws of physics , look at any modern tower block or building and you will see how the balconies jut out into thin air – perfect everyday examples of cantilevers in action , although it is simplest example of cantilever . This research lighting up cantilever structures as one of the beautiful and complicated structural designs , Sure , it is not easy to design or construct cantilever more than 10m and to be completely honest , more than 3 m , but also it is not impossible , all we need is more knowledge , science , and innovation , to show how structural engineering can make beautiful dream come true .

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