Dedication

To my Family

Acknowledgements

First I would like to thank without end our greatest ALLAH. Then I would like to thank my supervisor prof.Dr.Shawgy Hussein Abdalla who gave me great advice and help in this research . My thanks are due to Ham**di** group for their good typing. My thanks are due to any one who assisted by a way or another to bring this study to the light.

Abstract

We show the classification of the digit sets as product-forms. The spectral property of a class of cantor measures with consecutive digits and on $R^*$are given . The spectral structure of digit sets of self-similar tiles on R^1 with non-spectral problem for a class of planar self-affine measures in R^N with two-element digit set and decomposable digit sets are studied . The Mock Fouier series and existence of orthonormal bases of Certain Cantor-Moran measures are discussed. The spectrality of a class of infinite convolutions and Cantor Moran measures with three-element digit sets are obtained . The spectrality of Moran measures with four-element digit sets and one dimensional self-similar measures with consecutive digits are in vestigated.

الخلاصة

قمنا بتوضيح التصنيف لفئات الرقم كصبيغ-ناتج . تم اعطاء الخاصية الطيفية الي عائلة قياسات الكاتنور مع ارقام الصنف وعلي R^* . تم دراسة البناء الطيفي لفئات الرقم للبلاط المماثل الذاتي علي R مع المسالة غير الطيفية لاجل القياس النسيبي –الذاتي المستو في R^N مع فئة-رقم العنصر الثالث وفئات الرقم قابل للتحلل. تمت مناقشة متسلسلة فورير الزائفة ووجود الاساس المنتظم المتعامد لقياسات كانتور-موران المؤكدة . تم الحصول علي الطيفية الي عائلة الالتفافات اللانهائية وقياسات موران المؤكدة مع فئات رقم العنصر –الثالث . قمنا ببحث الطيفية لقياسات موران مع فئة –الرقم –العنصر –الرابع والقياسات المماثلة –الذاتية ذات البعد الواحد مع رقم الصف.

Introduction

Let A be an expanding matrix on R^s with integral entries. A fundamental question in the fractal tiling theory is to understand the structure of the digit set $D \subset Z^s$ so that the integral self-affine set $T(A, D)$ is a translational tile on R^s. We classified such tile digit sets D ⊂ Z by expressing the mask polynomial PD into product of cyclotomic polynomials. We consider equally-weighted Cantor measures µq,b arising from iterated function systems of the form b−1(x + i), i = 0, 1, · · · , q − 1, where q < b. We classify the (q, b) so that they have infinitely many mutually orthogonal exponentials in $L^2(\mu q, b)$. In particular, if q divides b, the measures have a complete orthogonal exponential system and hence spectral measures. Improving the construction in [DHS], we characterize all the maximal orthogonal sets Λ when q divides b via a maximal mapping on the q−adic tree in which all elements in Λ are represented uniquely in finite b−adic expansions and we can separate the maximal orthogonal sets into two types: regular and irregular sets. We study the structure of the digit sets *D* for the integral selfsimilar tiles *T*(*b*, *D*) (we call such a *D* a *tile digit set* with respect to *b*). So far the only available classes of such tile digit sets are the complete residue sets and the product-forms. Our investigation here is based on the spectrum of the mask polynomial P_D, i.e., the zeros of P_D on the unit circle. By using the Fourier criteria of self-similar tiles of Kenyon and Protasov, as well as the algebraic techniques of cyclotomic polynomials,

A probability measure in R^d is called a spectral measure if it has an orthonormal basis consisting of exponentials. We study spectral Cantor measures. Let $b \geq 2$ be a positive integer. Let *D* be a finite subset of Z and be a sequence of strictly increasing numbers. A Moran measure is a Borel probability measure generated by the Moran iterated function system (Moran IFS) .

We study spectral properties of the self-affine measure generated by an expanding integer matrix and a consecutive collinear digit set $D = \{0, 1, \ldots, q - 1\}v$ where and $q \geq 2$ is an integer. Some sufficient conditions for to be a spectral measure or to have infinitely many orthogonal exponentials are given. The self-affine measure associated with an expanding matrix and a finite digit set is uniquely determined by the self-affine identity with equal weight. The spectral and non-spectral problems on the selfaffine measures have some surprising connections with a number of areas in mathematics, and have been received much attention in recent years.

Let $\{ D_k \}_{k=1}^{\infty}$ be a sequence of digit sets in N and let $\{b_k\}_{k=1}^{\infty}$ be a sequence of integer numbers bigger than 1. We call the family a Moran iterated function system (IFS), which is a natural generalization of an IFS. For a finite set $D \subset Z$ and an integer $b \geq 2$, we say that (b, D) is compatible with a Hadamard matrix. Let $\{D_k\}_{k=1}^{\infty}$denote the uniformly discrete probability measure on *E*.

Let $\{d_n, p_n\}_{n=1}^{\infty}$ be a sequence of integers so that $0 < d_n < p_n$ for $n \geq 1$. The infinite convolution of probability measures with finite support and equal distribution is a Borel probability measure (Cantor–Moran measure).

The iterated function system with two-element digit set is the simplest case and the most important case in the study of self-affine measures. The one-dimensional case corresponds to the Bernoulli convolution whose spectral property is understandable. The higher dimensional analogue is not known, for which two conjectures about the spectrality and the non-spectrality remain open.

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