

بسم الله الرحمن الرحيم



**Sudan university of science and technology**  
**College of Petroleum and Mining engineering**  
**Department of petroleum engineering**



**Project Title:**

**Research Submitted to College of Petroleum and Mining Engineering**

**Partial Fulfillment of the Requirements for the Degree of B.Sc. in**

**Petroleum Engineering**

**Deconvolution method for pressure transient analysis Abu**

**Gabra formation Case Study- Sudan.**

**طريقة إزالة الالتفاف لتحليل بيانات الضغط المنتقل – دراسة على تكوين**

**أبو جابرة\_ السودان.**

**Prepared by:**

- **Mohaid Ali Hassan Alshiakh**
- **Mohammed Ali Babiker Mohammed Ali**
- **Momen Hassan Mohammed Albaid**
- **Omer Ahmed Mosmar Ahmed**

**Supervisor:**

**Dr. Altazi Khalid**

**November-2020**

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



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Project Supervisor : Dr.Altazi Khalid

Signature:.....

Head of Department: Eng. Abdelwhab Mohamed Fadoul.

Signature:.....

Dean of College: Dr. Elham Mohammed Mohammed Khair.

Signature:.....

Date: / / 2020



# الاستهلال

قَالَ تَعَالَى:

﴿وَيَسْأَلُونَكَ عَنِ الرُّوحِ <sup>ص</sup> قُلِ الرُّوحُ مِنْ أَمْرِ رَبِّي وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا ﴿٨٥﴾﴾

سورة الإسراء الآية (85)

## **Dedication**

To the spirit of our fathers, brothers, teachers, and academic supervisor **Dr. Altazi Khalid** for his kindness, gentleness, and generous. We won't stop asking Allah for him to forgive him and give him the highest levels of the paradise. To our fathers and mothers who taught us great lessons about life, guiding, motivation, innovation and support us along life's level. Without them, we would not become the people who we are today.

To our brothers and sisters who stand with us, encourage us and taught us the real meaning of helping other people with all we have. For future generations that hold future of the oil industry in Sudan. We are honor to offer you this modest work. Thanks all for giving us a chance to prove and improve our self through all levels of university life.

## **Acknowledgment**

Several sources of information have been used to make this project. We would like to express our sincere gratitude and appreciation to our academic supervisor Dr. **Altazi Khalid** for his consistent advice and receptiveness to opinions and ideas while we worked on this project. We would like to give a special thanks to for his support and guidance.

In addition, special thanks goes Mr. Osman Ahmed well test engineer in GNPOC.

# Abstract

Traditionally well testing is completed by analyzing transient pressure due to constant production rate. However, in the oil industry practice, engineer often has to deal with the transient pressure result from variable flowing rate history.

This traditional and conventional well testing approach is still being used. However, a new well testing tool called Deconvolution was starting to receive much attention, and has been emerging as a new tool of analyzing test data in the form of constant rate drawdown response. In other words, it transforms variable rate and pressure data into constant rate pressure response. Deconvolution techniques is a useful addition method to the well test analysis.

In this research, a brief introduction of conventional well test interpretation will be presented, followed deconvolution interpretation, and the results show that the deconvolution method can be applied on the drawdown period of single well test.

## التجريد

إن الطرق التقليدية لتحليل بيانات اختبارات الآبار تبنى على افتراض أن معدل السريان يكون ثابت وذلك من الصعب الحصول على معدل سريان ثابت إلا إذا تم إغلاق البئر وعندها يتم الحصول على معدل سريان يساوي صفر إذ تستخدم بيانات الضغط الناتجة منه لإجراء طرق التحليل التقليدية ولكن عملية إغلاق البئر للحصول على بيانات الضغط ومعدل سريان تحتاج لوقت طويل مما يزيد من تكاليف عملية اختبارات الآبار, لذا الطريقة الجديدة والمسماة بإزالة الالتفاف يمكن استخدامها في بيانات معدل سريان متغيرة إذ أنها تقوم بتحويل بيانات الضغط الناتجة من معدل سريان متغير إلى بيانات ضغط وكأنها مستخرجة من بيانات ضغط ذو معدل سريان ثابت وهذه الطريقة يمكن تطبيقها على بيانات الإنتاج العادية.

وفي هذا البحث تم تطبيق الطرق التقليدية على بئر ذات بيانات ضغط من معدل سريان ثابت (يساوي صفر) ولنفس البئر تم تطبيق طريقة إزالة الالتفاف لبيانات من معدل سريان متغير (**drawdown period**) وتم الحصول على نتائج متقاربة من الطريقتين أي أنه يمكن الاستفادة من طريقة إزالة الالتفاف لأجراء عملية التحليل إذ أنها توفر الوقت والتكلفة.



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## Abbreviations and Nomenclatures

<b>Abbreviations</b>	
MDH	Miller-Dyeshutshinson
ETR	Early time region
MTR	Middle time region
LTR	Late time region
NOC	National Oil company
IOC	International Oil company
PTA	Pressure Transient Analysis

Nomenclatures		
$\rho$	Density	$ML^{-3}$
k	Permeability	$L^2$
$\phi$	Porosity	-
$\mu$	Viscosity	$ML^{-1}T^{-1}$
$\nabla$	Deloperator	-
C	compressibility	$M^{-1}LT^2$
V	volume	$L^3$
$c_l$	Liquid compressibility	$M^{-1}LT^2$
P	Pressure	$ML^{-1}T^{-2}$
q	Flow rate	$L^3T^{-1}$
$c_\phi$	Formation Compressibility	$M^{-1}LT^2$
$c_t$	Total Compressibility	$M^{-1}LT^2$
$k_o$	Effective oil permeability	$L^2$
$k_{ro}$	Relative oil permeability	-
$s_o$	Oil saturation	-
$c_o$	Oil Compressibility	$M^{-1}LT^2$

$s_w$	Water saturation	-
$c_w$	Water Compressibility	$M^{-1}LT^2$
R	Radius	L
C1	Integral constant	-
$p_{wf}$	Bottom hole flowing pressure	$ML^{-1}T^{-2}$
$r_w$	Well radius	L
S	Skin factor	-
$r_{we}$	Effective well radius	L
C	Wellbore storage coefficient	$M^{-2}L^4T^2$
$\Delta V_{wb}$	Liquid volume change in the wellbore	$L^3$
$A_a$	Annulus area	$L^2$
$OD_T$	Outside tubing diameter	L
$ID_c$	Inside casing diameter	L
$C_D$	Dimensionless wellbore storage coefficient	-
$P_i$	Initial reservoir pressure	$ML^{-1}T^{-2}$
$r_D$	Dimensionless time	-
$P_D$	Dimensionless pressure	-
B	Formation volume factor	-
T	Time	T
$t_p$	Buildup period time	T
M	Slope of straight line	-
$\Delta p'$	Pressure Derivative	$ML^{-1}T^{-2}$
$P_u$	Unite rate pressure	$ML^{-1}T^{-2}$

# **Chapter 1**



# Chapter One

## Introduction

### 1.1 Pressure transient testing

Detailed reservoir information is essential to the petroleum engineer in order to analyze the current behavior and future performance of the reservoir.

A transient test is essentially conducted by creating a pressure disturbance in the reservoir and recording the pressure response at the wellbore bottom-hole flowing pressure as a function of time.

Pressure transient analysis is a critical feature in both reservoir and fluid characterization.

The need to obtain accurate data in a full well test or a fluid analysis, is extremely important. The accuracy and reliability of the data is essentially dependent on three main factors. Those factors can be summarized as; the scope of the pressure and rate data, the quality of the pressure and rate data, and finally the method applied for the analysis of the pressure and rate data.

Pressure transient testing is designed to provide the engineer with a quantitative analysis of the reservoir properties. It has long been recognized that the pressure behavior of a reservoir following a rate change directly reflects the geometry and flow properties of the reservoir. Information available from a well test includes:

1. Effective permeability.
2. Formation damage or stimulation.

3. Flow barriers and fluid contacts.
4. Volumetric average reservoir pressure.
5. Drainage pore volume.
6. Detection, length, capacity of fractures.
7. Communication between wells.

**1.1.1 The pressure transient tests used in the petroleum industry include:**

1. Pressure drawdown.
2. Pressure buildup.
3. Drill Stem.
4. Injectivity.

**1.1.2 The most common types of pressure transient test are:**

1. Pressure Draw down test.
2. Pressure Build up test.

**1.1.2.1 Pressure Drawdown Test:**

A pressure drawdown test is simply a series of bottom-hole pressure measurements made during a period of flow at constant producing rate. Usually the well is shut-in prior to the flow test for a period of time sufficient to allow the pressure to equalize throughout the formation to reach static pressure.

The fundamental objectives of drawdown testing are to obtain the average permeability of the reservoir rock within the drainage area of the well and to assess the degree of damage or stimulation induced in the vicinity of the wellbore.

through drilling and completion practices. Other objectives are to determine the pore volume and to detect reservoir inhomogeneities within the drainage area of the well.

During flow at a constant rate of the pressure behavior of a well in an infinite-acting reservoir during the unsteady-state flow period basically, well test analysis deals with the interpretation of the wellbore pressure response to a given change in the flow rate.

### **1.1.2.2 Pressure Buildup Test:**

The use of pressure buildup data has provided the reservoir engineer with one more useful tool in the determination of reservoir behavior. Pressure buildup analysis describes the buildup in wellbore pressure with time after a well has been shut in.

One of the principal objectives of this analysis is to determine the static reservoir pressure without waiting weeks or months for the pressure in the entire reservoir to stabilize.

Because the buildup in wellbore pressure will generally follow some definite trend, it has been possible to extend the pressure buildup analysis to determine:

1. Effective reservoir permeability.
2. Extent of permeability damage around the wellbore.
3. Presence of faults and to some degree the distance to the faults.
4. Any interference between producing wells.

5. Limits of the reservoir where there is not a strong water drive or where the aquifer is no larger than the hydrocarbon reservoir.

Certainly none of this information will probably be available from any given analysis, and the degree of usefulness of any of this information will depend on the experience in the area and the amount of other information available for correlation purposes.

Pressure buildup testing requires shutting in a producing well. The most common and the simplest analysis techniques require that the well produce at a constant rate, either from startup or long enough to establish a stabilized pressure distribution, before shut-in.

## **1.2 Assumption of pressure buildup and drawdown analysis:**

In pressure buildup and drawdown analysis the following assumptions, with regard to the reservoir, fluid and flow behavior, are usually made:

### **1. Reservoir:**

- Homogeneous.
- Isotropic.
- Horizontal of uniform thickness.

### **2. Fluid:**

- Single phase.
- Slightly compressible.
- Constant oil viscosity and formation volume factor.

### 3. Flow:

- Laminar flow.
- No gravity effects.

#### 1.3 Wellbore Storage Effect :

Unfortunately, the producing rate is controlled at the surface, not at the sand face. Because of the wellbore volume, a constant surface flow rate does not ensure that the entire rate is being produced from the formation. This effect is due to wellbore storage.

Consider the case of a drawdown test, when the well is first open to flow after a shut-in period, the pressure in the wellbore drops. This drop in the wellbore pressure causes the following two types of wellbore storage:

- i. Wellbore storage effect caused by **fluid expansion**.
- ii. Wellbore storage effect caused by **changing fluid level** in the casing tubing annulus.

Until storage effects are over, the pressure response alone will contain no useful reservoir information.

The only way to avoid wellbore storage effects is to shut-in the well as close to the reservoir as possible using a downhole shut-in tool.

#### 1.4 Problem statement:

The measured long-term pressures which combined test and production data are both achieved under variable condition. In practice it is impossible to keep constant flow condition for obtaining transient pressure. So the practical

data set is variable pressure transient rate. While current theoretical (conventional) methods for transient pressure analysis (PTA) in well testing are based on constant rate or pressure solutions which means before the transient analysis, the transient pressure data need to be normalized to that due to either a constant rate or constant pressure form.

### **1.5 Objectives:**

1. Used Conventional method to estimate the well and reservoir parameters from the data of the buildup period.
2. Used Deconvolution method to estimate the well and reservoir parameters from the data that obtained during production period.
3. Comparing between results of conventional and deconvolution methods.

## **Chapter 2**

## Chapter Two

### Literature Review and Theoretical Background

This chapter presents the literature study on deconvolution method along with theories used to analysis pressure transient data.

#### 2.1 Literature review

**ZhengShiyi and Wang Fei (2008)** designed to test the performance of the pressure rate deconvolution algorithm, Data of the simulated production history, which includes two short buildups and three drawdown periods. Pressure-rate deconvolution has been implemented for the whole test period. The result is the unit-rate pressure response because of deconvolution. A match of the equivalent constant-rate-pressure appears response reconstructed from the deconvolution algorithm with the simulated constant-rate transient pressure. They are almost identical.

**Mustafa onur and Fikri J. Kuchuk(2012)** considered a simulated well-test case for a vertical well with wellbore-storage effects in a closed homogeneous, isotropic reservoir. The data set is consistent with the deconvolution model. The number of pressure data points is 458 and the total duration of the test is 300 hours. The agreement between the true and reconstructed unit-rate drawdown responses and the match of measured pressure-derivative data with the computed pressure-derivative data are excellent. These results show that the deconvolution method works well.



**J. A. Cumming and D. A. Wooff (2013)** applied the multiple-well deconvolution methodology consists of three wells each producing for different durations over 2000 hour period the results of deconvolution are show that the pressure match is excellent This indicates that the deconvolution solution reproduces the pressure data to a high degree of accuracy.

**V. Jaffreziec and A. C. Gringarten (2019)** applied the multiple-well deconvolution methodology sandstone dry gas reservoir developed with two horizontal wells with lengths of approximately 400 meters and located 1.6km apart. The production dataset spans 19800 hours (about 2 years and 3 months). The deconvolved derivatives for wells and the corresponding calculated pressure histories are compared to actual data. A very good match is achieved.

## 2.2 Theoretical Background

We will derive the linear hydraulic diffusivity equation, which is the fundamental equation in well test analysis. There are a number of important approximations and assumptions involved when deriving this equation. These assumptions includes:

1. Isothermal flow.
2. A single fluid phase.
3. Constant isotropic permeability.
4. Fluid viscosity independent of pressure.
5. Compressibility independent of pressure.
6. Low fluid compressibility.

### 2.2.1 The diffusivity equation:

The starting point for deriving the diffusivity equation is the continuity equation for single-phase flow, which is an expression of conservation of mass in a volume element:

**mass in – mass out = change in mass**

$$-\nabla \cdot (\rho q) = \frac{\partial}{\partial t} (\rho \phi) \dots\dots\dots(2-1)$$

Here the nabla symbol  $\nabla$  represents the Del operator  $\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$ ,  $\rho$  is the fluid density,  $\phi$  is the porosity, and  $q$  is the volumetric fluid flux.

The volumetric fluid flux is related to the gradient in pore pressure via Darcy's law:

$$q = -\frac{k}{\mu} \nabla p \dots\dots\dots (2-2)$$

Inserting (2-2) into (2-1), and assuming constant permeability,  $k$ , and pressure independent viscosity,  $\mu$ , we get:

$$\frac{k}{\mu} \nabla \cdot (\rho \nabla p) = \frac{\partial}{\partial t} (\rho \phi) \dots\dots\dots (2-3)$$

We may expand the derivatives of the product on both sides of Eq. (2-3), which gives:

$$(\nabla \rho \cdot \nabla p + \rho \nabla^2 p) = \rho \frac{\partial}{\partial t} \phi + \phi \frac{\partial}{\partial t} \dots\dots\dots (2-4)$$

First, investigate the left hand side of Eq. (2-4):

Compressibility  $c$  is a measure of the relative volume change as a response to a pressure change:

$$c = \frac{-1}{v} \frac{\partial v}{\partial p} \dots\dots\dots (2-5)$$

Where  $V$  is the volume and the liquid compressibility may be used to convert derivatives of density into derivatives of pressure:

$$c_l = \frac{1}{\rho} \frac{\partial \rho}{\partial p} \dots\dots\dots (2-6)$$

Then:

$$\frac{\partial \rho}{\partial p} = \rho c_l \dots\dots\dots (2-7)$$

Thus, we see that the first term on the left hand side of Eq. (2-4) is:

$$\nabla \rho \cdot \nabla p = \frac{\partial \rho}{\partial p} \nabla p \cdot \nabla p = \rho c_l |\nabla p|^2 \dots\dots\dots (2.8)$$

In addition, since this term is proportional to the compressibility it may be ignored in the low compressibility limit. The left hand side of Eq. (2-4) is then simply to:

$$\rho \frac{k}{\mu} \nabla^2 p \dots\dots\dots (2-9)$$

Moreover, the formation compressibility:

$$c_\phi = \frac{1}{\phi} \frac{\partial \phi}{\partial p} \dots\dots\dots (2-10)$$

Then:

$$\frac{\partial \phi}{\partial p} = \phi c_\phi \dots\dots\dots (2-11)$$

The right hand side of Eq. (2-4) can be expressed in terms of the time derivative of pressure by applying the chain rule:

$$\rho \frac{\partial}{\partial t} \phi + \phi \frac{\partial}{\partial t} \rho = \rho \frac{\partial \phi}{\partial p} \frac{\partial}{\partial t} p + \phi \frac{\partial \rho}{\partial p} \frac{\partial}{\partial t} p \dots\dots\dots (2-12)$$

Inserting (2-7) and (2-11) into (2-12):

$$\rho \frac{\partial}{\partial t} \phi + \phi \frac{\partial}{\partial t} \rho = \rho \phi (c_\phi + c_l) \frac{\partial}{\partial t} p = \rho \phi c_t \frac{\partial}{\partial t} p \dots\dots\dots (2-13)$$

Where:

$c_t$  = total compressibility in  $psi^{-1}$ .

Equating left hand side (2-9) and right hand side (2-13), and dividing by  $\rho \phi c_t$  we get:

$$\frac{k}{\mu c_t} \nabla^2 p = \frac{\partial}{\partial t} p \dots\dots\dots (2-$$

14)

This is the (hydraulic) diffusivity equation, which is the fundamental equation in well testing, and the quantity

$$\eta = \frac{k}{\mu c_t} \dots\dots\dots (2-15)$$

$\eta$  is called the (hydraulic) diffusivity.

Using the  $\eta$  notation for the diffusivity, we get the following simplified diffusivity equation:

$$\eta \nabla^2 p = \frac{\partial p}{\partial t} \dots\dots\dots (2-16)$$

### 2.2.1.1 Diffusivity equation in oil reservoirs

As derived, the equation is thus only valid for reservoirs that contain a single low-compressible fluid phase that is for water reservoirs. However, the validity of the equation may be extended to oil reservoirs at irreducible water saturation  $S_{wi}$ .

The irreducible water does not flow, but it influences the total compressibility, so the diffusivity equation is valid for oil reservoirs at  $S_{wi}$  provided the following:

Permeability is replaced by oil permeability:

$$k_o = k_{ro}(S_{wi})k$$

Viscosity is the oil viscosity  $\mu_o$

Total compressibility is defined as:

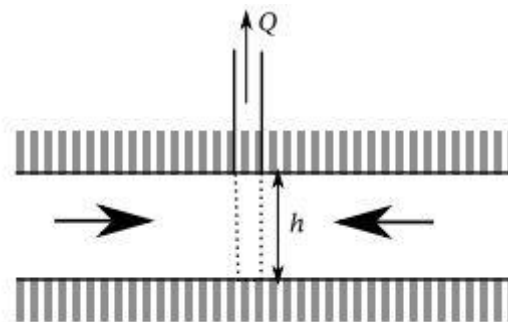
$$c_t = c_{\emptyset} + S_o c_o + S_w c_w$$

In this case, the equation takes the form

$$\frac{k_o}{\mu_o \phi c_t} \nabla^2 p = \frac{\partial p}{\partial t} \dots \dots \dots (2-17)$$

### 2.2.1.2 Vertical fully penetrating well – Radial flow

We will consider a vertical fully penetrating well in a reservoir of constant thickness, as illustrated in **Fig. (2.1)** In this case, it is natural to use cylinder coordinates.



**Figure (2.1):** Vertical fully penetrating well in a reservoir of constant thickness.

(Advanced Reservoir Engineering , Tarek Ahmed.)

The general form of the Laplace operator on the pressure field in cylinder coordinates is  $\nabla^2 p$

$$\nabla^2 p = \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \right] P \dots \dots \dots (2-18)$$

Where  $r$  is the radius,  $\theta$  is the angle, and  $z$  is the height.

For a fully penetrating well in an isotropic medium flow is independent of angle and height so that we have  $\frac{\partial}{\partial \theta} = 0$  and  $\frac{\partial}{\partial z} = 0$ . The diffusivity equation (2-16) is then

$$\eta \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{\partial p}{\partial t} \dots \dots \dots (2-19)$$

### 2.2.2 Steady state solution for Diffusivity equation

We will investigate the steady state solution of radial flow for **Eq.(2-19)**. Except for early times in a well test, this solution describes the pressure profile around a vertical fully penetrating well. Steady state means that the right hand side of is zero, so the steady state pressure profile can be found by solving:

$$\frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = 0 \dots\dots\dots (2-20)$$

We may integrate Eq. (2-20) and get:

$$\frac{\partial p}{\partial r} = C_1 \frac{1}{r} \dots\dots\dots (2-21)$$

Where:

$C_1$  is an integration constant. Furthermore, using integration by substitution to integrate both sides from the well radius  $r_w$  to  $r$ , we get:

$$p - p_{wf} = -C_1 \ln \left( \frac{r}{r_w} \right) \dots\dots\dots (2-22)$$

Where:

$p_{wf}$  = the well pressure in psi.

Darcy's law tells us that the volumetric flux is proportional to the pressure gradient:

$$q = -\frac{k}{\mu} \nabla p \dots\dots\dots (2-23)$$

The volumetric fluid flux  $q$  is defined as volumetric rate per area, so given the total downhole (reservoir) well production rate, we have

$$q = \frac{Q}{2\pi r h} \dots\dots\dots (2-24)$$

Where  $h$  is the perforation height (which in our case equals the height of the reservoir). The integration constant is determined by inserting the flux from **Eq. (2-24)** and the pressure derivative, from **Eq. (2-21)** into Darcy's law:

$$C_1 = \frac{Q\mu}{2\pi kh} \dots\dots\dots (2-25)$$

Which when inserted into **Eq. (2-23)** gives the general steady state solution for radial flow:

$$p = p_{wf} + \frac{Q\mu}{2\pi kh} \ln\left(\frac{r}{r_w}\right) \dots\dots\dots (2-26)$$

In general, the difference in pressure (pressure drop) from  $r_1$  to  $r_2$  is given by:

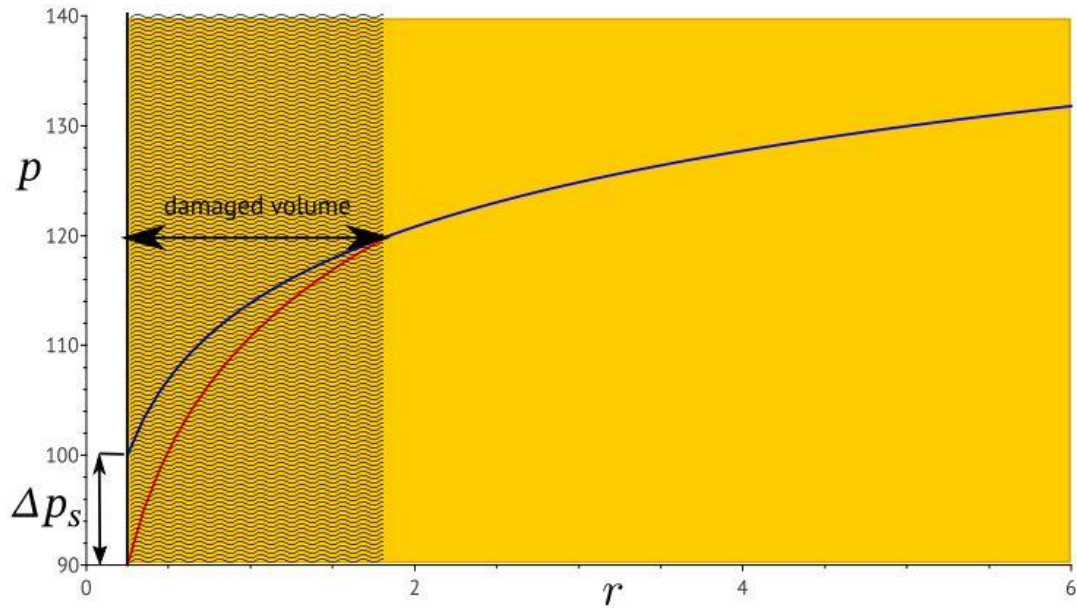
$$p(r_1) - p(r_2) = \frac{Q\mu}{2\pi kh} \ln\left(\frac{r_1}{r_2}\right) \dots\dots\dots (2-27)$$

### 2.2.3 Skin:

The formation volume close to the wellbore typically has altered properties compared to the surrounding reservoir. Of highest importance for well productivity is an altered permeability and the effect of this alteration on productivity is called skin. Skin is typically caused by formation damage because of drilling and production, but can also be the result of reduced mobility due to multiphase flow. Intentional improved permeability due to well treatments and hydraulic fracturing also contribute to skin, but the positive results of these treatments result in a negative skin in contrast to the normal positive skin due to formation damage.

As shown in **Fig. (2-2)** the effect of the skin is an additional pressure drop compared to a well without skin. The effect can be described quantitatively by the dimensionless skin factor,  $S$





**Figure (2.2):** Pressure profile around a well with skin. Actual pressure profile in red, ideal profile without skin in blue.

(Adel Salem (2011), Well testing)

$$\Delta P_s = \frac{Q\mu}{2\pi kh} S \dots\dots\dots (2-28)$$

Where  $s$  is called the skin factor and defined as:

$$S = \left[ \frac{k}{k_{skin}} - 1 \right] \ln \left( \frac{r_{skin}}{r_w} \right)$$

In addition, adding the extra pressure drop to the steady state solution (2-26) gives

$$p = p_{wf} + \frac{Q\mu}{2\pi kh} \left[ \ln \left( \frac{r}{r_w} \right) + S \right] \dots\dots\dots (2-29)$$

Typically, Positive Skin Factor,  $s > 0$  is considered as damaged zone near the wellbore Negative Skin Factor,  $s < 0$  is indicates an improved wellbore condition.

The equivalent wellbore radius can be expressed in terms of the skin factor:

$$r_{we} = r_w e^{-s} \dots\dots\dots (2-30)$$

## 2.2.4 Wellbore storage Coefficient

The effects of wellbore storage can be quantified in terms of the wellbore storage factor  $C$ , which is defined as:

$$C = \frac{\Delta V_{wb}}{\Delta p} \dots\dots\dots (2-31)$$

Where:

$C$  = wellbore storage volume in STB\psi.

$\Delta V_{wb}$  = change in the volume of fluid in the wellbore in STB.

The above relationship can be applied to mathematically represent the individual effect of wellbore fluid expansion and falling (or rising) fluid Level, to give:

### 2.2.4.1 Wellbore Storage Effect Due to Fluid Expansion

$$C = V_{wb} c_{wb} \dots\dots\dots (2-32)$$

Where:

$V_{wb}$  = total wellbore fluid volume in  $ft^3$ .

$c_{wb}$  = average compressibility of fluid in the wellbore in  $\psi^{-1}$ .

### 2.2.4.2 Wellbore Storage Effect Due to Changing Fluid Level

If  $A_a$  is the cross-sectional area of the annulus ( $ft^2$ ), and  $\rho$  is the average ( $lb_m/ft^3$ ) fluid density in the wellbore, the wellbore storage coefficient is given by :

$$C = \frac{144A_a}{5.615\rho} \dots\dots\dots (2-33)$$

With:

$$A_a = \frac{\pi[(ID_C)^2 - (OD_T)^2]}{4(144)} \dots\dots\dots (2-34)$$

Where:

$OD_T$  = outside diameter of the production tubing in inch.

$ID_C$  = inside diameter of the casing in inch.

To determine the duration of the wellbore storage effect, it is convenient to express the wellbore storage factor in a dimensionless form as :

$$C_D = \frac{0.894C}{\phi h c_t r_w^2} \dots\dots\dots (2-35)$$

Where:

$C_D$  = dimensionless wellbore storage factor.

$C$  = wellbore storage factor in STB\psi.

$c_t$  = total compressibility coefficient in  $\text{psi}^{-1}$ .

$r_w$  = wellbore radius in ft.

$h$  = thickness in ft.

### 2.2.5 Dimensionless analysis

Well test analysis often makes use of the concept of the dimensionless variables in solving the diffusivity equation. The importance of dimensionless variables is that they simplify the diffusivity equation and its solution by combining the reservoir parameters (such as permeability, porosity, etc.) and thereby reduce the total number of unknowns.

To introduce the concept of the dimensionless pressure drop solution, consider the Darcy equation for radial flow at any radius of the reservoir:

$$Q = \frac{2\pi kh[p_i - p(r,t)]}{\mu \ln\left(\frac{r}{r_w}\right)} \dots\dots\dots (2-36)$$

Rearrange the above equation to give:

$$\frac{[p_i - p(r, t)]}{\left(\frac{Q\mu}{2\pi kh}\right)} = \ln\left(\frac{r}{r_w}\right) \dots\dots\dots (2-37)$$

It is obvious that the right hand side of the above equation has no units (dimensionless) and, accordingly, the left-hand side must be dimensionless. Since the left-hand side is dimensionless, and  $[p_i - p(r, t)]$  has the units of psi, it follows that the term  $\left(\frac{Q\mu}{2\pi kh}\right)$  has units of pressure. Any pressure difference divided by  $\left(\frac{Q\mu}{2\pi kh}\right)$  is a dimensionless pressure. Therefore, Equation can be written in a dimensionless form as:

$$P_D = \ln(r_D) \dots\dots\dots (2-38)$$

Where

$$P_D = \frac{[p_i - p(r, t)]}{\left(\frac{Q\mu}{2\pi kh}\right)}$$

and

$$r_D = \frac{r}{r_w}$$

In transient pressure analysis, the dimensionless pressure is always a function of dimensionless time that is defined by the following expression:

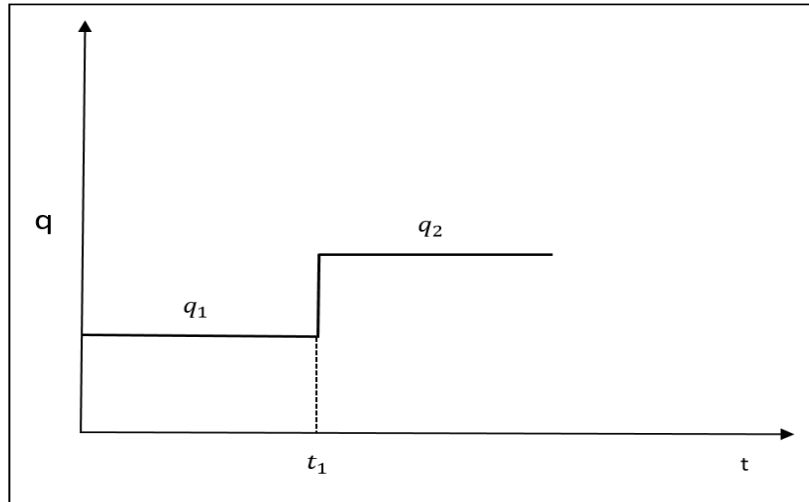
$$t_D = \frac{kt}{\phi\mu c_t r_w^2} \dots\dots\dots (2-39)$$

The above dimensionless groups ( $P_D, t_D$  and  $r_D$ ) can be introduced into the diffusivity equation **Eq.(2-19)** to transform the equation into the following dimensionless form:

$$\eta \frac{1}{r_D} \frac{\partial}{\partial r_D} \left( r_D \frac{\partial P_D}{\partial r_D} \right) = \frac{\partial}{\partial t_D} P_D \dots\dots\dots (2-40)$$

## 2.2.6 Principle of super-position

The pressure variations due to several flow rates are equal to the some of the pressure drops due to each of the different flow rates. This property is called superposition.



**Figure (2.3)** Diagram for two flow rate.

(John Lee, Well Testing)

If  $p_i - p(t) = \frac{q\beta\mu}{2\pi kh} p_D(t)$  is the pressure drop due to a flow rate  $q$ , beginning at time,  $t = 0$ .

The diagram shown in **Fig.(2.3)** can be considered as the sum of:

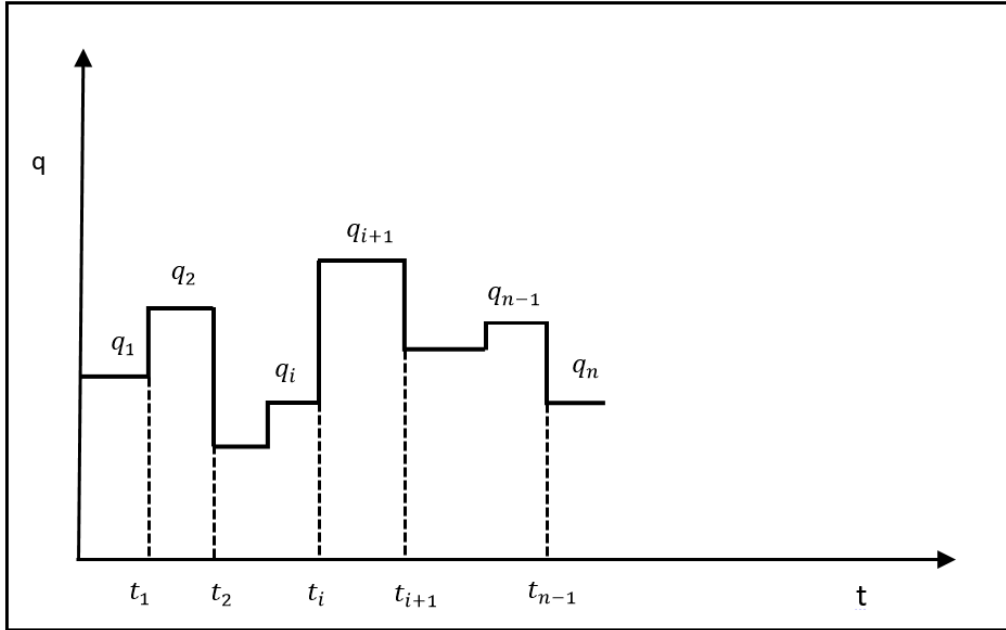
A production of a flow rate  $q_1$  since  $t = 0$ , and a production at flow rate  $(q_2 - q_1)$  since  $t = t_1$ .

A pressure variation due to the flow rates is equal to

$$p_i - p(t) = \frac{q_1\beta\mu}{2\pi kh} p_D(t) + \frac{(q_2 - q_1)\beta\mu}{2\pi kh} p_D(t - t_1) \dots\dots\dots (2-$$

41)

In case of Multi rate testing



**Figure (2.4)** Multi rate testing

(John Lee, Well Testing)

$$p_i - p(t) = \frac{q_1 \beta \mu}{2\pi k h} \sum_{i=1}^n (q_i - q_{i-1}) p_D(t - t_{i-1}) \dots\dots\dots (2-42)$$

With  $q_0 = 0$  and  $t_0 = 0$ .

## 2.3 Pressure transient analysis methods

### 2.3.1 Introduction

Due to the transient nature of a pressure front moving through the reservoir, the different classes of obtained reservoir properties are intrinsically linked to the time after the change in well rates. The different reservoir properties are organized according to time in Table (2-1).

**Table (2-1):** Time of measurement versus type of measurements

Early time	Middle time	Late time
Near wellbore	Reservoir	Reservoir boundaries
Skin. Wellbore storage. Fracture.	Permeability. Heterogeneity. Dual porosity. Dual permeability.	Reservoir volume. Faults (sealing / none-sealing). Pressure Boundaries.

--	--	--

These properties can be calculated by several methods. These methods are:

1. Conventional Analysis (Semi-log analysis) method.
2. Type Curve matching method.
3. Derivative method.
4. Deconvolution method.

### **2.3.2 Conventional Analysis (Semi-log analysis) method:**

#### **2.3.2.1 Horner analysis**

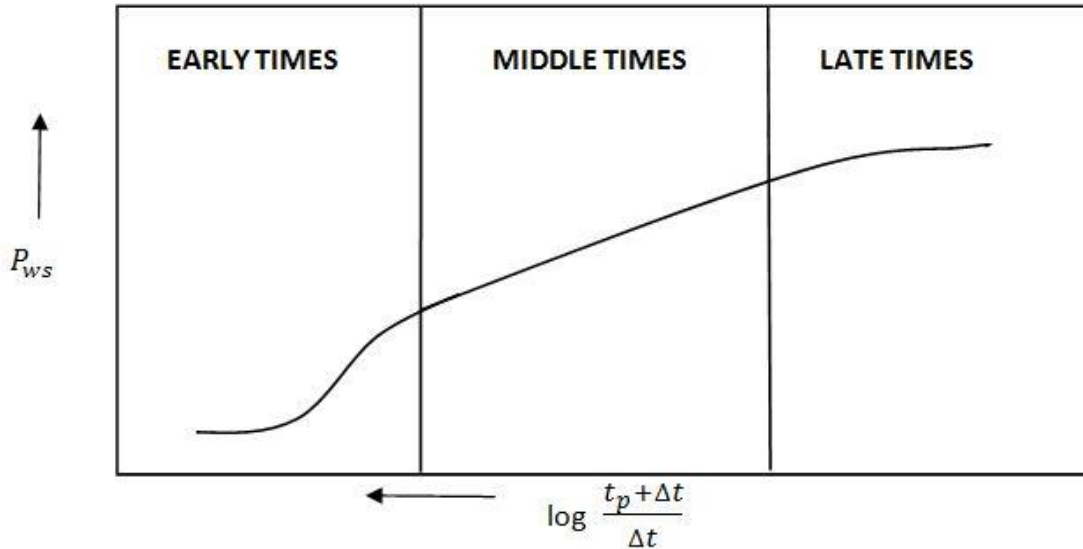
##### **2.3.2.1.1 Assumption**

- 1- Single flow rate.
- 2- Constant flow rate start and Zero.
- 3- Radial Flow.

##### **2.3.2.1.2 Derivation**

We will derive the well pressure for a well test where both the production time  $t_p$  and buildup time  $\Delta t$  is short enough to employ the transient infinitely acting solution. The expression for well pressure during buildup will contain two terms: One for a well with constant rate  $Q$  starting at time  $t = 0$  and one for a well with constant rate  $-Q$  starting at time  $t = t_p$ . Apart from the period dominated by wellbore storage and skin effects just after start of shut-in.

When we test a well, we obtain a curve with complicated shape instead of single straight line for all time. Based on the concept of radius of investigation we can divide a buildup curve into three region as shown in **Fig.(2.5)**.



**Figure (2.5) :Buildup curve region**

(John Lee, Well Testing)

$$p_i - p_{ws} = (Q)(p_i - p_{wf}) + (-Q)(p_i - p_{wf}) \dots\dots\dots (2-43)$$

$$p_{ws} =$$

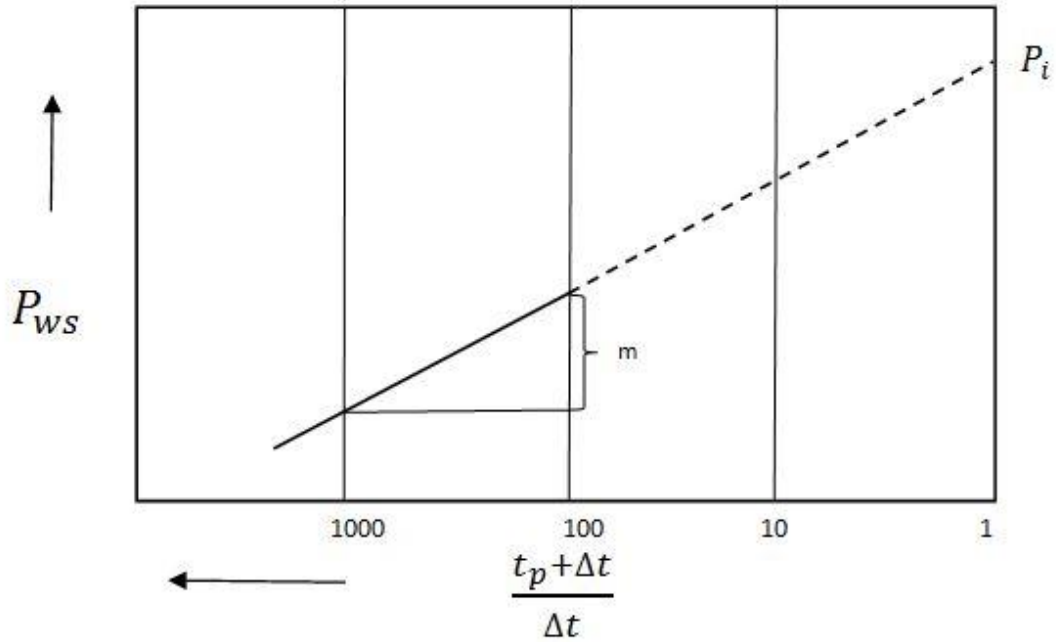
$$p_i - \frac{162.6QB_o\mu}{kh} \left[ \log \left( \frac{k(t_p + \Delta t)}{\phi\mu c_t r_w^2} \right) - 3.23 + 0.87S \right] - \frac{162.6Q_oB_o\mu}{kh} \left[ \log \left( \frac{kt_p}{\phi\mu c_t r_w^2} \right) - 3.23 + 0.87S \right] \dots\dots\dots (2-44)$$

Expanding this equation and canceling terms,

$$p_{ws} = p_i - \frac{162.6QB_o\mu}{kh} \left[ \log \left( \frac{k(t_p + \Delta t)}{\Delta t} \right) \right] \dots\dots\dots (2-45)$$

The pressure buildup equation was introduced by Horner (1951) and is commonly referred to as the Horner equation. **Eq.(2-45)** suggests that a plot of  $p_{ws}$  versus  $\frac{(t_p + \Delta t)}{\Delta t}$  would produce a straight-line relationship with intercept  $p_i$  and slope of  $-m$  **Fig.(2.6)**.





**Figure (2.6).**:Horner plot concept

(Adel Salem (2011), Well testing)

Where:

$$m = - \frac{162.6 Q_o B_o \mu}{kh}$$

Which gives the permeability:

$$k = 162.6 \frac{q\beta\mu}{mh} \dots\dots\dots(2-46)$$

When the permeability and pressure at start of shut in is known, the skin factor may be estimated based on the pressure at a specific time  $\Delta t$  after shut in.

$$S = 1.151 \left[ \frac{p_{1hr} - p_{wf}(\Delta t=0)}{m} - \log \left( \frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right] \dots\dots\dots (2-47)$$

The value of  $p_{1hr}$  must be taken from the Horner straight line. Typically, the straight line extrapolated pressure at  $\frac{(t_p + \Delta t)}{\Delta t} = 0$ .

### 2.3.2.2 MDH Analysis:

This method is called the Miller-Dyes Hutchinson or MDH plot. We will describe how the reservoir permeability and the skin factor of the well can be found by analyzing the well test data.

#### 2.3.2.2.1 Assumption

1. Infinite acting flow at shut-in.
2. The stable production period is very long.
4. Constant flow rate.
3. Radial flow.

#### 2.3.2.2.2 Derivation

The pressure response in an ideal drawdown test can be found using:

$$p_{wf} = p_i - \frac{162.6Q_oB_o\mu}{kh} \left[ \log \left( \frac{kt}{\phi\mu c_t r_w^2} \right) - 3.23 + 0.87S \right] \dots\dots\dots(2-48)$$

Rearranging this equation:

$$p_{wf} = p_i - \frac{162.6Q_oB_o\mu}{kh} \dots\dots\dots (2-49)$$

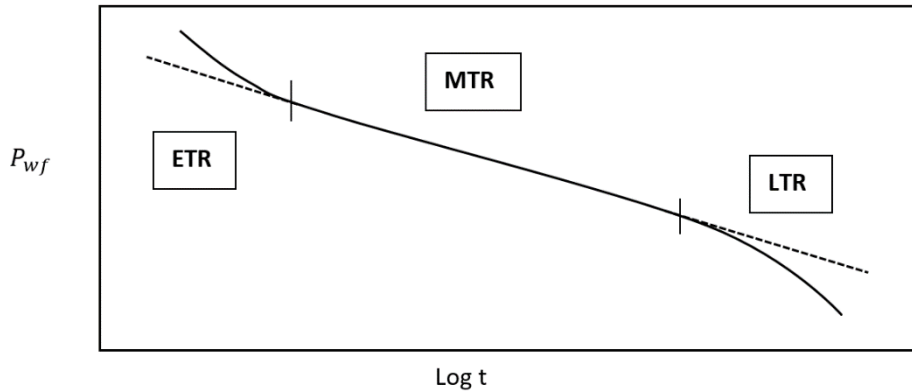
$$p_{wf} = m \log(t) + c \dots\dots\dots (2-50)$$

where

$$m = - \frac{162.6Q_oB_o\mu}{kh}$$

And  $C$  are constants independent of time. We see from **Eq. (2-**

**49)** that if we plot the well pressure as a function of  $\log(t)$  (semi-log plot), we should see a straight line with slope  $m$  at MTR **Fig(2.7)**.



**Figure (2.7) :MDH curve region**

(John Lee, Well Testing)

Permeability can be estimated based on the slope, of the line,  $m$ .

$$k = 162.6 \frac{q\beta\mu}{mh} \dots\dots\dots (2-51)$$

Once the permeability is known, the skin can be estimated from the fitted straight-line intercept or value on the straight line at any other point in time of  $p_{1hr}$  that can be found on the extension of the straight line at  $\log t$  (1 hr)

$$S = 1.151 \left[ \frac{p_i - p_{1hr}}{m} - \log \left( \frac{k}{\phi\mu c_t r_w^2} \right) + 3.23 \right] \dots\dots\dots (2-52)$$

In addition, reservoir pore volume can be estimated by plotting bottom hole pressure vs. time on ordinary Cartesian graph then:

$$V = \frac{-0.234q\beta}{c_t \left( \frac{\partial p_{wf}}{\partial t} \right)} \dots\dots\dots (2-53)$$

### 2.3.3 Type Curve matching method:

A type curve is a graphical representation of the theoretical solutions to flow equations. The type curve analysis consist of finding the theoretical type curve that “matches” the actual response from a test well and the reservoir when subjected to changes in production rates or pressure. The match can be found

graphically by physically superposing a graph of actual test data with a similar graph of type curves and searching for a type curve that provides the best match.

They are presented in terms of dimensionless variables rather than real variables. The reservoir and well parameters such as permeability and skin can then be calculated from the dimensionless parameters defining that type curve.

Type curves printed on tracing or translucent paper can be placed over log-log data plotted to the same graph size. Comparisons between the shapes of the actual data and the type curve can help to identify the reservoir model.

This process can now be done electronically using commercially available well test analysis software.

### 2.3.3.1 Concepts of type curves matching:

By properly selecting the plotting parameters, the effect of all these variables on pressure can be presented in compact form. Most dimensionless variables used in well testing arise logically from the equations which describe reservoir fluid flow.

$$\log P_D = \log [P_i - P_{wf}] + \log \frac{kh}{141.2q\mu\beta} \dots\dots\dots (2-54)$$

$$\log t_D = \log t + \log \frac{0.0002637k}{\phi\mu C_t r_w^2} \dots\dots\dots (2-55)$$

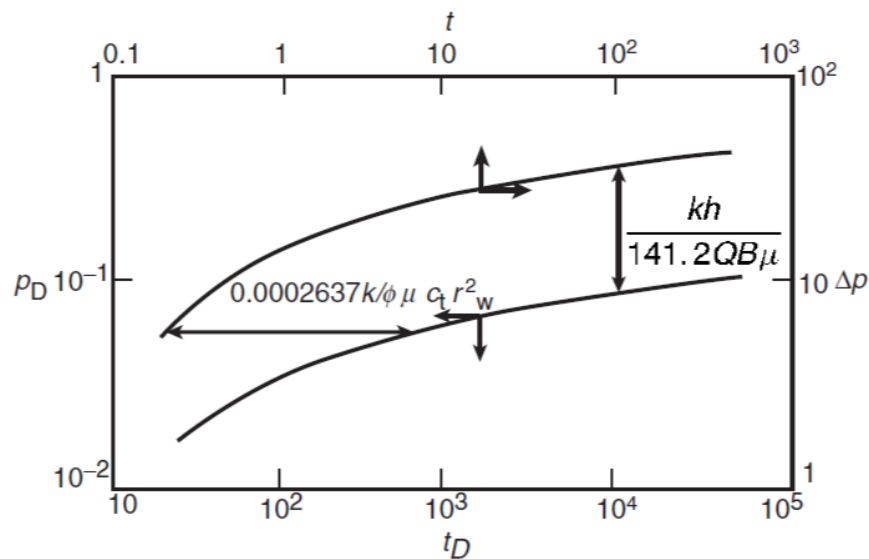
Hence, a graph of  $\log(\Delta p)$  vs.  $\log(t)$  will have an identical shape to the graph of  $\log(P_D)$  vs.  $\log(t_D)$  although the curve will be shifted by  $\log \left[ \frac{kh}{141.2q\mu\beta} \right]$  vertically in pressure and  $\log \left[ \frac{0.0002637k}{\phi\mu C_t r_w^2} \right]$  horizontally in time.

Not only these curves have same shape , but also if they are moved relative to each other until they match, the vertical and horizontal displacement required to achieve the match are related the constants in previous equation.

In particular, the vertical displacement required to achieve the match is related to the value of  $\frac{kh}{141.2q\mu\beta}$  further, the required horizontal displacement is related to the value of  $\frac{0.0002637k}{\phi\mu c_t r_w^2}$  , **Fig(2.8)**.

Once these constant are known, it is possible to determine reservoir properties such as k,and  $\phi$ .

This process of matching two curves through the vertical and horizontal displacement and determine the reservoir or well properties is called type curve matching.



**Figure (2.8)** vertical and horizontal displacement

(Adel Salem (2011), Well testing)

Many well test models have been established and many type-curves have been published since 1970s. The most commonly used type-curves in the world are:

### 2.3.3.2 Ramey type curves:

Fundamentally, a type curve is a pre-plotted family of pressure drawdown curves. The most fundamental of these curves is a plot of dimensionless pressure change,  $P_D$  vs. dimensionless time change  $t_D$ .

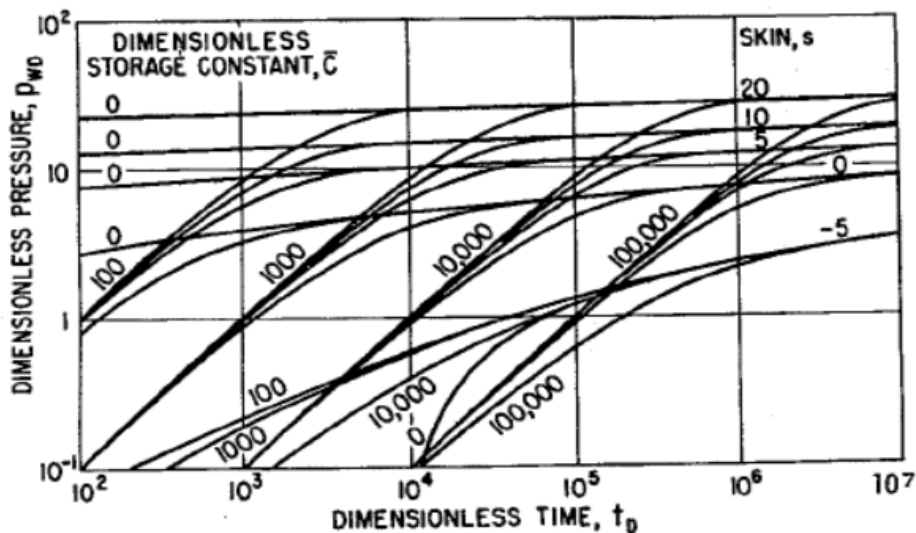
This curve has two parameters that distinguish the curves from one another: the skin factor,  $S$ , and a dimensionless wellbore storage constant  $C_{SD}$ .

$$P_D = \frac{0.00708kh}{q\mu\beta} [P_i - P_{wf}] \dots\dots\dots (2-56)$$

$$t_D = \frac{0.0002637kt}{\phi\mu C_t r_w^2} \dots\dots\dots (2-57)$$

It is presented in the **Fig.(2.9)** each curve in this graph represents the pressure transient data for a specific magnitude of formation skin and wellbore storage (WBS).

The magnitude of WBS is expressed by the dimensionless wellbore storage coefficient  $C_D$  was defined by **Eq. (2-35)**



**Figure (2.9):** Ramey Type Curve.

(Adel Salem (2011), Well testing)

Ramey type curve can be used to directly determine permeability and skin. Each of the curves in the graph represents the theoretical pressure behavior, which would be expected if the pressure drawdown test was run in a reservoir having the indicated skin a wellbore storage.

By applying matching techniques and getting the matching point which is:

$$(P_D)_M \text{ and } (\Delta P)_M \& (t)_M \text{ and } (t_D)_M$$

The following parameters can be calculate from this method:

Skin S read directly from the curve after matching.

WBS from log-log plot

$$C = \frac{q\beta t}{24(P_i - P_{wf})} \dots\dots\dots (2-58)$$

Dimensionless WBS from **Eq. (2-35)**

Permeability

$$k = \frac{141.2q\mu\beta (P_D)_M}{h (\Delta P)_M} \dots\dots\dots (2-59)$$

Porosity

$$\phi = \frac{0.0002637k (t)_M}{\mu C_t r_w^2 (t_D)_M} \dots\dots\dots (2-60)$$

### 2.3.3.3 Gringarten type curve matching:

By comparison the concluded two equations **Eq. (2-54)** and

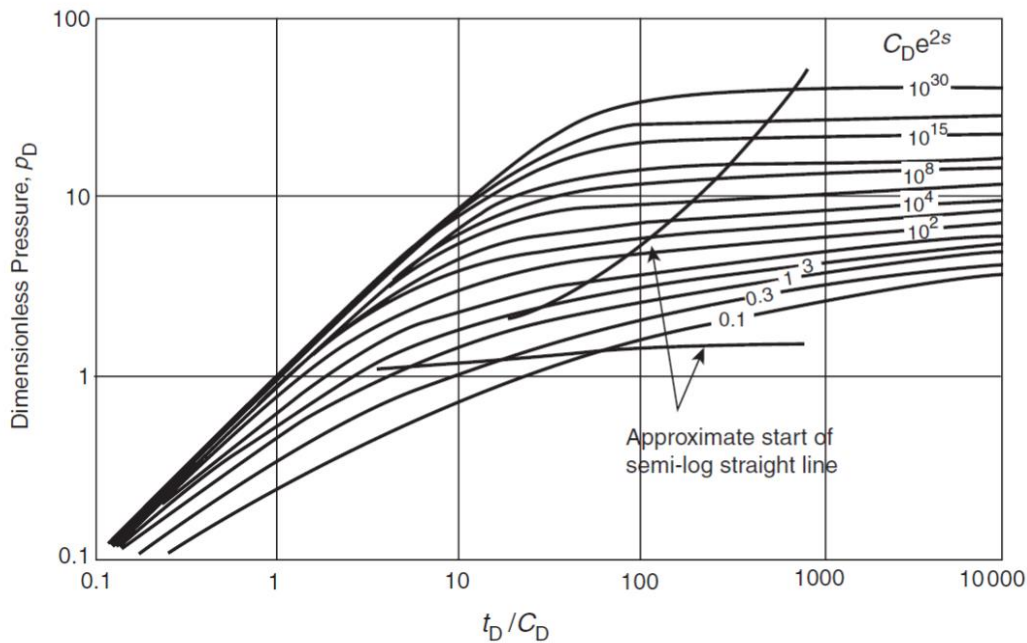
$$\log\left(\frac{t_D}{r_D^2}\right) = \log t + \log \frac{0.0002637k}{\phi \mu C_t r_w^2} \dots\dots\dots (2-61)$$

They indicate that a graph of  $\log(\Delta p)$  vs.  $\log(t)$  will have an identical shape to the graph of  $\log(P_D)$  vs.  $\log\left(\frac{t_D}{r_D^2}\right)$  although the curve will be shifted by  $\log\left[\frac{kh}{141.2q\mu\beta}\right]$  vertically in pressure and  $\log\left[\frac{0.0002637k}{\phi \mu C_t r_w^2}\right]$  horizontally in time.

When these two curves are moved relative to each other until they match the vertical and horizontal movements in mathematical term are given respectively by:

$$\left(\frac{P_D}{\Delta p}\right)_{MP} = \frac{kh}{141.2q\mu\beta} \text{ And } \left(\frac{t_D/r_D^2}{t}\right)_{MP} = \frac{0.0002637k}{\phi\mu C_t r_w^2}$$

The log-log type curve introduced by Gringarten shown in **Fig(2.10)** is based on presenting  $P_D$  as a function of  $t_D$  for specific values of  $C_D$  and  $S$ . This form of the type curve implies that the matching process should result in the same value of the permeability  $k$  from the pressure and time matches.



**Figure (2.10):**Gringarten Type Curve.

(Adel Salem (2011), Well testing)



## 2.3.4 Derivative method:

### 2.3.4.1 Definition:

The pressure derivative is essentially the rate of change of pressure with respect to the superposition time function.

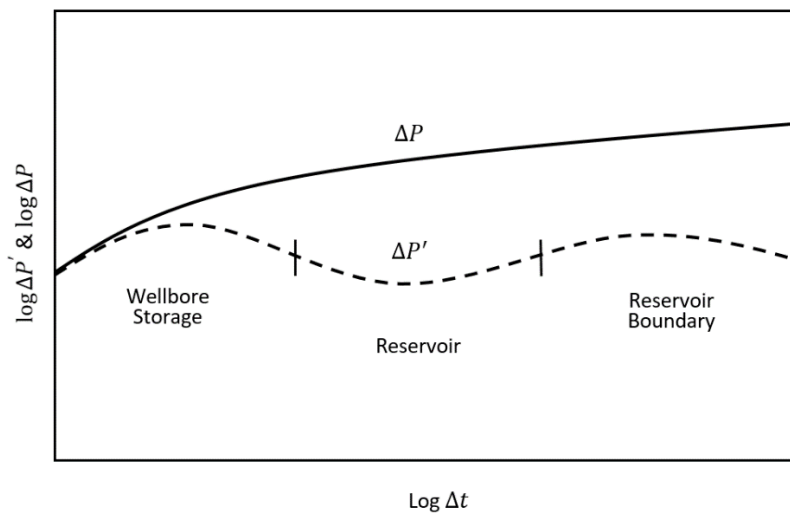
$$\Delta P' = \frac{d\Delta P}{d \ln \Delta t} = \Delta t \frac{d\Delta P}{d\Delta t} \dots\dots\dots (2-62)$$

Where:

$\Delta P'$  is pressure derivative.

$\Delta t$  is time measured from start of the transient test.

As pressure analysis,  $\Delta P'$  and  $\Delta P$  are plotted on log-log coordinates versus  $\Delta t$  as shown in **Fig(2.11)**.



**Figure (2.11):** log-log diagnostic plot.

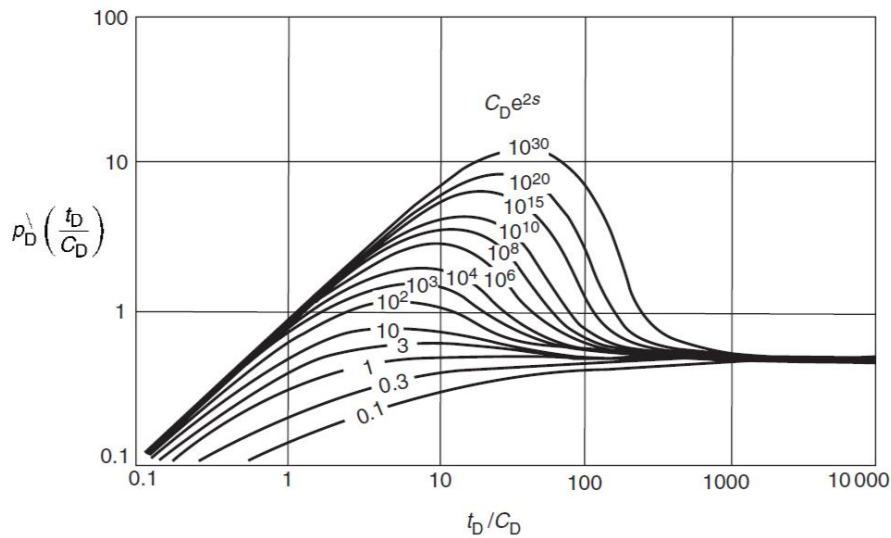
(Advance Reservoir Engineering, Tarek Ahmed)

### 2.3.4.2 Derivative type curve:

The pressure derivative is defined in term of  $P_D$  with respect to  $t_D/C_D$  as:

$$P'_D = \frac{d(P_D)}{d(t_D/C_D)} \dots \dots \dots (2-63)$$

The Gringarten type curve has been re-plotted in term of  $P'_D(t_D/C_D)$  vs.  $t_D/C_D$  in log-log scale as shown in **Fig.(2.12)**.



**Figure (2.12):** Pressure Derivative Gringarten type curve.

(Adel Salem (2011), Well testing)

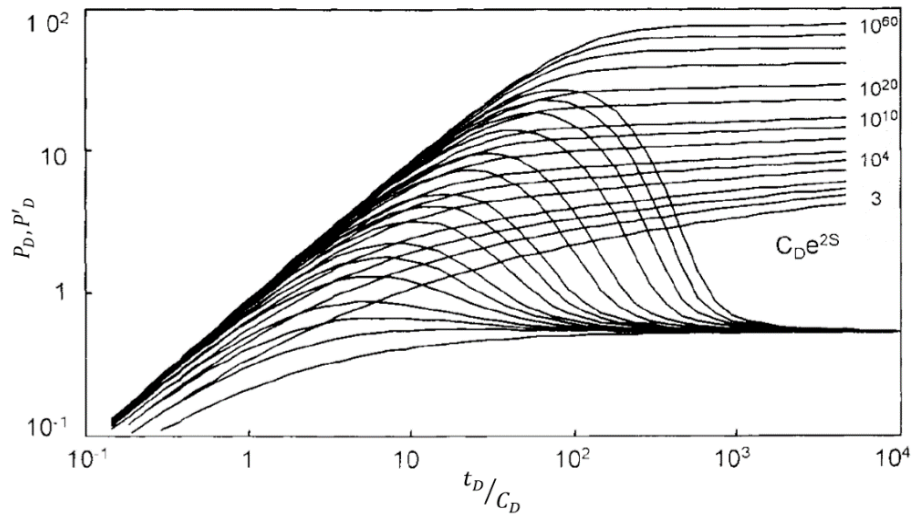
### 2.3.4.3 Combined type curve:

The main difficulty with the basic type curve procedure is uncertainty in the pressure match.

Thus, Gringarten's pressure type-curves and typical data curves (Bourdet's pressure derivative type-curves) are usually joined together and form a kind of compound type-curves.

Since the logarithmic derivative has the same units as pressure, we can graph the pressure and pressure derivative responses on a single scale.

The results of combining the two sets of type curves on the same graph is shown in **Fig.(2.13)**.



**Figure (2.13):** Combined type curve

(Modern Well Test Analysis, Roland N.Horne)

For each dimensionless pressure curve, referring to a particular value of the parameter  $C_D e^{2S}$ , there is a corresponding derivative curve of characteristic shape. At very early time, the pressure and its logarithmic derivative overlay on the unit slope diagonal (self-similarity). The beginning of the middle time region (MTR) is clearly indicated by the derivative becoming constant. in the case of wells with a positive skin The pressure derivative rises to a maximum and then falls sharply before flattening out for the MTR.

In case of wells with Negative skin The pressure derivative approaches a horizontal line.

The parameter for both pressure and derivative curves is  $C_D e^{2S}$  and the larger the value of  $S$  the greater the separation between a pressure response and the concomitant derivative response.

After matching, the permeability  $k$  can be calculated from pressure match using

**Eq. (2-59)**

And Calculate  $C_D$  from Time Match

$$C_D = \frac{0.0002637k}{\phi\mu c_t r_w^2} \left( \frac{t}{t_D/C_D} \right)_{M.P.} \dots\dots\dots (2-64)$$

Then Calculate  $S$  using  $C_D$  and value of  $C_D e^{2s}$

$$s = \frac{1}{2} \ln \left( \frac{C_D e^{2s}}{C_D} \right) \dots\dots\dots (2-65)$$

# **Chapter 3**

# **Chapter Three**

## **Methodology and General Procedures**

This chapter presents the deconvolution method concept and the steps and procedures that have been followed for estimating reservoir parameters by conventional and deconvolution method using saphir software. Before explain this process it is necessary to briefly explain saphir software and deconvolution method.

### **3.1 Deconvolution:**

Deconvolution is fundamentally a mathematical algorithm used to convert variable rate data to constant rate data .in other words it can be used to convert data into a single drawdown with constant rate this conversion yields to simplified analysis by allowing more of the same data to be analyzed. As a result of this approach attaining define conclusions about reservoir propertiesbecomes possible prior to this unclear and uncertain conclusions were often made deconvolution can be utilized on short build ups, which are often already conducted for well maintenance, leading to both clearer results and significant cost saving.

Essentially, the deconvolution tool allows you to extract more information about the reservoir from the same data. Despite this huge advantage, many welltest interpretations are still conducted without the use of advanced deconvolution software, thus limiting the value and efficiency of the data. Due to the importance of extracting good data from well test analysis, it is

fundamental that the use of deconvolution be emphasized as a means of improving the quality of pressure transient analysis where possible.

### 3.1.1 Assumptions of deconvolution:

- The basic assumption of all deconvolution techniques is the consistency of measured pressure and rate data with the linear Duhamel's model, which is based on the principle of superposition.
- The linearity of the system suggests only one well creating pressure perturbation to initially equalized region and static character of the parameters of reservoir and well.

### 3.1.2 Mathematical theory: Convolution and Deconvolution:

In the reservoir system the equation of the deconvolution is given by Duhamel's Integral which is function of time, where the pressure drop across the reservoir is the convolution product of rate and reservoir response.

$$\Delta p = p_i - p(t) = \int_0^t q(\tau) g(t - \tau) d\tau \dots\dots\dots (3-1)$$

where  $q(t)$  and  $p(t)$  are the measured flow rate and pressure at any place in the wellbore up to the wellhead, and  $p_i$  is the initial reservoir pressure. In this equation  $g$  is referred to as the impulse response of the reservoir system.

Therefore, in order to estimate the reservoir system response, the inversion of this convolution integral must be obtained and this mathematical problem is called deconvolution.

$$g(t) = dp_u/dt \dots\dots\dots (3-2)$$

$p_u$  is the unit rate pressure response of the reservoir system.

For the single well:

$$p(t) = p_i - \int_0^t q(t) \frac{dp_u(t - \tau)}{d\tau} d\tau$$

### 3.1.3 limitations of deconvolution:

- Deconvolution sets some limitations that, we cannot apply deconvolution if there is some interference from other wells nearby.
- The requirement for linearity of the system is the single-phase flow, which means that the downhole pressure for deconvolution must be higher than the reservoir bubble point pressure in oil reservoir.
- The initial uniformity of pressure, within the whole investigated part of reservoir and the well rate from all the production by this well, all the way from initial equilibrium state must be satisfied.
- We cannot apply deconvolution if there is aquifer or gas cap influence.

### 3.2 Saphir:

Saphir is the industry standard PTA software used by nearly all major IOC's, NOC's, Independents and Service Companies.

It's simple user interface and workflow allows for fast training and self-learning for occasional users.

For the advanced user, it offers a unique combination of analysis tools, analytical models and numerical models, which can connect to other dynamic data application for full field history matching.

Saphir can load an unlimited number of gauges, rates, pressure and other data in almost any format including ASCII, Excel.



### **3.2.1 Methodology for Deconvolution on Saphir:**

Pressure transient behavior analysis usually initiates with the investigation of the test data on different analysis, derivative, superposition, or Cartesian Plots. From these plots, a united and more recognizable picture can be built which will enable us to understand the main features of the test transient pressure behavior. Deconvolution will be used in several cases to analyze the test data, through Saphir NL. It offers a unique combination of analysis tools, analytical models and numerical models. Different cases will be studied, and for each case we will have different inputs.

According to the workflow of the software, first the rates and the durations are entered, followed by the creation of the test design. After that, the history plots and the pressure derivatives will be extracted. Then, for each case, deconvolution will be applied, and the results will be studied. In case it does work, we will take a look at the extra benefits that it provides with respect to the normal conventional way. In case it doesn't work, the main reason behind its malfunction will be stated, and the solution, in case of any, will be demonstrated, with some recommendations given.

Two deconvolution algorithms are implemented into the software and can be selected by the user as options:

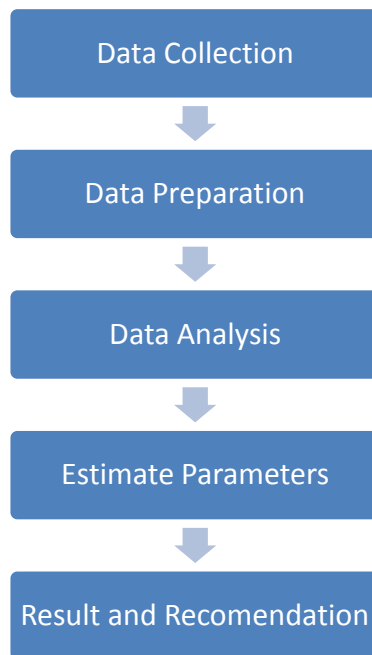
1- Deconvolution on one reference period and the end of the other periods: this method allows the definition of a convergence time on the log-log plot beyond which the derivatives exhibit a similar response. This results in a single

deconvolution response, matching with only a reference period prior to this convergence time and on all the extracted periods after the convergence time.

2. Separate deconvolutions with common  $P_i$  (Levitan et al): It is possible to perform a deconvolution for a single buildup if we combine the shut-in data with a known value of initial pressure. The idea of Levitan method is to perform one deconvolution for each buildup with a common value of initial pressure. If this value of initial pressure is correct, the late time behavior of all the buildups should be coherent. ,

### 3.3 Project Procedures:

A simple process chart was created and followed during this research as in **Fig.(3-1)**.



**Figure (3-1):** Procedures process chart.

### 3.3.1 Data Collection:

#### 3.3.1.1 Case study one:

The well (R1) located in Abu Gabra formation, three zones of the well had been tested between the periods 31-03-2015 to 28-04-2015.

#### 3.3.1.2 Case studytwo:

The well (R2)located in Abu Gabra formation,The well had been tested between the periods 01-04-2008 to 04-04-2008.

#### 3.3.1.3 Case study three:

The well (R3) located in Abu Gabra formation,The well had been tested between the periods 07-02-2013 to 14-02-2013.

The test measure the change in pressure and flow rate as function in time and this ourdata for estimation the characteristics.

### 3.3.2 Input data:

The following parameters are input data that can be obtained from laboratoryexperiments, PVT analysis or well logging.

#### 3.3.2.1 Casestudyone:

The required input data are summarized in the table (3-1):

**Table (3-1):** main input data (case one)

<b>Parameters</b>	<b>Value</b>	<b>Units</b>
$\beta$	1.24	bbl/STB
$\mu$	11.5	cp
$c_t$	12E-06	$psi^{-1}$

$r_w$	0.583	ft
h	9	m
$\emptyset$	19	%
$s_w$	56	%

The saturation and porosity can be obtained from the well logging interpretations as shown in **fig.(3-2)**.

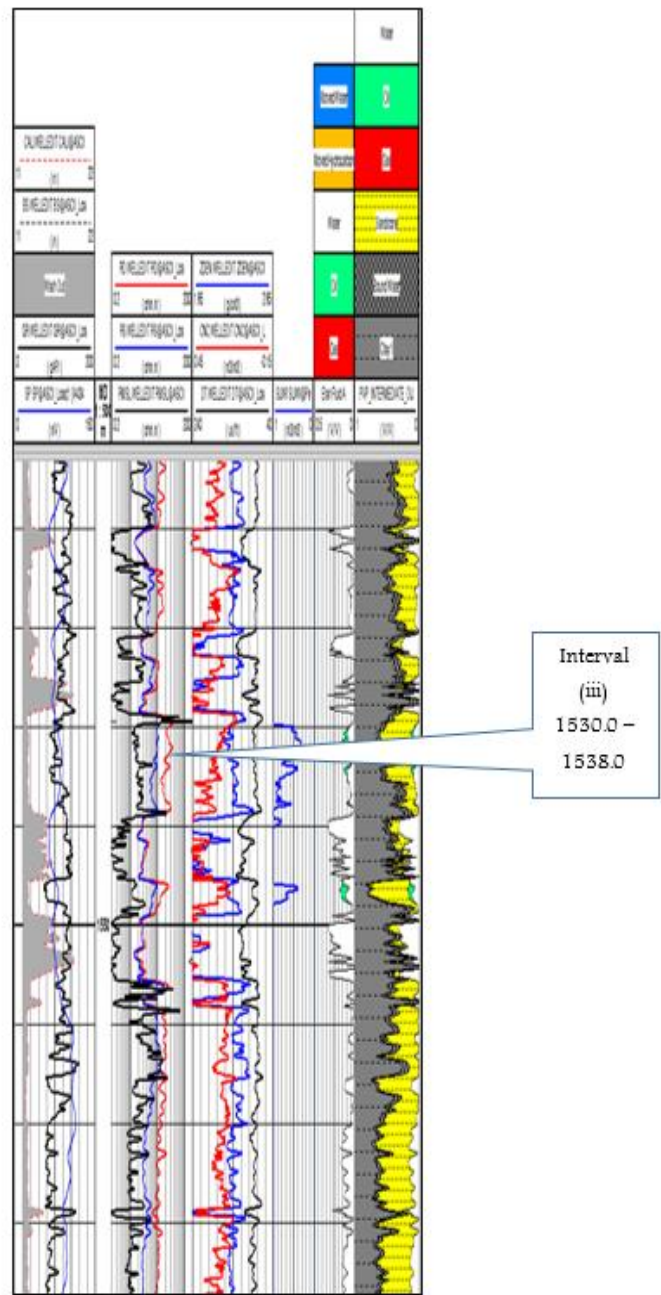


Figure (3-2) show the well logging of well (R)

### 3.3.2.2 Casestudytwo:

The required input data are summarized in the table (3-2):

**Table (3-2):** main input data (case two)

Parameters	Value	Units
$\beta$	1.24	bb1/STB
$\mu$	11.5	cp
$c_t$	12E-06	$psi^{-1}$
$r_w$	0.3	ft
h	10	m
$\phi$	10	%
$s_w$	56	%

### 3.3.2.3 Casestudythree:

The required input data are summarized in the table (3-3):

**Table (3-3):** main input data (case three)

Parameters	Value	Units
$\beta$	1.04	bb1/STB
$\mu$	3	cp
$c_t$	3E-06	$psi^{-1}$
$r_w$	0.4	ft
h	3	m
$\phi$	18	%
$s_w$	65	%

## 3.3.3 Data preparation:

### 3.3.3.1 Flow rate data:

Prepare the flow rate data from excel sheet of well test operation by take the average of the flow rates at time of any change in the choke size.

### 3.3.3.1.1 Case study one:

**Table (3-3):** Flow rate data (case one)

Flow rate (STB\ d)	Time (hr)
0	1
0	1
0	13.4167
10	0.942
0	0.183333
0	15.1283
605	5.83333
987	5.91667
1166	6
1683	6
1753	4.7047
0	32.7796

### 3.3.3.1.2 Case study two:

**Table (3-4):** Flow rate data (case two)

Flow rate (STB\ d)	Time (hr)
0	9.75
0	.25
320.277	1
0	1.5
456.137	6
1320.42	6.25
1191.73	5.75
1210.22	2.01
0	29.99

### 3.3.3.1.3 Case study three:

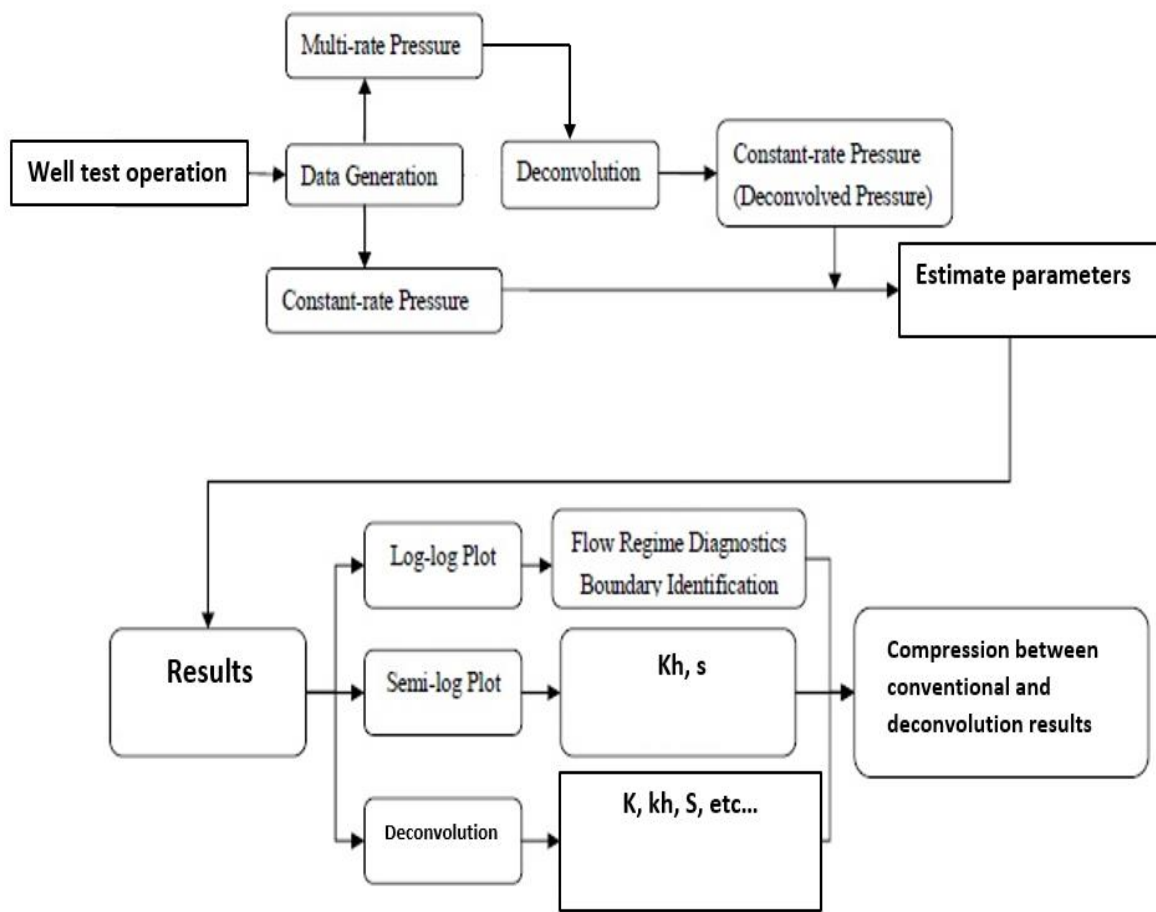
**Table (3-4):** Flow rate data (case three)

Flow rate (STB\ d)	Time (hr)
0	1
0	1
0	0.233333
0	1
137.495	23.695
0	4.245

706.534	48.75
0	36.255

### 3.3.3.2 Pressure data:

The pressure data for the three case studies were taken from gauge reports of each well.



**Figure(3-3):**Methodology diagram

# **Chapter 4**



# Chapter Four

## Results and Discussion

### 4.1 Introduction:

This section discusses the well test interpretation based on well test data acquired during the operations. The purpose of the analysis was to determine the reservoir properties such as permeability and skin factor, initial pressure and productivity index using conventional and new deconvolution method to prove that deconvolution algorithm work well with drawdown period.

The interpretation was carried out using SAPHIR well test analysis software.

### 4.2 Case Study one:

#### 4.2.1 Input main data in saphir software:

Main parameter, well data and PVT properties shown in table (3-1) had been entered to the software as shown in the **fig.(4-1)** and **fig(4-2)**

**First analysis: main options and parameters**

Field	Value	Unit
Name	Analysis 1	
Type	Standard	
Reference well	Tested Well	
Multi-layer	<input type="checkbox"/>	
Well radius	0.583	ft
Pay zone	9.00000	m
Rock compressibility	3.00000E-6	psi <sup>-1</sup>
Porosity	0.19	
Top reservoir depth	1530	m

**Figure(4-1):**Main parameters input (case one)

### First analysis: analytical parameters

Linearized PVT properties

Use pseudos

Formation volume factor B: 1.24000 B/STB

Viscosity  $\mu$ : 11.5000 cp

Total compressibility

Total compressibility ct: 1.20000E-5 psi<sup>-1</sup>

Sw: 0.56 Fraction

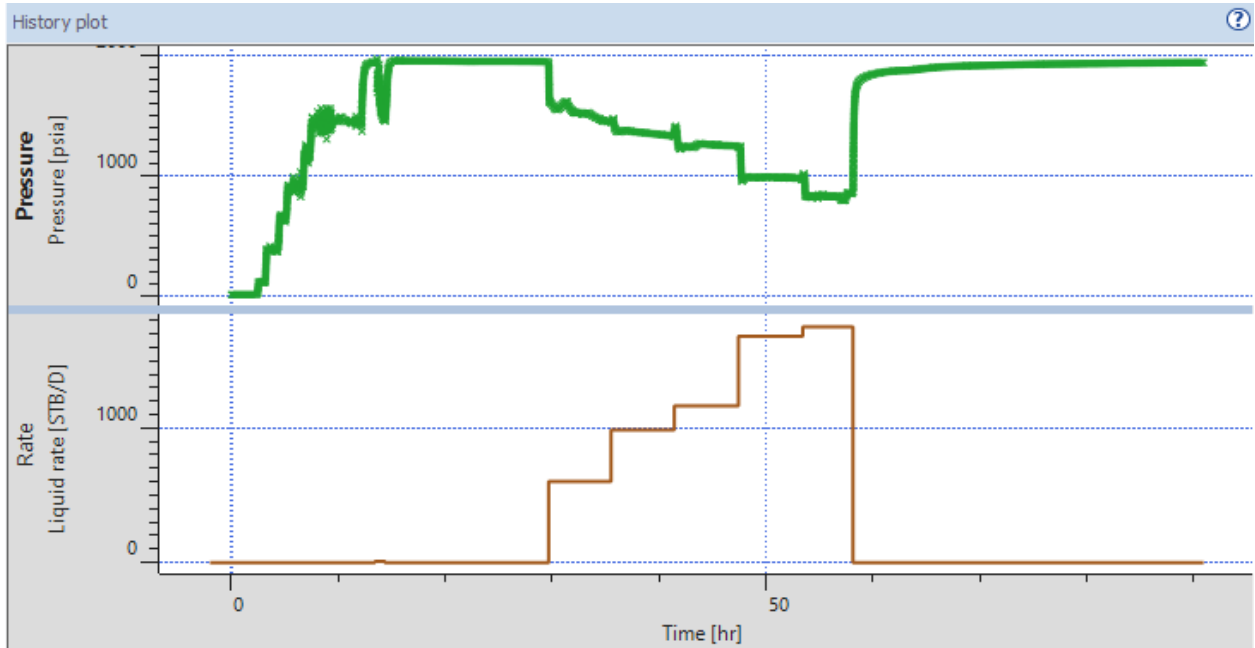
**Figure(4-2):**Main parameters input (case one)

#### 4.2.2 Model selection:

For the test design, it is constitute of a constant wellbore storage, a vertical well model, a homogenous reservoir model and an infinite boundary model, which are also considered to be the default test design of Saphir.

#### 4.2.3 Flow rate and pressure data:

**Fig.(4-3)** show the history plot of pressure and flow rate data which include two drawdown periods and two buildup periods.

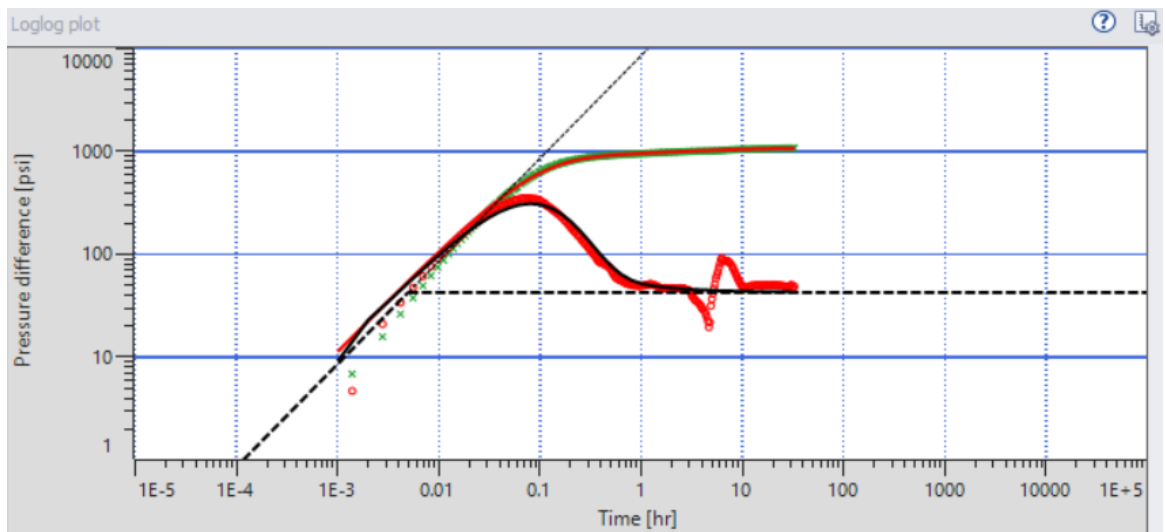


**Figure (4-3):** Production History (case one)

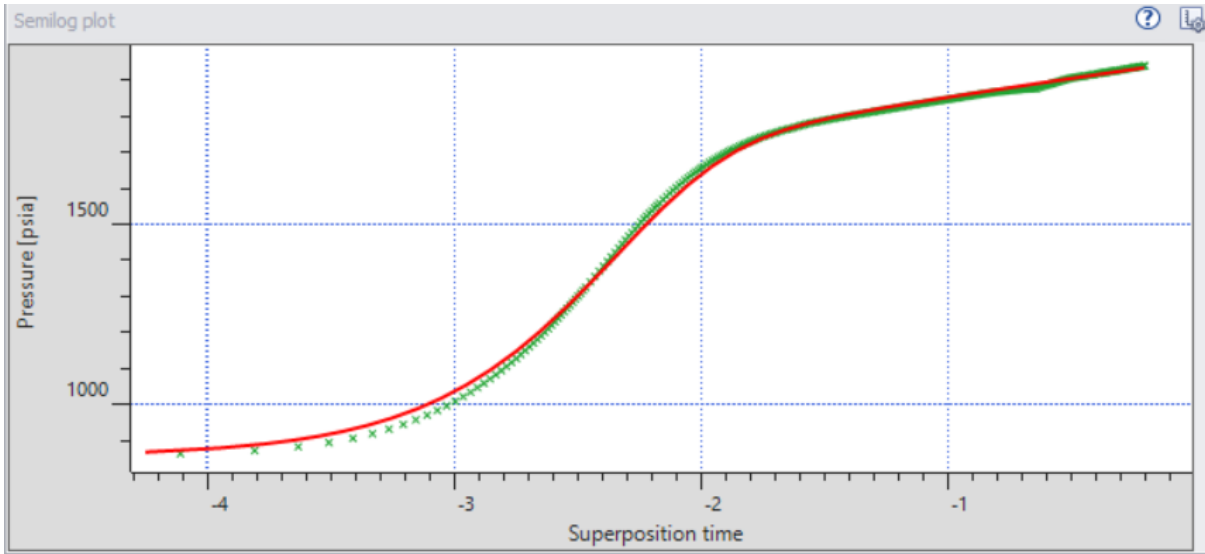
#### 4.2.4 Data analysis:

##### 4.2.4.1 Conventional analysis of buildup:

The conventional analysis has been implemented in the second build up period and good match is seen in **history data** with **log-log** and **semi log plot**. match is very good as shown in **fig(4-4)** and **fig(4-5)**.



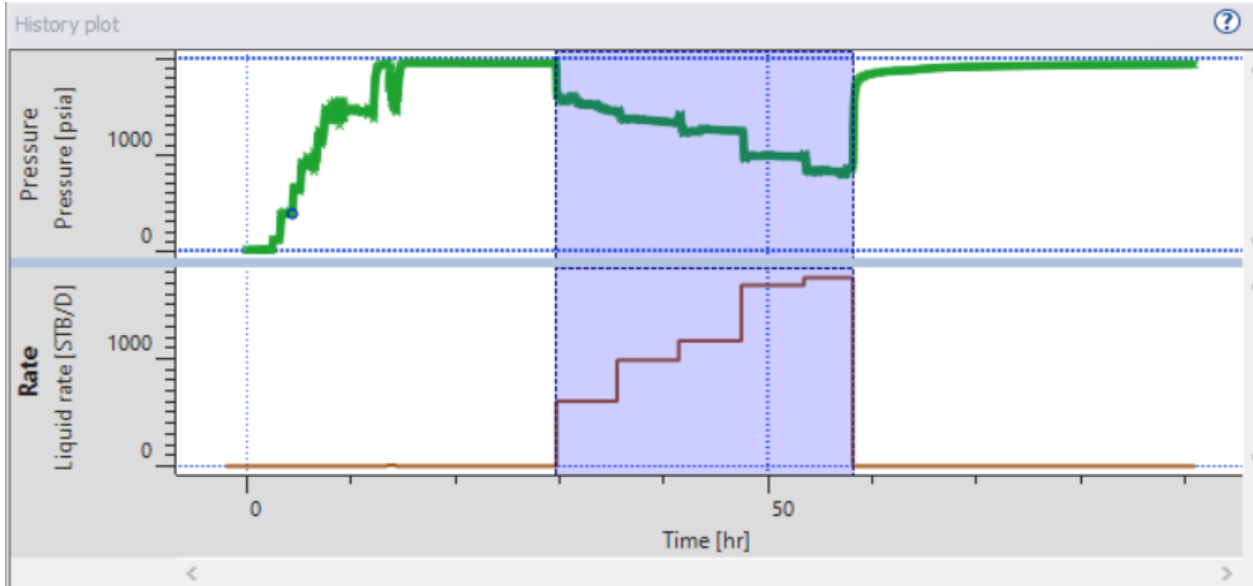
**Figure (4-4):** conventional log log plot for buildup period (case one)



**Figure (4-5):** Semi log plot for buildup period (case one)

**4.2.4.2 Conventional analysis for drawdown period:**

**Fig(4-6)** shown the drawdown period where the conventional and deconvolution analysis are obtained.

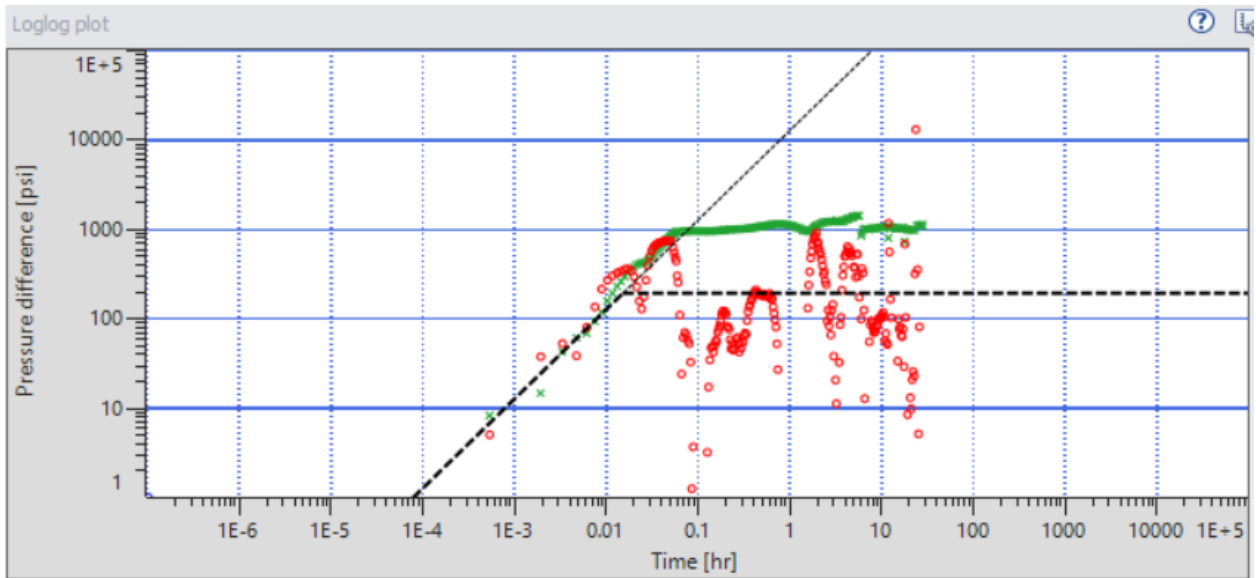


**Figure (4-6)** drawdown period (case one)

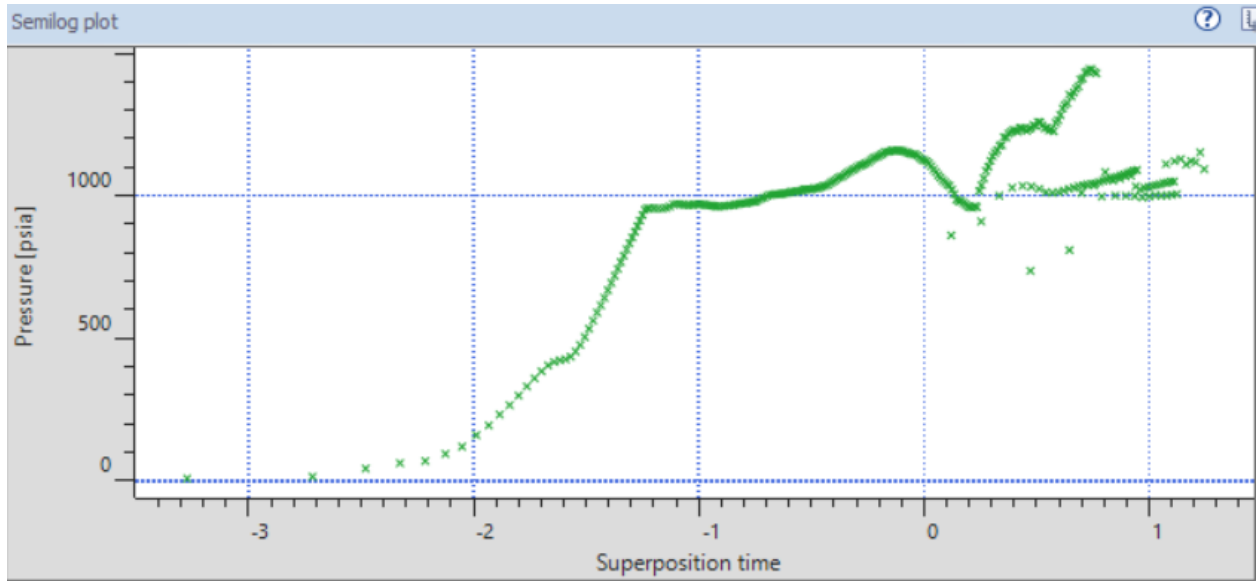
**Fig (4-7)** represent the conventional analysis of drawdown period

As we can see the data can not be used for any transient analysis to provide any result from it. And this shows why people usually use buildup data. In consequence what has been written in chapter two.

We see that for any conventional pressure transient analysis we need a constant flow rate and this is not actually exist in this case and that is why we hereby use deconvolution as a second way of solving this problem and to get more information from exiting result, as we can see in next section.



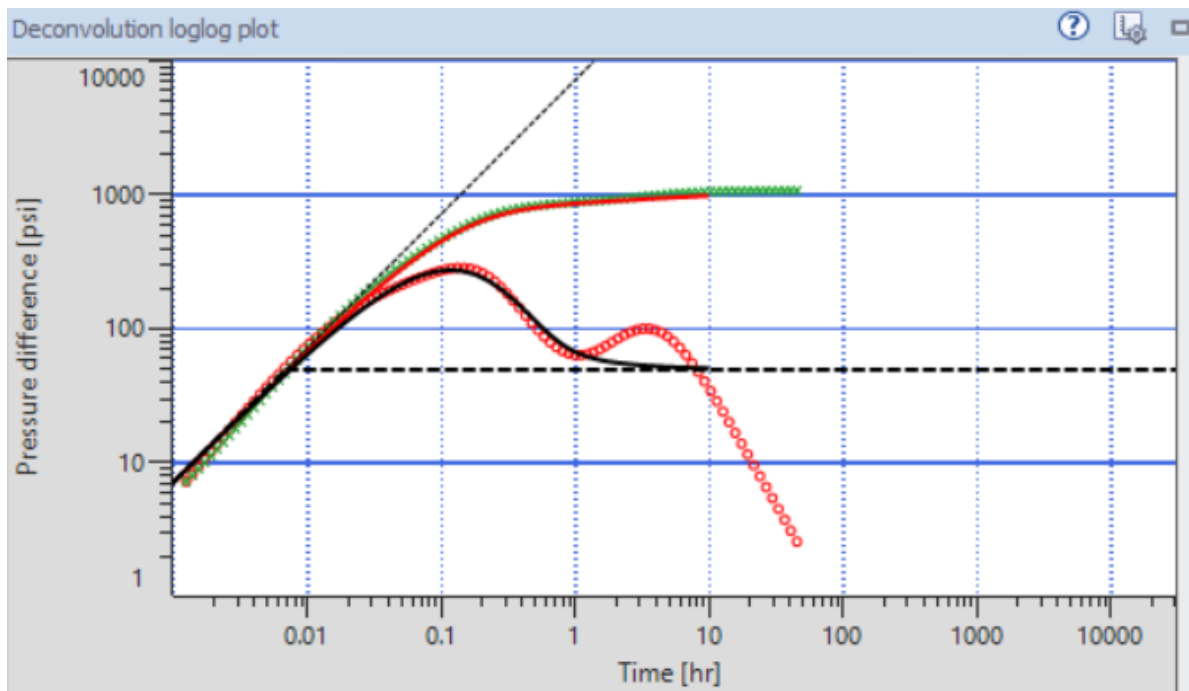
**Figure (4-7)** log-log plot of conventional drawdown period



**Figure (4-8)** semi log plot of drawdown period

#### 4.2.4.3 Deconvolution analysis

The deconvolution algorithm has been implemented in the second drawdown period. The Deconvolution log-log plot shown in **fig.(4-9)**, good match is seen between production data and model.



**Figure (4-9):** Deconvolution log-log plot

#### 4.2.5 Interpretation Results:

**Table (4-1):** Interpretation Result from conventional and deconvolution (case one)

Parameters	conventional	Deconvolution	unit
$kh/\mu$	3124.94	3101.67	<i>mD.ft/cp</i>
Kh	35936.8	35669.2	mD.ft
$k/\mu$	119.06	118.174	<i>mD/cp</i>
K	1369.19	1359	mD
S	3.22426	3.65	
C	0.01049	0.0118	<i>bbl/psi</i>
$P_i$	1943.45	1917.69	psi
PI	1.57	1.56	<i>(STB/d)/psi</i>
$R_{inv}$	2385.84	3718	ft

After we compare between the result of two methods, we prove that we can use production (drawdown) period to get parameters of the reservoir and well using deconvolution method.

#### 4.3Case Study two:

This case is investigating a different well which consist of a different buildup period and different zone.

##### 4.3.1 input main data:


Main parameter, well data and PVT properties shown in **table (3-2)** had been entered to the software as shown in the **fig.(4-10)** and **fig(4-11)**

**Step 2 - First analysis: main options and parameters**

Name:

Type:  Standard  Interference  Minifrac

Reference well:

Multi-layer 

**Test parameters**

Well radius:  ft

Pay zone:  m

Rock compressibility:  psi<sup>-1</sup>

Porosity:

Top reservoir depth:  m

**Figure(4-10):** Main parameters input (case two)



**Step 4 - First analysis: analytical parameters**

**Linearized PVT properties**

Use pseudos

Formation volume factor B:  B/STB

Viscosity  $\mu$ :  cp

---

**Total compressibility**

Total compressibility ct:  psi<sup>-1</sup> + -  
x =

Sw:  Fraction

---

**Multiphase flow**

Use Perrine

Use Kr 

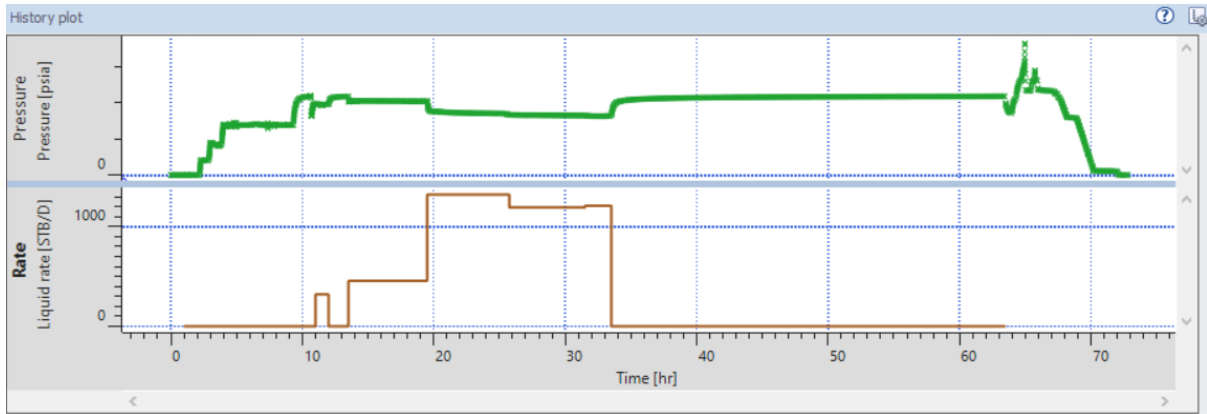
**Figure(4-11):** Main parameters input (case two)

### 4.3.2 Model selection:

For the test design, it is constituted of a constant wellbore storage, a vertical well model, a homogenous reservoir model and an infinite boundary model, which are also considered to be the default test design of Saphir.

### 4.3.3 Flow rate and pressure data:

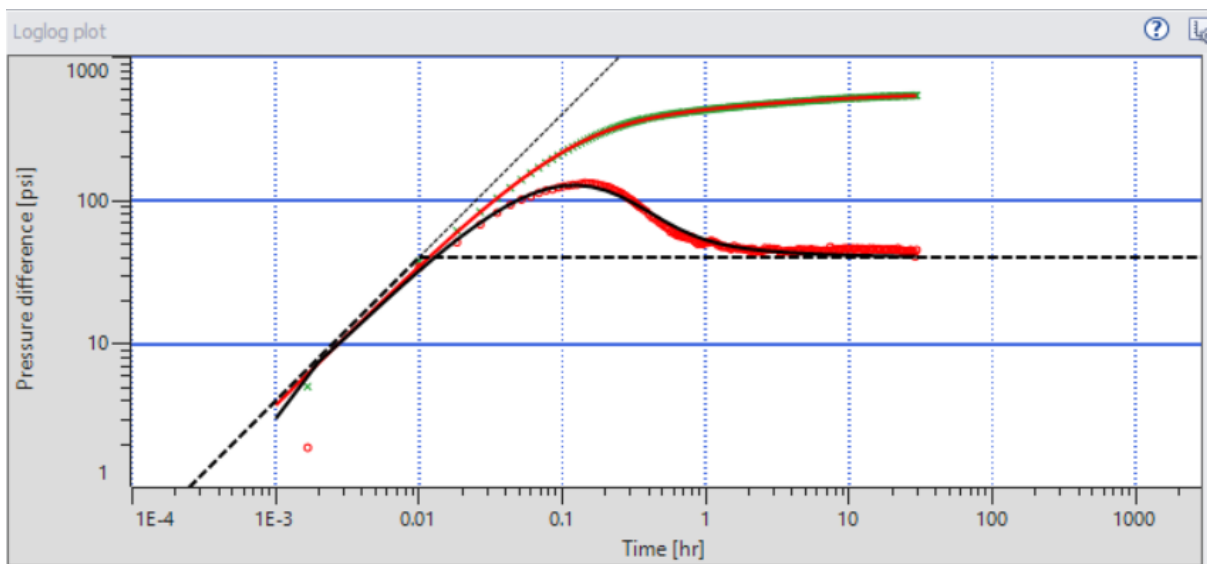
**fig(4-12)** show the history plot of pressure and flow rate data which include two drawdown periods and two buildup periods.



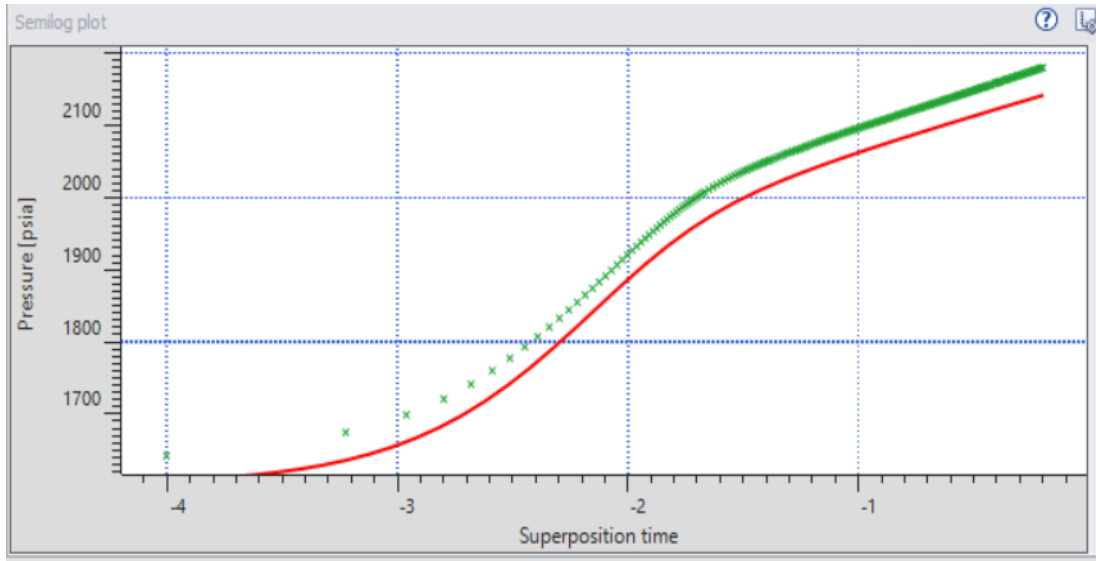
**Figure (4-12):**Production History (case two)

### 4.3.4 Data analysis:

#### 4.3.4.1 Conventional analysis for buildup:



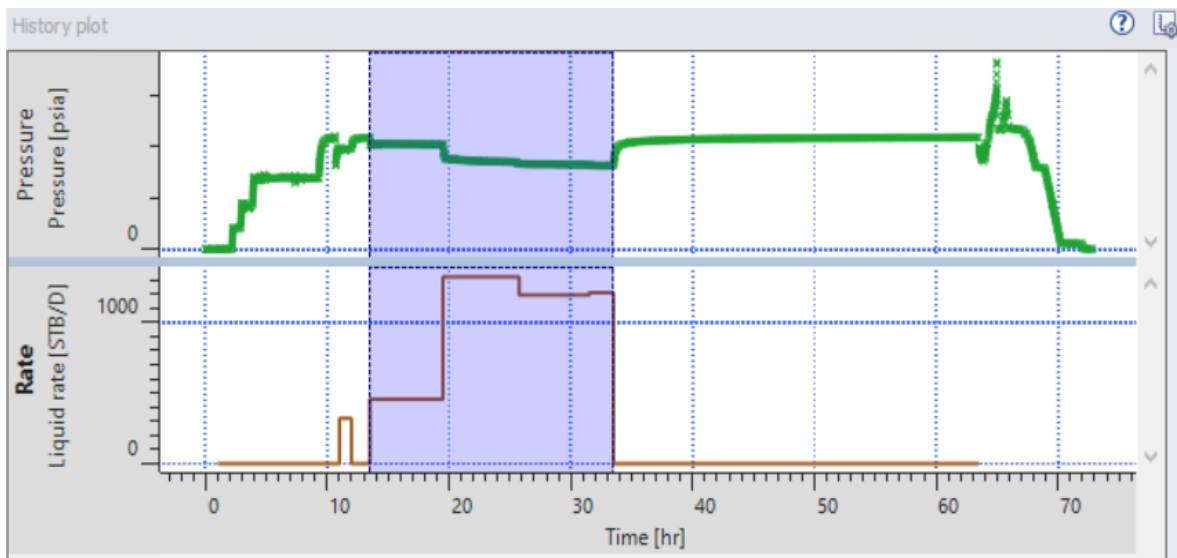
**Figure(4-13):** conventional log log plotfor buildup period (case two)



**Figure(4-14):** Semi log plot for buildup period (case two)

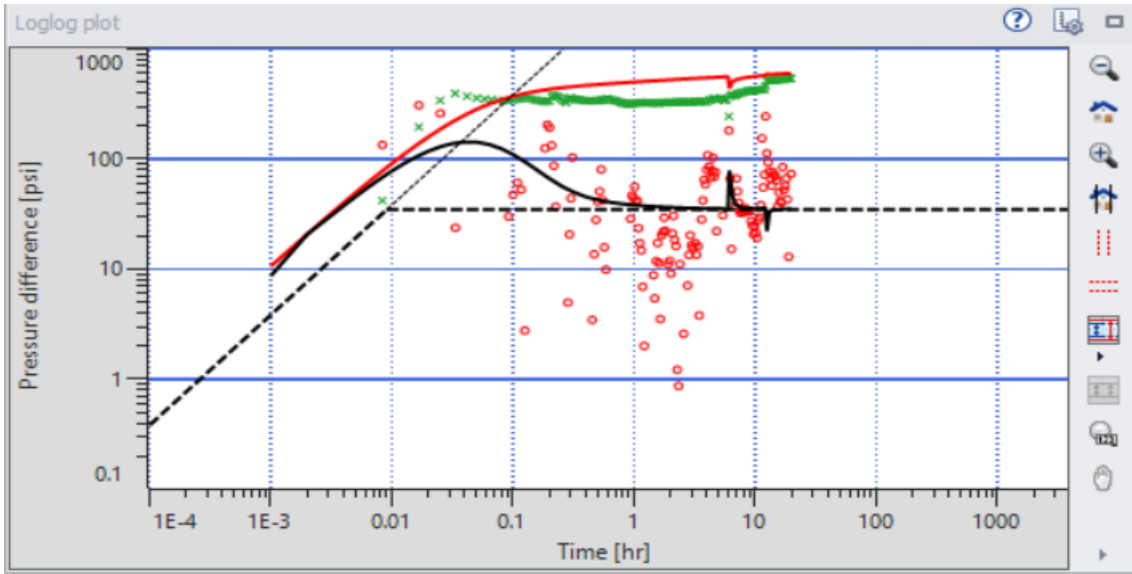
#### 4.3.4.2 Conventional analysis for drawdown period:

**Fig(4-15)** shown the drawdown period where the drawdown data is obtain, and the second tests are implemented.



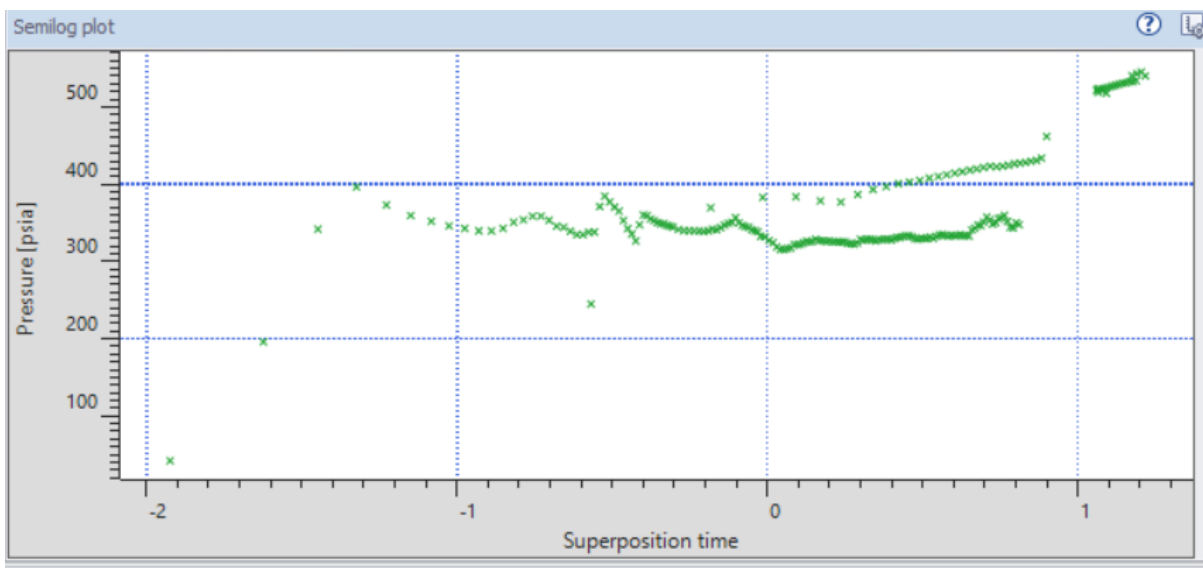
**figure (4-15):**drawdown period(case two)

**Fig(4-16)** represent the conventional analysis of drawdown data , as we see in the figure the curve are useless and we can't get a result from it.



**Figure(4-16):**conventionallog-log plot drawdown period(case two)

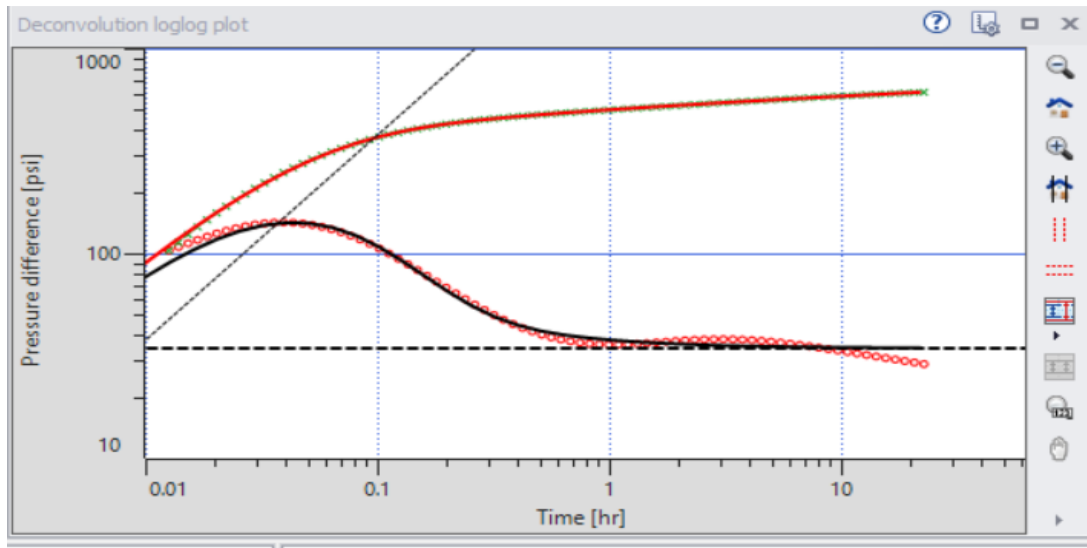
As we can see the data can not be used again for any transient analysis to provide any result from it. And this shows why people usually use buildup data. We see that for any conventional pressure transient analysis we need a constant flow rate and this is not actually exist in this case and that is why we hereby use deconvolution as a second way of solving this problem and to get more information from exiting result, as we can see in next section.



**Figure (4-17):**semi log plot of drawdown period(case two)

#### 4.3.4.3 Deconvolution analysis

The deconvolution algorithm has been implemented in the second drawdown period. The Deconvolution log-log plot shown in **fig(4-18)**, good match is seen between production data and model.



**Figure(4-18):**Deconvolution log-log plot (case two)

#### 4.3.5 Interpretation Results:

**Table (4-2):** Interpretation Result from conventional and deconvolution (case two)

Parameters	Conventional	Deconvolution	unit
$kh/\mu$	2117.37	2452.46	<i>mD.ft/cp</i>
Kh	2117.37	2452.46	mD.ft
$k/\mu$	51	60.418	<i>mD/cp</i>
K	51	60.418	mD
S	-2.66	-1.6	
C	0.0131	0.0046	<i>bbl/psi</i>

$P_i$	2160.68	2160.68	psi
PI	0.6	0.6	$(STB/d)/psi$
$R_{inv}$	7062.07	12135	ft

Concept has been proved in a Sudanese wells which open a wide gate for more transient analysis for a data that has not been quite used in the past and that has been ignored because the conventional way can not use it.

From this research we actually provide new solution which allow the engineers in Sudan to utilize this data and get more information especially for any special case like when the buildup data has issues.

#### **4.4 Case three:**

The third case we selected (R3)to make the problem even more difficult by selecting a well with swab report in sequence of drawdown period which make the rates even more noised.

##### **4.4.1 input main data :**

Main parameter, well data and PVT properties shown in **table (3-3)** had been entered to the software as shown in the **fig.(4-19)** and **fig(4-20)**

**Step 2 - First analysis: main options and parameters**

Name: Analysis 1

Type:  Standard  Interference  Minifrac

Reference well: Tested Well

Multi-layer

Test parameters

Well radius: 0.4 ft

Pay zone: 3.00000 m

Rock compressibility: 3.00000E-6  $\text{psi}^{-1}$

Porosity: 0.18

Top reservoir depth: 2248 m

< Back Next > Cancel

**Figure (4-19):**Main parameters input (case three)

**Step 4 - First analysis: analytical parameters**

Linearized PVT properties

Use pseudos

Formation volume factor B: 1.04000 B/STB

Viscosity  $\mu$ : 3.00000 cp

Total compressibility

Total compressibility ct: 3.00000E-6  $\text{psi}^{-1}$

Sw: 0.65 Fraction

Multiphase flow

Use Perrine

Use Kr

< Back Next > Cancel

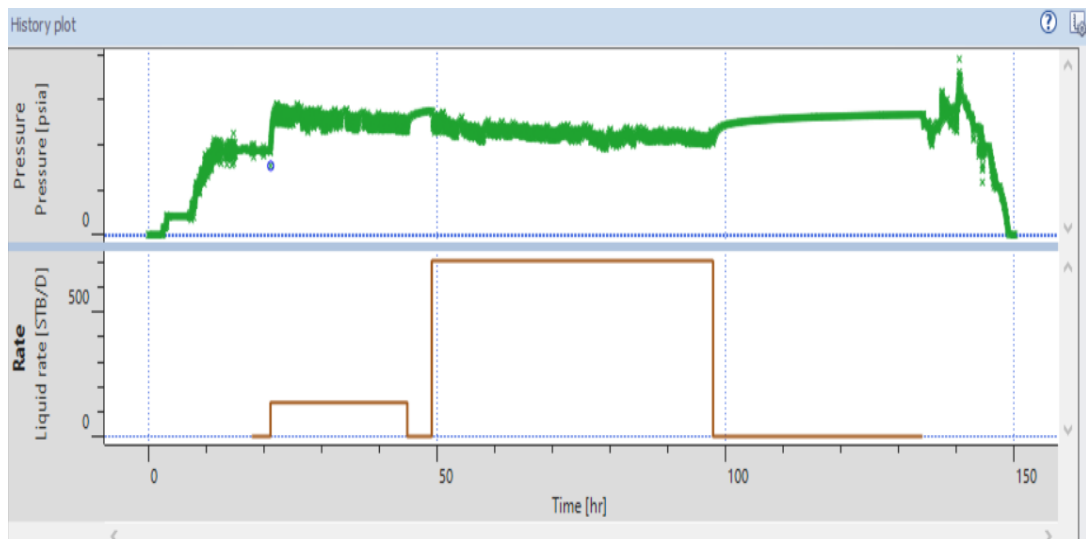
**Figure (4-20):** Main parameters input (case three)

#### 4.4.2 Model selection:

For the test design, it is constituted of a constant wellbore storage, a vertical well model, a homogenous reservoir model and an infinite boundary model, which are also considered to be the default test design of Saphir.

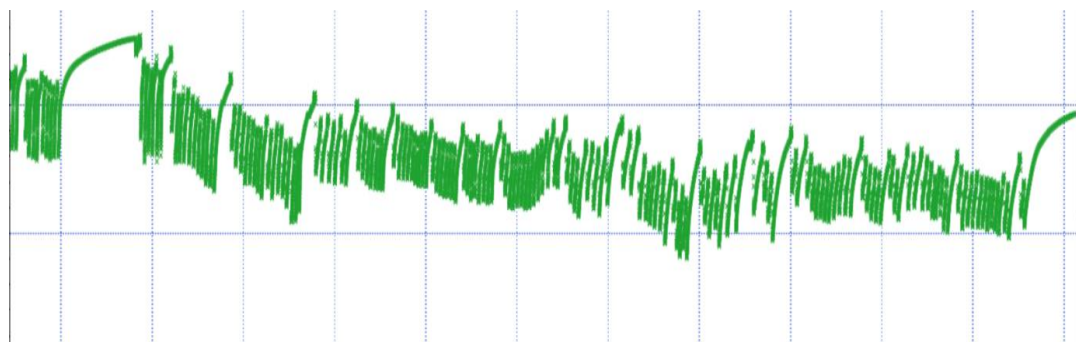
#### 4.3.3 Flow rate and pressure data:

fig(4-21) show the history plot of pressure and flow rate data which include two drawdown periods and two buildup periods.



**Figure (4-21):** Production History (case three).

The figure below show the effect of the swab in the pressure response



**Figure (4-22):** pressure response from swab



#### 4.4.4 Data analysis:

##### 4.4.4.1 Conventional analysis for buildup:

The conventional analysis has been implemented in the second build up period and good match is seen in history data with log-log and semi log plot. match is very good as shown in fig(4-23) and fig(4-24).

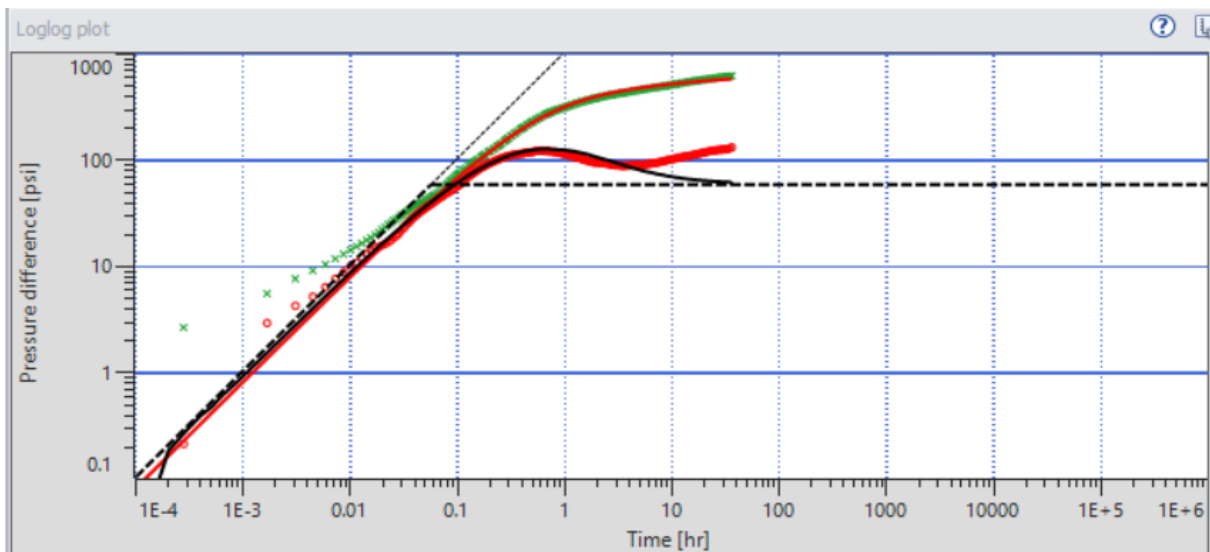


Figure (4-23): conventional log log plotfor buildup period (case three)

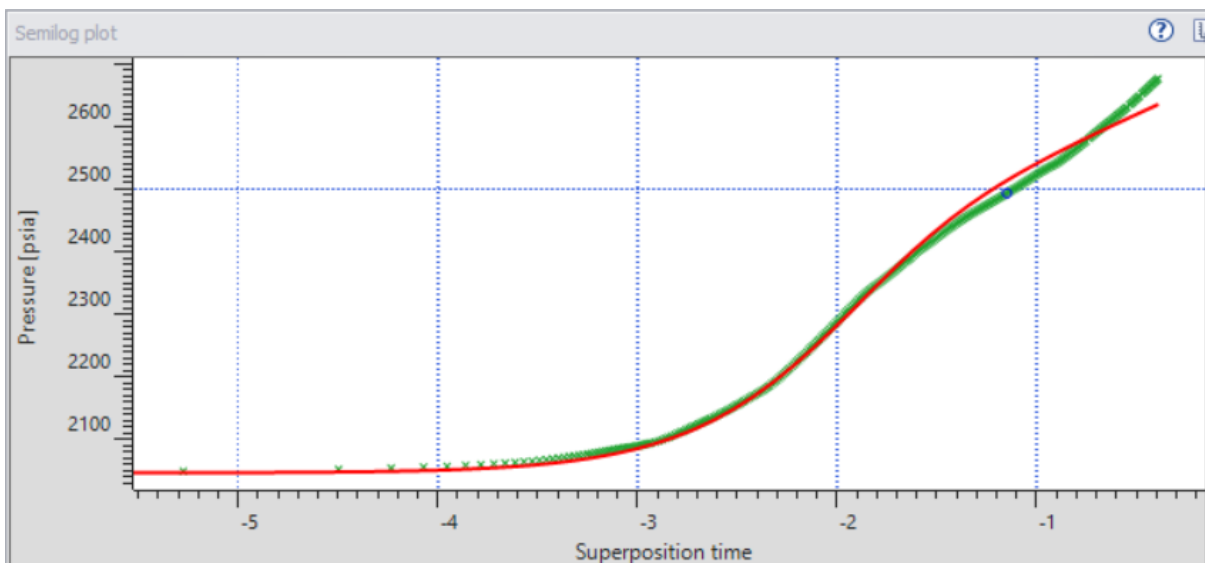
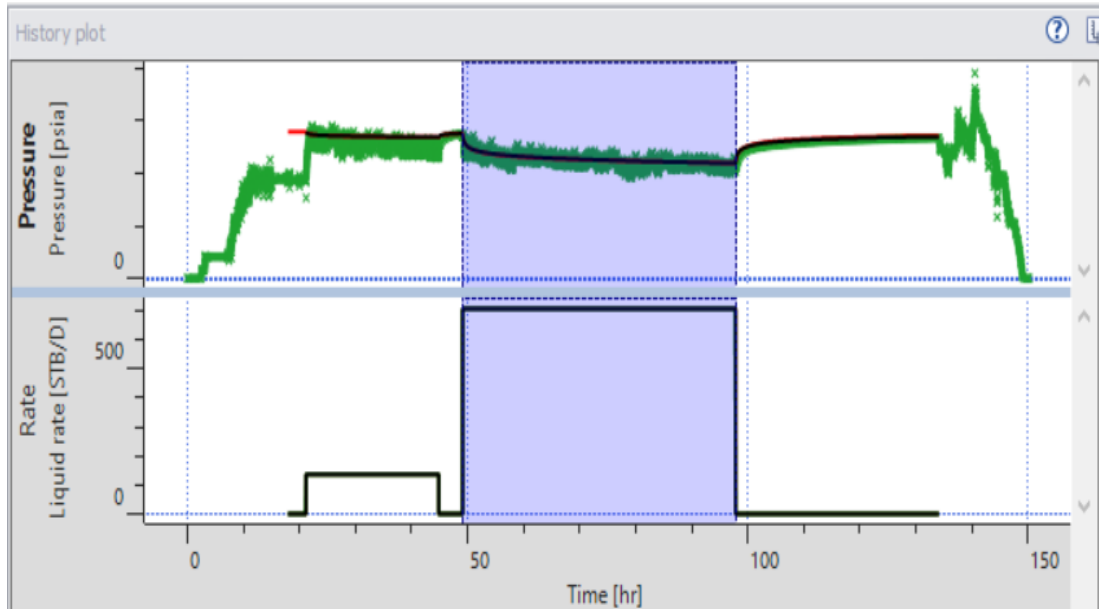


Figure (4-24): Semi log plot for buildup period (case three)

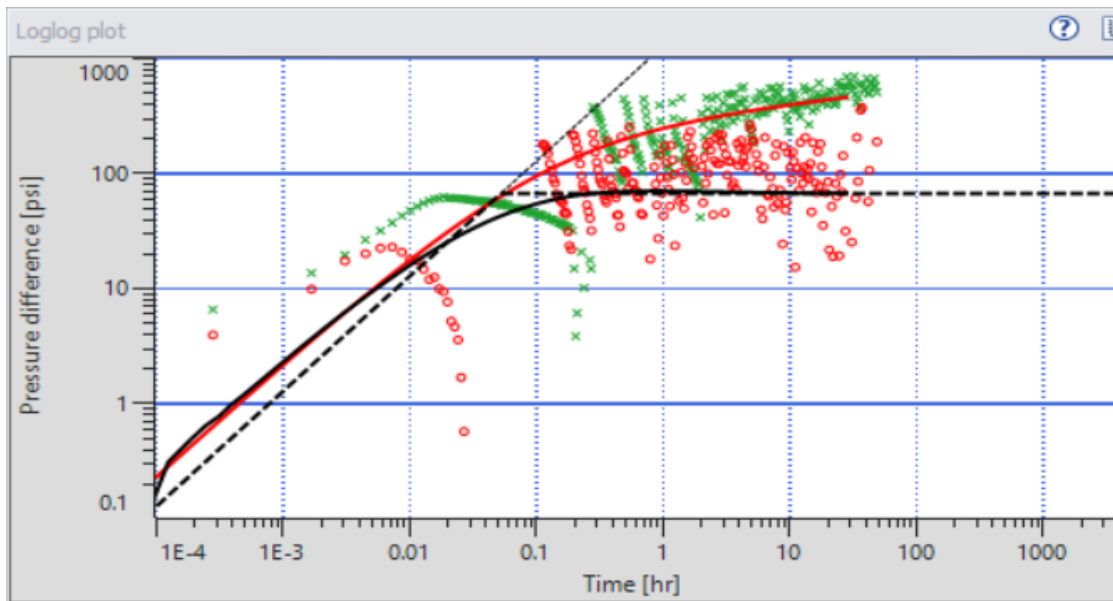
#### 4.4.4.2 Conventional analysis for drawdown period:

Fig(4-25) shown the drawdown period where the drawdown data is obtain, and the second tests are implemented.

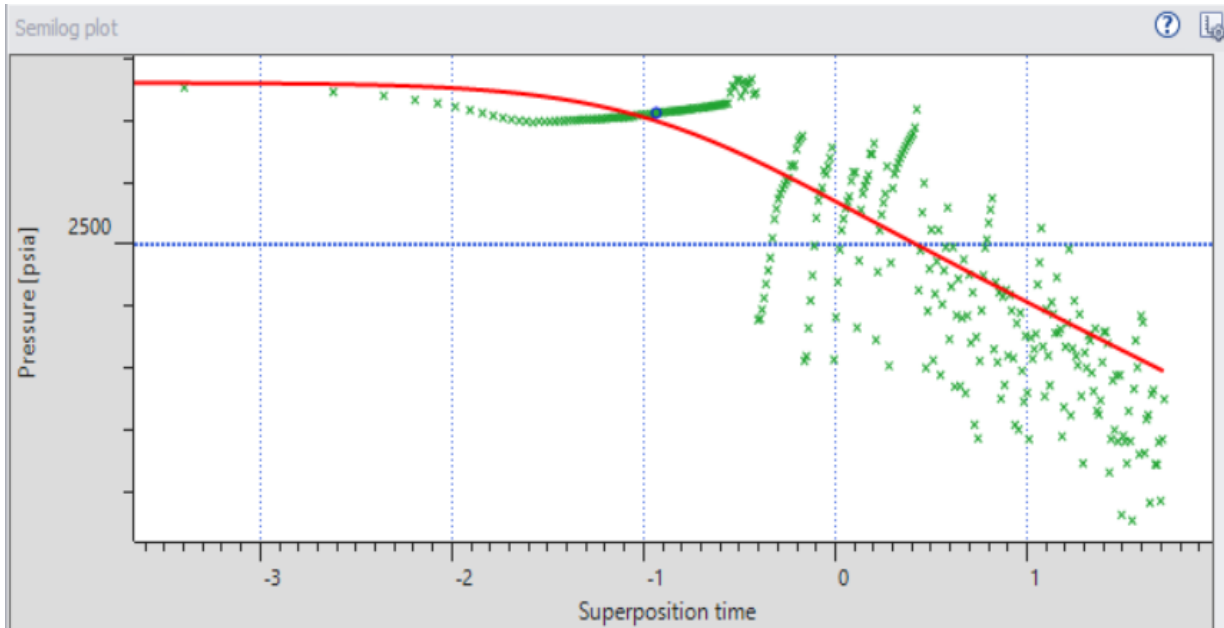


figure(4-25):drawdown period(case three)

Fig(4-26) represent the conventional analysis of drawdown data , as we see in the figure the curve are useless and we can't get a result from it.



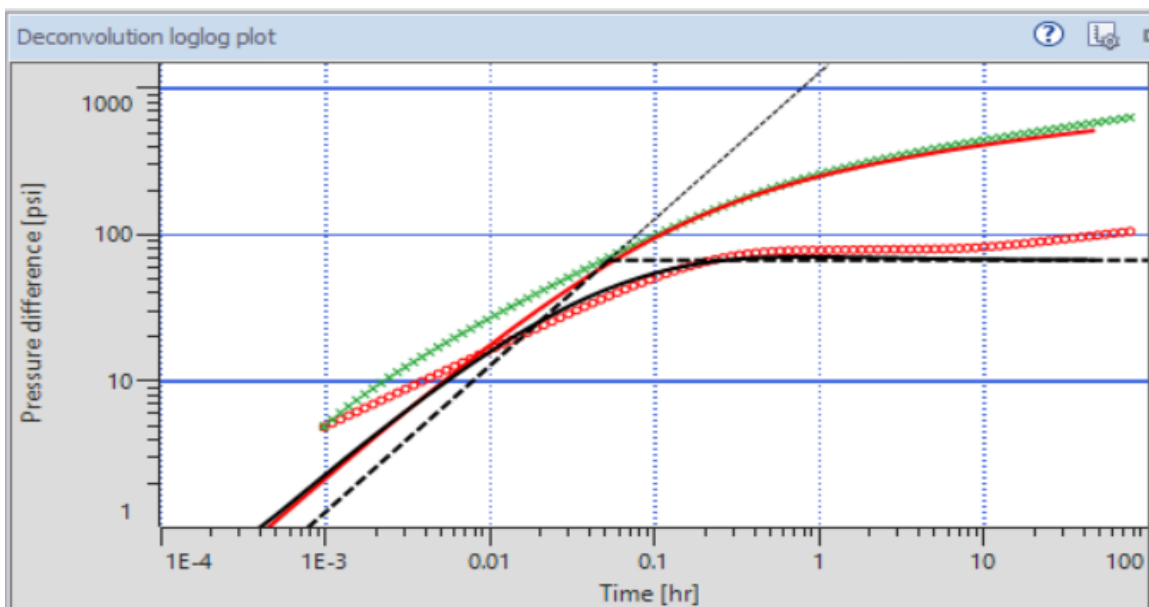
Figure(4-26): conventional log-log plot drawdown period(case three)



**Figure (4-27):**semi log plot of drawdown period(case three).

### 4.4.3 Deconvolution analysis

The deconvolution algorithm has been implemented in the second drawdown period. The Deconvolution log-log plot shown in **fig(4-28)**, good match is seen between production data and model.

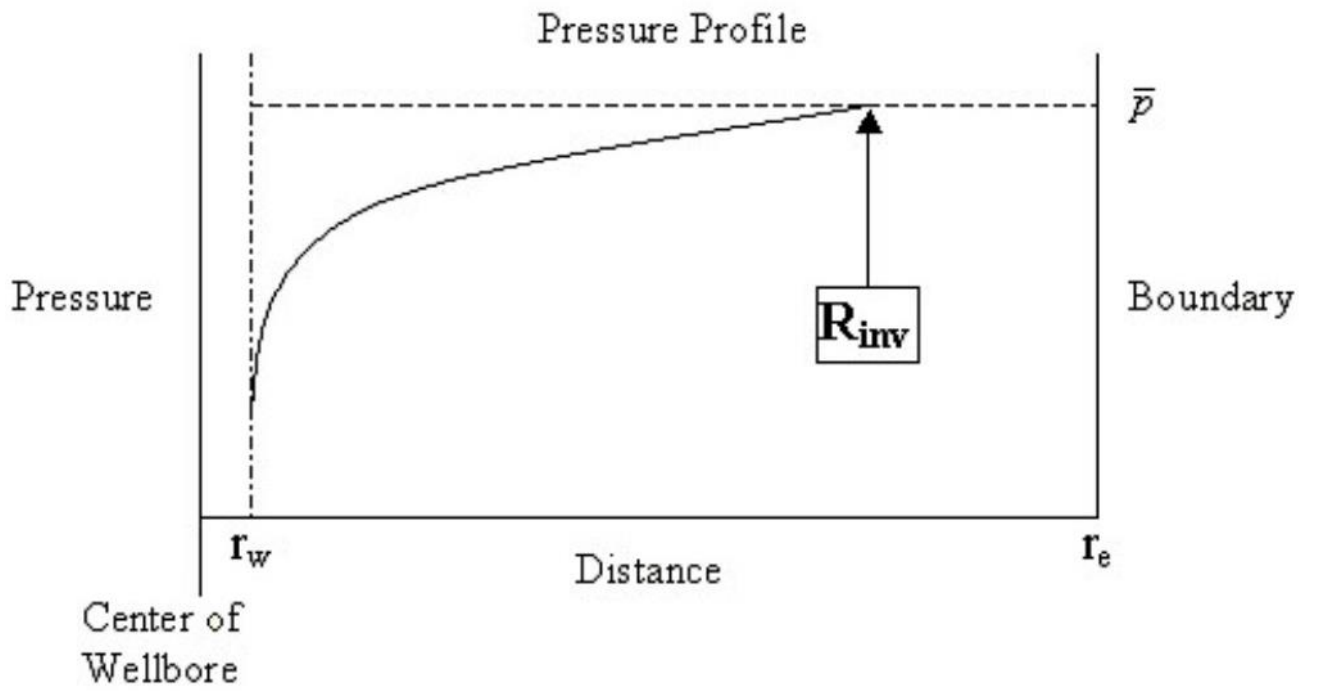


**Figure (4-28):**Deconvolution log-log plot (case three).

**Table (4-3):** Interpretation Result from conventional and deconvolution (case three)

<b>Parameters</b>	<b>conventional</b>	<b>Deconvolution</b>	<b>unit</b>
$kh/\mu$	873.944	770.638	<i>mD.ft/cp</i>
Kh	2621.83	2311.91	mD.ft
$k/\mu$	113.852	109.42	<i>mD/cp</i>
K	341.557	327.719	mD
S	-2.475	-3.98	
C	0.0365	0.0127	<i>bbl/psi</i>
$P_i$	2690.21	2791.78	psi
PI	0.38	0.43	<i>(STB/d)/psi</i>
$R_{inv}$	1071.51	1217.36	ft

As we can see there are large different between radius of investigation of two method, which is related to the higher production rate as compared with a zero production rate in the buildup, where by the buildup were results in a smaller radius of investigation . As shown in **fig.(4-29)**, and this open new gate to investigate deeper in the reservoir, this our own judgment which required further investigation (which has to be consisted with the geology represented in the geological map and the structural map)



**Figure (4-29):** Radius of investigation

(Energy Resources Conservation Board, Theory and Practice of the Testing of Gas Wells)

## **Chapter 5**

# Chapter Five

## Conclusion and Recommendation

### 5.1 Conclusion:

- Threedifferent case studies of well test analysis were conducted, based on oil rate.
- The analysis is done using two method conventional method and deconvolution method.
- The first case study is conduct in single well with no swab to know the feasibility of deconvolution method in analysis of drawdown period.
- The second case is conduct in single well with different in shut in period and different zone know the feasibility of deconvolution method in analysis of drawdown period.
- The third case study is conduct in single well with swab (noise) to know the feasibility of deconvolution method in analysis of drawdown period. in case of swab data.
- Concept has been proved in a Sudanese wells which open a wide gate for more transient analysis for a data that has not been quite used in the past and that has been ignored because the conventional way can not use it.From this research we actually provide new solution which allow the engineers in Sudan to utilize this data and get more information especially for any special case like when the buildup data has issues

## **5.2 Recommendation:**

From the result that we get from this research, we recommended:

- Use production history data to estimate the parameters of the well and reservoir using deconvolution method.
- The coming studies of deconvolution method in Sudan must be focus in multi well deconvolution (interference test)



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