بسم الله الرحمن الرحيم

Sudan University of Science and Technology

Collage of Graduate Studies

Quantum Frictional Model to Derive Statistical Physics For Fluid and Super Fluid Laws

النموذج الكمى الإحتكاكى لاستنباط قوانين الفيزياء الإحصائية للموائع والموائع الفائقة

Thesis Submitted for fulfillment of the Requirements
for Degree of Doctor of Philosophy in physics

Submitted By:
Mohammed Awadalkareim Yousef Balal
Supervisor:
Mubarak Dirar Abdalla
Co-Supervisor:
Ahmed Hassan Alfaki`
November 2019

بِسْمِ اللهِ الرَّحْمٰنِ الرَّحِيْمِ

الآيــة

قال تعالى:

{مَا خَلَقْنَا السَّمَوَاتِ وَالأَرْضَ وَمَا بَيْنَهُمَا لاعِبِينَ (38)

sمَا خَلَقْنَاهُمَا إِلاَّ بِالْحَقِّ وَلَكِنَّ أَكْثَرَهُمْ لا يَعْلَمُونَ } (39)

صدق الله العظيم

الدخان (38-39)

DEDICATION

I dedicate this work to…

my mother and father

my wife and sons

my brother and sister and

my teachers

Acknowledgement

Praise to Allah who gave me the idea patience to accomplish this work.

I own adept of gratitude to my supervisor Professor Mubarak Dirar, for suggesting this work, for this Keenness to flow this project.

My gratitude goes to staff of Science College of Sudan University for their continuing help.

Finally, Avery special thank to my beloved family and my friend. Without this entire people, this work could not have been done.

Abstract

Schrodinger equation for particle in a finite media with uniform potential inside abox has been solved the solution which is based on the fact that the particle exists gives complex and cosine wave function with energy relations different from that of the ordinary sine solution.

Maxwell distribution law has been also found using the expression for the wave function in a frictional medium, quantum energy average and integration by parts, another approach has been tackled using the general expression for quantum average and the ordinary differentiation.

Using Maxwell distribution Quantum law, and the Newtonian energy relation continuity and momentum fluid equation was done by differentiation the number density with respect to time and to coordinate. The momentum equation derivation requires the coefficient of the energy in the exponential power is equal to the thermal kinetic energy. This conforms with the statically value proposed by Maxwell distribution but with a positive sign. This number density function can successfully describes lasing. This is since it predict population inversion and intensity of amplified light. These fluid derived equations can be suitable for superfluid’s, since they are free from frictional term and conforms with statistical physics and quantum laws.

مستخلص

حُلت معادلة شرودنجر للجسيمات داخل وسط محدود مع جهد موحد داخل صندوق . الحل الذي يستند على حقيقة وجود الجسيمات يعطى دالة موجة جيب التمام المركبة مع علاقات طاقة مختلفة عن تلك المتعلقة بحلول الجيب العادى. تم ايضا الحصول على توزيع ماكسويل من تعبير دالة الموجة في وسط احتكاكى ومتوسط كمية الطاقة بالتكامل الجزيئ ، مع معالجة التعبير العام لمتوسط الكم بالتفاضل العادى .

أشتُقت معادلة الاندفاع والاستمرار من توزيع ماكسويل الكمى بمفاضلة كثافة الجسيمات بالنسبة للزمن والموضع . حيث اقتضى اشتقاق معادلة الاندفاع أن يكون معامل الطاقة فى الدالة الأسية مساويا للطاقة الحرارية الحركية . وهذا يتوافق مع القيمة الثابتة التى اقترحها توزيع ماكسويل ولكن باشارة موجبة. كما اتضح ان دالة كثافة العدد تصف بنجاح الفعل الليزرى وذلك لانها تصف الانقلاب السكانى وشدة الضوء المضخم . هذة المعادلات المستنبطة للمائع يمكن أن تكون مناسبة للموائع الفائقة لخلوها من حد الاحتكاك واتساقها مع قوانين الفيزياء الإحصائية والموائع .

TABLE OF CONTENTS

Chapter Two

Theoretical Back ground

Chapter Three

Literature Review

Chapter Four

The model of fluid and lasing equations from quantum and statistical equation

Chapter one

Introduction

1.1 Fluids:

Fluids is a state of matter that describes liquids and gases, Super-fluidity is a state of matter in which the matter behaves like a fluid with zero viscosity, where it appears to exhibit the ability to self-propel and travel in a way that defies the forces of gravity and surface tension. Superfluity is found in astrophysics, high energy physics, and theories of quantum gravity. The phenomenon is related to Bose-Einstein condensation, but neither is a specific type of the other, not all super-fluids are Bose-Einstein condensates [1, 2].

1.1.1 Super-fluidity of liquid helium:

Super-fluidity was originally discovered in liquid helium, by Pyotr Kapitsa and John F. Allen .It has since been described through phenomenology and microscopy theories. Inliquid helium-4, the super-fluidity occurs at far higher temperatures than it does in helium-3. Each atom of helium-4 is a boson particle, by virtue of its integer spin. A helium-3 atom is a fermions particle; it can form bosons only by pairing with itself at much lower temperatures. This process is similar to the electron pairing in superconductivity [1, 2, 3].

1.1.2 Ultracold atomic gases:

Super-fluidity in an ultra-cold fermionic gas was experimentally proven by Wolfgang Ketterle and his team who observed quantum vortices in ^{6}Li at a temperature of 50 nk at MIT in April [2] Such vortices had previously been observed in an ultra-cold bosonic gas using ^{87}Rb in 2000, [3] and more recently in two-dimensional gases.[4] As early as 1999 Lene Hau created such a condensate using sodium atoms [5] for the purpose slowing light, and later stopping it completely .[6] Her team then subsequently used this system of compressed light[7] to generate the super-fluid analogue of shock waves and tornadoes . These dramatic excitations result in the formation of solitonis that in turn decay into quantized vortices created far out equilibrium in Paris of opposite circulation revealing directly the process of super-fluid breakdown in Bose-Einstein condensates. With a double light-roadblock setup, we can generate controlled collisions between shock waves resulting in completely unexpected, nonlinear excitations. We have observed hybrid structures consisting of vortex rings embedded in dark solitonic shells. The vortex rings act as ' phantom propellers ' leading to very rich excitation dynamics. [8].

1.1.3 Super-fluidity in astrophysics:

The idea that super-fluidity exists inside neutron stars was first proposed by Arkady Migdal [9][10] By analogy with electrons inside superconductors forming Cooper pairs due to electron-lattice interaction, it is expected that nucleons in a neutron star at sufficiently high density and low temperature can also from Cooper pairs due to the long-range attractive nuclear force and lead to super-fluidity and superconductivity [11].

1.1.4 Super-fluidity in high-energy physics and quantum gravity:

Super fluid vacuum theory (SVT) is an approach in theoretical physics and quantum mechanics where the physical vacuum is viewed as super-fluid. The ultimate goal of the approach is to develop scientific modes that unify quantum mechanics with gravity. This makes SVT a candidate for the theory of quantum gravity and an extension of the standard Model. It is hoped that development of such theory would unify into a single consistent model of all fundamental interactions, and to describe all known interactions and elementary particles as different manifestations of the same entity, super-fluid vacuum [10].

1.2 Newton's Laws with Relation to Quantum and statistical Physics:

The concept of field is related to force. Force is related to momentum time .momentum is a valuable tool for predicting the future of physical system because it related to physical quantity that controlled a system, like force and energy. According to Newton's second law the resultant force is equal to the rate of change of momentum. The momentum is a conserved vector quantity. Scalar quantity, and to forces, which are only conserved locally. In quantum mechanics the energy cannot be continuous for a particle. The energy of quantum particle can have minimum value but cannot be zero. The importance of momentum in quantum mechanics results from the uncertainty principle and De Broglie hypothesis. Also the equation of motion as Schrödinger equation can be extracted from it {1, 2, and 3}.

The momentum also plays an important role in statistical physics. In a dynamical system theory, a phase space is a space in which all possible states of a system are represented, with each possible state corresponding to unique point in the phase space. For mechanical systems, the phase space usually consists of all possible values of position and momentum variables of {4, 5}. In statistical mechanics, any choice of a generalized coordinates for the position defines conjugate generalized momentum which together define coordinates on the phase space the momentum representation, wave functions are Fourier transforms of the equivalent real-space wave functions. The continuity equation governs the conservation of mass, charge and probability of any closed system.

This equation involves the spatial distribution of the flux density that is related to the temporal variation of the particle density (charge, mass). Ordinary, this equation is derived from the equation of motion. The motion of any continuous charge/mass distribution can be thought of as a continuum (field or fluid). The continuity equation guarantees that there is no loss or gain of such quantities. This equation provides us with information about the system. The information is carried from one point to another by a particle (field wave) [6, 7].

The continuity and momentum beside lasing equations are derived from Newton's laws. Different attempts were made to derive statistical laws and lasing equations [8, 9, 10].

This work is conceived with new derivation of this these equations using quantum wave function and Maxwell distribution. This is done in section (2), sections (3) and (4). Are conserved with discussion and conclusion.

Quantum mechanics is fundamental theory in physics which describes nature at the smallest scales of energy levels of atoms and subatomic particles equation that describes the changes over time of a physical system which is affected by the surrounding. This equation is considered as a back bone of quantum mechanics, which succeeded in describing the behavior of single particle but, but it fails to describe quantum system of many –body, because of the complex interaction between particles. As a consequence, the wave function of the system is complicated in nature having a large amount of information. On the other hand statistical mechanics is a branch of physics that uses method of probability theory and statistics to describe atoms and elementary particles. It uses mathematical tools for dealing with large amounts of particle in the physical system in solving physical problems. It can descried a wide of fields that consists of randomly be having particles. Its applications include electronics, laser and material science it is main purpose is to clarify the properties of matter in aggregate, in terms of the microscopic properties of individual constituents of the system. These properties of statistical physics make it in close link with fluids a fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces like liquids and gases. This branch of physics has two parts static and dynamic fluid. In fluid dynamic, the equation of motion is performed by principles of conservation in physics like mass, energy and momentum conservation, Bernoulli's equation is one of important equations describing fluid in motion, which can be obtained by principles of energy and momentum conservation. There are equivalence between the laws of fluid, quantum and classical mechanics because all of them depend on principles of conservations, and deals with very small particles. This encourages doing this work which is devoted to derive fluid continuity and momentum equation from quantum and statistical laws. This is done in section (3), section (2) is conserved with deriving and expression of the wave function in a homogenous media, and discussion and conclusion are done in sections (4).

1-3 Quantum Mechanics:

In 1900 Max Planck introduced the concept of quantum energy. He argued that the energy exchange between an electromagnetic wave of certain frequency and matter occurs only in integer multiples of a quantities which is proportional to the frequency, this energy is called quanta [1, 2].

In 1905 Einstein provided a powerful consolidation to Black's quantum concept. In trying to understand the photoelectric effect, he posited that the light is made of discrete bits of energy called photons [3, 4].

After discovering atomic nucleus by Rutherford's experimental in 1911, and combining Rutherford's atomic model, Blank's quantum concept, and Einstein's photons, Bohr introduced in 1913 his model of the hydrogen atom. Compton made an important discovery in 1923 that gave the most conclusive confirmation for the corpuscular aspect of light. By scattering X-rays with electrons, he confirmed that the X- ray photons behave like. De Broglie introduced in 1923 another powerful new concept that classical physics could not reconcile: he postulate that not only does radiation exhibit particle like behavior but, conversely, material particles themselves display wave-like behavior. This concept was confirmed experimentally in 1927 by Davisson and Germer; they showed that interference patterns, a property of waves, can be obtained with material particles such as electrons [5, 6, 7].

Heisenberg present first formulation called matrix mechanics to develop atomic structure in 1925, the second formulation, called wave mechanics, was due to Schrodinger 1926. In 1927 Max Born proposed his probabilistic interpretation of wave mechanics: he took the square moduli of wave functions that are solutions of Schrodinger equation and he interpreted them as probability density [8, 9, 10, and 11].

Combining special relativity with quantum mechanics, Dirac derived in 1928 an equation which describes the motion of electrons [5, 12, 13].

1.4 Literature Review:

The theory of the condensate of a weakly interacting Bose gas is developed. The condensate is described by a wave function normalized to the number of particles. It obeys a nonlinear self-consistent field equation. The solution in the presence of a rigid wall with the boundary condition of vanishing wave function involves a de Broglie length. This length depends on the mean potential energy per particle. The self-consistent field term keeps the density uniform except in localized spatial regions. In the hydrodynamical version, a key role is played by the quantum potential. A theory of quantized vortices and of general potential flows follows immediately. In contrast to classical hydrodynamics, the cores of vortices are completely determined by the de Broglie length and all energies are finite. Non-stationary disturbances of the condensate correspond to phonons, rotons, vortex waves etc. They can exchange momentum with rigid boundaries. This is compatible with the vanishing of the wavefunction at a boundary. This condition fully determines the dynamics of the system. These points are illustrated by considering the motion of a foreign ion in a Bose gas, a rotating container of fluid, and the Landau criterion for Super-fluidity [3].

This paper presents an attempt of explaining the phenomenon of Super-fluidity on the basisof the theory of degeneracy of a non-perfect Bose-Einstein gas. [4].

By using the method of the second quantization together with an approximation procedurewe show that in the case of the small interaction between molecules the low excited states of thegas can be described as a perfect Bose-Einstein gas of certain “quasi-particles” representingthe elementary excitations, which cannot be identified with the individual molecules.The special form of the energy of a quasi-particle as a function of its momentum is shown to beconnected with the Super-fluidity. [6]

We consider the superfluid phase transition that arises when a Feshbach resonance pairing occurs in a dilute Fermi gas. We apply our theory to consider a specific resonance in potassium (40 K), and find that for achievable experimental conditions, the transition to a superfluid phase is possible at the high critical temperature of about $0.5TF$. Observation of Super-fluidity in this regime would provide the opportunity to experimentally study the crossover from the superfluid phase of weakly coupled fermions to the Bose-Einstein condensation of strongly bound composite bosons. [9].

A theory is developed to describe grid turbulence in a superfluid in the case where the normal fluid is held stationary, as would be the case for super fluid He $3-B$ in which the normal fluid is very viscous. The theory is a straightforward development of earlier work, reviewed by Vinen and Niemela [5] and it shows that on large length scales the turbulence is strongly damped by mutual friction. A comparison is made with recent work by Volovik and his colleagues [6].which was developed while our work was in progress [2].

1.5 Research Problem:

Quantum, statistical physics and fluid laws seems to be not correlated although they all describe huge number of particles there is no full theory for relating them to each other.

(1-6) Aim of the Work:

The aim of the work is to construct quantum model based on Newton's laws to relate quantum, statistical physics and fluids laws to each other.

1.7 Thesis Layout:

The thesis consists of four chapters. Chapter one is the introduction. Chapter two is the theoretical background. Chapter three and four are concerned with the literature review and the contribution.

Chapter Two

Theoretical Background

2.1 Introduction:

In this chapter one will be derive the basic equations of motion for fluid, conservation of mass and conservation of momentum, Maxwell distribution, and superfluid's.

2.2 Fluid laws:

In this section one will derive the partial differential equation representing conservation of mass in a fluid flow, the so-called continuity equation .we will then, in a sense, "work backwards" to recover an integral form, often called the "control-volume" form, that can be applied to engineering calculations in an approximate, but very useful, way we will then consider some specific examples of employing this equation [17, 18].

2.2.1 Derivation of the continuity equation:

We begin this section with the general statement of conservation of mass, and arrive at the differential form of the continuity equation via a straightforward analysis involving application of the general transport theorem and Gauss's theorem [18, 19, 20].

2.2.2 Conservation of mass:

We start by considering a fixed mass m of fluid contained in an arbitrary region$R(t)$. As we have already hinted, we can identify this region with a fluid element, but in some cases we will choose to associate this with a macroscopic domain. In either case, the boundary $S(t) of R(t)$ can in general move with time. Any such region is often termed a system, especially in thermodynamics contexts, and it might be either open or closed. From our point of view it is only important that it have fixed mass; it does not matter whether it is the same mass at all times – only that the amount is the same[21].

It is convenient for our purposes to relate the mass of the system to the density of the fluid comprising it via

$$m = \int_{R(t)} \rho dV \qquad (2.2.1)$$

We emphasize that $R(t)$and ρ may both change with time, but they must do so in a way that leaves m unchanged if we are to have conservation of mass. An example of this might be a balloon filled with hot air surrounded by cooler. As heat is transferred from the balloon to its surroundings, the temperature of the air inside the balloon will decrease, and the density will increase (equation of state for a perfect gas). At the same time the size of the balloon will shrink, corresponding to a change in R (t). But the mass of air inside the balloon remains constant- at least if there are no leaks [21].

We can express this mathematically as:

$$\frac{dm}{dt} = \frac{d}{dt} \int_{R(t)} \rho dV = 0 \qquad (2.2.2)$$

That is, conservation of mass simply means that the time rate of change of mass of a system must be zero [17, 18 19].

<u>**(2.2.3)The differential continuity equation:**</u>

We now recall that region R (t) was arbitrary (i.e., it can be made arbitrarily small-within the confines of the continuum hypothesis), and this implies that the integrand must be zero everywhere within R (t). If this were not so (e.g., the integral is zero because there are positive and negative contributions that cancel), we could subdivide R (t) into smaller regions over which the integral was either positive or negative, and hence violating the fact that it is actually zero. Thus, we conclude that

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}\left(\rho v_y\right) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (2.2.3)$$

This is the differential form of the continuity equation, the expression for mass conservation in a flowing system [21, 22].

2.2.4 Momentum balance-the Navier-Stokes equations:

In this section we will derive the equations of motion for incompressible fluid flows, i.e., the Navier-Stokes (N-S) equations. We begin by stating a general force balance consistent with Newton's second law of motion, and then formulate this specifically for a control volume consisting of a fluid element. Following this we will employ the Reynolds transport theorem which we have already discussed, and an argument analogous to that used in deriving the continuity equation to obtain the differential form of the momentum equation. We then develop a multi-dimensional form of Newton's law of viscosity to evaluate surface forces papering in this equation and finally arrive at the N-S. Equations [21, 22, 23].

2.2.5 A basic force balance; Newtown's second of motion:

We begin by recalling that because we cannot readily view fluids and consisting of point masses, it is not appropriate to apply Newton's second law of motion in the usual form$F = ma$. Instead, we will use a more general form expressed in words as

{Time rate of change of momentum of a material region}= {sum of forces acting on the material region}

The somewhat vague terminology "material region" is widely used, and herein it will usually be simply a fluid element. But later when we develop the control-volume momentum equation the material region will be any region of interest in a given flow problem. We also remark that we are employing the actual version of Newton's second law instead of the one usually presented in elementary physics. Namely, if we recall that momentum is mass times velocity, e.g., mu in ID, then the general statement of Newton's second law is

$$F = \frac{d(mv)}{dt} \tag{2.2.4}$$

Which collapses to the usual $F = ma$ in the case of point masses that are independent of time. At this point it is worthwhile to recall the equation for conservation of mass, equation (2.3), which we write here in the abbreviated form

$$\rho_t = (\rho u)_x + (\rho v)_y + (\rho w)_z = 0 \tag{2.2.5}$$

Containing the dependent variables $\rho, u, v \ and \ w$. It will be convenient to express the momentum equations in terms of these same variables, and to this end we first observe that the product, e.g.,ρu, is momentum per unit volume (since ρ is mass per unit volume). Thus, yet another alternative expression of Newton's second law is

$$\frac{F}{v} = \frac{d}{dt}(\rho u) \tag{2.2.6}$$

Or force per unit volume is equal to time-rate of change of momentum per unit volume. We are now prepared to develop formulas for the left-and right-hand sides formula given above [28, 29, 30 31].

2.2.6 Time-Rate of change of momentum:

As was the case in deriving the differential equation representing conservation of mass, it will agian be convenient here to choose a fluid region corresponding to a fluid element. In contrast to what was done earlier, we will restrict our region R(t) to be a fluid element from the start . If, in addition, we utilize an Eulerian view of the fluid flow we recognize that the substantial derivative should be employed to represent acceleration or, in our present case, to calculate the time-rate of change of momentum. As noted above, it is convenient for later purposes to consider the momentum per unit volume, rather than the momentum itself; so for the x component of this we would have

$$\frac{D}{Dt}\int_{R(t)} \rho u dV$$

The equivalent of mass × acceleration .Then for the complete velocity vector **U** we can write

$$\frac{\mathrm{D}}{\mathrm{Dt}}\int_{\mathrm{R(t)}} \rho \mathbf{U} \mathrm{dV} \ \{\text{time rate of change ofmomentum vector}\}(2.2.7)$$

We remind the reader that application of the substantial derivative operator to a vector is accomplished by applying it to each component individually, so the above expression actually contains three components, each of the form of that for x momentum[36,37].

2.2.7 Sum of forces:

We next consider the general form of the right hand side of the word equation given earlier, via, the sum of forces acting on the material region (fluid element in the present case). There are two main types of forces to analyze:

i) Body forces acting on the entire region R (t), denoted

$$\int_{R(t)} \boldsymbol{F}_B dV \qquad (2.2.8)$$, and

ii) Surface forces

$$\int_{S(t)} \boldsymbol{F}_S dA \qquad (2.2.9)$$

Acting only on the surface S (t) of R (t).

It is useful to view the surface S (t) as dividing the fluid into two distinct regions: one that is interior to S (t), i.e., R (t), and one that is on the outside of s(t). This implies that when we focus attention on R (t) alone, as it will be convenient to do, we must somehow account for the fact that we have discarded the outside which interacts with R (t). We do this by representing these effects as surface forces acting on S (t).

2.3 The Maxwell Boltzmann Distribution:

Scottish physicist James clerk Maxwell developed his kinetic theory of gases in 1859. Maxwell determined the distribution of velocities among the molecules of a gas. Maxwell's finding was later generalized in 1871 by a German physicist, Ludwig Boltzmann to express the distribution of energies among the molecules [32, 33, 34].

Maxwell in the gas assuming to consist of billions of molecules moving rapidly at random , colliding with each other and the walls of the container. This was qualitatively consistent with the physical properties of gases, if we accept the notion that raising the temperature causes the molecules to move faster and collide with the walls of the container more frequently.

Maxwell made four assumptions:

1) The collisions between molecules conserve energy.

2) The diameter of the molecules is much smaller than the distance between them.

3) The positions and velocities of the molecules are initially at random.

4) The molecules move between collisions without interacting as a constant speed in a straight line.

We will derive the Maxwell Boltzmann distribution, which will prove useful information about the energy.

Why use statistical mechanics to predict molecule behavior? Why not just calculate the motion of the molecules exactly?

Even though we are discussing classical physics, there exists a degree of "uncertainty" with respect to the fact that the motion of every single particle at all times cannot be determined in practice in any large system.

Even if we were only dealing with one mole of gas, we would still have to determine characteristics of 6×10^{23} molecules!!

Maxwell's theory was based on statistical averages to see if the microstates, (I.e. measurable, observable) could be predicted from the microstates.

In these section it was determined that the thermal equilibrium is established when the temperature of the subsystem are equal. So…

What is the nature of the equilibrium distribution for a system of N non-interacting gas particles? Consider the simplest case, a system of N non-interacting classical gas particles.

-Classical system:

. There are no restrictions on how many particles can be put into any one state simultaneously

. The particles are distinguishable, i.e. each particle is labeled for all time

First, well need to determine the number of microstate within any given, i.e. the number of ways in which N objects can be arranged into n distinct groups, also called the multiplicity function.

The multiplicity function- determines the number of ways in which N objects can be arranged into n containers.

Consider the first twelve letters of the alphabet: $a\ b\ c\ d\ e\ f\ \ g\ h\ i\ j\ k\ l$

Arrange the letters into 3 containers without replacement. Containers 1 holds 3 letters, containers 2 holds 4 letters, and containers 3 holds 5 letters.

$$|-\ -\ -| \qquad |-\ -\ -\ -| \qquad |-\ -\ -\ -\ -\ |$$

For the 1st slot, there are 12 possibilities.

For the 2nd slot, there are 11 possibilities.

For the 3rd slot there are 10 possibilities.

Etc …

There are **12!** Possibilities arrangements if the containers are ignored.

The multiplicity function determined the number of ways in which N objects can be arranged into n containers. Since we care about the containers but we don't care about the order of the letters within each container, we divide out the number of arrangements within each given container, resulting in:

$$\frac{12!}{3!\,4!\,5!} = 27{,}720$$

There are 27,720 ways of partitioning 12 letters into the 3 containers

In general, the number of distinct arrangements of N particles into n groups containing $N_1, N_2, \dots, N_i \dots N_n$ objects becomes:

$\frac{N!}{N_1!N_2!\dots N_i!\dots,N_n!}$ Where N_i is the number of objects in container I

The multiplicity function.

$$Q(N_1, N_2 \dots N_i \dots N_n) = \frac{N!}{N_1!N_2!\dots N_i!\dots,N_n!} = \frac{N!}{\sum_{i=1}^{n} N!}$$

In general, the possible arrangements of N_i particles into g_i sub-containers is: ${g_i}^{N_i}$

Therefore, if our system has a particular state that has a particular degeneracy, there is an additional multiplicity of ${g_i}^{N_i}$ for that particular state. Therefore the total multiplicity function for a collection of classical particles is:

$$Q(N_1, N_2 \dots N_i \dots N_n) = \left\{\frac{N!}{\sum_{i=1}^{n} N!}\right\} \sum_{i=1}^{n} {g_i}^{N_i} \qquad (2.3.1)$$

There are two physical constraints on our classical system:

1- The total number of particles must be conserved

$$\emptyset = \sum_{i=1}^{n} N_i = N$$

2- The total energy of the system must be conserved

$$\psi = \sum_{i=1}^{n} E_i N_i = U$$

From previous slide:

$$Q(N_1, N_2 \dots N_i \dots N_n) = \left\{\frac{N!}{\sum_{i=1}^{n} N!}\right\} \sum_{i=1}^{n} {g_i}^{N_i}$$

This equation will be easier to deal with if we take the logarithm of both sides:

$$\ln Q = \ln N! + \sum_{i=1}^{n} N_i \ln g_i - \sum_{i=1}^{n} \ln N_i! \qquad (2.3.2)$$

Applying starling's approximation, for large:

$$\ln x! \approx x \ln x - x \qquad (2.3.3)$$

$$\ln Q N \ln N - N + \sum_{i=1}^{n} N_i \ln g_i - \sum_{i=1}^{n} N_i \ln N_i + \sum_{i=1}^{n} N_i \qquad (2.3.4)$$

In order to maximize, we need to make use of Lagrange multipliers and constraints 1 and 2:

$$\frac{\partial}{\partial N_j} \ln Q + \alpha \frac{\partial \emptyset}{\partial N_j} - \beta \frac{\partial \varphi}{\partial N_j} = 0 \qquad (2.3.5)$$

Substituting in $\ln Q$ and constraints 1 and 2:

$$\frac{\partial}{\partial N_j} \left(N \ln N - N + \sum_{i=1}^{n} N_i \ln g_i - \sum_{i=1}^{n} \mathrm{N_i} \ln N_i + \sum_{i=1}^{n} N_i\right) + \alpha \frac{\partial}{\partial N_j}\left(\sum_{i=1}^{n} N_i\right) - \beta \frac{\partial}{\partial N_j}\left(\sum_{i=1}^{n} E_i N_i\right) = 0 \qquad (2.3.6)$$

Taking the derivative, noting that N is const ant and the only terms that are nonzero are when i= j:

$$\ln g_i - \left(1\,.\ln N_j + N_j\,.\frac{1}{N_j}\right) + 1 + \alpha - \beta E_j = 0 \qquad (2.3.7)$$

$$\ln g_i - \ln N_j + \alpha - \beta E_j = 0 \qquad (2.3.8)$$

$$\ln \frac{N_j}{g_j} = \alpha - \beta E_j \qquad (2.3.9)$$

$$f(E_j) = \frac{N_j}{g_j} = e^{\alpha - \beta E_j} \qquad (2.3.10)$$

The equation (2-20) called Maxwell Boltzmann distribution. Energy To find β, we need to determine the total number of particles, N, in the system and the total energy, E, of the system .

If the states are closely spaced in energy, they form a quasi continuum and the total number of particles, N, is given by:

$$N = \int_0^\infty f(E)D(E)dE$$

Where $D(E)$ is the density of states function.

From previous slide:

$$N = \int_0^\infty f(E)D(E)dE$$

$$f(E) = \frac{N_j}{g_j} = e^{\alpha - \beta E} \qquad (2.3.11)$$

$$D(E) = \frac{m^{3/2}\sqrt{2E}}{\hbar^3 \pi^2} \qquad (2.3.12)$$

$$N = \frac{m^{3/2}\sqrt{2E}}{ħ^3\pi^2} e^{\alpha} \int_0^{\infty} \sqrt{E} e^{-\beta E}\, dE \qquad (2.3.13)$$

Using the stander integral:

$$\int_0^{\infty} x^n e^{-\alpha x}\, dx = \frac{\Gamma(n+1)}{a^{n+1}} \qquad (2.3.14)$$

$$E = \frac{m^{\frac{3}{2}}\sqrt{2}}{ħ^3\pi^2} e^{\alpha} \frac{\Gamma(\frac{5}{2})}{\beta^{\frac{5}{2}}} \qquad (2.3.15)$$

The average energy per particle is given as:

$$\frac{\bar{E}}{N} = \frac{3}{2} K_B T \qquad (2.3.16)$$

$$K_B = 8.6175 \times 10^{-5} \frac{ev}{K}$$

Where K_B is the Boltzmann constant.

Equation the average energy per particle with the ratio of our equations for E and N:

$$\frac{E}{N} = \frac{\frac{m^{\frac{3}{2}}\sqrt{2}}{ħ^3\pi^2} e^{\alpha} \frac{\Gamma(\frac{5}{2})}{\beta^{\frac{5}{2}}}}{\frac{m^{\frac{3}{2}}\sqrt{2}}{ħ^3\pi^2} e^{\alpha} \frac{\Gamma(\frac{3}{2})}{\beta^{\frac{3}{2}}}} = \frac{3}{2} K_B T$$

$$= \frac{\bar{E}}{N} \qquad (2.3.17)$$

$$\frac{\Gamma(\frac{5}{2})}{\beta^{\frac{5}{2}}} \frac{\beta^{\frac{3}{2}}}{\Gamma(\frac{3}{2})} = \frac{3}{2} K_B T \qquad (2.3.18)$$

(1+z)=z (z)

$$\frac{3}{2\beta} = \frac{3}{2}K_B T$$

$$\beta = \frac{1}{K_B T} \tag{2.3.19}$$

Now we have β but we still need α.

$$\alpha = \frac{\mu}{K_B T}$$

Where μ is the chemical potential. We will find out that the chemical potential, μ, is exactly equal to the Fermi Energy, E_F leaving us with an α of:

$$\alpha = \frac{E_F}{K_B T}$$

From previous slides:

$$f(E_j) = \frac{N_j}{g_j} = e^{\alpha - \beta E_j} \tag{2.3.20}$$

$$\beta = \frac{1}{K_B T}$$

Substituting α and β into$f(E_j)$:

$$f(E_j) = \frac{N_j}{g_j} = e^{\frac{E_F - E_j}{K_B T}} \tag{2.3.21}$$

From previous silde:

$$f(E_j) = \frac{N_j}{g_j} = e^{\frac{E_F - E_j}{K_B T}}$$

Reversing terms in the numerator of the exponent:

$$f\left(E_j\right)=\frac{N_j}{g_j}=e^{\frac{-(E_j-E_F)}{K_BT}} \tag{2.3.22}$$

The distribution gives the number of particles in the j state, where the j^{th} state has degeneracy, g_j

If we want to find the probability of finding the particle in the j^{th} state, we need to normalize, well start by dividing the number of particles in the j^{th} state. N_j, by the total number of particles, N.

Using constraint 1 and f (E):

$$N=\sum_{j=1}^{n} N_j=e^{\alpha}\sum_{j=1}^{n} g_j e^{-E_j/K_BT} \tag{2.3.23}$$

Solving fore^{α}:

$$e^{\alpha}=\frac{N}{\sum_{j=1}^{n} g_j e^{-E_j/K_BT}}$$

Substituting e^{α}into$f(E_j)$:

$$f\left(E_j\right)=N_j=g_j e^{\alpha} e^{-E_j/K_BT} \tag{2.3.24}$$

$$N_j=\frac{N g_j e^{-E_j/K_BT}}{\sum_{j=1}^{n} g_j e^{-E_j/K_BT}}$$

Normalizing:

$$\frac{N_j}{N}=\frac{g_j e^{-E_j/K_BT}}{\sum_{j=1}^{n} g_j e^{-E_j/K_BT}}$$

From previous slide:

$$\frac{N_j}{N} = \frac{g_j e^{-E_j/K_BT}}{\sum_{j=1}^{n} g_j e^{-E_j/K_BT}}$$

This normalized distribution is the probability,P_j, of finding the particle in the j^{th} stat with energyE_j:

$$P_j = \frac{N_j}{N} = \frac{g_j e^{-E_j/K_BT}}{\sum_{j=1}^{n} g_j e^{-E_j/K_BT}} \qquad (2.3.25)$$

Since j is just a dummy index, the probability of a particle having energy E_r is:

$$P_r = \frac{g_r e^{-E_r/K_BT}}{\sum_{j=1}^{n} g_j e^{-E_j/K_BT}}$$

If the degeneracy factor is 1, i.e. no repeatable arrangements, the probability become:

$$P_r = \frac{e^{-E_r/K_BT}}{\sum_{j=1}^{n} e^{-E_j/K_BT}} \qquad (2.3.26)$$

The mean value of a physical observable:

$$\bar{y} = \frac{\sum_r P_r\, y_r}{\sum_r P_r} \qquad (2.3.27)$$

The mean energy of a system can be determined by:

$$\bar{E} = \frac{\sum_r e^{-E_r/K_BT}}{\sum_r e^{-E_r/K_BT}} \qquad (2.3.28)$$

The classical partition function:

$$z = \sum_r e^{-E_r/K_BT} \quad ; \quad \beta = \frac{1}{K_BT} \quad \rightarrow \quad z = \sum_r e^{-\beta E_r}$$

We can use the classical partition function to easily calculate the mean value of the energy.

First, we need to note the following relationships:

$$\sum_r E_r\, e^{-\beta E_r} = -\sum_r \frac{\partial}{\partial\beta} e^{-\beta E_r}$$

$$-\sum_r \frac{\partial}{\partial\beta} e^{-\beta E_r} = -\frac{\partial}{\partial\beta}(\sum_r e^{-\beta E_r}) = -\frac{\partial}{\partial\beta} z$$

Substituting into the mean energy equation:

$$\bar{E} = \frac{-\frac{\partial}{\partial\beta} z}{z}$$

$$\bar{E} = -\left(\frac{1}{z}\right)\frac{\partial}{\partial\beta} z$$

The mean energy equation:

$$\bar{E} = -\frac{\partial}{\partial\beta}(\ln z) \qquad (2.3.29)$$

2.4 Maxwell's solution for wave propagation in a conductive medium:

According to GSR model the linear energy is given by

$$E = g_{00}^{1/2}\beta m_\circ c^2 + g_{00}^{-1/2} c\alpha.p$$

$$= \beta m_\circ c^2\left(1 + \frac{V}{m_\circ c^2}\right)$$

$$+ c\left(1 - \frac{V}{m_\circ c^2}\right)\alpha.p \qquad (2.4.1)$$

Where

$g_{00}^{1/2} = 1 + \frac{V}{m_\circ c^2} \quad , g_{00}^{-1/2} = 1 - \frac{V}{m_\circ c^2}$

Multiplying both side of equation (2.4.1) by ψ gives

$$E\psi = c\left(1 - \frac{V}{E}\right)\alpha . p\psi + \left(1 + \frac{V}{E}\right)\beta m_\circ \psi$$

$$E^2\psi = cE\left(1 - \frac{V}{E}\right)\alpha . p\psi + E\left(1 + \frac{V}{E}\right)\beta m_\circ \psi$$

$$\therefore E^2\psi = c(E - V)\alpha . p\psi + (E + V)\beta m_\circ c^2\psi \qquad (2.4.2)$$

$$E^2\psi = c\alpha . pE\psi - cV\alpha . p\psi + \beta m_\circ c^2 E\psi + \beta m_\circ c^2 V\psi$$

Where

$$E - \hat{\mathrm{H}} = i\hbar\frac{\partial}{\partial t} \quad , \quad \text{and} \quad p = \hat{p} = \frac{\hbar}{i}\vec{\nabla} \qquad (2.4.3)$$

From equation (2.4.2) and (2-4-3)

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = c\hbar^2\alpha . \vec{\nabla}\left(\frac{\partial\psi}{\partial t}\right) - c\frac{\hbar}{i}V\alpha . \vec{\nabla}\psi + i\hbar\beta m_\circ c^2\left(\frac{\partial\psi}{\partial t}\right) + \beta m_\circ c^2 V\psi \qquad (2.4.4)$$

From (2-4-4), by suggesting a solution

$$\psi = u(r)e^{-i\omega_\circ t} = ue^{-i\omega_\circ t} = ue^{\frac{iE}{\hbar}t}$$

$$\frac{\partial\psi}{\partial t} = -i\omega_\circ\psi \quad , \quad \frac{\partial^2\psi}{\partial t^2} = -i\omega_\circ\psi \qquad (2.4.5)$$

A direct substitution of (2.4.5) in (2.4.4) gives

$$\hbar^2\omega_\circ\psi = c\hbar^2\alpha\omega_\circ . \nabla\psi + ic\hbar V\alpha . \vec{\nabla}\psi - \beta m_\circ c^2\hbar\omega_\circ . \psi + \beta m_\circ c^2 \mathrm{V}\psi \qquad (2.4.6)$$

Where

$$E = \hbar\omega_\circ \ , \qquad E_0 = m_\circ c^2$$

$$E^2\psi = c\hbar E\alpha . \nabla\psi + ic\hbar V\alpha . \nabla\psi - \beta 3EE_0\psi + \beta E_0 V\psi \ (2.4.7)$$

$$e^{-i\omega_\circ t}(E^2 u) = e^{-i\omega_\circ t}(c\hbar E\alpha . \nabla u + ic\hbar V\alpha . \nabla u) + e^{-i\omega_\circ t}(\beta E + \beta V)E_0 u \quad (2.4.8)$$

This time decaying exponential term can be cancelled on both sides to get

$$(E^2-\beta(E+V)E_0)u = c\hbar(E+iV)\alpha.\nabla u \qquad (2.4.9)$$

This can be written as

$$c_1 u - c_2 V u = c_3\alpha.\nabla u + ic_4 V\alpha.\nabla u \qquad (2.4.10)$$

Where

$$c_1 = E^2-\beta E E_0 \ , c_2 = +\beta E_0 \, , c_3 = \ c\hbar E$$

$$c_4 = c\hbar \qquad (2.4.11)$$

Travelling wave solution

$$\psi = Ae^{i(Kr-\omega t)} \qquad (2.4.12)$$

$$\psi = e^{-i\omega t}u = e^{-i\omega t}u(\underline{r}) \qquad (2.4.13)$$

$$u = Ae^{iKr} \ , \qquad \nabla u = iKu \qquad (2.4.14)$$

$$c_1 u - \ c_2 V u = iK.[a](c_3 + ic_4 \mathrm{V})\mathrm{u} \qquad (2.4.15)$$

Equation coefficients of u and Vu, yields

$$c_1 = iK.\alpha c_3 \, , \qquad K = \frac{-ic_1}{c_3\alpha\cos\theta} = -iK_0$$

From equation (2.4.11)

$$K = \frac{-i(E-\beta E_0)}{c\hbar\alpha\cos\theta} = -iK_0 \qquad (2.4.16)$$

$$c_2 = -i^2 k.\alpha c_4 = k.\alpha c_2 \qquad (2.4.17)$$

$$k.\alpha = k\alpha\cos\theta = \frac{c_2}{c_4}$$

$$k = \frac{c_2}{c_4\alpha\cos\theta}$$

$$K = \frac{\beta E_0}{c\hbar\alpha\cos\theta} = K_1 \qquad (2.4.18)$$

The first expression for$k$ in equation (2.4.12) where $k = -ik_0 \; gives$

$$\psi = Ae^{k_0 r}e^{-\omega t} \qquad (2.4.19)$$

The second expression for k in equation (2.4.12) where $k \to k_1$ gives

$$\psi = e^{i(k_1 r-\omega t)} = u = Ae^{ik_1 r} \qquad (2.4.20)$$

Consider the outer most shell where electrons occupy this sell when the radius of the atom is a. In this case

$$|u(a)|^2 = 1$$

$$u(a) = 1$$

$$\cos k_1 a + i\sin k_1\, a = 1 \qquad (2.4.21)$$

Thus

$$\cos k_1 a = 1\ ,\sin k_1\, a = 0\ \ ,k_1 a = 2\pi n$$

There fore

$$k_1 = \frac{2\pi n}{a} \qquad (2.4.22)$$

Thus the momentum is given by

$$p = \hbar k_1 = \frac{\hbar n}{a} \qquad (2.4.23)$$

Hence the energy takes the form

$$E^2 = c^2P^2 + m_0^2c^4 \qquad (2.4.24)$$

$$E^2 = \frac{c^2\hbar^2n^2}{a^2} + m_0^2c^4 \qquad (2.4.25)$$

This liner energy is given by

$$E = c\alpha.p + \beta m_0c^2$$

$$E = \frac{c\alpha\hbar n}{a} + \beta m_0c^2 \qquad (2.4.26)$$

It is very interesting to note that the velocity is given by

$$V_0 = \lambda_0 f = \frac{\omega}{k_0} = \frac{\omega c_3 \cos\theta}{c_1} \tag{2.4.27}$$

Becomes infinite when

$$c_1 = E(E - \beta E_0) = 0$$

$$E = \beta E_0 \tag{2.4.28}$$

Where equation (2.4.27) gives

$$V_0 = \infty \tag{2.4.29}$$

In this case equation (2.4.2.16) gives

$$k_0 = 0 \tag{2.4.30}$$

Thus equation (2.4.19) become in the form

$$\psi = \mathrm{A}e^{i\omega t} \tag{2.4.31}$$

This represent a stationary oscillating wave. Fortunately equations (2.4.29) and (2.4.31) describe the behavior of biophotons which are stationary waves that spread themselves simultaneously through the surrounding media [44, 45].

2.5 Super Fluids:

Superfluidity is closely related to Bose-Einstein condensation. In a phenomenological level, superfluid can flow through narrow capillaries or slits without dissipating energy. Superfluid does not possess the shear viscosity. The superfluid of liquid ^{4}He, below the so-called λ-point, was discovered by Kapitza and independently by Allen and Misener. Soon after Landau explained that if the excitation spectrum satisfies certain criteria, the motion of the fluid does not cause the energy dissipation. These Landau criteria are met by the Bogoliubov excitation spectrum associated with the Bose-Einstein condensate consisting of an interacting Bose gas and thus establish the first connection between superfluidity and BEC. The connection between the two phenomena is further established in a deeper level through the relationship between irrotationality of the superfluid and the global phase of the BEC order parameter. This is the first subject of this chapter. The second subject of this chapter is the rotational properties of the irrotational superfluid, with special focus on the quantized vortices.[11,13,14]

2.5.1 Landau's criteria of super fluidity:

Landau's theory of super fluids is based on the Galilean transformation of energy and momentum. Let E and **P** be the energy and momentum of the fluid in a reference frame K. If we try to express the energy and momentum of the same fluid but in a moving frame $K^/$, which has a relative velocity v with respect to a reference frame K, we have the following relations:

$$P^/ = P - mv, \tag{2.5.1.1}$$

$$E^/ = \frac{|P^/|^2}{2m} = \frac{1}{2m}|P - mv|^2$$

$$= E - P.v + \frac{1}{2}m|v|^2 \tag{2.5.1.2}$$

where$E = \frac{|P^/|^2}{2m}$ and M is the total mass of the fluid.

We first consider a fluid at zero temperature, in which all particles are in the ground state and flowing along a capillary at constant velocity v. If the fluid is viscous, the motion will produce dissipation of energy via friction with the capillary wall and decrease of the kinetic energy. We assume that such dissipative processes take place through the creation of elementary excitation, which is the Bogoliubov quasi-particle for the case of an interacting Bose gas. Let us first describe this process in the reference frame K which, rather confusingly, moves with the same velocity v of the fluid. In this reference frame, the fluid is at rest. If a single elementary excitation with a momentum p appears in the fluid, the total energy of the fluid in the reference frame K is $E_0 + \varepsilon(\mathrm{p})$, where E_0 and $\varepsilon(\mathrm{p})$ are the ground state energy and the elementary excitation energy. Let us move to the moving frame $K^/$, in which the fluid moves with a velocity v but the capillary is at rest. In this moving frame K^0 which moves with the velocity

$-v$ with respect to the fluid, the energy and momentum of the fluid are given, setting $v = -v$ in (2.5.2.2) and (2.5.2.1), by

$$p^{/} = p + mv \quad (2.5.2.3)$$

$$E^{/} = E_0 + \varepsilon(\mathrm{p}) + \mathrm{p}.v + \frac{1}{2}mv^2 \qquad (2.5.2.4)$$

The above results indicate that the changes in energy and momentum caused by the appearance of one elementary excitation are $\varepsilon(p) + p.v$ and p, respectively. Spontaneous creation of elementary excitations, i.e. energy dissipation, can occur if and only if such a process is energetically favorable. This means if the energy of an elementary excitation, in the moving frame K^0 where the capillary is at rest, so that a thermal equilibrium condition is satisfied, is negative:

$$\varepsilon(p) + p.v < 0 \qquad (2.5.2.5)$$

The dissipation of energy occurs. The above condition is satisfied when $|v| > \frac{\varepsilon(p)}{|p|}$ and $p.v < 0$, i.e. when the elementary excitation has the momentum p opposite to the fluid velocity v and the fluid velocity $|v|$ exceeds the critical value,

$$v_c = min_p \frac{\varepsilon(p)}{|p|} \qquad (2.5.2.6)$$

where the minimum is calculated over all the values of p. If instead the fluid velocity v is smaller than (2.5.2.6), then no elementary excitation will be spontaneously formed. Thus, the Landau's criteria of superfluidity is summarized as the relative velocity between the fluid and the capillary is smaller than the critical value,$v < v_c$.

One can easily conclude that the weakly interacting Bose gas at zero temperature satisfies the Landau's criteria of super fluidity and that the critical velocity is given by the sound velocity. Strongly interacting fluids such as liquid ^{4}He also fulfil the Landau criteria but in this case the critical velocity is smaller than the sound velocity due to the complicated excitation spectrum. It is easily understood that the critical velocity decreases with the decrease in the particle-particle interaction and disappears in the limit of an ideal gas because $v_c = min_p \frac{\varepsilon(p)}{|p|} = 0$ for $(p) = \frac{p^2}{2m}$.The particle-particle interaction is a crucial requirement in the appearance of superfluidity.

2.5.2 Superfluidity at finite temperatures:

Let us next consider a uniform Bose-Einstein condensed fluid at a finite temperature. We assume the thermodynamic properties of the system are described by the Bogoliubov quasi-particles in thermal equilibrium distributions. According to the above argument, no new excitations can be created directly by the condensate due to the motion of the superfluid with respect to the capillary. However, the quasi-particles are excited thermally and the fluid associated with the quasi-particles is not superfluid but normal fluid. These elementary excitations can collide with the capillary walls and dissipate their energies and momenta. Thus, we have the two fluid components at a finite temperature: a superfluid without viscosity and a normal fluid with viscosity. Collisions establish thermodynamic equilibrium in the normal fluid in the frame where the capillary is at rest (capillary frame).

If the energy and momentum of the quasi-particle are ε(p) and p in the frame where the superfluid is at rest (superfluid frame), the energy of the same quasi-particle in the capillary frame becomes $\varepsilon(p) + p.v_s$where v_s is the relative velocity of the superfluid and the capillary. The Bogoliubov quasi-particles obey the thermal equilibrium distribution in the capillary frame (not in the superfluid frame). Thus, the quasi-particle population is given by

$$N_P = \frac{1}{exp\left[\frac{\varepsilon(p) + p.v_s}{K_B T}\right] - 1} \qquad (2.5.2.1)$$

If$\varepsilon(p) + p.v_s > 0$, i.e$|v_s| < min_p \frac{\varepsilon(p)}{|p|}$ the quasi-particle population N_p is positive for all values of p. Therefore, we can conclude the coexistence of the two fluids becomes possible. Notice that the condition for the positive N_P , $v_s < min_p \frac{\varepsilon(p)}{|p|}$ is identical to the Landau's criteria of superfluidity[15,16,17].

Chapter Three

Literature Review

3.1 Introduction:

Many attempts were made to modify Schrodinger equation to describe bulk matter [1, 17, 18, and 19]. Same of them uses the approach of complex many body problem [20, 21, 22], will some uses expression of energy in a frictional medium [23, 24, 25]. Some attempts link quantum laws with statistical physics and fluid or superfluid laws [26, 27,31 and 41]. Some of these attempts are exhibited here.

3.2 Potential Dependent Frictional Schrodinger Equation:

By treating particles as harmonic oscillator is obtained the friction energy related to the momentum. The energy and the corresponding Newtonian operator is found. This result in a new Schrodinger equation accounting for the effect of friction. This new equation shows that the energy and mass are quantized, if one treats particles as strings. The radioactive decay law and collision probability is also derived [2, 9,24,36].

3.2.1 Schrodinger equation for frictional medium:

According to Plank and de Broglie hypothesis the quantum quanta are treated as wave packets.

Pure waves is a wave packet consisting of single wave having specific wave length .while a localized particle is a wave packet having a very large of interfering waves having different wave lengths . This means that any quantum system is a single or aggregate of oscillators. Moreover, according to string theory matter building blocks are treated as vibrating string. Motivated by all there hypothesis, the energy dissipated by fraction can be derived consider now a fractional force E_f in terms of mass m, relaxation time τ and velocity v to be

$$E_f = \frac{mv}{\tau} \tag{3.2.1.1}$$

Considering matter building blocks as oscillators

$$v = v_\circ e^{i\omega t} \tag{3.2.1.2}$$

Thus, the displacement is given by:

$$x = \int v dt = v_\circ \int e^{i\omega t}\, dt = \frac{v_\circ}{i\omega} e^{i\omega t} = \frac{v}{i\omega} \tag{3.2.1.3}$$

The total dissipative energy E_f is given by:

$$\begin{aligned} E_f &= \int E_f \,.dx = \frac{m}{i\omega t}\int v\, dv = \frac{mv^2}{2i\omega t} = \frac{-imv^2}{2\omega t} = \frac{-i}{\omega t}\left(\frac{1}{2}mv^2\right) \\ &= \frac{-i}{\omega t}\left(\frac{p^2}{2m}\right) \end{aligned} \tag{3.2.1.4}$$

But according to Newtonian mechanics the total energy can be expressed in terms of the kinetic and potential energy V in the form

$$E = K + V = \frac{p^2}{2m} + V \tag{3.2.1.5}$$

Thus according to Eq. (3-3-5) and Eq. (3-3-4) E_f is given by

$$E_f = \frac{-i}{\omega t}(E - V) \tag{3.2.1.6}$$

But using plank hypothesis the energy E is given by

$$E = \hbar\omega \tag{3.2.1.7}$$

In view of Eqs. (3.3.6) and (3.3.7) the frictional energy is given by

$$E_f = \frac{-i}{\hbar\omega t}(E - V) = \frac{i\hbar}{\tau E}(V - E)$$

$$E_f = \frac{i\hbar}{\tau}\left(\frac{V}{E} - 1\right) \tag{3.2.1.8}$$

Thus the Hamiltonian classical relation for a particle in a fractional medium is given by

$$\begin{aligned} E = H = \frac{P^2}{2m} + V + \frac{i\hbar}{\tau}\left(\frac{V}{E} - 1\right) \\ = \frac{P^2}{2m} + V + \frac{i\hbar}{\tau}\left(\frac{V - E}{E}\right) \end{aligned} \tag{3.2.1.9}$$

Therefore

$$E^2 = \left(\frac{P^2}{2m} + V\right)E + \frac{i\hbar}{\tau}(V - E) \tag{3.2.1.10}$$

To find the Schrodinger equation corresponding to this relation multiplies both sides of Eq. (3-3-10) by ψ to get:

$$E^2\psi = \left(\frac{P^2}{2m} + V\right)E\psi + \frac{i\hbar}{\tau}(V - E)\psi \tag{3.2.1.11}$$

Considering the wave function

$$\psi = Ae^{\frac{i}{\hbar}(px - et)} \tag{3.2.1.12}$$

Hence

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi$$

$$E\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\frac{i^2}{\hbar^2}E^2\psi \qquad (3.2.1.13)$$

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = E^2\psi \qquad (3.2.1.14)$$

Similarly differentiating the wave function respect to x yields

$$\frac{\partial\psi}{\partial X} = \frac{i}{\hbar}P\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = P\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = \frac{i^2}{\hbar^2}P^2\psi$$

$$-\hbar^2\frac{\partial^2\psi}{\partial x^2} = -\hbar^2\nabla^2\psi \qquad (3.2.1.15)$$

Thus inserting Eqs. (3.2.1.13), (3.2.1.14) and (3.2.1.15) into Eq. (3.2.1.11) yields

$$-\hbar^2\frac{\partial^2\psi}{\partial x^2} = -\left(\frac{\hbar^2}{2m}\nabla^2 + V\right)i\hbar\frac{\partial\psi}{\partial t} + \frac{i\hbar}{\tau}\left(-i\hbar\frac{\partial\psi}{\partial t} + V\psi\right)$$

$$-\hbar^2\frac{\partial^2\psi}{\partial x^2} = i\hbar\left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{\tau}\frac{\partial\psi}{\partial t} + \frac{i\hbar}{\tau}V\nabla \quad (3.2.1.16)$$

3.2.2 Harmonic oscillator solution:

To see how fraction force consider the solution of Eq. (3.2.1.12) in the form

$$\psi = e^{-i\frac{E}{\hbar}tu(v)} = f(t)u(v) = fu$$

$$\frac{\partial\psi}{\partial t} = -i\frac{E}{\hbar}f(u)$$

$$\frac{\partial^2\psi}{\partial x^2}=\frac{i^2E^2}{ħ^2}\,fu=-\frac{E^2}{ħ^2}fu \qquad (3.2.2.1)$$

A direct substitution in Eq. (3.2.1.16) gives

$$E^2fu=iħ\left(-\frac{ħ^2}{2m}\nabla^2u+Vu\right)f\left(\frac{-iE}{ħ}\right)-i\frac{Eħ^2}{ħ\tau}fu$$
$$+i\frac{ħ}{\tau}Vfu \qquad (3.2.2.2)$$

Dividing both sides of Eq. (3.2.2.2) by f yields

$$E^2u=+E\left(-\frac{ħ^2}{2m}\nabla^2u+Vu\right)-i\frac{Eħ}{\tau}u+i\frac{ħ}{\tau}Vu \qquad (3.2.2.3)$$

Dividing both sides of Eq. (3.2.2.3) by +E yields

$$\left(E+\frac{iħ}{\tau}\right)u=-\frac{ħ^2}{2m}\nabla^2u+V\left(1+\frac{iħ}{E\tau}\right)u$$

$$-\frac{ħ^2}{2m}\nabla^2u+c_1Vu=E_1u \qquad (3.2.2.4)$$

Where

$$c_1=1+\frac{iħ}{E\tau}$$

$$E_1=E+\frac{iħ}{\tau} \qquad (3.2.2.5)$$

For harmonic oscillator one finds

$$V=\frac{1}{2}kx^2 \qquad (3.2.2.6)$$

Thus substituting this expression in Eq. (3.2.2.4) gives

$$-\frac{\hbar^2}{2m}\nabla^2 u + c_1\frac{1}{2}kx^2 = E_1 u \qquad (3.2.2.7)$$

Let now

$$k_\circ = c_1 k \qquad (3.2.2.8)$$

Therefore equation (3.2.2.7) became

$$-\frac{\hbar^2}{2m}\nabla^2 u + \frac{1}{2}k_\circ x^2 = E_1 u \qquad (3.2.2.9)$$

Thus substituting Eq. (3.2.2.5) into Eq. (3.2.2.9) gives

$$E_1 = E + \frac{i\hbar}{\tau} = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (3.2.2.10)$$

$$E = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{i\hbar}{\tau} \qquad (3.2.2.11)$$

The frequency is given according to Eq. (3.2.2.8) and Eq. (3.2.2.5) to be

$$k_{\circ=} m\omega^2$$

$$c_1 k = \left(1 + \frac{i\hbar}{\tau E}\right)k = m\omega^2$$

$$\left(E + \frac{i\hbar}{\tau E}\right)k = m\omega^2 E \qquad (3.2.2.12)$$

Thus

$$E = \left(\frac{m\omega^2}{k} - 1\right)\frac{i\hbar}{\tau}^{-1} \qquad (3.2.2.13)$$

From (3.2.1.12) and (3.2.1.13)

$$0 = -\frac{m\omega^2}{k} + \left(n + \frac{1}{2}\right)\hbar\omega$$

$$m = \left(1 + \frac{i}{\tau(n + \frac{1}{2})}\right)\frac{k}{\omega^2} \qquad (3.2.2.14)$$

Thus, from Eq. (3.2.2.14) one finds the mass is quantized

3.2.3 Radioactive decay low and collision probability:

Consider now Eq. (3.2.1.16) for constant potential V_o

Using the separation of variables let the wave function Ψ be in the form

$$\psi(r,t) = f(t)u(r) = fu \qquad (3.2.3.1)$$

A direct substitution of equation (3.2.3.1) in equation (3.2.1.16) gives

$$-\hbar^2 u\frac{\partial^2 f}{\partial t^2} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_\circ\right)u\left(i\hbar\frac{\partial f}{\partial t}\right) + \frac{i\hbar}{\tau}V_\circ uf + \frac{\hbar^2}{\tau}u\frac{\partial f}{\partial t}$$

Thus

$$\left(-\hbar^2 u\frac{\partial^2 f}{\partial t^2} - \frac{i\hbar}{\tau}V_\circ f - \frac{\hbar^2}{\tau}\frac{\partial f}{\partial t}\right)u = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_\circ\right)u\left(i\hbar\frac{\partial f}{\partial t}\right) \qquad (3.2.3.2)$$

Divide both sides of Eq. (3.2.3.2) by fu to get

$$\left(i\hbar\frac{\partial f}{\partial t}\right)^{-1}\left(-\hbar^2 u\frac{\partial^2 f}{\partial t^2} - \frac{iV_\circ\hbar}{\tau}f - \frac{\hbar^2}{\tau}\frac{\partial f}{\partial t}\right) = \frac{1}{u}\left(\frac{\hbar^2}{2m}\nabla^2 + V_\circ\right)u = E_\circ \qquad (3.2.3.3)$$

Taking the time part of Eq. (3.2.3.3) only gives

$$-\hbar^2\frac{\partial^2 f}{\partial t^2} - \frac{iV_\circ\hbar}{\tau}f - \frac{\hbar^2}{\tau}\frac{\partial f}{\partial t} = i\hbar E_\circ\frac{\partial f}{\partial t} \qquad (3.2.3.4)$$

Consider the case when the potential vanishes

$$V_\circ = 0 \qquad (3.2.3.5)$$

Hence

$$-\hbar^2\frac{\partial^2 f}{\partial t^2}-\frac{\hbar^2}{\tau}\frac{\partial f}{\partial t}=i\hbar E_\circ\frac{\partial f}{\partial t} \qquad (3.2.3.6)$$

Consider now a solution

$$f=Ae^{-\frac{i}{\hbar}Et}$$

$$\frac{\partial f}{\partial t}=-\frac{i}{\hbar}Ef$$

$$\frac{\partial^2 f}{\partial t^2}-\frac{i^2}{\hbar^2}E^2f=-\frac{E^2}{\hbar^2}f \qquad (3.2.3.7)$$

Inserting Eq. (3.2.3.7) in Eq. (3.2.3.6) yields

$$E^2f+\frac{i\hbar}{\tau}Ef=i\hbar E_\circ\left(-\frac{i}{\hbar}Ef\right) \qquad (3.2.3.8)$$

Dividing both sides of Eq. (3.2.3.8) by f gives

$$E^2+\frac{i\hbar}{\tau}E=E_\circ E \qquad (3.2.3.9)$$

Rearranging both sides of Eq. (3.2.3.9) gives

$$E^2=\left(E_\circ-\frac{i\hbar}{\tau}\right)E \qquad (3.2.3.10)$$

Dividing both sides of Eq. (3.2.2.4) by E gives

$$E=\left(E_\circ-\frac{i\hbar}{\tau}\right) \qquad (3.2.3.11)$$

Inserting Eq. (3.2.3.1) in Eq. (3.2.37) gives

$$f=Ae^{-\frac{i}{\hbar}\left(E_\circ-\frac{i\hbar}{\tau}\right)t}=Ae^{-\frac{t}{\tau}}e^{-\frac{i}{\hbar}E_\circ t}$$

Hence

$$f=Ae^{-\frac{t}{\tau}}e^{-\frac{i}{\hbar}E_\circ t} \qquad (3.2.3.12)$$

Since the probability and number of particles are given by

$$n = |f|^2 = ff^{/} = A^2 e^{-\frac{2t}{\tau}} \qquad (3.2.3.13)$$

Eq. (3.2.3.13) is the ordinary radioactive decay low with

$$\lambda = \frac{2}{\tau} \quad , \quad n_\circ = A^2 \qquad (3.2.3.14)$$

i.e.

$$n = n_\circ e^{-\lambda t} \qquad (3.2.3.15)$$

This expression also gives collision probability p with

$$p = n \quad , \quad p_\circ = A^2$$

$$\tau_\circ = \frac{\tau}{2} \qquad (3.2.3.16)$$

To get

$$p = p_\circ e^{-\frac{t}{\tau_\circ}} \qquad (3.2.3.17)$$

Eq. (3.2.3.17) is the ordinary collision probability relation [47, 48, and 49].

3.3 Quantum Schrodinger String Theory for Frictional Medium and Collision:

Maxwell's equation for decaying wave due to friction, beside aclassical and quantum expression for oscillating string energyare used to derive a useful expression for particle energy in frictional resistive media. This expression is used to derive Schrodinger equation for oscillating string in resistive media. This new quantum reduces to the ordinary. Schrodinger equation in the absence of friction it also gives collision probability similar to that obtained by transport equation. This new equation is used to derive an expression for energy lost by friction by the vibrating string. This energy is shown to be quantized [30,33,50].

3.3.1 Maxwell's Equations for Time Decaying Wave in Resistive Medium:

Consider an electromagnetic wave enters a medium of conductivity σ, and electric polarization $\dot{P}$. Maxwell's equations for this medium are given by

$$-\nabla^2 E - ᾓ\sigma\, \partial E/\partial t + ᾓ\varepsilon\, \partial^2 E/\partial t^2 = -ᾓ\, \partial^2 \dot{P}/\partial t^2 \quad (3.3.1.1)$$

The electric field intensity decays in this case and can be described by the relation

$$= E_0 e^{-\mu_k t} e^{i(kx-\omega t)} \qquad (3.3.1.2)$$

The corresponding displacement is given by

$$x = x_0 e^{-\mu_k t} e^{i(kx-\omega t)} = \frac{x_0}{E_0} E \qquad (3.3.1.3)$$

The electric polarization terms is defined to be

$$\dot{P} = enx = en\frac{x_0}{E_0} E \qquad (3.3.1.4)$$

With the aid of equations (3.3.1.2) and (3.3.1.4) of equation

(3.3.1.1) becomes

$$K^2 + \frac{{\mu_k}^2}{c^2} - \frac{\omega^2}{c^2} - ᾓ\sigma\tilde{\alpha} + \frac{2i\mu_k\,\omega}{c^2} - iᾓ\sigma\omega$$
$$= \frac{en\omega^2 ᾓ_0\, x_0}{E_0} \qquad (3.3.1.5)$$

Equation (3.3.1.5) can be simplified by using the relation

$$K = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{\omega}{c} \qquad (3.3.1.6)$$

And by assuming

$$\mu_k \ll c \qquad (3.3.1.7)$$

Where c is the speed of light in vacuum which is large. Thus

equation (3.3.1.5) becomes

$$-\text{ῃ̈̀}\sigma\mu_k + \frac{2i\mu_k\,\omega}{c^2} - i\text{ῃ̈̀}\sigma\omega = en\omega^2\text{ῃ̈̀}\frac{x_0}{E_0} \qquad (3.3.1.8)$$

Comparing real parts on both sides of equation (3.3.1.8) yields

$$\sigma\mu_k = -en\omega^2\mu\text{ῃ̈̀}\frac{x_0}{E_0} \qquad (3.3.1.10)$$

3.3.2 Friction Coefficient and Relaxation:

It is quite natural to relate frictional coefficientμ_k to the relaxation time τ. This due to the fact that by physical intuition are one can deduce that shorter the relaxation time, the bigger frictional coefficient. This can show also mathematics, by using the expression of energy dissipated by friction, which for oscillation particle is given by

$$E_f = \frac{m}{\tau} = V_e \qquad (3.3.2.1)$$

$$E_f = \frac{m\omega A^2}{2\tau} = \frac{m\omega^2 A^2}{2\tau\omega} = \frac{E}{\tau\omega} \qquad (3.3.2.2)$$

Where the effective displacement and velocity υ , are related to the maximum displacement by A by

$$x_e = \frac{A}{\sqrt{2}} \qquad V_e = \omega x_e = \frac{\omega A}{\sqrt{2}}$$

Where the classical energy of the oscillator is

$$E = \frac{1}{2}\omega^2 A^2 \qquad (3.3.2.3)$$

The oscillator frequency for classical and quantum system is the same. Thus one can write the quantum oscillator energy as

$$E = \hbar\omega \qquad (3.3.2.4)$$

Thus inserting (3.2.2.3) in (3.2.2.1) yields

$$E_f = \frac{\hbar E}{\tau\hbar\omega} = \frac{\hbar}{\tau} \qquad (3.3.2.5)$$

3.3.3 Derivation of frictional Schrodinger equation on the basis of frictional energy equation:

Ordinary Schrodinger is based on the postulates. The first postulate is related to the nature of micro particles. In this case the wave function takes the form

$$\psi = Ae^{i(kx-\omega t)} \qquad (3.3.3.1)$$

Using the fact that

$$P = \hbar k \quad \text{And } E_0 = \hbar\omega \qquad (3.3.3.2)$$

Therefore

$$\psi = Ae^{\frac{i}{\hbar}(Px-Et)} \qquad (3.3.3.3)$$

The second postulate is based on the classical expression of energy

$$E = \frac{P^2}{2m} + V \qquad (3.3.3.4)$$

Using these tow postulates one can derive Schrodinger equation, where

$$E_0\psi = \frac{P^2}{2m}\psi + V\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = E_0\psi$$

$$\frac{P^2}{2m}\psi = \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

Thus in three dimensions

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar^2}{2m}\nabla^2\psi + V\psi \qquad (3.3.3.5)$$

The expression of energy Eqn. (3.3.3.4) in the presence of fiction is given, with the aid of equation (3.2.3.2) to be

$$E = E_0 + E_f = \frac{P^2}{2m} + V - \frac{i\hbar}{\tau} \qquad (3.3.3.6)$$

Multiply both sides by ψ, one gets

$$E\psi = \frac{P^2}{2m}\psi + V\psi + \frac{i\hbar}{\tau}\psi \qquad (3.3.3.7)$$

But

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$\frac{\partial\psi}{\partial x} = \frac{i}{\hbar}P\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{P^2}{\hbar^2}\psi$$

In three dimensions

$$-\hbar^2\nabla^2\psi = P^2\psi \qquad (3.3.3.8)$$

Inserting Eqn. (3.2.3.8) in Eqn. (3.2.3.7), one gets

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi - i\frac{\hbar}{\tau}\psi \qquad (3.3.3.9)$$

Which is the Schrodinger equation or resistive media. One can solve this to do let equation for harmonic oscillator

$$\psi = f(t)u(r) \qquad (3.3.3.10)$$

Inserting in Eqn. (3.3.3.9) yields

$$i\frac{\hbar}{\mathrm{f}}\frac{\partial f}{\partial t} = -\frac{\hbar^2}{2mu}\nabla^2 u + V - i\frac{\hbar}{\tau} = E_0$$

Hence

$$i\hbar\frac{\partial f}{\partial t} = E_0 f \qquad (3.3.3.11)$$

For harmonic oscillator string vibrating in one dimension the potential is given by

$$V = \frac{1}{2}Kx^2$$

Thus Eqn. (3.2.3.9) reads

$$-\frac{\hbar^2}{2m}\nabla^2 u + \frac{1}{2}Kx^2 u = \left(E_0 + i\frac{\hbar}{\tau}\right)u = Eu \qquad (3.3.3.12)$$

This is the ordinary harmonic oscillator equation, with have quantized energy

$$E = \left(n_1 + \frac{1}{2}\right)\hbar\omega \qquad (3.3.3.13)$$

For Eqn. (3.2.3.9) one can suggest the solution

$$f = Ae^{-i\mu_{k0}\,t} \tag{3.3.3.14}$$

$$\mu_{k0}\,\hbar = E_0$$

The periodicity condition requires

$$f(t+T) = f(t) \tag{3.3.3.15}$$

Hence from Eqn. (3.3.3.13)

$$Ae^{-i\mu_{k0}\,(t+T)} = Ae^{-i\mu_{k0}\,t}$$

$$Ae^{-i\mu_{k0}\,t} = Ae^{-i\theta} = cos\theta - i\ sin\theta = 1$$

$$\theta = \mu_{k0}\,T = 2n_2\pi$$

$$\mu_{k0} = \frac{2}{T}\pi\frac{2\pi n_2}{T}n_2 = n_2\omega$$

Eqn. (3.2.3.14)

$$E_0 = n_2\omega\hbar \tag{3.3.3.16}$$

In view of equation (3.3.3.12)

$$E = E_0 - i\frac{\hbar}{\tau} \tag{3.3.3.17}$$

Using Eqns. (3.3.3.13), (3.2.3.16) and (3.3.3.17) yields

$$\left(n_1 + \frac{1}{2}\right)\hbar\omega = n_2\omega\hbar + i\frac{\hbar}{\tau}$$

$$\frac{i}{\tau} = \left[(n_2 - n_1) - \frac{1}{2}\omega\right] = \left[-n - \frac{1}{2}\right]\omega$$

$$\tau^{-1} = -\frac{\left(n + \frac{1}{2}\right)}{i}\omega = i\omega\left(n + \frac{1}{2}\right) \tag{3.3.3.18}$$

The physical meaning of complex relaxation can be known from Eqn. (3.3.3.6)

$$E = E_0 + i\frac{\hbar}{\tau} = E_0 + E_f$$

Thus the energy lost due to friction is given by

$$E_f = -i\frac{\hbar}{\tau} = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (3.3.3.19)$$

The minus sign indicates that the energy is lost by the particle.

3.4 Quantum and Generalized Special Relativistic Model for Electron Charge Quantization:

Explanation of electron self-energy and charge quantization is one of the challenging problems facing quantum electrodynamics. In this work one quantizes electron and elementary particles charges on the basis of electromagnetic Hamiltonian in a curved space-time at vacuum stage of the universe, using quantum spin angular momentum and Klein-Gordon equation beside generalized special relatively. Electron charge is found to be quantized and the electron self-energy is finite. The radius of the electron is also found [31,47,75].

3.4.1 Electromagnetic Hamiltonian in curved space time and vacuum energy:

According general relativity (GR) any energy form cause space to be curved. Thus electromagnetic field can cause space to be curved. According to GR, the time-component of the metric is gives by

$$g_{00} = -\left(1 + \frac{2\emptyset_g}{c^2}\right) \qquad (3.4.1.1)$$

Where $\emptyset_g$is the gravity potential per unit mass and is related to electric potential $\emptyset$ and electron charge e through the relation [3.4.1.13]

$$\emptyset_g = \frac{U}{m} = \frac{e\emptyset}{m} \qquad (3.4.1.2)$$

Thus equation (3.4.1.1) becomes

$$g_{00} = -\left(1 + \frac{2\emptyset_g}{mc^2}\right) \qquad (3.4.1.3)$$

At early stage of the universe electric charge is generated due to the electromagnetic (e.m) field at vacuum stage. This requires minimizing the Hamiltonian (H) w.r.t electric potential $\emptyset$ to find the electric charge and see how it is generated. Since the Hamiltonian part representing charge itself can be neglected as for as they are independent of $\emptyset$. The charge field interactions are neglected for simplicity. One also assumes electric charge to be at rest. This means that the magnetic field is not generated.

Therefore

$$A_\circ = \emptyset \;\; , \;\; A_i = 0 \;\; , \;\; i = 1,2,3,\ldots \qquad (3.4.1.4)$$

To find the Hamiltonian in curved space, one generalized the space one [3.4.1.14] [3.4.1.15]

$$H = \eta^{002}\varepsilon_i(\partial_i A_0 - \partial_0 A_i)^2 \qquad (3.4.1.5)$$

To be written in a curved space in the form [3.4.1.16][3.4.1.17]

$$H = g_{00}^2\varepsilon_i(\partial_i A_0 - \partial_0 A_i)^2 \qquad (3.4.1.6)$$

From equation (3.2.1.3) one gets

$$H = \left((1 + \frac{2e\emptyset}{mc^2}\right)^2 (\nabla\emptyset)^2 \qquad (3.4.1.7)$$

Thus minimization condition requires:

$$\frac{\partial H}{\partial\emptyset} = 2(\left(1 + \frac{2e\emptyset}{mc^2}\right)\left(\frac{2e}{mc^2}\right)(\nabla\emptyset)^2 = 0$$

$$1 + \frac{2e\emptyset}{mc^2} = 0, \;\; \emptyset = \frac{-2mc^2}{2e} \qquad (3.4.1.8)$$

Assuming the mass energy to be resulting from electric field energy density E_d where $E_d = \varepsilon_\circ E^2$ Inside electron of radius r_0, one gets

$$mc^2 = \varepsilon_0 E^2 \frac{4}{3}\pi r_0^3 = \frac{\varepsilon_0 e^2}{16\pi^2 \varepsilon_0^2 r_0^4}\frac{4}{3}\pi r_0^3 = \frac{e^2}{12\pi\varepsilon_0 r_0} \tag{3.4.1.9}$$

The vacuum energy potential which results from electric charge becomes

$$U_v = -e\emptyset = \frac{e^2}{12\pi\varepsilon_0 r_0} \tag{3.4.1.10}$$

according to a vacuum energy potential which takes the form [3.4.1.18]

$$U_v = \rho_v \left[\left(\frac{\pi^2 n^2}{x_0^2 n_0^2}\right) + \omega^2\right]^{-3} \tag{3.4.1.11}$$

Thus combining Equations (3.4.2.10) and (3.4.2.11) yields

$$\frac{e^2}{12\pi\varepsilon_0 r_0} = \left[\left(\frac{\pi^2 n^2}{x_0^2 n_0^2}\right) + \omega^2\right]^{-3}$$

Thus the electric charge is given by

$$e = \left[\left(\frac{\pi^2 n^2}{x_0^2 n_0^2}\right) + \omega^2\right]^{-\frac{3}{2}} (12\pi\varepsilon_0 r_0)^{1/2} \tag{3.4.1.12}$$

Setting to be equal to zero, for simplification. The electric charge is given by

$$e = (12\pi\varepsilon_0 r_0)^{\frac{1}{2}} (\frac{x_0 n_0}{n\pi})^3 \tag{3.4.1.13}$$

r_0 is the electron radius and x_0 is the universe radius. Thus the electron radius can be found by assuming that the electron energy results from its spinning, where the spin angular momentum is given by

$$L_s = \hbar[s(s+1)]^{1/2} = \frac{\sqrt{3}\hbar}{2} \tag{3.4.1.14}$$

Where for electron

$$s = \mp\frac{1}{2} \tag{3.4.1.15}$$

At vacuum stage we choose minimums lower value.

$$L_s = \frac{1}{2}\hbar \tag{3.4.1.16}$$

Assume that rest mass is neglected in relativistic expression to get

$$mc^2 = E = cp \tag{3.4.1.17}$$

$$mc = p \tag{3.4.1.18}$$

The same relation can hold for Newtonian mechanics by considering wave nature of electrons, where the maximum velocity v_m is related to the effective value v through the relations

$$v = \frac{v_m}{\sqrt{2}} \tag{3.4.1.19}$$

By assuming

$$p = mv$$

Thus the Newtonian expression for free particle takes the form

$$E = \frac{1}{2}mv_m^2 = mv^2 = \frac{m^2v^2}{m} = \frac{p^2}{m} \tag{3.4.1.20}$$

If one believes in relativistic energy mass relation, one gets

$$mc^2 = E = \frac{p^2}{m}$$

Thus one gets:

$$mc^2 = p^2 \ , \ mc = p \tag{3.4.1.21}$$

Since the momentum p is related to L according to the

$$p = mv = \frac{mvr_0}{r_0} = \frac{L_s}{r_0} \tag{3.4.1.22}$$

It follows from equation (3.4.2.21) that

$$\frac{L_s}{r_0} = mc$$

Using equation (3.4.1.16) one gets

$$r_0 = \frac{L_s}{mc} = \frac{\hbar}{2mc} \qquad (3.4.1.23)$$

Substituting the values of h, m and c, the electron radius can be calculated. The electric charge is assumed to be born at very early stages of the universe where vacuum exist and the minimum radius is x_0 where [3.4.1.18]

$$x_0 = 26.635 \times 10^{-3} m$$

The electric charge is numerically given by $= 1.6 \times 10^{-19} C$. It can be obtained by adjusting the quantum numbers n and n_0 to be

$$\frac{n}{n_0} = \frac{\pi}{x_0}[\left(\frac{e}{12\pi\varepsilon_0 r}\right)^2]^{1/3} \qquad (3.4.1.24)$$

Similarly, the charges of quarks and charged leptons can be found by adjusting the quantum numbers n and n_0

Equation (2.4.1.10) shows that vacuum energy is repulsive due to the existence of positive sign. This can form with cosmological models, which suggests repulsive vacuum energy. Inflation models suggest also very large vacuum energy. If one believes in this model, such that

$$\emptyset = \frac{U_v}{m_0} \to \frac{c^2}{2} \qquad (3.4.1.25)$$

In this case according to generalized special relativity model the electron mass is given by

$$m = m_0\left(1 - \frac{2\emptyset_g}{c^2}\right) \to \text{Iarge} \qquad (3.4.1.26)$$

Assume for simplicity

$$m = m_0 \times 10^{13} \times 9 \times 10^{-31} = 9 \times 10^{-18} kg \qquad (3.4.1.27)$$

From Equations (3.4.1.16), (3.4.1.21) and (3.4.1.22) the electron radius can be given to be

$$r_0 = \frac{\hbar}{2mc} = \frac{h}{4\pi mc} = \frac{6.63 \times 10^{-34}}{4\pi \times 9 \times 10^{-18} \times 3 \times 10^8}$$

$$r_0 = 1.954 \times 10^{-26}\, m \qquad (3.4.1.28)$$

Which is quite reasonable as far as nucleus or proton radius for very light atoms are

$$r_b = 10^{-14} \qquad r_p = 10^{-16}$$

3.5 Classical Newtonian model for destruction of superconductors by magnetic field:

Newton second law is used to describe the destruction of super conductivity for type 1 & type 2.The electron is assumed to be affected by external electric and magnetic field as well as the internal magnetic field. The conductivity and resistance depends on the internal as well as external magnetic field. For type 1 the super conducting state is destroyed when the external magnetic field exceeds the maximum internal field. For type 2 the superconductivity is destroyed partially in the region where the local maximum field is the lowest, and enters completely when the external field exceeds the maximum local internal field [32,58,77].

3.5.1 Classical Model to Describe Magnetic Destruction of Super Conductivity by the External Magnetic Field:

The equation of motion of the electron moving inside matter under the action of external electric and magnetic field intensities E and density B is given by

$$m\frac{dv}{dt} = eE + B_i ev - Bev \tag{3.5.1.1}$$

Where B_i is the internal field since the electrons move with constant velocity, hence

$$\frac{dv}{dt} = 0$$

And

$$(B - B_i)ev = eE \tag{3.5.1.2}$$

Therefore the velocity is given by

$$v = \frac{eE}{(B - B_i)e} \tag{3.5.1.3}$$

Thus the current density takes the form

$$J = nev = \frac{ne^2}{(B - B_i)e} \tag{3.5.1.4}$$

Where n stand for the charge density. The conductivity σ is thus given by

$$\sigma = \frac{ne}{B - B_i} \tag{3.5.1.5}$$

$$\rho = \frac{(B - B_i)}{ne} \tag{3.5.1.6}$$

Where the internal field can be written in terms of the field per atom as

$$B_i = n_a B_a \tag{3.5.1.7}$$

B_a Here represents the magnetic field induced by one atom, while n_a represent the number of diamagnetic atoms that induces magnetic field that opposes the external field B.

The conductivity becomes zero in type I, due to the fact that the induced internal field density B_i increases and when the external flux density increases such that always B_i equals $B, i. e$

$$B_i = B \tag{3.5.1.8}$$

According to equations (3.4.1.5) and (3.4.1.6)

$$\sigma = \infty \tag{3.5.1.9}$$

Thus the material becomes superconducting till all atoms N_a in a unit volume become magnetized. But when the external field B exceeds the maximum value

$$B_{im} = B_a N_a \tag{3.5.1.10}$$

i.e. when

$$B > B_{im} \tag{3.5.1.11}$$

In this case no more atoms can be magnetized to oppose and cancel$B$. In this case

$$\sigma = \frac{ne}{B - B_{im}} = finite$$

$$\rho = \frac{B - B_{im}}{ne} \neq 0 \tag{3.5.1.12}$$

And the whole superconductor becomes conducting

For type II superconductor, one can assume that there are regions where the densities of atoms are law and equalsN_{aL}. In such regions the increase of B increase B_i till

$$B_{iL} = N_{aL} B_a \tag{3.5.1.13}$$

When B is less than B_{iL}

$$B = B_i \tag{3.5.1.14}$$

And an equation (3.4.1.5) and (3.4.1.6)

$$\sigma = \infty$$

$$\rho = 0 \tag{3.5.1.15}$$

But when

$$B > B_{iL}$$

$$\sigma = \frac{ne}{B - B_{iL}} \neq \infty$$

$$\rho = \frac{B - B_{iL}}{ne} \neq 0 \tag{3.5.1.16}$$

And these regions become ordinary conductors, while other regions are still superconductors. The same hold for other regions with higher concentration than N_{aL}. They become ordinary conductors, when B exceeds their local maximum internal field. This process continues till the external field enters regions where the densities of atoms are high and equalsN_{ah}. In this region the increase of B increases B_i till

$$B = B_{ih} \tag{3.5.1.17}$$

In these case when

$$B = B_i \leq B_{ih} \tag{3.5.1.18}$$

$$\sigma = \infty$$

$$\rho = 0 \tag{3.4.1.19}$$

According to equations (3.5.1.5) and (3.5.1.6). But when

$$B > B_{ih} \tag{3.5.1.20}$$

In this case

$$\sigma = \frac{ne}{B - B_{ih}} \neq \infty$$

$$\rho = \frac{B - B_{ih}}{ne} \neq 0 \tag{3.5.1.21}$$

Thus when the external field B exceeds B_{iL} the SC becomes partially ordinary conductor in regions where the diamagnetic atoms have law density. Upon increasing B, such that

$$B > B_{il} \tag{3.5.1.22}$$

More regions become ordinary conductors, till all SC material become ordinary conductor when

Thus for type II, one have two critical magnetic fields B_{iL} and B_{ih}

Another explanation may also explain the behavior of type II SC. This approach is based on assuming that the matter density is homogeneous but the magnetic field of atoms are randomly oriented such that the net magnetic field in some regions is the lowest and in gradually increases and attains its maximum value in another region. According to this model the increase of external field in the lowest B_i value increases B_i according to the Langevin equation

$$w_l = \Delta \mathrm{w} = \frac{Be}{2m} \tag{3.5.1.24}$$

$$i = -\frac{ez_0 w_L}{2\pi} = -\frac{e^2 z_0 B}{4\pi m} \tag{3.5.1.25}$$

Where Z_0 is the number of electrons in the outer most sheet. By considering electrons moving in a circular orbit one gets the internal magnetic field in the form (here one assumes only outer most electrons can produce induced magnetic field)

$$B_a = \frac{\mu i}{2r} = \frac{\mu e^2 z_0}{8\pi m r} B \tag{3.5.1.26}$$

The internal field B_i is given by

$$B_i = \sum B_a \tag{3.5.1.27}$$

This internal field increases upon increasing the external one, till the electron kinetic energy exceeds electron binding energy E_b,. e.

$$\frac{1}{2} m v^2 > E_b \tag{3.5.1.28}$$

i.e

$$w > \frac{1}{r}\sqrt{\frac{2E_b}{m}}$$

The electron becomes free. In this case the electron will no longer revolve around the nucleus. Thus it cannot produce internal magnetic field. Thus the maximum produced atomic field is

$$b_{am} = \frac{\mu i_m}{2r}$$

Where the maximum current produced is

$$i = -\frac{ez_0 w_m}{2\pi}$$

Where

$$w_r > \frac{1}{r}\sqrt{\frac{2E_b}{m}}$$

Thus the internal field attains maximum value

$$B_{im} = \sum B_{am}$$

When B exceeds this maximum value in the region of lowest B_i, i.e.

$$B > B_{imL}$$

The resistivity will no longer vanishes according to equation (3.5.1.6), where

$$\rho = \frac{B - B_{imL}}{ne} > 0$$

The same hold for the region having maximum internal field value due to large orientation of magnetic field of atoms in the opposite direction of the external one. In such region the external magnetic field B is balanced by the internal oneB_{ih}, till electrons energy exceeds binding energy.

3.6 Energy-Momentum Relation and Eigen Equations In a Curved Space Time:

Using the expression of time and distance in a curved space time a useful expression of energy and momentum Eigen equation similar to that is a curved space is found. These relations can be used to derive the corresponding relations in the Euclidean space. The corresponding on of Energy-Momentum relations for both curved and Euclidian space gives a relation between energy and momentum typical to that obtained from the energy and momentum Eigen equations. The expression of mass in a curved space is similar to that of the generalized relativity [33,71,76].

The energy within the framework of the GSR and SR are given by:

$$g_{00}E^2 = g_{xx}P^2c^2 + g_{00}^2m_0^2c^4 \ ,$$

$$E_0^2 = P_0^2c^2 + m_0^2c^4 \tag{3.6.1}$$

Where E_0 is the ordinary SR energy. Thus the GSR energy E is given by

$$E = g_{00}^{-1/2}E_0 \tag{3.6.2}$$

The wave function in the curved space is thus

$$\psi = Ae^{(\frac{i}{\hbar})(Px_c - Et_c)} \tag{3.6.3}$$

Energy Eigen equation and time independent Schrodinger equation in the Euclidean space takes the form

$$i\hbar\frac{\partial\psi}{\partial t} = E_0\psi \tag{3.6.4}$$

Also the momentum Eigen equation in the Euclidian space is given by

$$\frac{\hbar}{i}\nabla\psi = \frac{\hbar}{i}\frac{\partial\psi}{\partial x} = P_0\psi \tag{3.6.5}$$

$$\psi = Ae^{(\frac{i}{\hbar})(\sqrt{g_{xx}}P_x - \sqrt{g_{00}}Et)}$$

Where

$$dt_c = \sqrt{g_{00}}dt \ , \qquad dx_c = \sqrt{g_{xx}}dx \tag{3.6.6}$$

Schrodinger equation in the curved space, where the time is denoted byt_c, can read

$$i\hbar\frac{\partial\psi}{\partial t_c} = i\hbar\frac{\partial\psi}{\sqrt{g_{00}}\partial t} = \frac{i\hbar}{\sqrt{g_{00}}}\left[-\frac{i}{\hbar}\sqrt{g_{00}}E\psi\right] = \frac{i\hbar}{\sqrt{g_{00}}}\frac{\partial\psi}{\partial t} = \frac{1}{\sqrt{g_{00}}}E\psi$$

$$= \frac{\sqrt{g_{00}}}{\sqrt{g_{00}}}E\psi = E\psi \qquad (3.6.7)$$

Thus

$$i\hbar\frac{\partial\psi}{\partial t_c} = E\psi \qquad (3.6.8)$$

But from (3)

$$i\hbar\frac{\partial\psi}{\partial t_c} = E\psi \qquad (3.6.9)$$

This is completely consistent with equation (3.6.8). Conversely from (3.6.6), (3.6.9) and (3.6.2)

$$i\hbar\frac{\partial\psi}{\partial t_c} = i\hbar\frac{\partial\psi}{\partial\sqrt{g_{00}}} = E\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = \sqrt{g_{00}}E\psi = E_0\psi$$

$$E_0 = \sqrt{g_{00}}E \qquad (3.6.10)$$

The momentum Eigen equation for the momentum in Euclidean space takes the form

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\psi = P_0\psi \qquad (3.6.11)$$

In a curved space, the momentum Eigen equation becomes

$$\frac{ħ}{i}\frac{\partial\psi}{\partial x_c} = P\psi$$

With

$$\partial x_c = \sqrt{g_{00}}dx \qquad (3.6.12)$$

Thus

$$\frac{ħ}{i}\frac{\partial\psi}{\sqrt{g_{00}}\partial x} = P\psi \qquad (3.6.13)$$

Thus, one can write

$$\frac{ħ}{i}\frac{\partial}{\partial x}\psi = \sqrt{g_{00}}\,P\psi \qquad (3.6.14)$$

Comparing this relation with (3.6.11) yields

$$P_0 = \sqrt{g_{00}}\,P \qquad (3.6.15a)$$

Where

$$P_0 = \sqrt{g_{00}}\,P \qquad (3.6.15b)$$

Thus equations (3.6.14) and (3.6.15.a) gives

$$\frac{ħ}{i}\frac{\partial}{\partial x}\psi = P_0\psi \qquad (3.6.15c)$$

This is the ordinary momentum Eigen equation in the Euclidian space. The velocity in a curved space is define to be

$$v = \frac{\sqrt{g_{00}}}{\sqrt{g_{00}}}\frac{dx}{dt} = \frac{\sqrt{g_{00}}}{\sqrt{g_{00}}}v_0 \qquad (3.6.16)$$

But the momentum in a curved space and Euclidean

$$P = mv \qquad (3.6.17)$$

$$P_0 = m_0 v_0$$

Using equation (3.6.15)

$$P = (g_{xx})^{1/2} P_0$$

Thus

$$mv = (g_{xx})^{-1} m_0 v_0$$

$$m = \frac{\sqrt{g_{00}}}{(\sqrt{g_{xx}})^2} m_0 = \frac{\sqrt{g_{00}}}{g_{xx}} m_0 \qquad (3.6.18)$$

Since in driving GSR, one assumes that

$$g_{xx} = 1 \qquad (3.6.19)$$

It follows that

$$M = g_{00} m_0 \qquad (3.6.20)$$

But the mass in GSR is given by

$$m = \frac{g_{00} m_c}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \qquad (3.6.21)$$

For the mass at rest

$$v = 0$$

$$m = \frac{g_{00} m_c}{\sqrt{g_{00}}} = \sqrt{g_{00}} m_0 \qquad (3.6.22)$$

This relation is consistent with equation.

To find the expression, which relates E to P in a curved space-time one uses the relation

$$c^2dt^2 = c^2g_{00}dt_0^2 - g_{xx}dx^2$$

$$\gamma^{-1} = \left(\frac{d\tau}{dt}\right) = \left[g_{00} - g_{xx}\frac{v_0^2}{c^2}\right] \qquad (3.6.23)$$

Thus

$$E = mc^2 = g_{00}\gamma m_0 = \frac{g_{00}m_{00}c^2}{\sqrt{g_{00} - g_{xx}\frac{v_0^2}{c^2}}} \qquad (3.6.24)$$

But from (16)

$$g_{xx}v_0^2 = g_{00}^2$$

$$E = \frac{g_{00}m_0c^2E}{\sqrt{g_{00}E^2 - g_{xx}P^2c^2}}$$

$$g_{00}E^2 - g_{00}P^2c^2 = g_{00}^2m_0^2c^4$$

$$g_{00}E^2 = g_{00}P^2c^2 + g_{00}^2m_0^2c^4 \qquad (3.6.25)$$

Setting

$$E_0^2 = g_{00}E^2P_0^2 = g_{00}P^2 \qquad (3.6.26)$$

One gets

$$E_0^2P_0^2c^2 + m_0^2c^4 \qquad (3.6.27)$$

However, when one replaces $v_0 by\ v$ in equation (3.6.23), one gets

$$\gamma^{-1} = \left[g_{00} - g_{xx}\frac{v_0^2}{c^2}\right]^{1/2} \qquad (3.6.28)$$

As a result, energy becomes

$$E=\frac{g_{00}m_0c^2}{\sqrt{g_{00}-g_{xx}\frac{v^2}{c^2}}}=\frac{g_{00}m_0c^2}{\sqrt{\frac{g_{00}m^2c^4-g_{xx}m^2v^2c^2}{m^2c^4}}}$$

$$=\frac{g_{00}m_0c^2}{\sqrt{\frac{g_{00}E^2-g_{xx}P^2c^2}{E^2}}}$$

$$E=\frac{g_{00}m_0c^2E}{\sqrt{g_{00}E^2-g_{xx}P^2c^2}}$$

$$g_{00}E^2-g_{xx}P^2c^2=g_{00}^2m_{00}^2c^4$$

$$g_{00}E^2=g_{xx}P^2c^2+g_{00}^2m_{00}^2c^4 \qquad (3.6.29)$$

By setting

$$E_0^2=g_{00}E^2P_0^2=g_{xx}P^2$$

$$E_0=g_{00}^{\frac{1}{2}}E\ ,\qquad P_0=\sqrt{g_{xx}}P \qquad (3.6.30)$$

One gets

$$E_0^2=P_0^2c^2m_0^2c^4 \qquad (3.6.31)$$

Where

$$m_0=g_{00}m_0 \qquad (3.6.32)$$

Harmonic oscillator in a curved space:

The Schrodinger equation in a curved space is given

$$i\hbar\frac{\partial\psi}{\partial t_c}=E\psi\quad ,\qquad i\hbar\frac{\partial\psi}{\partial\sqrt{g_{00}}}=E\psi$$

$$i\hbar\frac{\partial\psi}{\partial t}=\sqrt{g_{00}}E\psi=E_0\psi \qquad (3.6.33)$$

For harmonic oscillator in Euclidean space

$$E_0 = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (3.6.34)$$

$$(1 + x)^n \approx 1 + nx$$

For $x < 1$

$$\frac{2\emptyset}{c^2} = x \qquad (3.6.35)$$

$$(g_{00})^{-\frac{1}{2}} = (1 + \left(-\frac{1}{2}\right)\frac{2\emptyset}{c^2}$$

$$(g_{00})^{-\frac{1}{2}} = \left(1 - \frac{\emptyset}{c^2}\right) \qquad (3.6.36)$$

This approximation is justifiable since $\frac{2m\emptyset}{mc^2} < 1 \qquad 2V < mc^2$

Which means that the total energy is the greater than potential energy. Thus equation (3.6.30) gives

$$E = E_0(g_{00})^{-\frac{1}{2}} = E_0\left(1 - \frac{\emptyset}{c^2}\right) \qquad (3.6.37)$$

$$E = \left(n + \frac{1}{2}\right)\left(1 - \frac{\emptyset}{c^2}\right)\hbar\omega$$

$$E = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{\emptyset}{c^2}\left(n + \frac{1}{2}\right)\hbar\omega \qquad (3.6.38)$$

$$E = E_0\left(1 - \frac{\emptyset}{c^2}\right) = E_0\left(1 - \frac{m_0\emptyset}{m_0c^2}\right) = E_0\left(1 + \frac{V_0}{E_0}\right)$$

$$= E_0 + V_0 \qquad (3.6.39)$$

3.7 summary for other attempts:

Similar attempts and some which are not related directly to this work was done by others [34, 35, and 36]. For example in the work done An O (N) algorithm is proposed for calculating linear response functions of non-interacting electrons. This algorithm is simple and suitable to parallel- and vector- computation. Since it avoids $O(N^3)$ computational effort of matrix diagonalization, it requires only $O(N)$ computational efforts where N is the dimension of the statevector. The use of this $O(N)$ algorithm is very effective since otherwise we have to calculate large number of eigenstates, i.e., the occupied one-electron states up to the Fermi energy and the unoccupied states with higher energy. The advantage of this method compared to the Chebyshev polynomial method recently developed by Wang (L.W. Wang, Phys. Rev. B **49**, 10154 (1994);L.W. Wang, Phys. Rev. Lett. **73**, 1039 (1994)) is that our method can calculate linear response functions without any storage of huge statevectors on external storage. etal [47,49,57].

Also another work was done by with the use a variant of the method of separation of the method of separation We analyze the quantum dynamics of radiation propagating in a single mode optical fiber with dispersion, nonlinearity, and Raman coupling to thermal phonons. We start from a fundamental Hamiltonian that includes the principal known nonlinear effects and quantum noise sources, including linear gain and loss. Both Markovian and frequency-dependent, non-Markovian reservoirs are treated. This allows quantum Langevin equations to be calculated, which have a classical form except for additional quantum noise terms. In practical calculations, it is more useful to transform to Wigner or $+P$ quasi-probability operator representations. These result in stochastic equations that can be analyzed using perturbation theory or exact numerical techniques. The results have applications to fiber optics communications, networking, and sensor technology. etal [38,49,70].

A useful work was also done by With the use of a variant of the method of separation of variables, the initial value problem for the time-dependent linear Schrodinger¨ equation is solved exactly for a large class of potential functions related to multisoliton interactions in the vector nonlinear Schrodinger¨ equation. Completeness of states is proved for absolutely continuous initial data in L_1. Copyright © 1998 Elsevier Science B.V etal [41,61,65],

A new approach was also faceted by A new method is presented for the solution of the time dependent Schrodinger equation, expressed in polar or spherical coordinates. The radial part of the Laplacian operator is computed using a Fast Hankel Transform. An algorithm for the FHT is described, based on the Fast Fourier Transform. The accuracy of the Hankel method is checked for the two- and three-dimensional harmonic oscillator by comparing with the analytical solution. The Hankel method is applied to the system H + H, with Delves hyperspherical coordinates and is com- pared to the Fourier method. 6 1985 Academic Press, Inc. etal [43,55,59,62] .

Chapter Four

The model of fluid and lasing equations from quantum and statistical equation:

4.1 Introduction:

In this chapter quantum expression for the wave function in a frictional medium has been used to derive statistical physical laws, beside fluid laws. This link makes possibility to describe super fluid behavior.

(4.2) New Maxwell Quantum Distribution law and new Energy relation for particle in a medium:

Schrodinger equation for particle in a finite media with uniform potential was solved.

The solution which is based on the fact that the particle exists gives complex and cosine wave function with energy relations different from that of the ordinary sine solution.

Maxwell distribution law has been also found using the expression for the wave function in a frictional medium, quantum energy average and integration by parts, another approach has been tackled using the general expression for quantum average and the ordinary differentiation

Keywords: Schrodinger equation, finite medium, Maxwell quantum distribution, friction.

4.3Quantum and statistical laws to derive fluid and lasing equations:

The equation of motion of a particle moving in a field of potential V and a frictional medium with relaxation time T is given by :

$$mv\frac{dv}{dx} = \ mv\frac{dv}{dx}\frac{dx}{dt} = m\frac{dv}{dt} = -\frac{dV}{dx} - \frac{m}{\tau}v \qquad (4.3.1)$$

Considering the practical as vibrating string

$$x = x_\circ e^{-i\omega t}$$

$$v = x^{\circ} = -iwx \qquad (4.3.2)$$

Thus:

$$m\int vdv = -\int dV + \frac{m}{i\tau w}\int vdv$$

$$\frac{1}{2}mv^2 = -V + \frac{m}{2\tau wi}v^2 \qquad (4.3.3)$$

Sine the kinetic energy oscillator is k=$\frac{1}{2}mv^2$ and the potential energy is

$$V=\frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 = \frac{1}{2}mv^2 = \frac{1}{2}mv^2 = K \qquad (4.3.4)$$

On the other hand equation (3) yields

$$K + V + \frac{K}{\omega\tau i} = \text{constant} \qquad (4.3.5)$$

This constant of motion is the total energy E of the system, which takes the form

$$E = K + V - \frac{iK}{\omega\tau} = E_\circ - \frac{iK}{\omega\tau} \qquad (4.3.6)$$

Where the non-frictional energy takes the form

$$E_\circ = K + V = 2K \qquad (4.3.7)$$

Thus

$$E=E_\circ - i\frac{E_\circ}{wt} = K + V - \frac{iE_\circ}{\omega\tau} \qquad (4.3.8)$$

IF the relation time is assumed to be proportional to the periodic time T, such that T=$\alpha\tau$.

It follows that $E=E_\circ - i\frac{\alpha}{2\pi}E_\circ = E_\circ - i\alpha_\circ E_\circ = K + V - i\alpha_\circ E_\circ$ (4.3.9)

Therefore

$$E=\frac{P^2}{2m}+V-i\alpha_\circ E_\circ \qquad (4.3.10)$$

For particle in a box Schrodinger equation in a frictional medium is given by:

$$i\hbar\frac{\partial\varphi}{\partial t}= -\frac{\hbar^2}{2m}\nabla^2\varphi+V_\circ\varphi-i\alpha_\circ E_\circ\varphi \qquad (4.3.11)$$

Where $V_\circ$the potential barrier and T is a relaxation time .sing the method of separation of variables one can write:

$$\varphi=\varphi(r,t)=f(t)u(r)=fu \qquad (4.3.12)$$

A direct substitution in equation (4.3.11) gives:

$$i\hbar u\frac{df}{dt}=\left\{-\frac{\hbar^2}{2m}\nabla^2u+V_\circ u i\alpha_\circ E_\circ\right\}f$$

Rearranging gives

$$\frac{i}{f}\hbar\frac{df}{dt}+i\alpha_\circ E_\circ+V_\circ= -\frac{\hbar^2}{2mu}\nabla^2u=E_\circ \qquad (4.3.13)$$

$$\frac{i}{f}\hbar\frac{df}{dt}=(E_\circ-V_\circ-i\alpha_\circ E_\circ)f \qquad (4.3.14)$$

$$-\frac{\hbar^2}{2m}\nabla^2u=E_\circ u=Ef \qquad (4.3.15)$$

The solution of equation (4.3.14) can be given by :

$$f=A_1e^{-\frac{iEt}{\hbar}}=A_1e^{-\frac{i}{\hbar}(E_\circ-V_\circ-i\alpha_\circ)t}$$

$$=A_1e^{-\alpha_\circ E_\circ t}e^{-\frac{i}{\hbar}(E_\circ-V_\circ)t} \qquad (4.3.16)$$

Two possible solutions can be suggested for equation (4.3.15). In one of them

$$u=A_2\sin\alpha x \qquad (4.3.17)$$

To get : $\frac{\hbar^2}{2m}\alpha^2u=E_\circ u \qquad \alpha=\sqrt{\frac{2mE_\circ}{\hbar^2}}=\frac{\sqrt{2m_\circ E_\circ}}{\hbar}$ (4.3.18)

For a one dimensional box of length L just outside the box

$$|u|^2=|\varphi|^2=0 \qquad (4.3.19)$$

$$u(L) = \varphi(x = L) = A_2 \sin\alpha L = 0 \quad (4.3.20)$$

Thus : $\alpha L = n\pi$

Using (18) gives $E_\circ = \frac{\hbar^2\alpha^2}{2m} = \frac{\hbar^2\pi^2n^2}{2mL^2}$ (4.3.21)

$$E_\circ = \frac{\hbar^2 n^2}{8mL^2} \quad (4.3.22)$$

The other solution of equation (4.3.15) can be

$$u = A_3 e^{i\alpha x} \quad (4.2.23)$$

This with the aid equation (4.3.15), gives:

$$\frac{\hbar^2}{2m}\alpha^2 u = E_\circ u$$

This solution with be consist with the first solution and gives the same energy relation when are assumes that the probability is equal are at the bound any just inside the box at $(x = L)$

i.e.

$$u(L) = A_3 e^{i\alpha L} = A_3 \cos\alpha L + A_3 \sin\alpha L = 1 \quad (4.3.24)$$

Which requires:

$$A_3 \cos\alpha L = 1$$

$$A_3 \sin\alpha L = 0 \quad (4.3.25)$$

This requires:

$$\alpha L = 2n\pi \qquad \alpha = \frac{2n\pi}{L} \qquad A_3 = 1 \quad (4.3.26)$$

Thus according to equation (4.3.18) equation (4.3.16) gives

$$2mE_\circ = \hbar^2\alpha^2 = \frac{4\hbar^2\pi^2\hbar^2}{L^2}$$

$$E_\circ = \frac{\hbar^2 n^2}{2mL^2} \quad (4.3.27)$$

This equation is not completely conforming to equation (4.3.22), the full complete agreement requires rewriting (4.3.11) by suggesting

$$\alpha L = 2n\pi \quad (4.3.28)$$

Which also satisfies equation (4.3.27) to get again:

$$E_\circ = \frac{\hbar^2 \mathbf{n}^2}{2mL^2} \quad (4.3.29)$$

Another boundary condition can be obtained by suggesting that at the boundary just inside the box (medium) the probability of finding the particle inside the box is finite and is equal to $P_\circ$. This requires

$$|u(L)|^2 = P_\circ$$

$$u(L) = \sqrt{P_\circ} \qquad (4.3.30)$$

Thus: $u(L) = A_3 e^{i\alpha L} = A_3 \cos\alpha L + iA_3 \sin\alpha L + VP_\circ \qquad (4.3.31)$

$$A_3 \cos\alpha L = \sqrt{P_\circ} \quad , \quad \cos\alpha L = \frac{\sqrt{P_\circ}}{A_3} \quad , \quad \sin\alpha L = 0 \quad , \quad \alpha L = n\pi$$

This requires: $\frac{\sqrt{P_\circ}}{A_3} = \cos\alpha L = \pm 1 \qquad (4.3.32)$

Where for

$n = 1,3,5,7 \qquad \cos\alpha L = -1$

And for

$n = 0,2,4,6 \qquad \cos\alpha L = 1 \qquad (4.3.33)$

This requires

$$A_3 = \pm\sqrt{P_\circ} \qquad (4.3.34)$$

This means that u is real and is given by

$$u = A_3 \cos\alpha x \qquad (4.3.35)$$

But this is not consistent with the solution

$$u = A_2 \sin\alpha x \qquad (4.3.36)$$

Since they give different probability distributions. To have solutions typical to each other consider

$$u(x = L) = iP_\circ \qquad (4.3.37)$$

This satisfies

$$|u(L)|^2 = P_\circ \qquad (4.3.38)$$

And gives also

$$A_3 \cos\alpha L + iA_3 \sin\alpha L = i\sqrt{P_\circ} \qquad (4.3.39)$$

This means that

$$\cos\alpha L = 0 \qquad A_3\sin\alpha L = \sqrt{P_\circ} \tag{4.3.40}$$

This requires

$$\sin\alpha L = 1 \qquad A_3 = \sqrt{P_\circ} \tag{4.3.41}$$

$$\alpha L = (n + 1/2)\pi$$

$$\alpha = \frac{(n+\frac{1}{2})\pi}{L} \tag{4.3.42}$$

Which gives the energy in the form

$$E_\circ = \frac{(n+\frac{1}{2})^2\hbar^2}{8mL^2} \tag{4.3.43}$$

Another solution based on the existence of particles inside the box can be suqqested by assuming

$$u(x) = A_4\cos\alpha x \qquad ,\nabla u = \alpha A_4\sin\alpha x$$

$$\nabla^2 u = -\alpha^2 A_4 \cos\alpha x = -\alpha^2 u \tag{4.3.44}$$

Inserting this expression in equation (4.3.15) gives

$$\frac{\hbar^2}{2m}\alpha^2 = E_\circ u \qquad \alpha = \frac{\sqrt{2mE_\circ}}{\hbar} \tag{4.3.45}$$

Since just inside the box at (x=L) the box at $(x = L)$ the box the particle exist, it flows that

$$|u(x = L)|^2 = |A_4\cos\alpha L|^2 = P_\circ \tag{4.3.46}$$

Which means that the probability of existence of the particle is $P_\circ$. One of the possible solution is to suggest

$$u(L) = A_4\cos\alpha L = \sqrt{P_\circ} \quad , \quad A_4 = \sqrt{P_\circ} \tag{4.3.47}$$

$$\cos\alpha L = 1 \tag{4.3.48}$$

$$\alpha L = 2n\pi \tag{4.3.49}$$

According to equation (4.3.4)

$$\frac{L\sqrt{2mE_\circ}}{\hbar} = 2n\pi \quad , \qquad E_\circ = \frac{n^2h^2}{2mL^2} \tag{4.3.50}$$

This solution gives the same energy from as that proposed by the exponential solution with real wave function shown in equation(4.3.23), (4.3.27) {see equation (4.3.27)} In view of equations (4.3.12), (4.3.16) and (4.3.23) the wave function in excited state takes the form

$$\varphi = A_1e^{i\theta}e^{-\alpha E\tau} \tag{4.3.51}$$

Where the collision for time τ cause it to go in energy to be in an excited state with energy E and θ

$$\theta = \frac{(V_\circ - E_\circ)\tau + \alpha x_\circ}{\hbar} \tag{4.3.52}$$

Here one assumes that all particles are in the ground state with $(n = 0\,, E = 0)$

Thus when they are excited their energy is. Can write the wave function as:

$$\varphi = Ae^{-\alpha E} \tag{4.3.53}$$

Where $\alpha = \alpha_\circ\tau$ (4.3.54)

$$A_\circ = A_1e^{i\theta} \tag{4.3.55}$$

$$\overline{E} = \frac{\int_0^\infty \overline{\varphi}E\varphi dE}{\int_0^\infty \overline{\varphi}\varphi dE} = \frac{|A|^2\int_0^\infty E\, e^{-2\alpha E}dE}{|A|^2\int_0^\infty e^{-2\alpha E}\, dE} \tag{4.3.56}$$

Using the identity: $\int udv = uv - \int vdu$ (4.3.57)

With:

$\mathrm{u} = E \qquad\qquad dv = e^{-2\alpha E}du$

$$v = -\frac{1}{2\alpha}e^{-2\alpha E} \tag{4.3.58}$$

$$\int_0^\infty Ee^{-2\alpha E}dE = -\frac{E}{2\alpha}e^{-2\alpha E}\Big|_0^\infty + \frac{1}{2\alpha}\int_0^\infty e^{-2\alpha E}dE$$

$$= -\frac{\infty e^{-\infty}}{2\alpha} + \frac{0e^{-0}}{2\alpha} - \frac{1}{4\alpha^2}e^{-2\alpha E}\Big|_0^\infty$$

$$= -0 + 0 - \frac{1}{4\alpha^2}\{e^{-\infty} - e^{-0}\}$$

$$= \frac{1}{4\alpha^2} \tag{4.3.59}$$

Also:

$$\int_0^\infty e^{-2\alpha E}dE = -\frac{1}{2\alpha}e^{-2\alpha E}\Big|_0^\infty$$

$$= -\frac{1}{2\alpha}\{e^{-\infty} - e^{-0}\}$$

$$= \frac{1}{2\alpha} \tag{4.3.60}$$

Thus:

$$\bar{E} = \frac{2\alpha}{4\alpha^2} = \frac{1}{2\alpha} \tag{4.3.61}$$

In view of equation (4.3.53) the wave function takes the form

$$\varphi = Ae^{-\frac{E}{2\bar{E}}} \tag{4.3.62}$$

Thus the number of particle are given by

$$n = \varphi\bar{\varphi} = A^2e^{-\frac{E}{\bar{E}}}$$

$$= n_\circ e^{-\beta E} \qquad \beta = \frac{-1}{\bar{E}} \tag{4.3.63}$$

The statistical laws can also be found by using equation (4.3.56) to get

$$\bar{E} = \frac{\int_0^\infty \bar{\varphi} E \varphi dE}{\int_\circ^\infty \bar{\varphi} \varphi dE} = \frac{\int_0^\infty nEdE}{\int_0^\infty ndE} = \frac{\int_0^\infty EndE}{I} \qquad (4.3.64)$$

Where:

$$I = \int_0^\infty ndE \qquad (4.3.65)$$

But one can write equation (4.3.64) in the form:

$$\bar{E} = \frac{dLnI}{d\beta_\circ} = \frac{dLnI}{dI}\frac{dI}{d\beta_\circ} = \frac{1}{I}\frac{dI}{d\beta_\circ} \qquad (4.3.66)$$

This means that {compeer (4.3.64) and (4.3.66)}

$$\frac{dI}{d\beta_\circ} = \int EndE \qquad (4.3.67)$$

But from (4.3.66) $\int dLnI = \int \bar{E} d\beta_\circ$

$$ln\, I = \int \bar{E} d\beta_\circ + C_\circ \quad , \quad I = e^{\int \bar{E} d\beta_\circ} e^{C_\circ} = e^{C_\circ} e^{\int \bar{E} d\beta_\circ}$$

$$I = C_1 e^{\int \bar{E} d\beta_\circ} \qquad (4.3.68)$$

For constant $\bar{E}$

$$I = C_1 e^{\beta_\circ \bar{E}} \qquad (4.3.69)$$

When all values are near the average value

$$I = C_1 e^{\beta_\circ E} \qquad (4.3.70)$$

But from (4.2.64):

$$I = \int \frac{dI}{dE} dE = \int ndE \qquad (4.3.71)$$

Thus from (4.3.70) and (4.3.71):

$$n = \frac{dI}{dE} = \frac{C_1}{\beta_\circ} e^{\beta_\circ E} = Ae^{\beta_\circ E} \qquad (4.3.72)$$

But from (4.3.66) and (4.3.70)

$$\overline{E} = \frac{1}{I}\frac{dI}{d\beta_\circ} = \frac{1}{I}(\beta_\circ I) = \beta_\circ \qquad (4.3.73)$$

Hence from (4.272):

$$n = Ae^{-\frac{E}{\overline{E}}} \qquad (4.3.74)$$

Another approach is based on defining [see (4.3.67)]

$$\frac{dI}{d\beta_\circ} = S = \int EndE = \frac{1}{2}\int ndE^2 = \int \frac{dS}{dE^2} dE^2$$

This means that:

$$\frac{dS}{dE^2} = \frac{1}{2}n \qquad (4.3.75)$$

From (4.3.72) and (4.3.75)

$$\frac{dI}{dE} = 2\frac{dS}{dE^2} \qquad (4.3.76)$$

Thus

$$\frac{1}{2}\frac{dE^2}{dE} = \frac{dS}{dI} \qquad (4.3.77)$$

From (4.3.75):

$$E = \frac{dS}{dI} = \frac{dS}{\left(\frac{dI}{d\beta_\circ}\right)d\beta_\circ} = \frac{dS}{Sd\beta_\circ}$$

$$\int \frac{dS}{S} = \int E d\beta_\circ + C_3$$

$$\ln S = C_3 + \int E d\beta_\circ$$

$$S = e^{c_3} e^{\int E d\beta_\circ} = C_4 e^{\int E d\beta_\circ} \qquad (4.3.78)$$

Assuming E to be independent of $\beta_\circ$, one gets:

$$S = C_4 e^{E\beta_\circ} \qquad (4.3.79)$$

Thus:

$$n = 2\frac{dS}{dE^2} = 2\frac{dS}{dE}\frac{dE}{dE^2} = \beta_\circ C_4 e^{\beta_\circ E}\left(\frac{1}{E}\right) = \frac{C_4\beta_\circ e^{\beta_\circ E}}{E} \qquad (4.3.79)$$

$$n = g(E)e^{-\beta_\circ E} \qquad (4.3.80)$$

4.4 String model To Derive Continuity and Momentum Fluid Equations from Quantum and Maxwell Distribution Laws and Lasing Process:

Using Maxwell distribution Quantum law, and the Newtonian energy relation continuity and momentum fluid equation was done by differentiation the number density with respect to time and to coordinate. The momentum equation derivation requires the coefficient of the energy in the exponential power is equal to the thermal kinetic energy. This conforms with the statically value proposed by Maxwell distribution but with a positive sing.

This number density function can successfully describes lasing. This is since it predict population inversion and intensity of amplified light.

Key words: Maxwell statistical distribution, energy, continuity equation, momentum equation, Fluid, Energy.

4.5 String Model for Fluid Equations and Lasing:

Consider the particles as small vibrating strings with kinetic and potential energies given by

$$k = \frac{1}{2} m v_e^2 \tag{4.5.1}$$

$$V = \frac{1}{2} k x_e^2 = \frac{1}{2} m \omega^2 x_e^2 \tag{4.5.2}$$

Where the effective values are related to the maximum valves according to the relations

$$v_e = \frac{1}{\sqrt{2}}\, v_m \quad , \quad x_e = \frac{1}{\sqrt{2}}\, x_m \tag{4.5.3}$$

Since $x = x_m \sin \omega t$ (4.5.4)

$$v = x^{\circ} = \omega x_m \cos \omega t = v_m cos\omega t \tag{4.5.5}$$

$$v_m = \omega x_m \tag{4.5.6}$$

Thus:

$$v_e^2 = \frac{v_m^2}{2} = \frac{\omega^2 x_m^2}{2} = \omega^2 x_m^2 \tag{4.5.7}$$

$$x_e^2 = \frac{x_m^2}{2} \tag{4.5.8}$$

From equations (4.5.1) and (4.5.2):

$$\frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 x_e^2 = V \tag{4.5.9}$$

Thus the total energy E is given to be

$$E = K + V = 2K = m v_e^2 = m v^2 \tag{4.5.10}$$

One can also treat strings as subjected only to kinetic force such that its energy is related to the maximum velocity,

$$E = \frac{1}{2} m v_m^2 \tag{4.5.11}$$

Where m is the mas and v_m is assumed to represent the maximum velocity, such that the average velocity is given by:

$$v = \frac{v_m}{\sqrt{2}} \tag{4.5.12}$$

Hence from (4.5.11) and (4.5.12) one can write the energy to be given by:

$$E = m v^2 = m\left(v_x^2 + v_y^2 + v_z^2\right) \tag{4.5.13}$$

Since the momentum P is given by:

$$P = m v \tag{4.5.14}$$

Thus one can write

$$E = mv.v = P.v \tag{4.5.15}$$

In the x – direction

$$E = mv_x^2 = P_x v_x = P.v \tag{4.5.16}$$

Now multiply both sides by the Quantum expression for the particle density for resistive bulk matter which is given by:

$$n = Ae^{\alpha px-\beta ET} \tag{4.5.17}$$

To get

$$En = Pnv \tag{4.5.18}$$

Thus $\frac{\partial n}{\partial t} = -\beta En$

$$En = \frac{-1}{\beta}\frac{\partial n}{\partial t} \tag{4.5.19}$$

$$\frac{\partial nv}{\partial x} = v\frac{\partial n}{\partial x} + n\frac{\partial v}{\partial x} \tag{4.5.20}$$

$$\frac{\partial n}{\partial x} = \alpha Pn \tag{4.5.21}$$

Here one assumes in (4.5.17) that $P = mv$ is independent of coordinates thus

$$\frac{\partial P}{\partial x} = m\frac{\partial v}{\partial x} = 0 \qquad \frac{\partial v}{\partial x} = 0 \tag{4.5.22}$$

Thus equation (4.5.22) and (4.5.10) beside (4.5.11) gives

$$\frac{\partial nv}{\partial x} = \alpha Pnv \tag{4.5.23}$$

Since the matter density ρ is given by

$$\rho = mn \tag{4.5.24}$$

And assuming uniform fluid particles with constant mass m equation (4.5.19), (4.5.23) and (4.5.18) gives

$$-\frac{m}{\beta}\frac{\partial n}{\partial t} = \frac{m}{\alpha}\frac{\partial nv}{\partial x}$$

$$\frac{1}{\alpha}\frac{\partial \rho \mathrm{v}}{\partial \mathrm{x}} + \frac{1}{\beta}\frac{\partial \rho}{\partial t} = 0 \tag{4.5.25}$$

For:

$$\alpha = \beta \tag{4.5.26}$$

One gets the continuity equation

$$\frac{\partial \rho v}{\partial x} + \frac{\partial \rho}{\partial t} \tag{4.5.27}$$

The momentum equation can be derived also from number density expression of Maxwell distribution

$$n = Ae^{\beta E} \tag{4.5.28}$$

Where the mass density is given by

$$\rho = mn = mAe^{\beta E} = \rho_\circ e^{\beta E} \tag{4.5.29}$$

Where the energy is given by

$$E = \frac{1}{2}\,\rho v_x^2 + V \tag{4.5.30}$$

And

$$\frac{dE}{dx} = \frac{d(\frac{1}{2}\rho v_x^2)}{dx} + \frac{dV}{dx} \tag{4.5.31}$$

But from (4.5.29)

$$\frac{d\rho}{dx} = \rho_\circ .e^{\beta E} .\left\{\left(\beta\frac{\partial(\frac{1}{2}\rho v_x^2)}{\partial \mathrm{x}} + \beta\frac{\partial \mathrm{V}}{\partial \mathrm{x}}\right)\right\} \tag{4.5.32}$$

$$\frac{\partial \mathrm{\rho}}{\partial \mathrm{x}} = \beta\rho\left\{\frac{1}{2}v_x^2\frac{\partial\rho}{\partial x} + \frac{1}{2}\rho\frac{\partial v_x^2}{\partial x} + \frac{\partial V}{\partial x}\right\} = \mathrm{\rho}\beta\left\{\frac{1}{2}v_x^2\frac{\partial\rho}{\partial x} + \rho v_x\frac{\partial v_x}{\partial x} + \frac{\partial V}{\partial x}\right\} \tag{4.5.33}$$

Since the force F is real ted to the potential according to the relation:

$$F = -\frac{\partial V}{\partial x} \tag{4.5.34}$$

Rearranging (4.5.23) gives:

$$\{\frac{1}{\beta} - \frac{1}{2}\rho v_x^2\}\frac{\partial\rho}{\partial x} = \rho v_x^2\frac{\partial v_x}{\partial x} + \rho \tag{4.5.35}$$

But

$$\frac{\partial\rho v_x}{\partial t} = \frac{\partial\rho v_x}{\partial x}\frac{\partial x}{\partial t} = v_x\frac{\partial\rho v_x}{\partial x} \tag{4.5.36}$$

By using the laws of statistical physics:

$$\frac{1}{\beta} = kT = \frac{1}{2}\rho v_x^2 \tag{4.5.37}$$

Thus equation (4.5.35) gives:

$$\rho v_x\frac{\partial v_x}{\partial x} = -\frac{\partial V}{\partial x} = F \tag{4.5.38}$$

If one assumes that:

$$\frac{dv_x}{dx} = \frac{\partial v_x}{\partial x} \tag{4.5.39}$$

Then equation (4.5.38) gives:

$$\mathrm{\rho} v_x\frac{dv_x}{dx} = \rho\frac{\partial x}{\partial t}\frac{dv_x}{dx} = \rho\frac{dv_x}{dt} = F \tag{4.5.40}$$

Which is the ordinary momentum fluid equation .Another more direct approach can be obtained from equation (4.5.28) to get:

$$\rho = mn = \rho_\circ e^{\beta E} \tag{4.5.41}$$

Where that total differentiation ω . r. t. x gives:

$$\frac{d\rho}{dx} = \beta\rho_\circ e^{\beta E}\frac{dE}{dx} = \beta\rho\frac{dE}{dx} \tag{4.5.42}$$

Using the expression (4.5.20)

$$\frac{dE}{dx} = \frac{d\rho\frac{v_x^2}{2}}{dx} + \frac{dV}{dx} \tag{4.5.43}$$

$$\frac{\mathrm{d}(\rho\frac{v_x^2}{2})}{\mathrm{dx}} = \frac{1}{2}v_x^2\frac{d\rho}{dx} + \rho v_x\frac{dv_x}{dx} = \frac{1}{2}v_x^2\frac{d\rho}{dx} + \rho\frac{dx}{dt}\frac{dv_x}{dx} =$$
$$\frac{1}{2}v_x^2\frac{dy}{dx} +$$
$$\rho\frac{dv_x}{dt} \tag{4.5.44}$$

Assuming that the potential depends on x only:

$$\frac{\mathrm{dV}}{\mathrm{dx}} = \frac{\partial V}{\partial x} = -F \tag{4.5.45}$$

Inserting (4.5.44) and (4.5.45) in (4.5.43)

$$\frac{dE}{dx} = \frac{1}{2}v_x^2\frac{d\rho}{dx} + \rho\frac{dv_x}{dt} - F \tag{4.5.46}$$

With the aid of equation (4.5.46) equation (4.5.32) give:

$$\frac{1}{\beta}\frac{d\rho}{dx} = \frac{1}{2}\rho v_x^2\frac{d\rho}{dx} + (\rho\frac{dv_x}{dt} - F)\rho$$

$$(\frac{1}{\beta} - \frac{1}{2}\rho v_x^2)\frac{d\rho}{dx} = (\rho\frac{dv_x}{dt} - F)\rho \tag{4.5.47}$$

Thus

$$\frac{1}{\beta} = \frac{1}{2}\rho v_x^2 \quad , \quad \rho\frac{dv_x}{dt} = F \tag{4.5.48}$$

A third approach can also be tackled by differentiating (4.5.41) $\omega.r.t$ (t)
To get:

$$\frac{d\rho}{dt} = \beta\rho\frac{dE}{dt} \tag{4.5.49}$$

In view of equation (4.5.30)

$$\frac{dE}{dt} = \frac{1}{2}\frac{d\rho v_x^2}{dt} + \frac{dV}{dt} = v_x\rho\frac{dv_x}{dt} + \frac{dx}{dt}\frac{dV}{dx} + \frac{1}{2}v_x^2\frac{d\rho}{dt}$$
$$= \rho v_x\frac{dv_x}{dt} + \frac{1}{2}v_x^2\frac{d\rho}{dt} + v_x\frac{dV}{dx} \tag{4.5.50}$$

Inserting (4.5.50) in (4.5.49) gives

$$\frac{1}{\beta}\frac{d\rho}{dt}=\rho\left\{\rho v_x\frac{dv_x}{dt}+\frac{1}{2}v_x^2\frac{d\rho}{dt}+v_x\frac{dV}{dx}\right\}+\left\{\frac{1}{\beta}-\frac{1}{2}\rho v_x^2\right\}\frac{d\rho}{dt}=\rho v_x\left\{\rho\frac{dv_x}{dt}+\frac{dV}{dx}\right\} \tag{4.5.51}$$

One of the possible solutions is to set

$$\frac{1}{\beta}=\frac{1}{2}\rho v_x^2=kT \tag{4.5.52}$$

Thus equation (4.5.51) gives

$$\rho\frac{dv_x}{dt}=-\frac{dV}{dx}=-\frac{dV}{dx}=F \tag{4.5.53}$$

Which again represents the ordinary fluid momentum equation, according to equation (4.5.28) the light intensity is given by:

$$I=cnhf=cAe^{\beta E}(hf)=I_\circ e^{\beta E} \tag{4.5.54}$$

Thus at ground state (E=0)

$$n=A \tag{4.5.55}$$

But at excited state $(E=\infty)$, $(n\to\infty)$ (4.5.56)

Thus population inversion takes place. If one assumes that at ground state the energy is only due to potential part. Thus state the energy is only due to potential part.

Thus $E_\circ=V$ (4.5.57)

If it collide and gain kinetic energy:

$$E=E_\circ+\frac{1}{2}mv^2=E_\circ+\beta=E_\circ+hf \tag{4.5.58}$$

Thus from (4.5.28):

$$n=Ae^{\frac{E_\circ}{hf}}e^1=Ae^1e^{\frac{E_\circ\tau}{h}} \tag{4.5.59}$$

Where: $\Delta\tau\Delta E=h$, $\tau(hf)=h$

$$\tau=\frac{1}{f} \tag{4.5.60}$$

This means that n at excited state increases with life and relaxation time in E which agrees with laser theory.

4.6 Discussion:

Using the equation of motion of a particle in a frictional medium in equation (4.3.1) a useful expression for energy of string in a frictional medium was found in equation (4.3.8). This expression is used to find Schrodinger equation (4.3.11) in a frictional media. Using separation of variables for particle in a box subjected to constant potential the expressions of the wave functions were found in equation (4.3.16) and (4.3.17). Assuming that just outside the bulk matter (the box) no particles exist (see equation (4.3.20)). The energy is shown to be quantized, in this approach the spatial wave function. One can use an exponential spatial wave function to find the same energy expression by assuming that just inside the bulk matter the practical exists as shown by equations (4.3.24) a new energy expression (4.3.27) is found by assuming u to be real. The same expression in (4.3.50) can be obtained it u is a cosine function as shown by equation (4.3.44).However it u is imaginary as equation (4.3.37) shows another energy quantization expression is found in equation (4.3.43).

The statistical distribution Maxwell equation can be found by using the wave function (4.3.53) for frictional media. Using the quantum average (4.3.56)

The number of particles was found in equation (4.3.63) by using integration by parts. Another useful expression for Maxwell distribution was found use ordinary differentiation and the quantum expression for average physical quantity in (4.3.64) to get the number of particles in equations (4.3.63) and (4.3.81).

When deriving continuity equation deals only with the relation between the changes of fluid density with its motion. Thus the kinetic term is important and one can ignore the role of potential energy. This is done by assuming that particles as in the form of strings. In this care the potential energy effective value is equal to the effective value of the kinetic energy as shown by equation (4.5.9). Thus equation (4.5.10) gives the total energy, in terms of the kinetic energy. Another typical energy from can be also found by assuming the string energy to be purely kinetic and related to the maximum velocity as shown by equation (4.5.11). The energy relation (16) resembles (4.5.11).

Treating strings as travelling quantum waves moving in a medium , the number density is given in equation (4.5.17) as function of energy and momentum differentiating this expression $\omega.r.t$ space and time the ordinary continuity equation (4.5.27) for the fluid has been found . On the other hand the Maxwell distribution law was derived by some of this paper authors from quantum wave function. This expression (4.5.28) is used to derive the fluid momentum equation. This is done by using Newtonian energy density equation (4.5.31) , then multiplying both sides by m to find matter density (see equation (4.5.29) .A direct differentiation of the matter density partially $\omega.r.t$ (see equation (4.5.33)) .gives fluid momentum equation (4.5.40) This requires the parameter β to be equal to the kinetic thermal energy as shown by equation (4.5.37) .this conforms with what proposed in statistical physics . the same results can be obtained by differentiating the matter density ρ $\omega.r.t$ to x and t totally as shown by equations (4.5.48) and (4.5.53) .In all cases , equations (4.5.37) , (4.5.48) and (4.5.52) shows that the parameter β is equal to kinetic thermal energy . these results agree with that proposal by statistical mechanics .

However the distribution law in this model has exponential coefficient with a positive sign, which is a like that of Maxwell with a negative sing. But for fortunately this relation can describe light amplification and lasing process. This is very clear from equation (4.5.55) and (4.5.56), where the number of particles in the excited state is larger than the number in the ground state. This means that population inversion takes place.

The intensity relation in equation (4.5.54) and the number expression (4.5.59) show also that the light intensity increases when the life time of metastable state increases. This again agrees with the laser theories.

(4.7) Conclusion:

The probability distribution for particle in a box or a medium with constant potential was found for complex wave function as well as cosine wave function using the fact that the particle exists inside the medium. This gives new different probability distribution and different energy relations. The Maxwell distribution was found also by using wave function for frictional medium and quantum average as well as integration by parts. The same distribution was found by using the quantum energy average and ordinary differentiation laws.

Using quantum laws and statistical physical laws continuity equation and momentum were derived. A new statistical law capable of describing lasing process was also obtained.

It is very interesting to note that the fluid equations being derived from quantum and statistical physics laws and being free from frictional term can suitably describe super fluids.

References:

[1] "MIT physicists create new form of matter". Retrieved November 22, 2010.

[2] Grimm, R. (2005). "Low-temperature physics: A quantum revolution". Nature **435** (7045): 1035–1036. doi: 0.1038/4351035a. PMID

[3] Madison, K.; Chevy, F.; Wohlleben, W.; Dalibard, J. (2000). "Vortex Formation in a Stirred Bose-Einstein Condensate". Physical Review Letters **84** (5): 806–809. arXiv:cond-mat/9912015Bibcode 2000: RhRvL..84.806M doi: 10.1103/PhysRevLett.84.806. PMID .

[4] A. Serway, Emerity and James, Phy. For scientists and Engineer serway and John, Tewett, Jr. London UK, 2014.

[5] Burnett, K. (2007). "Atomic physics: Cold gases venture into Flatland". Nature Physics **3** (9): 589. Bibcode 32007NatPh...3..589B. doi:10.1038/nphys704.

[6] Hau, L. V.; Harris, S. E.; Dutton, Z.; Behroozi, C. H. (1999). "Light speed Work supported by the United States National Science Foundation under Grants GP13381 and GP27242(6720): 594–598. doi/10.1038/17561.

[7] "Lene Hau". Physicscentral.com. Retrieved 2013-02-10.

[8] Lene Vestergaard Hau (2003). "Frozen Light" (PDF). Scientific American: 44–51.

[9] Shocking Bose-Einstein Condensates with Slow Light.

[10] Migdal A. B. (1959). "Super-fluidity and the moments of inertia of nuclei". Nucl. Phys. **13** (5): 655–674. Bibcode1959:NucPh..13..655M .doi/ :29-5582(59)90264-0.

[11] Migdal A. B. (1960). Soviet Phys. JETP **10**: 176. Missing or empty

|title= (1help).

[12] Lombardo U. &. Schulze H.-J (2001). "Super-fluidity in Neutron Star Matter". Physics of Neutron Star Interiors. Lecture Notes in Physics **578**. Springer. pp. 30–53. arXiv:astro- ph/0012209 .doi: --/. -10. 1007/3 -540 -44578 -1_2. 1963The American Institute of Physics DOI: ./.http: //dx .doi. org/10 .1063/1.1703944.

[13] Received 30 July 1962 published online 22 December 2004.

[14] W. F. Vinen Phys. Rev. B 71, 024513 – Published 24 January 2005 work sported by the United States National Science Foundation under Grants GP13381 and GP27242**.**

[15] Academy of Sciences of the Ukrainian SSR and Moscow State University (Received October 12, 1946).

[16] M. Holland, S. J. J. M. F. Kokkelmans, M. L. Chiofalo, and R. WalserPhys. Rev. Lett. 87, 120406 – Published 31 August 2001.

[17] Griffiths, David J. . . Introduction to Quantum Mechanics (2nd ed.). 2004.

[18] Hagen Kleinert. Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets, 3rd ed. Singapore: World Scientific. Draft of 4th edition, 2004.

[19] George Mackey. The mathematical foundations of quantum mechanics. 2004.

[20] D. Greenberger, K. Hentschel, F. Weinert, eds. Compendium of quantum physics, Concepts, experiments, history and philosophy, Springer-Verlag, Berlin, Heidelberg. 2009.

[21] PEckert, Michael. The Dawn of Fluid Dynamics: A Discipline between Science and Technology. Wiley. 2006.

[22] Jump up to:[a] [b] Anderson, J. D. Fundamentals of Aerodynamics (4th ed.). London: McGraw–Hill. 2007.

[23] Landau, Lev Davidovich; Lifshitz, Evgenii Mikhailovich. Fluid Mechanics. London: Pergamon.1987.

[24] Chester, Marvin. Primer of Quantum Mechanics. John Wiley. (1987).

[25] Griffiths, David J. Introduction to Quantum Mechanics (2nd ed.). A standard undergraduate text. 2004.

[26] Hagen Kleinert. Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets, 3rd ed. Singapore: World Scientific. Draft of 4th edition. 2004..

[27] Curtright, T. L.; Zachos, C. K. "Quantum Mechanics in Phase Space". Asia Pacific Physics Newsletter. 2012.

[28] Laurendeau, Normand M. Statistical Thermodynamics: Fundamentals and Applications. New York: Cambridge University Press. 2005.

[29] Jump up to:[a b c] Pedlosky, Joseph. Geophysical fluid dynamics. Springer. pp. 10–13. ISBN 978-0-387-96387-7. 1987.

[30] George Mackey. The mathematical foundations of quantum mechanics. 2004.

[31] D. Greenberger, K. Hentschel, F. Weinert, eds. Compendium of quantum physics, Concepts, experiments, history and philosophy, Springer-Verlag, Berlin, Heidelberg. 2009.

[32] Eckert, Michael. The Dawn of Fluid Dynamics: A Discipline Between Science and Technology. Wiley. 2006.

[33] A.F.J Levi, Applied Quantum Mechanics, Cambridge University Press, California (2003).

[34] Michel Le Bellac, Quantum Mechanics, Cambridge University Press, Cambridge (2006).

[35] L.I .Schiff, Quantum Mechanics (McGraw Hill, Tokyo, 2015).

[36] R.M. Wald, Quantum Field Theory in Curved Space and Black Hole Thermodyics (Chicago. 1994).

[37] E.W. kolb and M.S. Turner, The early Universe (Presew, 1994).

[38] Polchinski, String Theory, Cambridge University Press, Cambridge (1998).

[39] Ghaga M.Elhindi etal, Superconductivity Itopping Processon The basic Generalized Statistical Physics, Inter. J .of Sci. Eng And App. Sci V2, I9, Sep (2016).

[40] Lutfi M. A etal, Schrodinger Quantum Equation from Classical and Quantum Harmonic Oscillator, Int, J of Eng. Sci. & Res, Tech. [Algadir, 5(2): Feb 2016).

[41] Rashida I. Abdelrahman, New Deraivation of Simple Josphson Effect Relation Using New Quantum
Mechanic Equation, Natural Science,8 ,85-88 (2016).

[42] Raymond A. Serway and John W. Jewett, Physics for Scientists and Engineers with Modern Physics, Compositor: Lachina Publishing Services 2014, 2010, 2008 by Raymond A. Serway, Library of Congress Control Number: 2012947242, ISBN-13: 978-1-133-95405-7, ISBN-10: 1-133-95405-7.

[43] Mubarak Dirar Abdallah, Nuha Abdelrahman Khalid, Zoalnoon Ahmed Abeid Allah and Sawsan Ahmed Elhouri Ahmed, Maxwell Equation And Lorentz Transformation Inacurved Space For Fields And Free Space On The Basis Of Maxwell’s Equations, Global Journal Of Engineering Science And Researches, 2018, ISSN 2348 – 8034

[44] David McMahon, Relativity Demystified, McGraw-Hill Companies, 2006.

[45] S. S. Schweber, An Introduction to Relativistic Quantum Field Theory,

Courier Corporation, 2011.

[46] Marshall L. Burns, Modern physics for science and engineering, (2012), ISBN: 978-0-9713134-4-6, produced in the United States of America.

[47] R. Adler , M. Basin and Shiffer, Introduction to General Relativity (MC Graw Hill Book

[48] J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics International Series in Pure and Applied Physics, McGraw-Hill, Inc, 1964.

[49] John D. Kraus, Danil A, Fleisch, Electromagnetic with application, fifth edition, McGraw- Hil Book Co. New York ,1999.

[50] Xun Shen and Roeland Van Wijk, Biophotonics Optical Science and Engineering for the 21st Century, Springer Science-H Business Media, 2005, ISBN-10: 0-387-24995-8.

[51] Fritz-Albert Popp, Properties of biophotons and their theoretical implications, Indian Journal of Experimental Biology , Vol. 41 , May 2003, pp.391 -402.

[52] Greiner. W, Relativistic Quantum Mechanics, Springer, Berlin, 3rd edition, (2000).

[53] Vinen and Niemela [J. Low Temp. Phys. 128, 167 (2002)],

[54] [G. E. Volovik, JETP Lett. 78, 533 (2003); G. E. Volovik, J. Low Temp. Phys. (to be published); and L’vov, Nazarenko, and Volovik, JETP Lett. (To be published)].

[55] S. Flüge. Practical quantum mechanics. Springer–Verlag, London. 1999.

[56] W. Pauli. General principles of quantum mechanics. Springer-Verlag, New York. 1980.

[57] W. Pauli. Selected topics in field quantization. MIT Press, Cambridge,

1973.

[58] Bradbury, T.C.: Theoretical Mechanics. Wiley, New York (1968) Sivardiere, J.: Eur. Phys. 7, 283 (1986).

[59] ’t Hooft G. and Nobbenhuis S., Class. Quantum Grav., 23 (2006) 3819.

[60] L. Lamata, J. León, T. Schätz and E. Solano, “Dirac Equation and Quantum Relativistic Effects in a Single Trapped Ion,” Physical Review Letters, Vol. 98, No. 25, 2007, Article ID: 253005. doi:10.1103/PhysRevLett.98.253005

[61] Sasan Ahmed. Mubarac Dirar. Ahmed Elhassan Lutfi M.A Algadir Mohamed Ismail “Quantization of Friction for Nano Isolated Systems” Phys.lett.81. No31435. 2015.

[62] Dekker H., On the phase space quantization of the linearly damped harmonic oscillator, Phys. A, 1979, V.95, 311–323.

[63] Bateman H., On dissipative systems and related variational principles, Phys. Rev., 1931, V.38, 815–819; Dekker H., Classical and quantum mechanics of the damped harmonic oscillator, Phys. Rep., 1981, V.80,

[64] C. L. Tang “Fundamental Quantum Mechanic for Solid StateElectronic and Optics” Cambridge University Press, UK, 2005.

[65] H. Kleinert. Path integrals in quantum mechanics, statistics and polymer physics. World Scientific, London. 1995

[66] A. F. J. Levi “Applied Quantum Mechanics” University of Southern California, Cambridge University Press, 2003.

[67] David Griffiths “Introduction to Quantum Mechanics” Second Edition, Person Education International, 2005.

[68] Ray J.R., Lagrangians and systems they describe – how not to treat dissipation in quantum mechanics, Am. J. Phys., 1979, V.47, 626–630.1–112.

[69] David McMahon“Quantum Field Theory Demystified ”McGraw-Hill

Companies, New York.2008.

[70] M. Blasone and P. Jizba. [quant-ph/0102128].

[71] G. ’t Hooft. In Basics and highlights of fundamental physics. Erice. 1999. [hep-th/0003005].

[72] Nesvizhevsky, V., et al.: Quantum states of neutrons in the Earth’s gravitational field. Nature 415, 297 (2002)

[73] Eastham, Derek. “Atomic Physics of Lasers” Philadelphia: Taylor and Francis, 1986.

[74] Kanai E., On the quantization of dissipative systems, Prog. Theor. Phys., 1948, 440–442;

[75] Professor Dr. Florian Scheck “Quantum Physics” University Mainz, New York, 1965.

[76] Mohamed Eltoum and Sawsan Ahmed“ The Quantization Expression of the Role of Relaxation Time on Optical
Absorption in Semiconductor” IJESRT. ISSN.2277. 2014.

[77] J. Anandan. In Quantum coherence. Edited by J. Anandan.
World Scientific, London. 1989. pp. 136–149.