Sudan University of Science and Technology

College of Graduate Studies

On The Solution of Spectral Local Linearization Method of Two Dimensional Steady Flow Over A non-Isothermal Wedge

دلالة حل طريقة الطيف محلية الخطية على إنسياب مستقر فوق حافة غير متساوية الحرارة

A Thesis Submitted In Partial Fulfillment Of The Requirement Of The Degree Of M.Sc. In Mathematic

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Contents

Abstract

In this thesis the problem of steady two-dimensional Laminar magneto-hydrodynamic forced convection flow over a non-isothermal wedge with effects of viscous dissipation and stress work is presented. The governing partial deferential equations are transformed into highly nonlinear dimensionless partial differential equations by introducing a suitable transformation and then they were solved numerically using spectral local linearisation method (SLLM). Results are presented to illustrate the effects of the controlling parameters; namely, Prandtl number and Eckert number on the fluid velocity, temperature and rate of heat transfer. Numerical data for the Nusselt number have been shown in graphical type for various governing parametric.

Abstract (Arabic):

المستخلص:

في هذه الاطروحة نتناول تدفق الانسياب الطبقي المغناطسي المستقر على قطع غير حراري تحت تأثير تبدد اللزوجة وقوة الاجهاد السطحي. المعادلات التفاضلية الجزئية الحاكمة حولت الى معادلات تفاضلية جزئية لا بعدية عالية اللاخطية من ثم حلت عدديا باستخدام طريقة الطيف محلية الخطية . النتائج التي تصف تأثيرات البارميترات الحاكمة مثل رقم برانتل ، رقم ايكارد على سرعة وحرارة المائع ومعامل انتقال الحراري .القيم العددية لرقم نسلت وضحت في صورة رسومات للبارميترات الحاكمة.

Dedication

To

my father . . . .

mother . . .

brothers . . . .

and sisters . . .

Thanks

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First of all, I am grateful to Allah for helping me to complete this thesis. I would like to take this opportunity to express my deepest gratitude to all who helped me directly and indirectly towards the succssful completion of this work. Foremost, Iwould like to express my sincere gratitude to my supervisor Dr. Faiz Awad for his guidance this study. Without his valuable assistance, this work would not have been completed. I would also like to thank Dr. Mohamed Abdelaziz and Dr. Mohamed Khabir for their support and cooperation. Special thanks to my friends who helped me (especially Khalda, Lemya, and my colleague Ekram). Finally, I would like to thank my parents, brothers and sisters for their unconditional love, support and encouragement.

Chapter 1

Introduction

Most problems in science and engineering are governed by nonlinear differential equations. When those equations are strongly nonlinear, exact solutions are not easily obtained and we often resort to approximate numerical solutions. There are many well established numerical schemes such as the Runge-Kutta schemes, the Keller-box method, the shooting method, and finite element and volume methods. The main disadvantage of numerical solutions, however, is that they may not give any insights into the structure of the solution, particularly when the problem involves many em bedded parameters. Numerical methods may also give discontinuous points on the solution curve, Paripour et al. [1]. Moreover, some numerical methods may not be stable or uniformly convergent. In such cases is often made to either the classical series method or other perturbation methods to find approximate analytical solutions. The main disadvantage of traditional perturbation methods, however, is that they require the presence of a large or small parameter in the problem to be solved. Recent analytical techniques include the Lyapunov artificial small parameter method, Lyapunov [2], the Adomain decomposition method, Adomian [3], the homotopy perturbation method, He [4, 5] and the homotopy analysis method, Liao [6, 7, 8, 9, 10].

These methods may not always be convergent or valid. For example, the Adomain decomposition method has a small convergence region, Jiao et al. [11]. In this study we use two innovative semi-numerical methods for solving strongly nonlinear systems that arise in the study of fluid flow problems.

The objective of this study is to demonstrate the spectral local linearisation method (SLLM) to systems of nonlinear ordinary and partial differential equations. The method, originally introduced by Motsa et al. [12, 13, 14], was used to solve nonlinear differential equations. Through the objective, we used to test the accuracy, computational efficiency and general validity of spectral relaxation technique in solving systems of highly nonlinear differential equations that arise in fluid flow problems.

Chapter 2

Spectral Local Linearisation Mehod

2.1 General governing equation system

Consider a system of m nonlinear ordinary differential equations in m unknowns functions $z_i(\eta)$ $i = 1, 2, 3, ..., m$ where η is the independent variable. The system can be written as a sum of it's linear $\mathbf{L}$ and nonlinear $\mathbf{N}$ components as

$$\mathbf{L}\left[z_1(\eta), z_2(\eta), ..., z_m(\eta)\right] + \mathbf{N}\left[z_1(\eta), z_2(\eta), ..., z_m(\eta)\right] = \mathbf{H}(\eta), \quad (\eta) \in (a, b), \tag{2.1}$$

subject to the boundary conditions

$$A_i\left[z_1(a), z_1(a), ..., z_m(a)\right] = K_{a,i}, \quad B_i\left[z_1(b), z_2(b), ..., z_m(b)\right] = K_{b,i}, \tag{2.2}$$

where A_i and B_i are linear operators and $K_{a,i}$ and $K_{b,i}$ are constants for $i = 1, 2, ..., m$. Define the vector Z_i to be the vector of the derivatives of the variable z_i with respect to the dependent variable η, that is

$$Z_i = \left[z_i^{(0)}, z_i^{(1)}, ..., z_i^{(n_i)},\right], \tag{2.3}$$

where $z_i^{(0)} = z_i$, $z_i^{(p)}$ is the p^{th} derivative of z_i with respect to η and n_i $(i = 1, 2, .., m)$ is the highest derivative order of the variable z_i appearing in the system of equations. In addition, we define $\mathbf{L}_i$ and $\mathbf{N}_i$ to be the linear and nonlinear operators, respectively, that operate on the Z_i for $i = 1, 2, .., m$, with these definitions, equation (2.1) and (2.2) can be written as

$$\mathbf{L}_i\left[Z_1, Z_2, ...Z_m\right] + \mathbf{N}_i\left[Z_1, Z_2, ...Z_m\right] = \sum_{j=1}^{m}\sum_{p=0}^{n_i} \alpha_{i,j}^{[p]} z_j^{(p)} + \mathbf{N}_i\left[Z_1, Z_2, ...Z_m\right] = \mathbf{H}_i, \tag{2.4}$$

where $\alpha_{i,j}^{[p]}$ are the constant coefficient of $z_j^{(p)}$, the derivative of z_j $(j = 1, 2, ...m)$ that appears in the i^{th} equation for $i = 1, 2, ..., m$. Noting that, for each variable z_i in the derivatives in the boundary conditions can at most be one less than the highest derivative of z_i in the governing system (2.1) we define the vector $\tilde{\mathbf{Z}}_i$ to be the vector of the derivatives of the variable z_i with respect to the dependent variable η from 0 up to $(n_i - 1)$, that is

$$\tilde{\mathbf{Z}}_i = \left[z_i^{(0)}, z_i^{(1)}, ..., z_i^{(n_i-1)}\right], \tag{2.5}$$

The boundary conditions(2.2) can be written as

$$A_\nu\left[\tilde{\mathbf{Z}}_1(a), \tilde{\mathbf{Z}}_2(a), ..., \tilde{\mathbf{Z}}_m(a)\right] = \sum_{j=1}^{m}\sum_{p=0}^{n_j-1} \beta_{\nu,j}^{[p]} z_j^{(p)}(a) = K_{a,\nu}, \quad \nu = 1, 2, ..., m_a, \tag{2.6}$$

$$B_\sigma\left[\tilde{\mathbf{Z}}_1(b), \tilde{\mathbf{Z}}_2(b), ..., \tilde{\mathbf{Z}}_m(b)\right] = \sum_{j=1}^{m}\sum_{p=0}^{n_j-1} \gamma_{\sigma,j}^{[p]} z_j^{(p)}(b) = K_{b,\sigma}, \quad \sigma = 1, 2, ..., m_b, \tag{2.7}$$

where $\beta_{\nu,j}^{[p]}$ and $\gamma_{\sigma,j}^{[p]}$ are the constant coefficients of $z_j^{(p)}$ in the boundary conditions, and m_a, m_b, are the total number of prescribed boundary conditions, at $x = a$ and

$x = b$ respectively. We remark that the sum $m_a + m_b$ is equal to the sum of the highest orders of the derivatives corresponding to the dependent variable z_i,that is

$$m_a + m_b = \sum_{i=1}^{m} n_i, \tag{2.8}$$

2.2 Spectral Local Linearisation Method (SLLM)

Let us consider a system of m nonlinear ordinary differential equations in m unknowns functions $z_i(\eta)$ $i = 1, 2, 3, \ldots.., m$ where η is the independent variable. The system can be written as a sum of it's linear $\mathbf{L}$ and nonlinear $\mathbf{N}$ components as

$$\mathbf{L}\left[z_1(\eta), z_2(\eta), \ldots., z_m(\eta)\right] + \mathbf{N}\left[z_1(\eta), z_2(\eta), \ldots., z_m(\eta)\right] = \mathbf{H}(\eta), \quad \eta \in [a, b], \tag{2.9}$$

To develop the iteration scheme, we apply local linearisation of N_i about $Z_{i,r}$ (the previous iteration) to the i^{th} non-linear equation assuming that all other $Z_{k,r}(k \neq i)$ are known. Thus, at the ith equation, N_i is linearised as follows

$$Ni[Z_1, Z_2, \ldots, Z_m] = N_i[Z_{1,r}, \ldots, Z_{m,r}] + \frac{\partial N_i}{\partial Z_i}[Z_{1,r}, Z_{2,r}, \ldots, Z_{m,r}](Z_i - Z_{i,r}), \tag{2.10}$$

$$L_i[Z_{1,r+1}, \ldots, Z_{m,r+1}] + \frac{\partial N_i}{\partial Z_i}[\ldots]Z_{i,r+1} = H_i + \frac{\partial N_i}{\partial Z_i}[\ldots]Z_{i,r} - Ni[Z_{1,r}, \ldots, Z_{m,r}] \tag{2.11}$$

where $[\ldots]$ denotes $[Z_{1,r}, Z_{2,r}, \ldots, Z_{m,r}]$ and $Z_{i,r+1}$ and $Z_{i,r}$ are the approximations of Z_i at the current and the previous iteration, respectively. Thus, starting from an initial approximation $Z_{1,0}, Z_{2,0}, \ldots, Z_{m,0}$, the proposed iterative scheme (2.9) is then solved as a loop until the system converges at a consistent solution for all the variables. To solve the iteration scheme (2.9), it is convenient to use the Chebyshev pseudo-spectral method as in previous section. For this reason the proposed method is referred to as the spectral local linearization iteration method (SLLM). Before applying the spectral method, it is convenient to transform the domain on which the governing equation is defined to the interval $[-1, 1]$ on which the spectral method can be implemented. We use the transformation $\eta = (b-a)(t+1)/2$ to map the interval $[a,b]$ to $[-1,1]$. The basic idea behind the spectral collocation method is the introduction of a differentiation matrix $\mathbf{D}$ which is used to approximate the derivatives of the unknown variables $z_i(\eta)$ at the collocation points as the matrix vector product

$$\frac{dZ_i}{d\eta} = \sum_{j=1}^{m}\sum_{k=0}^{\widehat{N}} \mathbf{D}_{lk} Z_i(t_k) = DZ_i, \quad l = 0, 1, ..., \widehat{N} \tag{2.12}$$

where $\widehat{N}+1$ is the number of collocation points (grid points), $\mathbf{D} = 2D/(b-a)$, and $Z = [z(t_0), z(t_1), ..., z(t_N)]^T$ is the vector function at the collocation points. Higher order derivatives are obtained as powers of $\mathbf{D}$, that is

$$Z_j^{(p)} = D_p Z_j. \tag{2.13}$$

Chapter 3

Two Dimensional Flow Over a Non-Isothermal Wedge

Consider the problem of the effects of viscous dissipation and stress work on the steady two-dimensional Laminar magneto-hydrodynamic forced convection flow over a non-isothermal wedge [15], all the fluid properties are assumed to be constat .Interducing the boundary Layer approximation, The governing equation for the continuity, momentum and energy can be written as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + U_\infty\frac{dU_\infty}{dx} + \frac{\sigma B_0^2}{\rho}(U_\infty - u), \tag{3.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{u}{C_p}\left(u_\infty\frac{du_\infty}{dx} + \frac{\sigma B_0^2}{\rho}u_\infty\right) + \frac{\sigma B_0^2}{\rho C_p}u^2. \tag{3.3}$$

The boundary conditions are defined as follows:

$$
\begin{aligned}
y = 0 : u = 0, \quad v = 0, \quad T = T_w, \\
y \longmapsto \infty : u \;=\; u_\infty, \; T = T_\infty,
\end{aligned}
\tag{3.4}
$$

where x and y are coordinates measured along and normal to the surface, respectively, u and v are the velocity components in the x and y directions, respectively, ν is the kinematic viscosity. $u_\infty = ax$ is the velocity of the potential flow outside the boundary layer, C is a positive number, σ is the electrical conductivity. B_0 is the externally imposed magnetic field in the $y-$direction, p is the density. The induced magnetic field and the Hall effect are neglected. T is the temperature of the fluid, α is the thermal diffusivity, C_p is the specific heat at constant pressure, a is a positive number, The subscripts w and ∞ refer to the plate surface and ambient conditions respectively. The stream function ψ is defined by $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, there fore, the continuity equation is automatically satisfied. Invoking the following dimensionless variables

$$
\xi = \frac{\sigma B_0^2 x}{\rho u_\infty}, \quad \eta = \frac{y}{x}\left(\frac{u_\infty x}{\nu}\right)^{\frac{1}{2}}, \quad f(\xi, \eta) = \frac{\Psi}{(u_\infty x \nu)^{\frac{1}{2}}}, \quad \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{3.5}
$$

By substituting equation (3.5) into Eqs. (3.2)-(3.3), yields

$$
f''' + \frac{1}{2} f f'' + \xi(1 - f') = \xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right), \tag{3.6}
$$

$$
\frac{1}{Pr}\theta'' + \frac{1}{2} f\theta' + Ec[(f'')^2 - \xi f' + \xi (f')^2] = \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right), \tag{3.7}
$$

subject to the boundary conditions

$$
\begin{aligned}
\eta = 0 : f' = 0, \quad f = 0, \quad \theta = 1 \\
\eta \longmapsto \infty : f' = 1, \quad \theta = 0.
\end{aligned}
\tag{3.8}
$$

In the foregoing equations, the primes denote the differentiation with respect to η. Here ξ is the magnetic parameter. $Pr = \nu/\alpha$ is the Prandtl number. $Ec = u_\infty^2/C_p[T_w - T_\infty]$ is the Eckert number.

Chapter 4

Numerical Experment

To solve equations (3.6) and (3.7) along with the boundary conditions (3.8), the spectral local linearisation method (SLLM) was used, see Motsa et al. [12, 13, 14]. This method is preferred since it has been shown to be accurate and generally easier to use compared to other common numerical methods such as finite differences.

We start by reducing the order of equation (3.6) from third to second order. To this end, we set $f' = u$, so that equation (3.6) becomes

$$u'' + \frac{1}{2}fu' + \xi - \xi u = \xi\left(u\frac{\partial u}{\partial \xi} - u'\frac{\partial f}{\partial \xi}\right), \tag{4.1}$$

$$f' = u. \tag{4.2}$$

The spectral local linearisation method(SLLM) approach is used to decouple the equations leading to a linear system which is subsequently solved using the Chebyshev spectral collocation method. The basic idea behind the SLLM stems from the combination of the Gauss-Seidel method for decoupling equations and the Newton-Raphson based quasi-linearisation. In this regard, linearisation in the momentum equation (4.1) is applied only in terms involving $u(\eta)$ and its derivatives. All other terms are assumed to be known from previous iterations. The terms involving $f(\eta)$ are assumed to be known from previous iteration while the updated solution for $u(\eta)$ at the current iteration is used. Similarly, in Eqn (3.7), only terms in $\theta(\eta)$ are linearised if there is nonlinear terms while terms in $f(\eta)$ and $u(\eta)$ are assumed to be now known at the current iteration (denoted by $(r+1)$). Thus applying the LLM on Eqns (3.7) and (4.1) gives

$$u''_{r+1} + a_{1,r}u'_{r+1} + a_{2,r}u_{r+1} + a_{3,r} = a_{4,r}\frac{\partial u_{r+1}}{\partial \xi}, \tag{4.3}$$

$$f'_{r+1} = u_{r+1}, \tag{4.4}$$

$$\theta''_{r+1} + b_{1,r}\theta'_{r+1} + b_{2,r} = b_{3,r}\frac{\partial \theta_{r+1}}{\partial \xi}, \tag{4.5}$$

where the primes denote partial derivatives with respect to η. The boundary conditions are given by

$$\begin{aligned} &f_{r+1}(\eta, 0) = 0, \quad u_{r+1}(\eta, 0) = 1, \quad \theta_{r+1}(\eta, 0) = 0, \\ &u_{r+1}(\eta, \infty) = 0, \quad \theta_{r+1}(\eta, \infty) = 0. \end{aligned} \tag{4.6}$$

The coefficients in (4.3) and (4.5) are defined as

$$\begin{aligned} a_{1,r} &= \frac{1}{2}f_{r+1} + \xi\frac{\partial f_r}{\partial \xi}, \; a_{2,r} = -\xi - \xi\frac{\partial u_r}{\partial \xi}, \; a_{3,r} = \xi + \xi u_r\frac{\partial u_r}{\partial \xi}, \; a_{4,r} = \xi u_r \\ b_{1,r} &= Pr\left(\frac{1}{2}f_{r+1} + \xi\frac{\partial f_{r+1}}{\partial \xi}\right), \; b_{2,r} = PrEc[u'^2_{r+1} - \xi u_{r+1} + \xi u^2_{r+1}], \; b_{3,r} = Pr\xi u_{r+1}. \end{aligned}$$

To solve the linearised system of (4.3) – (4.5) we employ the Chebyshev spectral collocation method to discretize in the η- direction and use an implicit finite difference method in the ξ-direction. To this end, we define the grid points on (η, ξ) as

$$\begin{aligned} &\eta_j = \cos\frac{\pi_j}{N_\eta}, \quad \xi^n = n\Delta\xi, \\ &j = 0, 1, ..., N_\eta; n = 0, 1, ..., N_\xi, \end{aligned} \tag{4.7}$$

where N_η, N_ξ are the total number of grid points in the η- and ξ-direction, respectively, and $\Delta\xi$ is the spacing in the ξ-direction. The finite difference scheme is applied with centering about a midpoint halfway between ξ^{n+1} and ξ^n. This midpoint is defined as $\xi^{n+\frac{1}{2}} = (\xi^{n+1} + \xi^n)/2$. The derivatives with respect to η are discretized in terms of the Chebyshev differentiation matrices. Applying the centering about $\xi^{n+\frac{1}{2}}$ to any function, say $f(\eta, \xi)$ and its associated derivative, we obtain

$$\begin{aligned} f(\eta_j, \xi^{n+1/2}) &= f_j^{n+1/2} = \frac{f_j^{n+1} + f_j^n}{2}, \\ \left(\frac{\partial f}{\partial \xi}\right)^{n+1/2} &= \frac{f_j^{n+1} - f_j^n}{\Delta\xi}. \end{aligned} \tag{4.8}$$

In applying the Chebyshev spectral collocation method, the continuous derivatives in the unknown functions are approximated by matrix-vector products of the so-called differentiation matrices at the collocation points. Before the spectral method is applied, the domain $\eta \in [0, \eta_\infty]$ is transformed to the domain $Y \in [-1, 1]$ by using the mapping $\eta = \eta_\infty(Y + 1)/2$. The basic idea behind the spectral collocation method is the introduction of a differentiation matrix D which is used to approximate the derivatives of the unknown variables f, u and h, at the collocation points Y_j $(j = 0, 1, ..., N_\eta)$.

$$\left.\frac{\partial f}{\partial \eta}\right|_{\eta=\eta_j} = \sum_{k=0}^{N_\eta} D_{jk} f(Y_k, \xi) = \mathbf{D}F, \quad j = 0, 1, ..., N_\eta, \tag{4.9}$$

where $N_\eta + 1$ is the number of collocation points, $\mathbf{D} = 2D/\eta_\infty$, where the matrix D is of size $(N_\eta + 1)(N_\eta + 1)$ and its entries are defined as

$$\begin{aligned} D_{jk} &= \frac{c_j(-1)^{j+k}}{c_k(Y_j - Y_k)}, \quad j \neq k;\ j, k = 0, 1, 2, 3, ..., N_\eta, \\ D_{kk} &= -\frac{Y_k}{2(1 - Y_k^2)}, \quad k = 1, 2, 3, ..., N_\eta - 1, \\ D_{00} &= \frac{2N_\eta^2 + 1}{6} = -D_{N_\eta N_\eta}, \end{aligned}$$

with

$$c_k = \begin{cases} 2, & k = 0, N_\eta; \\ 1, & -1 \leq k \leq N_\eta - 1. \end{cases} \tag{4.10}$$

which

$$\begin{aligned} F &= [f(Y_0, \xi), f(Y_1, \xi), ..., f(Y_{N_\eta}, \xi)]^T, \\ U &= [u(Y_0, \xi), u(Y_1, \xi), ..., u(Y_{N_\eta}, \xi)]^T, \\ \Theta &= [\theta(Y_0, \xi), \theta(Y_1, \xi), ..., \theta(Y_{N_\eta}, \xi)]^T, \end{aligned}$$

are the vector functions at the collocation points. In general, a derivative of orders for the function $f(\eta)$ can be transformed as

$$f^{(s)}(\eta) \longrightarrow \mathbf{D^s}F, \quad u^{(s)}(\eta) \longrightarrow \mathbf{D^s}U, \quad \theta^{(s)}(\eta) \longrightarrow \mathbf{D^s}\Theta, \tag{4.11}$$

where s is the order of the derivative. Thus, applying the spectral local linearisation method in η and finite difference method in ξ gives

$$\begin{aligned} A_1 U_{r+1}^{n+1} &= B_1 U_{r+1}^{n} + K_1, \\ A_2 F_{r+1} &= K_2, \\ A_3 \Theta_{r+1}^{n+1} &= B_3 \Theta_{r+1}^{n} + K_3, \end{aligned} \tag{4.12}$$

where

$$
\begin{aligned}
A_1 &= \frac{1}{2}\left(\mathbf{D}^2 + \mathbf{diag}[a_{1,r}]\mathbf{D} + \mathbf{diag}[a_{2,r}]\mathbf{I}\right) - \frac{1}{\triangle\xi}\mathbf{diag}[a_{4,r}] \\
B_1 &= -\frac{1}{2}\left(\mathbf{D}^2 + \mathbf{diag}[a_{1,r}]\mathbf{D} + \mathbf{diag}[a_{2,r}]\mathbf{I}\right) - \frac{1}{\triangle\xi}\mathbf{diag}[a_{4,r}] \\
K_1 &= a_{3,r} \\
A_2 &= \mathbf{D}, \quad K_2 = u_r \\
A_3 &= \frac{1}{2}\left(\mathbf{D}^2 + \mathbf{diag}[b_{1,r}]\mathbf{D}\right) - \frac{1}{\triangle\xi}\mathbf{diag}[b_{3,r}] \\
B_3 &= -\frac{1}{2}\left(\mathbf{D}^2 + \mathbf{diag}[b_{1,r}]\mathbf{D}\right) - \frac{1}{\triangle\xi}\mathbf{diag}[b_{3,r}] \\
K_3 &= b_{3,r}
\end{aligned}
$$

with boundary conditions

$$
\begin{aligned}
&U_{r+1}(\eta_0, \xi^n) = 0, \ U_{r+1}(\eta_{N_\eta}, \xi^n) = 1, \\
&F_{r+1}(\eta_{N_\eta}, \xi^n) = 0, \\
&\Theta_{r+1}(\eta_0, \xi^n) = 0, \ \Theta_{r+1}(\eta_{N_\eta}, \xi^n) = 0,
\end{aligned}
$$

In the above equations Θ, U and F correspond to the approximate values of $\theta(\eta, \xi)$, $u(\eta, \xi)$ and $f(\eta, \xi)$ at the collocation points. The approximate solutions for f and θ are obtained by solving (4.12). The convergence and stability of the iteration schemes are assessed by considering the norm of the difference in the values of the approximate functions between two successive iterations. Thus, for each iteration scheme, we define the following maximum error E at the $(r+1)th$ iteration:

$$
E = \max(\|F_{r+1} - F_r\|_\infty, \|U_{r+1} - U_r\|_\infty, \|\Theta_{r+1} - \Theta_r\|_\infty). \tag{4.13}
$$

The unknowns f, u and θ were iteratively calculated, for a given number of collocation points N_η, until the following criteria for convergence was satisfied at iteration r :

$$E \leq \epsilon, \tag{4.14}$$

where ϵ is the convergence tolerance level.

Chapter 5

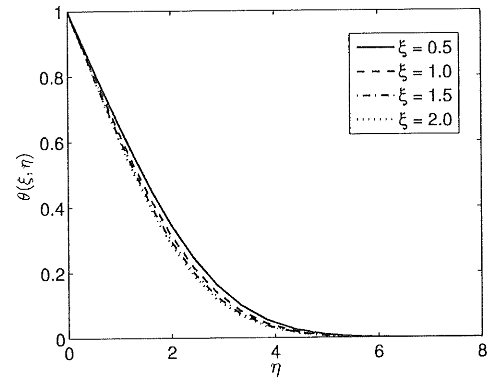
Results and Discussion

In this part we present the SLLM results for the solution of the governing equations (3.6)-(3.7). In all the spectral method based numerical simulations a finite computational domain. In order to obtain a clear insight into the physics of the problem, the effects of various governing parameters on the fluid properties and physical quantities are presented in Figs. 5.1 5.3.

Fig. 5.1 depicts change in the velocity and temperature profiles with ξ respectively. We observe that an increase in ξ leads to an increase in the momentum boundary layer thickness. Hence, there is enhancement in the velocity of the fluid along the surface. On the other hand fast fluid has less thermal boundary layer, thus reduces the temperature of the fluid.

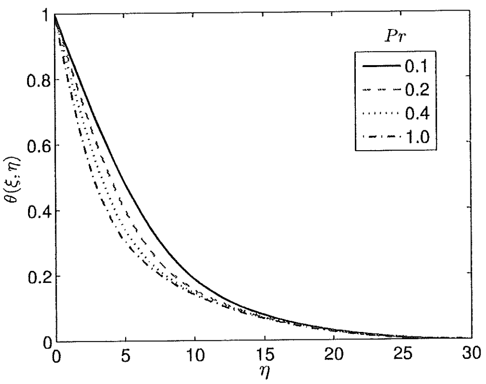
Fig. 5.2 shows the effects of the Prandtl number Pr and the Eckert number Ec on the temperature profile respectively. Prandtl number characterizes the ratio of thickness of the viscous and thermal boundary layers, therefore, the thermal boundary layer thickness is seen to decrease with increasing values of Pr, thus leading to temperature profile decreasing. On the other hand, the temperature profile increases with Ec due to increasing in the thermal boundary layer thinness.

Fig. 5.3 presented changes in the rate of heat transfer due to change in the Prandtl number Pr and the Eekert number Ec. The local Nusselt number enhances with increasing the Prandtl number Pr. The local Nusselt number decreases with increasing the Eckert number Ec.

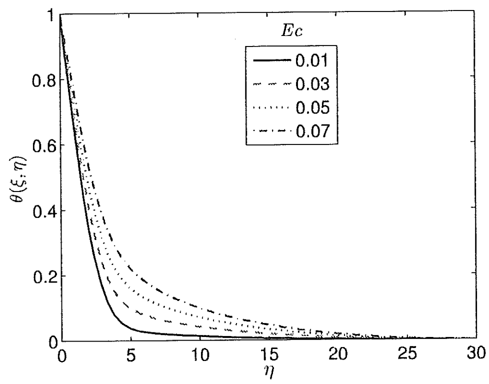


(a)

Figure 5.1: Velocity and temperature profiles for difference values of ξ with $Pr = 0.72$ and $Ec = 1$.

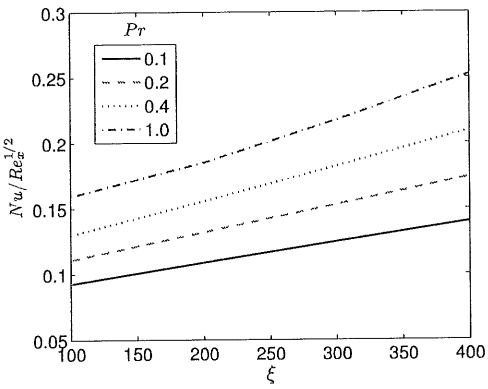


(a)

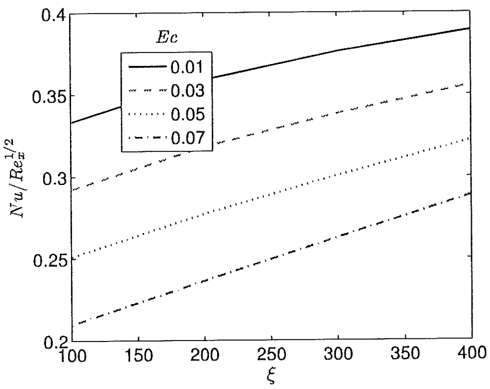


(b)

Figure 5.2: Effects of Prndtl number Pr and Eckerd number Ec on temperature profile respectively.



(a)



(b)

Figure 5.3: Effects of Prndtl number Pr and Eckerd number Ec on the rate of heat transfer respectively.

Chapter 6

Conclusion

In this work we have studied the two-dimensional Laminar magneto-hydrodynamic forced convection flow over a non-isothermal wedge. Assuming the boundary layer approximation in present of a suitable transformation a dimensionless nonlinear partial differential equations obtained from the governing equations with associate governing parameters such as the Prandtl parameter and Echert number. Effects of the governing parameters on heat transfer coefficients and fluid flow characteristics have been studied graphically.

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